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**Study of the nonlocal active sound control with preservation of desired field
in time domain**

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1 In the active sound control, a domain is protected from externally generated noise
2 via constructing secondary sound sources, which are called controls. These controls
3 are applied on the boundary of the shielded domain. Apart from the external noise, a
4 desired sound generated by interior sources should also be retained inside the shielded
5 domain. However, it turns out it is a challenge to preserve the internally generated
6 sound unaffected due to both the reverse effect of the controls on the input data and
7 their sparse distribution on the boundary. To take into account the reverse effect, an
8 innovative algorithm based on nonlocal control is implemented in the time domain
9 for the first time. Its real-time practical implementation may include preliminary
10 tuning to the real surrounding conditions. A number of test cases are considered
11 including external broadband noise and internal monochromatic desired sound. A
12 sensitivity analysis is carried out with respect to some key design parameters such as
13 density of sensors and controls as well as their respective geometrical displacement
14 from one another determined by the Hausdorff distance. It is demonstrated that the
15 nonlocal control provides the noise attenuation level which is not much sensitive to
16 the presence of the desired sound.

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I. INTRODUCTION

With more and more stringent noise control regulations in industry, high interest has been drawn to noise control during past few decades. The passive noise attenuation is achieved via implementation of insulation materials and can be quite efficient in application to bounded domains to be protected from high frequency noise. Meanwhile, the application of passive noise control becomes problematic if a low frequency noise propagates since long waves can decay much slower than short waves ([Hansen and Snyder, 1996](#)). In turn, the active sound control (ASC) has proved to be efficient for the attenuation of low-frequency noise ([Canevet, 1978](#)). In the ASC problems, a domain, which is usually bounded, is supposed to be shielded from noise generated outside by constructing secondary sound sources also known as controls. In the classical formulation, the ASC is based on the application of Huygens' principle ([Baker and Copson, 1939](#)). As a result, an anti-noise is generated by controls positioned at the perimeter of a domain to be shielded. In contrast to the passive sound control, ASC does not use any absorbing material to cancel noise. Practically, passive sound control is mostly applicable to the attenuation of mid- and high-frequency noise, whereas the ASC is better suitable for low frequency noise due to the space and time limitations on the control system ([Elliott and Nelson, 1990](#)). Therefore, the ASC can be regarded as an effective supplement to the passive sound control.

The conventional ASC control systems are based on one of the two principal algorithms with feed-back and feed-forward controls ([Elliott, 2001](#); [Elliott and Sutton, 1996](#); [Nelson and Elliott, 1991](#)). In the feed-back system, the noise signal is detected by a set of error sensors

implemented inside the shielded domain. The operation of feed-forward control is based on a predetermined strategy that the measurement of noise is completed before the controls (Kletschkowski, 2011). To realize the noise attenuation by the action of controls, the filtered-x least mean square (FxLMS) algorithm is commonly used for feed-back and feed-forward systems to adjust the noise signal (Hansen *et al.*, 2012). Some examples of current ASC research include headphones (Hansen *et al.*, 2012), ventilation and refrigeration systems (Tokhi *et al.*, 2002), aircrafts, elevators and car cabins (Bai and Chen, 2000; Broadbent, 1976; Landaluze *et al.*, 2003). In these applications the controls are implemented over the entire acoustically transparent boundary to be shielded. However, in some applications the controls can be applied only on a part of the boundary of a protected domain. For example, in (Zou *et al.*, 2010) the controls were mounted on the top of anti-noise barriers to attenuate noise from outdoor transformers.

It should be noted that the preservation of desired sound is usually not considered in the conventional ASC problems (Williams, 1984). In practice, especially for a feed-back system, it is problematic to differentiate immediately between noise and desired sound in the total acoustic field from the primary sources. It seems that for the first time the ASC problem with the existence of an interior component was analysed by Malyuzhinets in (Malyuzhinets, 1971). He obtained the solution to the ASC problem in a bounded domain for the case when the total field from only primary sources is available. This solution presumes a continuous distribution of the controls at the perimeter of the domain to be shielded. A discrete realization of this approach, which was more practical, was made in paper (Konyaev *et al.*, 1977).

Ryaben’kii obtained a solution to the stationary problem in a quite general formulation in discrete spaces (Ryabenkii, 1995, 2002). Similar to (Malyuzhinets, 1971), this solution presumes the knowledge of the primary field at the perimeter of the domain to be shielded. Thanks to a discrete form, it can be realized via a set of monopoles and dipoles. This opportunity was clearly demonstrated by Tsynkov (Tsynkov, 2003). In his paper the links between finite-difference surface potentials, solution from (Ryabenkii, 2002) and acoustic equations are shown. The developed technique is applied to the Helmholtz equations in (Loncaric *et al.*, 2001) where a link with the surface Calderón potentials was first provided. For the first time, the problem on ASC was considered in composite domains in (Ryaben’kii *et al.*, 2007).

The approach described above was extended to unsteady problems in (Ryaben’kii and Utyuzhnikov, 2006a, 2007, 2006b; Utyuzhnikov, 2009a, 2010a). In (Utyuzhnikov, 2010a), the operation of ASC was explicitly demonstrated via the exact analytical solution to the unsteady problem obtained in a quite general formulation. A desired sound was allowed to be present in the shielded domain. It was shown that ASC can be realized even for resonance regimes in bounded domains. Afterwards, Utyuzhnikov was the first who considered the solvability of the nonlinear problem via the apparatus of generalized nonlinear potentials (Utyuzhnikov, 2010b) introduced. The experimental realization of the potential-based approach applied to ASC was addressed in a number of publications (see, e.g., (Lim *et al.*, 2014, 2011, 2009; Ntummy and Utyuzhnikov, 2014, 2015)).

It is to be noted the key assumption for the control mentioned above is based on an apriori knowledge of the primary field generated by both undesired and desired sources.

This is hardly achievable in practice since in real-time realization the sound component generated by the controls is inevitable in the input data to be measured. As shown by Ryaben’kii and Utyuzhnikov, controls can double the desired sound outside the protected region (Ryaben’kii and Utyuzhnikov, 2007, 2006b). This makes the original field from the primary sources unattainable. In some practical applications, mostly related to the noise control in ventilation, this problem is partially resolved via directional measurements (see, e.g., (Huang *et al.*, 2011; Jakob and Moser, 2003a,b; Kwon and Park, 2011; Sieck and Lau, 2011; Wang *et al.*, 2016)). In (Ryaben’kii, 2011), it is proposed to solve an auxiliary local initial-boundary value problem across the control boundary to generate the control function. Although in a finite-difference formulation considered in the paper the algorithm is strict, its practical realization is very problematic. Indeed, it presumes a second-order numerical differentiation of noisy and contaminated input data. Moreover, for the algorithm it is important that the controls are supposed to be acoustically transparent and have no volume that is practically not realizable.

A new approach to tackle this problem was proposed by Utyuzhnikov in (Utyuzhnikov, 2009b) and (Utyuzhnikov, 2014). In these papers the proposed control is nonlocal. This means the operation of a control depends on the field along an entire control surface. The approach is based on the projection property of Calderón surface potentials. As a result, the contribution of the desired sound and secondary field is effectively subtracted from the total field. In this way, there is no need to predict desired sound thanks to the automatic separation of noise and desired sound from the total field. A practical algorithm for realization of the nonlocal active control was proposed in (Utyuzhnikov, 2017). The algorithm

is potentially applicable to realization in real-time regimes. It was implemented for the first time in the frequency domain by Zhou and Utyuzhnikov in (Zhou and Utyuzhnikov, 2020). In that paper it was demonstrated that the nonlocal control has an advantage over the local control even without the presence of desired sound.

The current paper is organized as follows. First, an introduction to the general formulation of ASC problem is provided in the next section. It presumes the presence of desired sound along with noise generated outside a protected area. Then, Section III addresses the local and nonlocal ASC algorithms. The nonlocal control is based on a potential-based algorithm. The problem of the reverse effect of the controls on the input data is discussed. The results of numerical experiments carried out for a few pure-tone sources are demonstrated in Section IV. A sensitivity analysis is considered with respect to different input parameters such as the distribution density of sensors and dynamics as well as their relative geometrical displacement. Finally, the test cases with a broadband noise are discussed in Section V which is followed by the Conclusion.

II. GENERAL FORMULATION OF THE ASC PROBLEM

We consider the following problem formulation. Introduce some domain $D_0: D_0 \subseteq R^3$. In addition, we introduce subdomain $D^+: D^+ \subset D_0$ with smooth enough boundary Γ . This subdomain is supposed to be shielded from the noise generated in domain $D^-: = D_0 \setminus \overline{D^+}$.

Assume the acoustic field in domain D_0 is described by the following linear initial-boundary-value problem (IBVP):

$$p_{tt} - a^2 \Delta p = f, \quad (1)$$

$$p(\mathbf{x}, 0) = p_t(\mathbf{x}, 0) = 0, \quad \mathbf{x} \in D_0, \quad (2)$$

$$p \in \Xi_{D_0},$$

where p is the sound pressure; a is the speed of sound; f corresponds to the acoustic sources; Ξ_{D_0} is a functional space such that the solution of IBVP (1) and (2) exists and is unique. If domain D_0 is unbounded, then space Ξ_{D_0} is determined by the Sommerfeld boundary conditions.

The acoustic source f presented by the right hand side in (1) is composed of noise sources f^- and desired sound sources f^+ :

$$f = f^+ + f^-, \quad (3)$$

$$\text{supp } f^+ \subset D^+,$$

$$\text{supp } f^- \subset D^-.$$

In turn, the total sound field p consists of desired sound p^+ and noise p^- :

$$p = p^+ + p^-. \quad (4)$$

Assume that we are going to protect domain D^+ from noise p^- . At the same time, it is required to retain the desired field p^+ unaffected there. It is worth noting that p^+ also includes any field reflected from the boundaries if it exists. The same definition is applicable

to p^- . Then, the ASC becomes an inverse source problem: find secondary sound sources G_0 on boundary surface Γ such that the solution to IBVP

$$p_{c|tt} - a^2 \Delta p_c = f + G_0 \quad (5)$$

$$\text{supp } G_0 \subset D \setminus D^+,$$

$$p_c(\mathbf{x}, 0) = p_{c|t}(\mathbf{x}, 0) = 0, \quad \mathbf{x} \in D_0,$$

$$p_c \in \Xi_{D_0}.$$

₁₂₈ coincides with p^+ in D^+ : $p_c = p^+$.

FIG. 1. Domain sketch.

129 III. ACTIVE SOUND CONTROL

130 A. Local sound control

131 Consider the control function as a linear combination of simple and double-layer sources:

$$G_0(\Gamma, t) = -a^2 \delta(\Gamma) \frac{\partial p}{\partial \mathbf{n}|_\Gamma} - a^2 p_\Gamma \frac{\partial \delta(\Gamma)}{\partial \mathbf{n}}, \quad (6)$$

132 where $\mathbf{n}$ is the outward normal to boundary Γ ; $\delta(\Gamma)$ represents the surface delta-function
 133 associated with Γ ; p_Γ is the restriction of p to Γ : $p_\Gamma = p(\Gamma, t)$; $p(\Gamma, t)$ corresponds to the
 134 total field from the primary sources at boundary Γ .

135 Source G_0 generates field p_l :

$$p_l(\mathbf{x}, t) = \int_{\Gamma, \tau} \left(\frac{\partial Gr}{\partial \mathbf{n}}(\mathbf{x}, t | \mathbf{y}, \tau) p_\Gamma(\mathbf{y}, \tau) - Gr(\mathbf{x}, t | \mathbf{y}, \tau) \frac{\partial p}{\partial \mathbf{n}|_\Gamma}(\mathbf{y}, \tau) \right) d\sigma d\tau, \quad (7)$$

136 where $\mathbf{n}$ is the outward normal to Γ ; Gr represents Green's function; $\mathbf{x}$ correspond to the
 137 observation points; the retarded time is considered in the convolution.

Equation (7) can be rewritten in a more compact form via the Calderón potentials ([Utyuzhnikov, 2017](#)):

$$p_l(\mathbf{x}, t) = P_{D+} \boldsymbol{\xi}_\Gamma(\Gamma, t). \quad (8)$$

138 Here, $\boldsymbol{\xi}_\Gamma = (p_\Gamma, \frac{\partial p}{\partial \mathbf{n}|_\Gamma})^T$ means the density of the potential.

139 In application to the wave equation with homogeneous initial conditions, the Calderón
 140 potential in domain Ω is determined by ([Utyuzhnikov, 2009a](#)):

$$P_{\Omega} \boldsymbol{\xi}_{\partial\Omega}(\mathbf{x}, t) = \int_{\partial\Omega, \tau} (Gr(\mathbf{x}, t \mid \mathbf{y}, \tau) \xi_{\partial\Omega|2}(\mathbf{y}, \tau) - \frac{\partial Gr}{\partial \mathbf{n}}(\mathbf{x}, t \mid \mathbf{y}, \tau) \xi_{\partial\Omega|1}(\mathbf{y}, \tau)) d\sigma d\tau, \quad (9)$$

141 where $\mathbf{x} \in \Omega$, $\boldsymbol{\xi}_{\partial\Omega} = (\xi_{\partial\Omega|1}, \xi_{\partial\Omega|2})^T$, $\mathbf{n}$ is the outward normal to $\partial\Omega$.

142 It is to be noted that for the ASC the initial data are not important. In a more general
143 formulation with nonhomogeneous initial conditions, the definition of the Calderón potential
144 can be found in (Petropavlovsky *et al.*, 2018).

145 The key property of the Calderón potential is a projection. It immediately follows from
146 the Green's formula (Petropavlovsky *et al.*, 2018; Utyuzhnikov, 2009a). Thanks to the
147 projection property, in the case of continuous surface control

$$P_{D^+} \boldsymbol{\xi}_{\Gamma}(\mathbf{x}, t) = p^-(\mathbf{x}, t), \quad \mathbf{x} \in D^+,$$

148 if $\boldsymbol{\xi}_{\Gamma} = (p_{\Gamma}, \frac{\partial p}{\partial \mathbf{n}}|_{\Gamma})^T$.

149 Thus, if the total field from the primary sources p_{Γ} is available, then G_0 generates anti-
150 noise and provides complete noise attenuation (see, e.g., (Utyuzhnikov, 2010a)).

151 The control is local because the surface integrals are approximated in a discrete form via
152 a set of monopoles and dipoles (Utyuzhnikov, 2017):

$$G_0(\Gamma, t) \approx -a^2 \sum_{i=1}^N \left[\frac{\partial p}{\partial \mathbf{n}}|_{\Gamma}(\mathbf{x}_i, t) \delta(\mathbf{x} - \mathbf{x}_i) + p_{\Gamma}(\mathbf{x}_i, t) \frac{\partial \delta}{\partial \mathbf{n}}(\mathbf{x} - \mathbf{x}_i) \right] \Delta\sigma_i, \quad (10)$$

153 where $\mathbf{x}_i \in \Delta\sigma_i$ ($i = 1, \dots, N$), $\mathbf{n}$ is the outward normal to Γ , $\Delta\sigma_i$ ($i = 1, \dots, N$) are
154 nonintersecting elementary areas σ_i which cover the entire surface Γ .

It is to be noted that from the projection property

$$P_{D^+}\boldsymbol{\xi}_\Gamma(\mathbf{x}, t) = P_{D^+}\boldsymbol{\xi}_\Gamma^-(\mathbf{x}, t), \quad (11)$$

155 where $\boldsymbol{\xi}_\Gamma^- = (p_\Gamma^-(\mathbf{x}_i, t), \frac{\partial p^-}{\partial \mathbf{n}}|_\Gamma(\mathbf{x}_i, t))^T$.

In turn

$$P_{D^+}\boldsymbol{\xi}_\Gamma^+(\mathbf{x}, t) = 0, \quad (12)$$

156 where $\boldsymbol{\xi}_\Gamma^+ = (p_\Gamma^+(\mathbf{x}_i, t), \frac{\partial p^+}{\partial \mathbf{n}}|_\Gamma(\mathbf{x}_i, t))^T$.

157 Thus, the control looks similar to the case when only noise is present. In the case of
 158 pure noise, an anti-noise can be generated locally along the entire perimeter of the protected
 159 region. However, this similarity takes place only in the case of continuous surface control. In
 160 other words, this presumes the validity of equations (11) and (12) that are actually nonlocal.
 161 In the case of a discrete approximation, the projection property is violated up to the error
 162 of approximation. This issue is discussed in more detail in the next section.

163 It is to be noted that if desired sound is present, a field equal to the desired acoustic
 164 field p^+ is simultaneously generated by control G_0 immediately outside the shielded domain
 165 (Utyuzhnikov, 2010a). As a result, the primary field is practically unavailable for real-time
 166 measurements unless the field is repeated in time. In order to remove the reverse effect of
 167 controls, a nonlocal control is introduced in the next section. It is to be noted that the
 168 reverse effect occurs only if a desired sound is allowed to be present in the shielded domain.

B. Nonlocal active control

To take into account the reverse effect of the controls, an algorithm of nonlocal active control was developed in (Utyuzhnikov, 2017). Unlike the algorithm introduced in the previous section, the input data $p_c(\Gamma^-, t)$ are assumed to be measured on the external side Γ^- of the boundary surface Γ . Thus, $p_c(\Gamma^-, t)$ is the total field generated from both the primary and secondary sources.

Practically, surfaces Γ and Γ^- should be separated. One can introduce domain D_-^+ : $D^+ \subset D_-^+$. Then, the boundary of D_-^+ can be selected as Γ^- (see Figure 1). Thus, on boundary surface Γ we obtain field $p_e^-(\Gamma, t)$ to be attenuated:

$$p_{e|\Gamma}^-(\mathbf{x}, t) = \int_{\Gamma^-, \tau} (Gr(\mathbf{x}, t | \mathbf{y}, \tau) \frac{\partial p_c}{\partial \mathbf{n}^-}(\mathbf{y}, \tau) - \frac{\partial Gr}{\partial \mathbf{n}^-}(\mathbf{x}, t | \mathbf{y}, \tau) p_c(\mathbf{y}, \tau)) d\sigma d\tau, \quad (13)$$

$$\mathbf{x} \in \Gamma, \mathbf{y} \in \Gamma^-,$$

where $\mathbf{n}^-$ is the outward normal to the external surface Γ^- .

In a more general formulation, the field p_e^- can be calculated via the Calderón potentials (Utyuzhnikov, 2017):

$$p_{e|\Gamma}^-(\mathbf{x}, t) = P_{D_-^+} \boldsymbol{\xi}_{\Gamma^-}(\Gamma^-, t). \quad (14)$$

Thanks to the projection property, ideally, i.e. in the case of continuous surface control, $p_{|\Gamma}^-(\mathbf{x}, t) = p_{e|\Gamma}^-(\mathbf{x}, t)$.

Thus, the control at the boundary Γ is represented by

$$G_0(\Gamma, t | p_{e|\Gamma}^-) = -a^2 \frac{\partial p_e^-}{\partial \mathbf{n}_{|\Gamma}} \delta(\Gamma) - a^2 \frac{\partial p_{e|\Gamma}^- \delta(\Gamma)}{\partial \mathbf{n}}. \quad (15)$$

Hence, we can extract the noise field from the total field $p_c(\Gamma^-, t)$ on external surface Γ^- due to the projection property of the surface potentials. As a result, this approach is able to remove the reverse effect of controls on the input data. The obtained field $p_e^-(\Gamma, t)$ on Γ can then be used as the input field for the local control in (6) to achieve noise attenuation. The entire control is non-local in space since its operation at each point of the control surface is based on the total field over the whole surface Γ^- . Besides, it is clear that the control is also nonlocal in time.

It is to be noted that in the general case Green's function is unknown. Therefore, practically control (13) cannot be applicable straightforward.

In order to approximate the integrals in (13), a discrete distribution of sensors and controls is considered in (Utyuzhnikov, 2017). The external surface Γ^- and boundary surface Γ are divided into small enough nonintersecting areas $\Delta\sigma_j^-$ and $\Delta\sigma_i$, respectively. Presume that $l_j^- = \sqrt{\Delta\sigma_j^-} \ll r_j = |\mathbf{y} - \mathbf{y}_j|$ ($j = 1, \dots, N^-$), $l_i = \sqrt{\Delta\sigma_i} \ll r_i = |\mathbf{x} - \mathbf{x}_i|$ ($i = 1, \dots, N$).

Next, (13) can be approximated as follows:

$$p_e^-(\mathbf{x}_i, t) \approx \sum_{j=1}^{N^-} \Delta\sigma_j^- \int_{\tau} \left(\frac{\partial p_c}{\partial \mathbf{n}^-}(\mathbf{y}_j, \tau) Gr(\mathbf{x}_i, t | \mathbf{y}_j, \tau) - p_c(\mathbf{y}_j, \tau) \frac{\partial Gr}{\partial \mathbf{n}^-}(\mathbf{x}_i, t | \mathbf{y}_j, \tau) \right) d\tau,$$

where $\mathbf{x}_i \in \Delta\sigma_i$ ($i = 1, \dots, N$), $\mathbf{y}_j \in \Delta\sigma_j^-$ ($j = 1, \dots, N^-$).

Then

$$G_0(\Gamma, t | p_{e|\Gamma}^-) \approx -a^2 \sum_{i=1}^N \Delta\sigma_i \left(\frac{\partial p_e^-}{\partial \mathbf{n}}(\mathbf{x}_i, t) \delta(\mathbf{x} - \mathbf{x}_i) + p_e^-(\mathbf{x}_i, t) \frac{\partial \delta}{\partial \mathbf{n}}(\mathbf{x} - \mathbf{x}_i) \right),$$

where $\mathbf{n}$ is the outward normal to Γ .

Then, the explicit formula for the nonlocal control reads :

$$\begin{aligned}
G_0 = & -a^2 \sum_{i=1}^N \Delta \sigma_i * \\
& \sum_{j=1}^{N^-} \Delta \sigma_j^- \int_t (A_{ij} \frac{\partial p_c}{\partial \mathbf{n}^-} + B_{ij} p_c)|_{\mathbf{y}_j, \tau} \delta(\mathbf{x} - \mathbf{x}_i) + (C_{ij} \frac{\partial p_c}{\partial \mathbf{n}^-} + D_{ij} p_c)|_{\mathbf{y}_j, \tau} \frac{\partial \delta}{\partial \mathbf{n}}(\mathbf{x} - \mathbf{x}_i) d\tau. \quad (16)
\end{aligned}$$

Here, transfer matrices A , B , C and D , which have dimensions $[N \times N^-]$, depend only on Green's function and the coordinates of sensors and controls if the geometry of boundary surfaces Γ and Γ^- is known. Thus, these matrices can be measured or calculated efficiently in advance with the use of the method of Difference Potentials (Petropavlovsky *et al.*, 2018; Ryaben'kii, 2012).

It is worth noting here that the entire algorithm is based on the projection property of the surface potentials. It presumes the controls and sensors are distributed over whole boundaries Γ and Γ^- , respectively. In case only on a selected part of the boundary is covered, the projection property can be either significantly violated or not valid.

C. Calculation of surface potentials

The unsteady surface potentials can be efficiently calculated with implementation of the impulse response function that represents an unit impulse applied to a small enough period of time. In the algorithm, the input field on external surface Γ^- is approximated by a set of basis functions $\Phi_k(\mathbf{y})$ and $\frac{\partial \Phi_k}{\partial \mathbf{n}^-}(\mathbf{y})$, $\mathbf{y} \in \Gamma^-$, $k = 1, \dots, N_b$, where $N_b \ll N^-$. Then, an impulse response function can be determined as follows:

$$v_k(\mathbf{x}_i, t) = \begin{cases} \sum_{j=1}^{N^-} \Delta\sigma_j^- \left(\frac{\partial\Phi_k}{\partial\mathbf{n}^-}(\mathbf{y}_j) Gr(\mathbf{x}_i, t|\mathbf{y}_j, 0) - \Phi_k(\mathbf{y}_j) \frac{\partial Gr}{\partial\mathbf{n}^-}(\mathbf{x}_i, t|\mathbf{y}_j, 0) \right), \\ \text{if } \min_j \tau_{ij} \leq t \leq \max_j \tau_{ij} \\ 0 \text{ else,} \end{cases} \quad (17)$$

208 where $\mathbf{x}_i \in \Delta\sigma_i$ ($i = 1, \dots, N$); $\mathbf{y}_j \in \Delta\sigma_j^-$ ($j = 1, \dots, N^-$); $\mathbf{n}^-$ is the outward normal to Γ^- ;
 209 $\tau_{ij} = \frac{|\mathbf{x}_i - \mathbf{x}_j|}{a}$ represents a retarded time.

The impulse response functions can be preliminary either calculated or measured. Next, at each time moment $t = t_p$ ($p = 0, 1, 2, 3, \dots$) density $\boldsymbol{\xi}_{\Gamma^-}(t_p)$ can be approximated by solving the following variation problem:

$$\min_{\alpha_k} = \|\boldsymbol{\xi}_{\Gamma^-}(t_p) - \sum_{k=1}^{N_b} \alpha_k(t_p) \boldsymbol{\eta}_{k\Gamma^-}\|_2, \quad (18)$$

210 where $\boldsymbol{\eta}_{k\Gamma^-} = (\Phi_k(\mathbf{y}), \frac{\partial\Phi_k}{\partial\mathbf{n}^-}(\mathbf{y}))^T$, $\mathbf{y} \in \Gamma^-$.

Then, the surface potentials can be calculated at boundary surface Γ :

$$P_{\Gamma\Gamma^-} \boldsymbol{\xi}_{\Gamma^-}(t) \approx \sum_{k=1}^{N_b} \sum_{l=0}^p \alpha_k(t_l) v_{k\Gamma}(t - t_l)(t_{l+1} - t_l). \quad (19)$$

211 Thus, potential (19) provides for control (15) noise field $p_{e|\Gamma}^-$ to be attenuated.

212 A more efficient strict way to calculate the unsteady Calderón potentials can be found
 213 in (Petropavlovsky *et al.*, 2018). It is based on the partition in time and the existence of
 214 lacunas.

215 In the next section, the results of numerical simulation are presented and discussed.

IV. RESULTS OF NUMERICAL EXPERIMENTS WITH ASC

In the setup of numerical simulation, virtual primary sources, sensors and secondary sources are implemented. As domain D^+ to be shielded we consider a cube with the edge length equal to 1 m . Assume the center of domain D^+ coincides with the origin of the Cartesian coordinate system. The boundary surface Γ^- corresponds to the sides of cube D_-^+ which is obtained by the expansion of cube D^+ with the factor of 1.4 in each direction. In this case, the Hausdorff distance between cubes D^+ and D_-^+ equals 0.2 m . Assume three virtual secondary sources are evenly distributed along each axis on each side of cube D^+ . In turn, six virtual sensors are implemented along each axis on each side of cube D^+ to measure the total acoustic field.

Three types of virtual primary sources are considered in the numerical simulations: pure tone, mixed sound of three monochromatic sources and broadband signal. In the pure tone test, a noise source with frequency of 50 Hz is situated outside the shielded domain D^+ at point with coordinate (1.5 m , 1.5 m , 0 m). In turn, a desired acoustic source with the frequency of 50 Hz is supposed to be at the center of D^+ . In addition, we also consider a mixed noise case that is composed of three noise point sources with the frequencies equal to 40 Hz , 70 Hz and 90 Hz , which are situated outside the shielded domain and have coordinates (1.5 m , 1.5 m , 0 m), (1 m , -1 m , 1 m) and (-1 m , 0.5 m , -1 m), respectively. Similarly, the desired mixed sound is composed of three point sources with the frequencies of 40 Hz , 70 Hz and 90 Hz . These point sources are situated inside domain D^+ and have coordinates (0.3 m , 0.3 m , 0.3 m), (-0.4 m , 0.1 m , 0.2 m) and (0.2 m , -0.3 m , -0.3 m),

237 respectively. In the numerical simulations, the sampling rate is chosen as much as 1 kHz to
 238 ensure the accuracy of approximation.

In the numerical experiments, an average noise attenuation is measured at N observation points that are evenly distributed in shielded domain D^+ . The level of noise attenuation is determined as follows:

$$A_s = \frac{1}{N} \sum_{k=1}^N 20 \log_{10} \left| \frac{p_k^-}{p_{c|k} - p_k^+} \right| dB, \quad (20)$$

239 where p_c corresponds to the total field while the control is on.

240 The noise attenuation achieved with the local and nonlocal controls is discussed in the
 241 next section. First, we consider the test case with a monochromatic noise source. Then,
 242 the case with a mixed noise field is addressed. It is followed by a sensitivity analysis with
 243 respect to the Hausdorff distance d_H between surfaces Γ and Γ^- as well as the density of
 244 sensors n_s and controls n_c . In the final section, we carry out numerical experiments with
 245 broadband noise.

246 A. Pure tone case

247 First, consider the conventional ASC algorithm based on the local control for the case of
 248 pure-tone primary sources. Presume that the total primary field is a priori known. In case
 249 the desired sound is not present, Figure 2(a) shows the 50 Hz noise field with and without
 250 shielding inside domain D^+ for time interval $[0 \text{ s}, 1 \text{ s}]$. The average noise attenuation of
 251 about 12 dB is achieved over time. Here and further, $N = 100$. Thus, the noise is effectively
 252 attenuated inside the shielded domain.

253 In turn, if the desired sound is present, the noise attenuation deteriorates even though the
254 primary field is available at the boundary surface Γ . As can be seen from Figure 2(b), the
255 noise field with ASC is only slightly attenuated. The corresponding average noise attenuation
256 drops to 2.5 dB , which is essentially worse than that in the previous case. The reason for
257 this can be interpreted as follows. As mentioned in the previous section, the local control is
258 determined by the total acoustic field at the perimeter Γ . The error in the approximation of
259 surface potentials in Eq. (11) leads to some violation of the projection property (12). This
260 error can be treated as an extra noise. Thus, the noise attenuation is deteriorated.

FIG. 2. Application of local control to a pure tone field with input primary field: (a) Noise signal (without desired sound), (b) Noise signal (with desired sound).

As mentioned above, practically the primary field is not available at the perimeter Γ if a desired sound is present due to the reverse effect of controls. Therefore, next a more realistic case that takes into account the reverse effect of controls on the input data is considered. Figure 3(a) shows that the noise is rather reinforced than attenuated with the input data based on the total field in the case of the local control. The primary field is the same as that in the previous example. As can be seen, the noise level grows with time under the effect of the control. This phenomenon can be explained as follows.

Control (6) doubles internally generated sound outside the shielded domain in the case of continuous control (see, e.g., (Utyuzhnikov, 2009a)). Since control (6) must be approximated by discrete control (10), projection (12) is only valid with some error. This error should effectively lead to generation of extra imaginary noise sources caused by the discrete control

that follows from the Huygens principle. The field generated by the imaginary sources is also reinforced outside the shielded domain as an internally generated sound. This effect is repeated and reinforced with time.

Next, consider the application of nonlocal control to a pure tone field. In contrast to the local control, the input data are measured on Γ^- . In case the desired field is not present, Figure 4 demonstrates the sound pressure achieved at 0.5 s. As can be seen, the sound pressure is efficiently attenuated inside the cubic shielded domain D^+ . Moreover, the amplitude of sound wave achieved with the nonlocal control for time interval $[0 \text{ s}, 1 \text{ s}]$ is similar to that obtained with the local control considered above (see Figures 2(a) and Table I).

TABLE I. Application of nonlocal control to a pure tone field with input total field.

The case without desired sound				The case with desired sound		
Amplitude, Pa		Attenuation,		Amplitude, Pa		Attenuation,
(without ASC)	(with ASC)	dB		(without ASC)	(with ASC)	dB
Noise	17.10	4.60	11.40	17	4.75	11.10

In turn, as can be seen in Table I, a good level of noise attenuation is still achieved by the nonlocal control even if the desired sound is present. The corresponding average noise attenuation of 11 dB basically retains the same as that without the existence of the desired sound source. The contribution of the desired field and extra noise caused by a discrete

approximation of surface potentials is basically eliminated from the total field on Γ^- . This is achieved thanks to the projection property of the surface potentials used.

FIG. 3. Reverse effect of local control with input total field. Pure tone case: (a) Noise level, (b) Level of noise attenuation.

FIG. 4. Sound pressure at 0.5 s. Pure tone case without desired sound.

It is to be noted that with the nonlocal control the level of noise attenuation is determined by the frequency of noise source and almost does not depend on the frequency of desired sound. This is confirmed by the result that the level of noise attenuation with and without desired sound is practically the same.

B. Mixed sound case

Next, consider the attenuation of noise composed of three noise point sources with frequencies of 40 Hz , 70 Hz and 90 Hz . In turn, there are three desired sound sources with the same frequencies if they are supposed to be present.

First, the case with local control is studied without the presence of desired sound. Assume that the primary field is always available at perimeter Γ over time. Then, as can be seen in Table II, the noise can be significantly attenuated. The level of noise attenuation varies between 7 dB and 13 dB at different time ($t = 0.015\ s, 0.02\ s, 0.025\ s, 0.03\ s, 0.035\ s$). In contrast to the attenuation of pure tone noise, the level of noise attenuation for the mixed case essentially depends on time. This happens because the level of noise attenuation depends on the frequency. It is clear that the higher frequency the lower level of noise attenuation occurs (Epain and Friot, 2007; Zhou and Utyuzhnikov, 2020).

In case the local control is used along with the presence of desired sound, the noise is reinforced similar to the monochromatic case. As shown in Table II, the level of noise attenuation falls to -6 dB at 0.03 s due to the additional noise caused by the discrete control. Over time, the negative level of noise attenuation varies between -6 dB and 4 dB .

If the primary field is not a priori available on surface Γ , the control affects the input data measured at Γ . Similar to the single tone case, this effect continuously grows over time as can be seen in Figure 5(a). Moreover, as Figure 5(b) demonstrates, the noise attenuation becomes practically impossible with the local control.

TABLE II. Application of local control to mixed sound case with input primary field.

t, s	The case without desired sound			The case with desired sound		
	Noise, Pa		Attenuation,	Noise, Pa		Attenuation,
	(without ASC)	(with ASC)	dB	(without ASC)	(with ASC)	dB
0.015	-33.45	8.48	12	-33.45	-35.38	-0.5
0.02	58.63	-20.41	9	58.6	-38.46	4
0.025	9.96	4.27	7	9.96	12.07	-2
0.03	37.06	-11.47	10	37.06	-75.55	-6
0.035	-39.86	9.29	13	-39.86	34.11	1

Afterwards, consider the nonlocal control applied to the mixed field. First, Table III demonstrates that the level of noise signal with the active control is significantly less than the original noise signal in case there are no interior sources. The peak of the noise attenuation reaches about 13 dB at 0.035 s . In turn, if the preservation of the interior desired sound in domain D^+ is required, the noise can significantly be reduced with the nonlocal control. Moreover, although the noise attenuation varies over time, the level of noise attenuation

318 achieved with preservation of interior sound is quite similar to that achieved without any
 319 desired sound. This can be expected because the input total field on external surface Γ^- is
 320 separated thanks to the projection property of the surface potentials used.

FIG. 5. Reverse effect of local controls on noise attenuation. Mixed sound: (a) Noise, (b) Level of noise attenuation.

TABLE III. Application of nonlocal control to mixed sound case with input primary field.

t, s	The case without desired sound			The case with desired sound		
	Noise, Pa		Attenuation,	Noise, Pa		Attenuation,
	(without ASC)	(with ASC)	dB	(without ASC)	(with ASC)	dB
0.015	-33.45	9.91	11	-33.45	9.56	11
0.02	58.63	-22.66	8	58.6	-21.52	9
0.025	9.96	-4.70	7	9.96	-4.73	7
0.03	37.06	-12.73	9	37.06	-12.10	10
0.035	-39.86	9.43	13	-39.86	9.35	13

C. Sensitivity analysis

As mentioned in the previous sections, the level of noise attenuation significantly depends on the errors in the discrete approximation of the surface potentials. Practically, both the sensors and controls (loudspeakers) cannot be implemented continuously. Therefore, it is important to discuss how the densities of sensors n_s and controls n_c affect the ASC performance. Besides, the Hausdorff distance d_H between the external surface Γ^- and boundary surface Γ has an effect on the noise attenuation. This section focuses on the sensitivity analysis with respect to these factors via some numerical experiments. In the tests, the pure tone and mixed primary fields are considered.

First, we study how the density of sensors affects the noise attenuation. Suppose that the number of controls along each axis N_c is equal to 3. In other words, 9 controls on each side and 54 controls in total are distributed on surface Γ . Several sets of sensors with $N_s = 3, 4, 5, 6$ are implemented on external surface Γ^- to detect the input field.

TABLE IV. Effect of the number of sensors N_s on attenuation of pure tone sound.

	N_s			
	3	4	5	6
Noise attenuation, dB	-9.83	4.34	7.53	11.10

As can be seen in Table IV and Figures 6, a poor noise attenuation is achieved with pure tone and mixed primary field in the case with $N_s = 3, N_c = 3$. The noise attenuation even

falls to -9.83 dB with the pure tone primary field in time interval $[0$ s, 1 s]. Besides, the level of noise attenuation occurs mostly in a negative area with the mixed primary field. Moreover, a satisfactory noise attenuation is not achieved when the number of sensors along each axis N_s is equal to 4 and 5. In turn, about 11 dB reduction in the noise level is achieved if N_s increases to 6. It is clear that the error in the discrete presentation of sensors (13) is reduced as the higher density of sensors is implemented. It turns out that the number of sensors along each axis N_s is required to be twice more than the number of controls along each axis N_c for the most efficient noise attenuation.

FIG. 6. Effect of the number of sensors N_s on attenuation of mixed sound.

Next, consider the sensitivity analysis with respect to the density of controls. Several sets of controls ($N_c = 2, 3, 4, 5$) are uniformly distributed on the boundary surface Γ while N_s is equal to 10. First, presume that the total primary field is known for ASC. Table V and VI show the effect of the number of controls on the attenuation of pure tone and mixed noise, respectively.

TABLE V. Effect of the number of controls N_c on attenuation of pure tone sound.

N_c	Noise attenuation, dB			
	local control	local control	Nonlocal control	Nonlocal control
	(without desired sound)	(with desired sound)	(without desired sound)	(with desired sound)
2	9.56	0.69	8.98	9.02
3	11.70	2.63	11.40	11.10
4	14.46	5.39	14.50	14.63
5	17.28	8.22	17.74	17.87

As noted above, if a desired sound is present the noise attenuation with the local control is much worse than that achieved without the presence of desired field. As shown in Table V, a good enough noise attenuation of 9.56 dB is achieved with the local control without the presence of desired sources if N_c equals 2. However, if the desired sound source is present, the noise attenuation drops to 0.69 dB . The deterioration of noise attenuation is about 9 dB . The same level of drop retains in case N_c increases to 3, 4 and 5. In turn, the nonlocal control allows us to keep a satisfactory level of noise attenuation despite the presence of the desired sound. Even in case N_c equals 2, the noise attenuation obtained with and without preservation of desired field corresponds to 9.02 dB and 8.98 dB , respectively. The level of

noise attenuation remains low sensitive to the presence of desired sound with the other sets of controls: $N_c = 3, 4, 5$.

In the mixed field case, the noise attenuation caused by the local control varies between 13 *dB* and 18 *dB* without the presence of desired sound if N_c equals 5 (see Table VI). In case we consider the preservation of desired sound, the deterioration in the noise attenuation grows with the increase of N_c . The peak of noise attenuation even falls to a negative value, about -12 *dB*, if N_c reduces to 2. This means that the mixed noise is enhanced rather than reduced if the local control is applied. In addition, the level of noise attenuation fluctuates more significantly with the lower density of controls. In turn, in case the nonlocal control is applied to the mixed field, as shown in Table VI, a good enough level of noise attenuation, 13 – 19 *dB*, is achieved. It retains the same regardless the presence of desired sources if N_c is equal to 5. Even if N_c decreases to 2, the noise attenuation is quite reasonable. The reason for this is similar to the pure tone case discussed above.

TABLE VI. Effect of the number of controls N_c on attenuation of mixed sound.

N_c	Interval of noise attenuation, <i>dB</i>			
	local control	local control	Nonlocal control	Nonlocal control
	(without desired sound)	(with desired sound)	(without desired sound)	(with desired sound)
2	5 – 11	-12 – -1	5 – 11	5 – 11
3	7 – 13	-7 – 4	7 – 13	7 – 13
4	10 – 16	-3 – 6	9 – 16	10 – 16
5	13 – 18	0 – 9	13 – 19	13 – 19

Finally, we consider how the ratio of Hausdorff distance d_H between surfaces Γ and Γ^- to wave length λ affects the noise attenuation achieved with the use of nonlocal control. Suppose that the number of controls N_c and sensors N_s is equal to 3 and 6, for the sake of comparison. First, Table VII presents the noise attenuation for the pure tone field with $\lambda = 10\text{ m}$ achieved for different Hausdorff distances ($d_H = 0.1\text{ m}, 0.2\text{ m}, \dots, 0.9\text{ m}$). A significant noise attenuation is obtained for the pure tone field if the ratio d_H/λ is above 0.02. The noise attenuation increases with the increase of ratio d_H/λ until it reaches a maximal value of 13.21 dB with $d_H/\lambda = 0.08$. Afterwards, the noise attenuation decreases slightly to 13.18 dB if the ratio d_H/λ increases to 0.09. Thus, optimal ratio d_H/λ is equal to 0.08 in this case. In turn, in case d_H/λ is equal to 0.01, a negative noise attenuation of about -5 dB is obtained in time interval $[0\text{ s}, 1\text{ s}]$. The deterioration in the noise attenuation occurs because the operation of nonlocal control becomes rather local in case the external surface Γ^- is too close to the boundary surface Γ . As demonstrated in (Zhou and Utyuzhnikov, 2020), a control point source operates locally in its some vicinity.

TABLE VII. Effect of the ratio between Hausdorff distance d_H and wave length λ on attenuation of pure tone sound.

	d_H/λ								
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Noise attenuation, dB	-4.91	11.10	11.88	12.07	12.22	12.43	12.83	13.21	13.18

Next, Figure 7 demonstrates the noise attenuation for the mixed primary fields achieved for different Hausdorff distances ($d_H = 0.1\text{ m}$, 0.15 m , 0.2 m , 0.25 m). The minimal wave length λ_{min} is used as the reference length. As can be seen, the mixed noise is effectively attenuated if the ratio d_H/λ_{min} equals 0.06. There is a slight increase of noise attenuation if d_H/λ_{min} decreases to 0.05. The level of noise attenuation retains if d_H/λ_{min} is equal to 0.04. In turn, the noise attenuation achieved with $d_H/\lambda_{min} = 0.03$ reaches its minimum of -8 dB . Thus, the optimal Hausdorff distance d_H between surfaces Γ and Γ^- is about 5% of the minimal wave length to be attenuated.

FIG. 7. Effect of the ratio between Hausdorff distance d_H and λ_{min} on attenuation of mixed sound.

In the final section, the technique is extended to the case of broadband noise.

V. BROADBAND SOUND

Consider now the case of broadband noise in the range below $100 Hz$. The coordinates of noise and desired sources, parameters of shielded domain, density of controls and sensors, Hausdorff distance are the same as those in the pure tone case. The considered desired sound wave is monochromatic with the frequency of $50 Hz$. Figure 8 represents the sound pressure obtained at $0.5 s$ with the preservation of desired sound. As can be seen, the controls are individually distinguishable at perimeter Γ . With the application of controls, the desired sound is preserved inside domain D^+ while the noise is shielded. It is to be noted here that the results presented for very low frequencies down to $0 Hz$ are rather formal since such frequencies are hardly achievable in practice.

If the desired sound is not present, the broadband noise with and without shielding inside domain D^+ for time interval $[0 s, 1 s]$ is shown in Figure 9(a). As can be seen, the broadband noise can also be effectively attenuated. In turn, despite the presence of the desired sound, as can be seen in Figure 9(b), the noise attenuation obtained with the nonlocal control is basically the same as that without the desired sound.

The corresponding frequency spectrums with and without preservation of desired sound are shown in Figures 9(c) and 9(d), respectively. At low frequencies, especially below $40 Hz$, a significant noise attenuation is reached. It decreases as the frequency increases. It should be noted that the noise with ASC is attenuated slightly or even enhanced when the frequency exceeds $90 Hz$. Next, Figures 9(f) and 9(e) present the spectrum of noise attenuation obtained with and without presence of desired sound, respectively. Above $3 dB$,

415 the noise attenuation can be reached with frequencies below 40 Hz . In turn, a negative level
 416 of noise attenuation, about -0.05 dB , is obtained at 100 Hz . As can be seen, the presence
 417 of desired sound weakly affects the level of noise attenuation. This is a key property of the
 418 nonlocal control.

FIG. 8. Sound pressure at 0.5 s . Broadband sound case with desired sound.

419 Next, consider the sensitivity analysis with respect to the ratio between the Hausdorff
 420 distance d_H and minimal wave length λ_{min} to be attenuated. Since the set up of the broad-
 421 band case is the same as that in pure tone case, the reference Hausdorff distance d_H and its
 422 corresponding ratio d_H/λ_{min} are equal to 0.2 m and 0.06 . Several tests with different ratios

d_H/λ_{min} ($d_H/\lambda_{min} = 0.05, 0.06, 0.07, 0.08$) are considered while the densities of controls N_c and sensors N_s remain the same as those in the monochromatic case ($N_c = 3, N_s = 6$). Figure 10 shows the frequency spectrum of noise attenuation reached with several sets of ratio d_H/λ_{min} in case the desired sound is present. First, a negative value of noise attenuation is obtained with $d_H/\lambda_{min} = 0.07$ and 0.08 if frequency is above 65 and 55 Hz , respectively. In turn, a better noise attenuation is achieved with ratio d_H/λ_{min} of 0.06 than that obtained with ratio d_H/λ_{min} of 0.07 and 0.08 in the frequency spectrum. In case ratio d_H/λ_{min} decreases to 0.05 , a negative peak of noise attenuation, about -6 dB , is observed at 50 Hz . This phenomenon occurs because the contribution of the desired sound is not completely removed from the total field on the external surface Γ^- . Moreover, the operation of controls becomes local since the external surface Γ^- is too close to the boundary surface Γ . Thus, the optimal ratio d_H/λ_{min} equals 0.06 in this case.

Afterwards, consider the sensitivity analysis with respect to the density of sensors N_s . Suppose that d_H/λ_{min} and N_c are equal to 0.06 and 3 , respectively. Figure 11 demonstrates the frequency spectrum of noise attenuation obtained with several sets of N_s ($N_s = 4, 5, 6, 7$) if the desired sound is present. In the cases of $N_s = 4, 5$ the density of sensors is not large enough such that the deterioration of noise attenuation occurs between -2 dB and 0.8 dB if the frequency exceeds 50 Hz . If N_s increases to 6 , even though a good enough noise attenuation is reached for frequencies below 50 Hz , the noise attenuation deteriorates to a negative value for frequencies above 95 Hz . In turn, a better noise attenuation, above 1 dB , is obtained with $N_s = 7$ for higher frequencies in comparison with $N_s = 6$.

FIG. 9. Application of nonlocal control on broadband sound case with input primary field: (a) Noise signal (without desired sound), (b) Noise signal (with desired sound); (c) Frequency spectrum (without desired sound); (d) Frequency spectrum (without desired sound); (e) Noise attenuation (without desired sound); (f) Noise attenuation (with desired sound).

FIG. 10. Effect of the ratio between Hausdorff distance d_H and λ_{min} on attenuation of broadband noise.

444 Finally, consider how the density of controls N_c affects the noise attenuation. Assume
 445 that d_H/λ_{min} and N_s are equal to 0.06 and 6, respectively. Figure 12 presents the frequency
 446 spectrum of noise attenuation reached with several sets of the density of controls N_c ($N_c =$
 447 2, 3, 4, 5) in case the desired sound is present. A satisfactory noise attenuation is reached in
 448 the frequency range up to 100 Hz if N_c equals 5. Significant noise attenuation is reached at
 449 very low frequencies. In turn, the noise attenuation deteriorates to around 0 dB at higher
 450 frequencies. Finally, a negative level of noise attenuation is obtained with $N_c = 2$ if the
 451 frequency is above 80 Hz.

FIG. 11. Effect of the number of sensors N_s on attenuation of broadband noise.

FIG. 12. Effect of the number of controls N_c on attenuation of broadband noise.

452 VI. CONCLUSION

453 The applicability of the nonlocal potential-based algorithm for a real-time noise attenua-
 454 tion has been demonstrated via a series of numerical experiments in the time domain. It is
 455 shown that the noise attenuation achieved with a local control deteriorates with the presence
 456 of desired sound even if the total primary field at the perimeter of the domain to be shielded
 457 is available. This result occurs because a discrete set of controls is not able to provide a
 458 strict projection property of the surface potentials. In addition, the real-time ASC becomes
 459 unrealistic with the local control if a desired sound component is present and to be retained.
 460 In this case the noise level grows with time because of the reverse effect of the discrete
 461 control. As has been shown, the nonlocal control enables us to retain a significant noise
 462 attenuation even if the input field from the primary sources is unavailable and limited only
 463 to the total field from the primary and secondary sources. It turns out that with the non-
 464 local control the level of noise attenuation is mostly determined by the undesired incident
 465 field and weakly depends on the desired interior sound with the same level of intensity. This
 466 property is not realized with the local control.

467 The sensitivity analysis demonstrates that the density of sensors generally plays a more
 468 important role than the density of controls in the case of nonlocal control. On average, it is
 469 required to double the density of controls with respect to the density of sensors. In practical
 470 applications, the Hausdorff distance between the surfaces with sensors and controls is not
 471 allowed to be much less than the minimal wavelength to be attenuated. As demonstrated
 472 on different test cases including a broadband noise, for low enough frequencies the nonlocal

473 control provides practically the same level of noise attenuation regardless the presence of
474 desired sound.

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