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Event-Triggered Anti-Disturbance Attitude Control for Rigid Spacecrafts with Multiple Disturbances[†]

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SUMMARY

This paper is devoted to the problem of attitude control for rigid spacecraft subject to multiple disturbances. Due to the limited communication and storage resource, an event-triggered anti-disturbance attitude control approach is proposed for the attitude control system (ACS) with multiple disturbances. Different from some existing event-triggered attitude control methods, the multiple disturbances are considered and can be divided into an uncertain modeled disturbance and a norm bounded equivalent disturbance. A disturbance observer is designed to estimate the modeled disturbance. Based on the framework of disturbance observer-based control (DOBC), the proposed event-triggered anti-disturbance controller can guarantee that the ACS converges toward a small invariant set, which successfully avoids continuous communication and Zeno phenomenon. Finally, simulation results are given to demonstrate the efficiency of the proposed approach. Copyright © 2010 John Wiley & Sons, Ltd.

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KEY WORDS: Spacecraft, Attitude Control System, Disturbance Observer, Event-triggered Control

1. INTRODUCTION

Spacecrafts refer to all kinds of aircraft carrying out special missions such as exploration, development, and utilization of space (see [1]). The spacecraft system is generally divided into two categories: special subsystem (payload) and support subsystem (general load). The support subsystem of spacecraft often includes structural subsystem, thermal control subsystem, power subsystem, attitude and orbit control subsystem, radio measurement and control subsystem, computer subsystem, etc. An attitude control system (ACS) is used to maintain or change the attitude of the spacecraft. Precise attitude plays a vital role in accomplishing scientific missions. Commonly used attitude control methods include three-axis attitude control, spin stability, gravity gradient stability, and so on. Elegant attitude control results have been proposed based on various methods such as PD (proportional-derivative) control ([2]), robust control ([3]), finite-time attitude tracking ([4, 5, 6]), anti-disturbance control ([7, 8, 9]), adaptive control ([10]), sliding mode control (SMC) ([11, 12]), back-stepping control ([13, 14]), fixed-time control ([15]), etc.

The structure and weight of the low-cost spacecraft limit the onboard communication bandwidth, storage resource, power. However, it is worth mentioning that most of the existing attitude control

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methods are developed in the continuous-time framework. These control approaches are time-triggered in a periodic sampling on the digital platform. When there is no external disturbance or the state variation is very small, the time-triggered periodic control method is a waste of storage and communication resources ([16, 17]). This has a great impact on the working efficiency of the spacecraft. In [18], a nonlinear event-triggered feedback control algorithm was proposed to ensure system exponential stable for ACS of the rigid spacecraft without disturbances. An event-triggered SMC attitude stabilization approach was addressed for a rigid spacecraft with disturbances and model uncertainties in [19]. In [20], an event-triggered controller with discrete triggered times was proposed to guarantee that attitude control errors converge to a small invariant set. In [21], an event-triggered inverse optimal control based on backstepping design was presented to realize the globally exponentially stable. For a rigid spacecraft with uncertainties, external disturbances, and input saturation, an adaptive SMC with periodic event-triggering strategy was proposed to guarantee the ACS to be ultimately bounded in [22]. An event-triggered fault tolerant control (FTC) approach based on a learning observer was presented for the spacecraft subject to disturbances and actuator faults with less updating frequency ([23]). For the spacecraft subject to uncertainty, external disturbances, and actuator faults, an event-triggered adaptive FTC based on the framework of the backstepping was addressed to reduce the communication burden in [24]. As we know, various types of disturbances commonly exist in the ACS of spacecraft including modeling error, space environmental disturbance, sensor noise, actuator error, etc. Nowadays, anti-disturbance control is much more challenging and has received a considerable amount of attention in the control field (see [25, 26, 27, 28, 29, 30] and references therein). To reject the vibration of the flexible appendages, a PD controller combining with disturbance-observer-based control (DOBC) was designed in [7]. A composite controller based on an adaptive sliding mode disturbance observer was formulated to compensate the system uncertainty in finite-time ([31]). For satellites subject to multiple disturbances, input saturation, and actuator faults, an anti-windup FTC method combining with disturbance observer was proposed in [8]. Different from the above-mentioned results, a novel event-triggered anti-disturbance control approach is presented for ACS of a rigid spacecraft with multiple disturbances in this paper. Compared with the existing anti-disturbance attitude control results ([7, 8, 9]), the dynamic and kinematic equations in this paper can be obtained without the assumption that Euler angles are very small. In addition, compared with references [18, 19, 20, 21], a rigid spacecraft subject to multiple disturbances is considered rather than the counterpart without disturbance or with an equivalent disturbance. Firstly, the dynamic and kinematic equations subject to multiple disturbances are described by the quaternion. Then, based on the framework of DOBC, a novel event-triggered anti-disturbance attitude control method is investigated. The proposed event-triggered controller design can guarantee that the closed-loop system converge towards to a small invariable set without Zeno phenomenon. Finally, simulation results are presented to illustrate the effectiveness of the proposed approach.

This paper is organized as follows. In section II, the dynamic and kinematic equations of a rigid spacecraft subject to multiple disturbances are introduced. In section III, disturbance observer is designed to estimate the modeled disturbance. Then, an event-triggered controller is constructed for a spacecraft. Simulation examples are provided in section IV to demonstrate the efficiency of the proposed method. The concluding remarks are given in section V.

Notation: In this paper, the Euclidean norm of a vector $s(t)$ is defined by $\|s(t)\|^2 = s^T(t)s(t)$. The matrix norm is defined as the Frobenius norm. $R^{m \times n}$ represents the $m \times n$ dimensional real matrix. A real symmetric matrix $P > (\geq) 0$ denotes P being a positive definite (positive semi-definite) matrix. The superscripts "T" and "-1" stand for matrix transposition and matrix inverse, respectively. I is used to denote an identity matrix with proper dimension. Matrices, if not explicitly stated, are supposed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by *. The symbol $\text{sym}(\cdot)$ represents $\text{sym}(\Xi) := \Xi + \Xi^T$, and $\text{diag}\{a\}$ is a diagonal matrix with the elements of variable a as the diagonal elements.

2. PROBLEM DESCRIPTION

The dynamic equation of a rigid spacecraft is considered as follows (see [32]).

$$J\dot{\omega} + \omega^\times J\omega = u + d_1 + d_2 \quad (1)$$

where $J \in \mathbb{R}^{3 \times 3}$ represents the positive definite inertia moment. $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T$ is the angular velocity vector. u represents three-axis control torque. d_2 represents the norm bounded disturbance including space environmental torques, modeling error, etc. The notation $\omega^\times$ for a vector ω is the skew-symmetric matrix, which can be represented by the following form

$$\omega^\times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad (2)$$

Reaction wheels are widely used in the ACS of a three-axis stabilized satellite. The reaction wheel will generate a periodic vibration in the operating process, such as harmonic disturbances or their superposition. Then, the modeled disturbance of actuator can be denoted by an exo-system

$$\begin{cases} \dot{x}(t) &= Ax(t) \\ d_1(t) &= Cx(t) \end{cases} \quad (3)$$

where $x(t)$ is the state variable of the exogenous system. $A \in \mathbb{R}^{p \times p}$, and $C \in \mathbb{R}^{3 \times p}$ are the known parameter matrices.

The kinematic equations of spacecrafts can be described by the quaternion.

$$\dot{q} = \frac{1}{2}(q^\times + q_0 I_3)\omega \quad (4)$$

$$\dot{q}_0 = -\frac{1}{2}q^T \omega \quad (5)$$

where I_3 represents the 3×3 identity matrix. $(q_0, q^T) = (q_0, q_1, q_2, q_3) \in \mathbb{R} \times \mathbb{R}^3$ is the unit quaternion that denotes the attitude of satellites in the body frame with respect to the inertial frame. The unit quaternion can be calculated that

$$q_0^2 + q^T q = q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (6)$$

In order to deal with the problem conveniently, we give the following assumptions.

Assumption 1: It is assumed that the change rate of the modeled disturbance d_1 is bounded.

$$\|\dot{d}_1\| \leq \bar{D}_1 \quad (7)$$

Assumption 2: The equivalent disturbance d_2 is norm bounded, namely:

$$\|d_2\| \leq \bar{d}_2 \quad (8)$$

Assumption 3: Based on the process of attitude measurement and determination for spacecraft, the attitude quaternion $[q_0, q^T]^T$ and angular velocity ω can be assumed to be available.

Assumption 4: The pair (A, C) is observable.

Remark 1. The parameter matrices of an exo-system in (3) can be determined by parameter identification methods or measurement. For instance, the matrices A and C of a harmonic signal can be determined by measuring the frequency with sensors. Different from the existing anti-disturbance attitude control results of spacecraft ([7, 8, 9]), the assumption that Euler angles are very small can be removed in the dynamic and kinematic equations in this paper. In addition, the anti-disturbance control problem of the event-triggered control is firstly considered for spacecraft subject to multiple disturbances.

3. MAIN RESULTS

3.1. Disturbance Observer

In this subsection, a disturbance observer will be presented to estimate the modeled external disturbance $d_1(t)$. The reduced-order disturbance observer is designed as

$$\begin{cases} \hat{x}(t) &= r(t) - LJ\omega \\ \dot{r}(t) &= (A + LC)(r(t) - LJ\omega) + L(-\omega^\times J\omega + u) \\ \hat{d}_1(t) &= C\hat{x}(t) \end{cases} \quad (9)$$

where $\hat{x}(t)$ and $\hat{d}_1(t)$ are the estimations of $x(t)$ and $d_1(t)$, respectively. Matrix L is the observer gain to be determined later. $r(t)$ is an auxiliary variable. The estimation error of disturbance is defined as $e_x(t) = x(t) - \hat{x}(t)$. From equations (1), (3) and (9), the estimation error equation of modeled disturbance satisfies

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= (A + LC)e_x(t) + Ld_2(t) \end{aligned} \quad (10)$$

The following lemma will be used in the subsequent control law design.

Lemma 1 ([21]). For any vectors $a, b \in R^n$ and matrix $M \in R^{n \times n}$, $M = M^T > 0$,

$$2a^T b \leq a^T M a + b^T M^{-1} b \quad (11)$$

In this section, the objective is to design an event-triggered anti-disturbance controller to guarantee that the closed-loop system (1), (4) and (5) is stable in the presence of multiple disturbances. The structure of controller is formulated as

$$u(t) = \frac{1}{2}\omega^\times J\omega - \hat{d}_1 - K_1\omega - K_2q \quad (12)$$

where K_1 and K_2 are controller gains to be determined later. Substituting the controller (12) into the system (1), it can be seen that

$$J\dot{\omega} = -\frac{1}{2}\omega^\times J\omega - K_1\omega - K_2q + Ce_x + d_2 \quad (13)$$

The designed approach of the time-triggered anti-disturbance attitude control can be formulated as follows.

Theorem 1. For parameters $\alpha, \alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0$ and matrix $Q > 0$, if there exist matrices $P > 0, L, M, K_1$ and K_2 satisfying

$$\Xi_1 = \begin{bmatrix} \frac{1}{2}J & \frac{\alpha}{2}J & 0 \\ * & K_2I & 0 \\ * & * & P \end{bmatrix} > 0 \quad (14)$$

$$\Xi_2 = \begin{bmatrix} \frac{1}{2}J & \frac{\alpha}{2}J & 0 \\ * & (n+1)K_2I & 0 \\ * & * & P \end{bmatrix} > 0 \quad (15)$$

$$\Xi_3 = \begin{bmatrix} K_1I - \alpha_1I - \frac{1}{2}\alpha J & \frac{\alpha}{2}K_1 & -\frac{C}{2} \\ * & \alpha(K_2 - \alpha_2)I & -\frac{\alpha C}{2} \\ * & * & Q \end{bmatrix} > 0 \quad (16)$$

and

$$PA + MC + A^T P + C^T M^T + \alpha_3 M^T M < -Q \quad (17)$$

then with the controller (12) and gain $L = P^{-1}M$, the controlled system consisting of equations (1), (4), and (5) is ultimately stable.

Proof. Select the candidate of Lyapunov function as

$$V(t) = \frac{1}{2}\omega^T J\omega + K_2[(1 - q_0)^2 + q^T q] + \alpha q^T J\omega + e_x^T P e_x \quad (18)$$

Because q_0 satisfies the unit constraint (6), the following inequalities hold with the condition that $-\frac{n-1}{n+1} \leq q_0 \leq 1$.

$$q^T q \leq (1 - q_0)^2 + q^T q \leq (n + 1)q^T q \quad (19)$$

where $n \geq 1$. Then one has

$$X^T \Xi_1 X \leq V(t) \leq X^T \Xi_2 X \quad (20)$$

where $X^T = [\omega^T \quad q^T \quad e_x^T]$.

By choosing appropriate parameters α and K_2 , the matrices Ξ_1 and Ξ_2 are positive definite. It is verified that $V(t) > 0$ holds for all arguments. Along with the trajectories of (4), (5) and (13), one obtained that

$$\begin{aligned} \dot{V}(t) &= \omega^T J\dot{\omega} + K_2[2(q_0 - 1)\dot{q}_0 + 2q^T \dot{q}] + \alpha q^T J\dot{\omega} + \alpha \omega^T J\dot{q} + 2e_x^T P \dot{e}_x \\ &= \omega^T (-\frac{1}{2}\omega^\times J\omega - K_1\omega - K_2q + Ce_x + d_2) + K_2[-(q_0 - 1)q^T \omega + q^T (q^\times + q_0 I_3)\omega] \\ &\quad + \alpha q^T (-\frac{1}{2}\omega^\times J\omega - K_1\omega - K_2q + Ce_x + d_2) + \frac{1}{2}\alpha \omega^T J(q^\times + q_0 I_3)\omega \\ &\quad + e_x^T [P(A + LC) + (A + LC)^T P]e_x + 2e_x^T P L d_2 \end{aligned}$$

The property of operator $\times$ implies that

$$q^T q^\times = [0 \quad 0 \quad 0], q^T \omega^\times = -\omega^T q^\times, \omega^T q^\times J\omega = -\omega^T J q^\times \omega,$$

then we have

$$\begin{aligned} \dot{V}(t) &= -K_1\omega^T \omega + \omega^T C e_x + \omega^T d_2 + \frac{1}{2}\alpha q_0 \omega^T J\omega - \alpha K_1 q^T \omega - \alpha K_2 q^T q \\ &\quad + \alpha q^T C e_x + \alpha q^T d_2 + e_x^T [P(A + LC) + (A + LC)^T P]e_x + 2e_x^T P L d_2 \end{aligned}$$

From Lemma 1, it can be concluded that

$$\begin{aligned} \dot{V}(t) &\leq -(K_1 - \alpha_1)\omega^T \omega - \alpha(K_2 - \alpha_2)q^T q + (\frac{1}{4\alpha_1} + \frac{\alpha}{4\alpha_2} + \frac{1}{\alpha_3})\bar{d}_2^2 \\ &\quad + \omega^T C e_x - \alpha K_1 q^T \omega + \frac{1}{2}\alpha q_0 \omega^T J\omega + \alpha q^T C e_x \\ &\quad + e_x^T [P(A + LC) + (A + LC)^T P + \alpha_3 L^T P P L]e_x \\ &= -(K_1 I - \alpha_1 I - \frac{1}{2}\alpha q_0 J)\omega^T \omega - \alpha(K_2 - \alpha_2)q^T q + \omega^T C e_x + \alpha q^T C e_x - e_x^T Q e_x + f \\ &\leq -X^T \Xi_3 X + f \end{aligned}$$

where α_1, α_2 and α_3 are selected parameters,

$$f = (\frac{1}{4\alpha_1} + \frac{\alpha}{4\alpha_2} + \frac{1}{\alpha_3})\bar{d}_2^2 \quad (21)$$

According to the comparison lemma [33], one has

$$V(t) \leq e^{-\kappa t} V(0) + \int_0^t e^{-\kappa(t-\tau)} f(\tau) d\tau \quad (22)$$

where

$$\kappa = \frac{\lambda_{\min}(\Xi_3)}{\lambda_{\max}(\Xi_2)} \quad (23)$$

Based on the Lyapunov theory, it can be seen that the error system (1) is stable in the presence of norm bounded disturbances $d_2(t)$ and modeled disturbance $d_1(t)$. This completes the proof.

Remark 2. In [18], q_0 is restricted to be $0 \leq q_0 \leq 1$. As can be seen from the proof of theorem 1, the range of the unit quaternion is determined by the constant n . Therefore, the range of quaternion in this paper is larger than the counterpart in [18].

3.2. Design of Event-Triggered Anti-Disturbance Controller

This subsection is to design an event-triggered anti-disturbance controller for rigid spacecraft subject to multiple disturbances. The event-triggering mechanism is designed as

$$\tau(t) = u(t_i), \quad t \in [t_i, t_{i+1}) \quad (24)$$

with

$$t_0 = 0, \quad t_{i+1} = \inf\{t | t > t_i, \|e_\tau(t)\|^2 > \nu e^{-\mu t} + \rho\}, i = 0, 1, 2, \dots \quad (25)$$

where

$$e_\tau(t) = \tau(t) - u(t) \quad (26)$$

is the accumulated error of the control signal, $\nu > 0$, $\mu > 0$ and $\rho > 0$ are parameters to be determined.

Substituting the event-triggered anti-disturbance controller (24) into the system (1), it can be seen that

$$J\dot{\omega} = -\frac{1}{2}\omega^\times J\omega - K_1\omega - K_2q + Ce_x + d_2 + e_\tau \quad (27)$$

Theorem 2. For parameters $\alpha, \alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0$ and matrix $Q > 0$, if there exist matrices $P > 0, L, M, K_1$ and K_2 satisfying the following inequalities (14), (15), (17) and

$$\Xi_4 = \begin{bmatrix} K_1I - \alpha_1I - \frac{1}{2}\alpha J - \gamma_1I & \frac{\alpha}{2}K_1 & -\frac{C}{2} \\ * & \alpha(K_2 - \alpha_2 - \gamma_2)I & -\frac{\alpha C}{2} \\ * & * & Q \end{bmatrix} > 0 \quad (28)$$

then with the event-triggered anti-disturbance controller (24) and gain $L = P^{-1}M$, the controlled system consisting of equations (1), (4), and (5) is uniformly ultimately bounded. In addition, there is no Zeno behavior in the event triggering time sequence.

Proof. The time derivative of the Lyapunov function V defined in (18) satisfies that

$$\begin{aligned} \dot{V}(t) &= -K_1\omega^T\omega + \omega^T Ce_x + \omega^T d_2 + \frac{1}{2}\alpha q_0\omega^T J\omega - \alpha K_1 q^T\omega - \alpha K_2 q^T q \\ &\quad + \alpha q^T Ce_x + \alpha q^T d_2 + e_x^T [P(A + LC) + (A + LC)^T P]e_x + 2e_x^T PLd_2 + \omega^T e_\tau + q^T e_\tau \\ &\leq -(K_1I - \alpha_1I - \frac{1}{2}\alpha q_0J)\omega^T\omega - \alpha(K_2 - \alpha_2)q^T q + \omega^T Ce_x + \alpha q^T Ce_x \\ &\quad - \alpha K_1 q^T\omega + \alpha q^T e_\tau + \omega^T e_\tau - e_x^T Qe_x + f \\ &\leq -(K_1I - \alpha_1I - \frac{1}{2}\alpha q_0J - \gamma_1I)\omega^T\omega - \alpha(K_2 - \alpha_2 - \gamma_2)q^T q + \omega^T Ce_x + \alpha q^T Ce_x \\ &\quad - \alpha K_1 q^T\omega - e_x^T Qe_x + f + (\frac{1}{4\gamma_1} + \frac{\alpha}{4\gamma_2})\|e_\tau\|^2 \\ &\leq -X^T \Xi_4 X + f + (\frac{1}{\gamma_1} + \frac{1}{\gamma_2})\|e_\tau\|^2 \\ &\leq -\kappa_0 V + f_0 \end{aligned}$$

where

$$\kappa_0 = \frac{\lambda_{\min}(\Xi_4)}{\lambda_{\max}(\Xi_2)} \quad (29)$$

$$f_0 = (\frac{1}{\gamma_1} + \frac{1}{\gamma_2})(\nu + \rho) + f \quad (30)$$

According to the comparison lemma [33], it can be obtained that

$$\begin{aligned} V(t) &\leq e^{-\kappa_0 t} V(0) + \int_0^t e^{-\kappa_0(t-\tau)} f_0 d\tau \\ &= e^{-\kappa_0 t} V(0) + \frac{f_0}{\kappa_0} (1 - e^{-\kappa_0 t}) \\ &= e^{-\kappa_0 t} (V(0) - \frac{f_0}{\kappa_0}) + \frac{f_0}{\kappa_0} \\ &\leq e^{-\kappa_0 t} V(0) + \frac{f_0}{\kappa_0} \end{aligned}$$

Therefore, the variable X satisfies

$$\|X\| \leq \sqrt{\frac{e^{-\kappa_0 t} V(0) + \frac{f_0}{\kappa_0}}{\lambda_{\min}(P_1)}} = R \quad (31)$$

The next step is to prove that there is no Zeno behavior in the event triggering time sequence. From (27), it can be obtained that

$$\begin{aligned} \|\dot{\omega}\| &= \|J^{-1}[\frac{1}{2}\omega^\times J\omega - K_1\omega - K_2q + Ce_x + d_2 + e_\tau]\| \\ &\leq \|J^{-1}\|[\frac{1}{2}\|\omega^\times\|\|J\|\|\omega\| + K_1\|\omega\| + K_2\|q\| + \|Ce_x\| + \bar{d}_2 + \|e_\tau\|] \\ &\leq \|J^{-1}\|[\frac{\sqrt{2}}{2}\|J\|R^2 + K_1R + K_2R + \|C\|R + \bar{d}_2 + \nu + \rho] \\ &= R_0 \end{aligned} \quad (32)$$

On any interval $[t_i, t_{i+1})$, we have,

$$\begin{aligned} \dot{e}_\tau(t) &= -\dot{u}(t) = -\frac{1}{2}\omega^\times J\dot{\omega} - \frac{1}{2}\dot{\omega}^\times J\omega + \dot{d}_1(t) + K_1\dot{\omega} + K_2\dot{q} \\ &= K_1\dot{\omega} + \frac{K_2}{2}(q^\times + q_0I_3)\omega - C[(A + LC)e_x + Ld_2] + \dot{d}_1 - \frac{1}{2}\omega^\times J\dot{\omega} - \frac{1}{2}\dot{\omega}^\times J\omega \\ &\leq K_1\|\dot{\omega}\| + \frac{K_2}{2}(\|q^\times\| + 1)\|\omega\| + \|C(A + LC)\|\|e_x\| + \|CL\|\bar{d}_2 + \bar{D}_1 \\ &\quad + \frac{1}{2}\|\omega^\times\|\|J\|\|\dot{\omega}\| + \frac{1}{2}\|\dot{\omega}^\times\|\|J\|\|\omega\| \\ &\leq K_1\|\dot{\omega}\| + \frac{K_2}{2}(\sqrt{2}\|q\| + \sqrt{3})\|\omega\| + \|C(A + LC)\|\|e_x\| + \|CL\|\bar{d}_2 + \bar{D}_1 \\ &\quad + \frac{\sqrt{2}}{2}\|\omega\|\|J\|\|\dot{\omega}\| + \frac{\sqrt{2}}{2}\|\dot{\omega}\|\|J\|\|\omega\| \end{aligned} \quad (33)$$

Using (33), it can be derived as follows.

$$\begin{aligned} \frac{d[e_\tau^T e_\tau]}{dt} &= 2e_\tau^T \dot{e}_\tau \\ &\leq 2[K_1\|\dot{\omega}\| + \frac{K_2}{2}(\sqrt{2}\|q\| + 1)\|\omega\| + \|C(A + LC)\|\|e_x\| + \|CL\|\bar{d}_2 + \bar{D}_1 \\ &\quad + \frac{\sqrt{2}}{2}\|\omega\|\|J\|\|\dot{\omega}\| + \frac{\sqrt{2}}{2}\|\dot{\omega}\|\|J\|\|\omega\|]\|e_\tau\| \\ &\leq 2[K_1R_0 + \frac{K_2}{2}(\sqrt{2}R + 1)R + \|C(A + LC)\|R + \|CL\|\bar{d}_2 + \bar{D}_1 \\ &\quad + \frac{\sqrt{2}}{2}R\|J\|R_0 + \frac{\sqrt{2}}{2}R_0\|J\|R](\nu + \rho) \\ &= \varpi \end{aligned} \quad (34)$$

For $t \in [t_i, t_{i+1})$, it can be seen that

$$e_\tau^T(t)e_\tau(t) \leq \vartheta(t) \quad (35)$$

Hence, the following relationship is established.

$$\dot{\vartheta}(t) = \varpi, t \in [t_i, t_{i+1}) \quad (36)$$

It can be concluded that

$$\vartheta(t) = (t - t_i)\varpi \quad (37)$$

That is to say

$$\vartheta(t_{i+1}) = (t_{i+1} - t_i)\varpi = \nu e^{-\mu t_{i+1}} + \rho \quad (38)$$

We have

$$\varpi(t_{i+1} - t_i) > \rho \quad (39)$$

This results in that

$$t_{i+1} - t_i > \frac{\rho}{\varpi} \quad (40)$$

The inter-execution time has a constant positive lower bound. Therefore, there is no Zeno behavior in the event triggering time sequence.

This completes the proof.

Remark 3. The gains of the disturbance observer and controller can be determined in theorem 2 based on the convex optimization algorithm.

4. SIMULATION EXAMPLES

In this section, we consider a spacecraft operating in a circular orbit. The three-axis inertia matrix is given by

$$J = \begin{bmatrix} 40 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} \quad (41)$$

The initial angular velocity is $\omega(0) = [1 \ 0 \ -1]^T$ (rad/sec), and the initial attitude quaternion is $q(0) = [0.3 \ -0.2 \ -0.3 \ 0.8832]^T$. The equivalent disturbance d_2 is supposed to be a random signal with upper bound 1. The modeled disturbance d_1 in (3) is a periodic signal caused by vibration and selected as

$$d_1 = \begin{bmatrix} 1.5 \sin 7t + 3 \cos 8.4t \\ 1.5 \cos 7t + 3 \sin 8.4t \\ 3 \sin 7t + 1.5 \cos 8.4t \end{bmatrix}$$

The parameter matrices are denoted as

$$A = \begin{bmatrix} 0 & -7 & 0 & 0 \\ 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8.4 \\ 0 & 0 & -8.4 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1.5 & 0 & 3 & 0 \\ 0 & 1.5 & 0 & 3 \\ 3 & 0 & 1.5 & 0 \end{bmatrix}$$

For $\alpha = 1$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\gamma_1 = 0.1$, $\gamma_2 = 0.2$, $\gamma_3 = 49$, and positive definition matrix

$$Q = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad (42)$$

the gains of the disturbance observer (9) and the state feedback controller (12) are concluded that

$$L = \begin{bmatrix} -0.0645 & -0.0062 & -0.2019 \\ 0.0061 & -0.1494 & 0.0077 \\ -0.1320 & 0.0051 & -0.0298 \\ -0.0037 & -0.1699 & -0.0132 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 44.7962 & 0.4000 & 0.3000 \\ 0.4000 & 37.1296 & 0.4667 \\ 0.3000 & 0.4667 & 36.4629 \end{bmatrix}$$

Figure 1. Response curves of attitude quaternion.

Figure 2. Response curves of angular velocities.

Figure 3. Response curves of angular velocities without disturbance observer.

$$K_2 = \begin{bmatrix} 47.3944 & 0 & 0 \\ 0 & 47.3944 & 0 \\ 0 & 0 & 47.3944 \end{bmatrix}.$$

Figure 4. The responses of disturbance $d_1(t)$, its estimation and error.

Figure 5. The responses of disturbance $d_2(t)$, its estimation and error.

The response curves of attitude quaternion are demonstrated in Fig. 1, and angular velocities are shown in Fig. 2. The response curves of angular velocities without disturbance observer are depicted in Fig. 3. From Figs. 1-3, the attitude and angular velocity of the closed-loop system are stable, which demonstrates that the proposed event-triggered anti-disturbance controller can reject the modeled disturbance in presence of multiple disturbances.

Figs. 4-6 demonstrate the responses of the modeled disturbance $d_1(t)$, its estimation and the estimation error. From Figs. 4-6, the proposed disturbance observer can effectively estimate the modeled disturbance $d_1(t)$.

The proposed event-triggered anti-disturbance controller is depicted in Fig. 7. The time-triggered anti-disturbance controller is shown in Fig. 8. From Figs. 7-8, the event-triggered control reduces the waste of communication and computation resources for the ACS.

The inter-event time of the proposed event-triggered controller is shown in Fig. 9.

The curve of the event-triggered controller without disturbance observer is shown in Fig. 10. The inter-event time without disturbance observer is demonstrated in Fig. 11. From Figs. 9-11, the trigger times of the proposed controller is more frequency than the counterpart of the event-triggered controller without disturbance observer. However, from Figs. 2-3, it can be seen that the proposed event-triggered controller with disturbance observer has a better anti-disturbance ability and attitude accuracy than the event-triggered controller without disturbance observer.

Figure 6. The responses of disturbance $d_3(t)$, its estimation and error.

Figure 7. Curves of the event-triggered control torques.

Figure 8. Curves of the time-triggered control torques.

Figure 9. Response curve of inter event time with disturbance observer.

Figure 10. Curve of the event-triggered control torques without disturbance observer.

Figure 11. Response curve of inter event time without disturbance observer.

5. CONCLUSION

In this paper, the problem of event-triggered anti-disturbance attitude control is investigated for spacecraft with multiple disturbances. There are the following features of the proposed method

- Firstly, a new dynamic equation subject to multiple disturbances is considered instead of lumping all the disturbances and errors together as a stochastic noise or bounded variable. The multiple disturbances are divided into two parts including modeled disturbance and the equivalent norm bounded disturbance.
- Secondly, an event-triggered anti-disturbance attitude controller is designed for the concerned ACS with both disturbance rejection and attenuation performance. A disturbance observer is designed to estimate the modeled disturbance. Based on the framework of DOBC, the proposed controller can guarantee that the ACS converges toward a small invariant set without Zeno phenomenon.

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