Characteristics of Reversed Propagations Generated using Time Reversal Mirrors and Instantaneous Time Mirrors in Electromagnetics

Crystal T. Wu, Nuno M. Nobre, Emmanuel Fort, Graham D. Riley, and Fumie Costen

Abstract – The possibility of Instantaneous Time Mirror is explored in electromagnetic waves through the manipulation of a medium’s complex relative permittivity, causing a sudden disruption to the waves’ speed. The quality of the reversed foci obtained through Instantaneous Time Mirror and its classical alternative, Time Reversal Mirror, is reported in both air and water. We show how careful setup of both techniques can produce similar results. We adopt a visually-guided qualitative metric, but also quantitative metrics to describe accuracy and resolution, including a newly-proposed method to assess the multi-dimensional resolution of a focus.

1. Introduction

The principle of time reversal invariance allows a physical process to flow in both positive and negative temporal directions [1]. This can be thought of as if time were going backwards, akin to a video being played backwards. The process of time reversal in physical waves was shown to be possible by introducing a disruption in time or space [2]. The usual approach is through the use of a TRM (Time Reversal Mirror) [3]. TRM relies on transceivers to record the wave field on a spatial boundary, then, the digitally recorded waves are time reversed and re-emitted from the same spatial boundary. The alternative approach, known as ITM (Instantaneous Time Mirror) [4], manipulates a time boundary to create time reversed propagations without any transceivers.

The concept of ITM has been discussed theoretically [5] and tested in 1D (one-dimensional) simulations for EM (electromagnetic) waves to improve fundamental understanding of temporal boundaries [6]. Practical experimentation demonstrated ITM in water waves [4], induced through a sudden change of the effective gravity, which in turn altered the wave’s propagation speed. This ITM technique produced a new set of reversed wave fields that converged to the source location without using transceivers. Our work builds on top of the theoretical concept behind the water-based experiments in [4]. Through computer simulations, we explore the possibility of using ITM in EM waves in 2D (two-dimensions) via rapid manipulation of a medium’s complex relative permittivity. Thus, our time-reversed waves are generated by a change of the wave propagation speed. In practice, one can imagine this could be achieved by rapidly modifying properties of the propagation medium which are known to affect the electric permittivity: e.g. humidity, pressure, and temperature [7-9]. These challenges are, however, out of the scope of this paper. Instead, we focus on studying ITM theoretically in EM waves.

In theory, ITM and TRM work under the same principle but exploit different possibilities of the Cauchy initial or boundary conditions, respectively [4, 10]. Our interest here lies in the quality of the reversed foci generated by their use. Simulations of two sources placed at varying distances from each other were conducted independently in a dispersive lossy material, water, and in a nondispersive lossless material, air. Specifically, we compare results of our ITM reversed foci to the known quality of TRM reversed foci, produced using setups described in Section 3. In this paper, we present qualitative and quantitative evaluations of the reversed propagations generated using TRM and ITM, and show how they can be carefully manipulated to closely mimic each other.

2. Analytical Metrics

Various methods have been used to describe spatial resolution. These include: 1D FWHM (Full Width at Half Maximum) across the target [11, 12]; lateral resolution [13, 14]; and the minimum size of a detectable reflective source [15, 16]. While these methods present 1D information on the resolution, resolution characteristics in other dimensions are often neglected. Here, we present an additional analytic metric for spatial resolution that considers the multi-dimensionality of a focus.

On the basis of 1D resolution metrics, we propose multi-dimensional resolution to be expressed by a normalized contour plot, in which an area (or volume) can be computed based on any chosen relative intensity. Visually, the width of a peak in all directions in a space can be captured. This method allows us to relate spatial resolutions holistically in all orientations, and is suitable for studies conducted in multi-dimensional spaces with a complex scenario, such as a study with a human phantom involving asymmetry and different tissues.

In this paper, the foci of reversed propagations are
characterized. We explore different features of ITM and TRM, and investigate the limiting distance between two sources for which the same two targets are distinguishable by the reversed waves. An example is also provided where the new multi-dimensional resolution analysis reveals more information than the 1D resolution metrics.

3. Multi-Focal Simulation Model

The point sources were each modeled as a soft source, simulating Hertzian dipole antennas [17]. The excitation current was the first derivative of a Gaussian pulse with a central frequency of 3.0 GHz, covering a range approximately from 0.1 GHz to 7.5 GHz. The two sources were identical and placed equally offset from the center of the simulation space. The TRM transceivers also implemented soft sources, but the excitation current varied depending on the experiment, as detailed below.

The numerical modeling of ITM and TRM was conducted using the transverse magnetic mode of the 2D FDTD (Finite Difference Time Domain) method with a one pole Debye model for the permittivity [17]:

\[ \varepsilon_r = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + i\omega \tau_\sigma} = \frac{\sigma}{\omega \varepsilon_0}, \]

where \( \varepsilon_r \) is the complex relative permittivity, \( \varepsilon_\infty \) the optical permittivity, \( \varepsilon_s \) the static permittivity, \( i \) the imaginary unit, \( \omega \) the angular frequency, \( \tau_\sigma \) the relaxation time, \( \sigma \) the conductivity, and \( \varepsilon_0 \) the vacuum permittivity. The simulation took place in a 1000 mm \( \times \) 1000 mm space of a single medium, air or water, with absorbing boundary conditions [18]. To ensure validity of our 2D model, the temporal step \( \Delta t \) satisfies the stability condition in [19], given as: \( \Delta t \leq \delta / (v_{\text{max}} \sqrt{2}) \), where \( \delta \) is the length of a cell, and \( v_{\text{max}} \) the maximum wave phase velocity. \( \delta \) is 1/10 of the minimum wavelength for numerical accuracy in the spatial derivatives [19]. For air: \( \delta = 1 \) mm and \( \Delta t = 1.92 \) ps; for water: \( \delta = 0.48 \) mm and \( \Delta t = 0.93 \) ps. The Debye parameters used are shown in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \varepsilon_\infty )</th>
<th>( \varepsilon_s )</th>
<th>( \sigma ) (S/m)</th>
<th>( \tau_\sigma ) (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Water</td>
<td>74.87</td>
<td>5.10</td>
<td>0.115</td>
<td>6.50</td>
</tr>
</tbody>
</table>

Previous research showed time reversed waves can resolve multi-focal targets, provided the distance between the sources is at least a wavelength [22]. However, the quality of the reversed wave is affected by various factors owing to the nature of TRM and ITM, whose results cannot, in general, be directly compared [2]. Despite their different nature, we show TRM transceivers can be manipulated to produce visually similar results to ITM. We also examine limitations imposed by the dis-

Figure 1: Arrangement of the transceivers for TRM in (a) air and (b) water are denoted by black dots. Example source locations are shown as red asterisks.
Figure 2: Multi-focal target simulation in air showing TRM’s reverse excitation current can be manipulated to closely reproduce ITM reverse wave fields. The sources are 40 mm apart.

Figure 3: Multi-focal target simulation in water showing ITM’s disruption moment can be chosen to closely match TRM reverse wave fields. The sources are 40 mm apart.

Figure 4: Multi-focal target simulation in water showing ITM’s resolution power for the chosen frequency range.

Figure 5: Multi-focal target simulation in air showing the value of using multiple metrics to assess the focus quality.
distance between the two sources, and compared them in air and water to probe attenuation effects in TRM and ITM.

In Figure 2 to Figure 5, numerical results of the ‘convergence moment’ from the reversed propagations are presented. For ITM, this moment can be approximated by doubling the time at which the speed disruption is introduced. For TRM, this would be at a time after re-emission comparable to the time it took the initial field to reach the transceivers. We define the convergence moment by looking for the least distance between the reversed foci and the targets, with the smallest 50% contour area, in a time window around the predictions above.

We report whether the reversed waves converge to distinguishable foci: qualitatively, through a 1D projection on the line passing through both sources, shown on the right of Figure 2 to Figure 5; and quantitatively, via the 2D contour area at 50% of the peak’s amplitude, on the left of Figure 2 to Figure 5. The accuracy of the reversed foci relative to the sources’ sites is also shown.

In the 2D illustrations, local maxima are denoted as black crosses and the source locations are at the intersections of the white dashed lines. For the 1D projections, the source locations are outlined by black dashed lines. From Figure 3, the TRM results use the displacement current as the transceiver sources, and from Figure 4, ITM results use a speed disruption when the forward waves were approximately at the TRM transceiver locations.

Figure 2 and Figure 3 present results utilizing different attributes of TRM and ITM, respectively. In Figure 2, by using the derivative of the reversed electric displacement field in TRM, the resulting reversed wave field resembles that produced in ITM. Here, it is mostly a field sign issue but we demonstrated that the collected signal can be operated on and tuned to our advantage.

Figure 3 shows the implementation time of the ITM speed disruption affects the resolution area of the 50% contour. The earlier the disruption, the smaller the area, most likely due to attenuation in water. The contour area of the TRM example provided resembles ITM, when the time of ITM disruption coincides with the distance of the TRM transceivers to the sources. Note forward waves are also observable in some ITM results in the 1D perspective of Figure 3 and Figure 4. Artifacts caused by the finite number of TRM transceivers are observable as well.

The quality of the reversed propagations as we vary the distance between the sources is reported under three criteria in Figure 4 and Figure 5: (1) resolution, i.e. 2D contour area at 50% intensity of the normalized electric field, (2) accuracy, i.e distance from reversed foci to targets, and (3) 1D qualitative observation of the two foci. We expected the two targets to be resolvable to the order of a wavelength. The wavelengths corresponding to the wave’s central frequency were approximately 100 mm in air and 11.6 mm in water, and 40 mm in air and 4.8 mm in water for the highest frequency, but our results suggest a limit around 30 mm to 40 mm in air and 20 mm in water for both time reversal approaches. Although the resolvable distance between the two sources is smaller in water due to the shorter wavelengths, it is higher than predicted. We believe this is caused by the loss of high frequency components to attenuation\(^1\).

In Figure 5, we compare our area-based resolution and 1D qualitative methods. For the 30 mm case, two foci can be observed in 1D but the 50% contour reveals inaccuracy: the two foci missed the targets and their peaks blended together. This could, however, also be seen by using the 1D FWHM. Perhaps the study of EM waves in a human phantom provides a stronger case where analysis solely from a 1D perspective may be misleading. Figure 6a is an example where the 1D FWHM in distinct orientations would be different at the convergence moment. In Figure 6b, the absolute maximum of the electric field over the whole 2D space does not fall on the target. A 1D perspective analysis would instead show a relative peak, i.e. a ‘false’ focus. By using 2D contour graphs, we can quantify the focus area at a fraction of the absolute peak’s amplitude and provide anisotropic geometrical insight.

**5. Summary and Conclusion**

We simulated ITM-reversed waves on a simple, symmetrical scenario and our general observations agree with [2, 4]. We showed EM waves obtained through ITM can produce distinct foci and that, with manipulation, TRM can visually mimic wave fields generated by ITM even in the presence of dispersive and lossy media. We illustrated the benefits of a multi-dimensional resolution metric on heterogeneous media like the human body. Future work will explore attenuation-compensation methods for ITM in dispersive lossy media.

\(^1\)Time reversal invariance is known to break in conductive lossy media [11, 12]. Compensation methods for ITM are prospective research.
6. References


Crystal T. Wu is with Department of Computer Science, The University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom; e-mail: crystal.wu@manchester.ac.uk

Nuno M. Nobre is with Department of Computer Science, The University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom; e-mail: nunomiguel.nobre@manchester.ac.uk

Emmanuel Fort is with Institut Langevin, ESPCI Paris, PSL University, CNRS UMR 7587, 1 rue Jussieu, Paris, 75005, France; e-mail: emmanuel.fort@espci.fr

Graham D. Riley is with Department of Computer Science, The University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom; e-mail: graham.riley@manchester.ac.uk

Fumie Costen is with Department of Electrical and Electronic Engineering, The University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom; e-mail: fumie.costen@manchester.ac.uk