AERODYNAMIC AND AEROACOUSTIC ANALYSIS OF COUNTER ROTATING OPEN ROTORS

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Science and Engineering

2020

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Copyright

Acknowledgements

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Abstract

Due to their potential to significantly reduce aircraft emissions, there is a renewed interest in Counter Rotating Open Rotors (CROR). However, there are concerns surrounding their noise emissions. In an effort to address this, the present work focuses on developing and applying numerical tools to study noise-reducing strategies for CROR. In particular, the work is primarily focussed on the application to general aviation aircraft.

A number of low-order tools have been developed in order to consider a large design space. The low-order tools are used to undertake a CROR design study utilising an optimisation routine. A number of CROR designs were identified that successfully reduced noise at take-off whilst simultaneously meeting a number of other additional design constraints.

A number of novel CROR configurations are proposed to reduce noise in the terminal area. Firstly, it is proposed to lock either the fore or the aft rotor during take-off, climb-out and approach conditions, with the operative rotor delivering the total thrust. During cruise, the locked rotor is restarted to realise the high efficiency of the CROR. It is hypothesised that locking a blade row will reduce the interaction component that dominates the CROR noise footprint in the terminal area.

The low-order design tools are first used to investigate the potential of the novel configurations over a wide design space. The low-order models identified a number of blade count combinations offering reductions in noise relative to a baseline CROR.

High-order simulations were subsequently carried out to support these predictions. uRANS CFD, coupled to an acoustic solver, was used to confirm the potential of the proposed concepts. In particular, noise reductions of $\sim 3.6$ dB(A) were observed for the locked-fore configurations, whilst the locked-aft configuration offered $\sim 7.9$ dB(A) reductions relative to a baseline configuration. These noise reductions came at the cost of $\sim 5\%$ penalty in efficiency during take-off. However, the cruise efficiency remains unchanged.
Declaration

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I would firstly like to thank Mr Mike Newton for the financial support that has allowed me to carry out the work presented within the thesis. I would like to thank my supervisor Dr Filippone for the support in getting me this far. I would like to acknowledge the assistance given by Research IT, and the use of the HPC Pool. Furthermore, I would also like to acknowledge the use of EPCC’s Cirrus HPC service and the N8 regional HPC service. Finally, I wish to thank Prof. Barakos and the members of the CFD laboratory at the University of Glasgow for the use of the HMB3 solver for the present work.
Publications

Journal Papers


Papers in Conference Proceedings

D. A. Smith, A. Filippone, G. N. Barakos, “High-Order Modelling of Open Rotors”, Proceedings of the MACE PGR Conference, 26 March 2019, Manchester UK,

Presentations without Proceedings

D. A. Smith, A. Filippone, “Counter Rotating Open Rotors for General Aviation”, UK Vertical Lift Network Annual Technical Workshop, 22-23 May 2017
Nomenclature

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>BEM</td>
<td>Boundary Element Method</td>
</tr>
<tr>
<td>BEMT</td>
<td>Blade Element Momentum Theory</td>
</tr>
<tr>
<td>BET</td>
<td>Blade Element Theory</td>
</tr>
<tr>
<td>BPF</td>
<td>Blade Passage Frequency = $\frac{N\Omega}{2\pi}$</td>
</tr>
<tr>
<td>CAA</td>
<td>Computational AeroAcoustics</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CROR</td>
<td>Counter Rotating Open Rotors</td>
</tr>
<tr>
<td>EPNL</td>
<td>Effective Perceived Noise Level</td>
</tr>
<tr>
<td>SPL</td>
<td>Sound Pressure Level</td>
</tr>
<tr>
<td>SRP</td>
<td>Single Rotation Propeller</td>
</tr>
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Roman Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$A$</td>
<td>Blade element area</td>
<td>$[m^2]$</td>
</tr>
<tr>
<td>$c$</td>
<td>Blade element chord</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Speed of sound</td>
<td>$[m/s]$</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Blade element drag coefficient</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$c_l$</td>
<td>Blade element lift coefficient</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$C_a$</td>
<td>Blade element axial force coefficient</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Blade element normal force coefficient</td>
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</tr>
<tr>
<td>$C_p$</td>
<td>Rotor power coefficient = $\frac{p}{\rho n^2 D^2}$</td>
<td>$[-]$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>$C_Q$</td>
<td>Rotor torque coefficient $\frac{Q}{\rho n^2 D^5}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Rotor thrust coefficient $\frac{T}{\rho n^2 D^4}$</td>
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</tr>
<tr>
<td>$D$</td>
<td>Rotor diameter</td>
<td>[m]</td>
</tr>
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<td>$\mathcal{D}$</td>
<td>Drag acoustic source term</td>
<td>[-]</td>
</tr>
<tr>
<td>$e$</td>
<td>Specific energy of fluid element $e$</td>
<td>[J/kg]</td>
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<tr>
<td>$E$</td>
<td>Total energy (per unit volume) of fluid element $E$</td>
<td>[J/m$^3$]</td>
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<td>$f$</td>
<td>Frequency</td>
<td>[Hz]</td>
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<tr>
<td>$f_D$</td>
<td>Chordwise drag coefficient distribution</td>
<td>[-]</td>
</tr>
<tr>
<td>$f_L$</td>
<td>Chordwise lift coefficient distribution</td>
<td>[-]</td>
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<tr>
<td>$f_T$</td>
<td>Chordwise thickness distribution</td>
<td>[-]</td>
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<tr>
<td>$F_c$</td>
<td>Centrifugal force</td>
<td>[N]</td>
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<td>$F_{i,v}$</td>
<td>Inviscid/viscous flux vectors</td>
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</tr>
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<td>$F_T$</td>
<td>Prandtl's tip/hub loss factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$g$</td>
<td>Axial spacing between rotors</td>
<td>[m]</td>
</tr>
<tr>
<td>$G_A$</td>
<td>A-weighting filter gain</td>
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</tr>
<tr>
<td>$j$</td>
<td>Complex variable, $\sqrt{-1}$</td>
<td>[-]</td>
</tr>
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<td>$J$</td>
<td>Rotor advance ratio $\frac{V_\infty}{nD}$</td>
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<tr>
<td>$\mathcal{J}_\nu(Z)$</td>
<td>Bessel function of order $\nu$ and argument $Z$</td>
<td>[-]</td>
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<tr>
<td>$k_1,k_2$</td>
<td>Harmonics</td>
<td>[-]</td>
</tr>
<tr>
<td>$k_x,k_y$</td>
<td>Chordwise wave numbers representing non-compactness</td>
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</tr>
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<td>$l_{\text{ref}}$</td>
<td>Reference length</td>
<td>[m]</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Turbulent length scale</td>
<td>[m]</td>
</tr>
<tr>
<td>$L_A$</td>
<td>A-weighted SPL</td>
<td>[dB]</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>Lift acoustic source term</td>
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<tr>
<td>$m$</td>
<td>Mass</td>
<td>[kg]</td>
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<tr>
<td>( m )</td>
<td>Mass flow rate</td>
<td>[kg/s]</td>
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<td>( M_\infty )</td>
<td>Free-stream Mach number = ( V_\infty / c_0 )</td>
<td>[-]</td>
</tr>
<tr>
<td>( M_t )</td>
<td>Tip Mach number = ( \Omega R / c_0 )</td>
<td>[-]</td>
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<tr>
<td>( M_r )</td>
<td>Relative Mach number = ( V_r / c_0 )</td>
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</tr>
<tr>
<td>( n )</td>
<td>Rotational frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>( n )</td>
<td>Unit normal</td>
<td>[-]</td>
</tr>
<tr>
<td>( N )</td>
<td>Rotor blade count</td>
<td>[-]</td>
</tr>
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<td>( p )</td>
<td>Pressure of fluid element</td>
<td>[N/m²]</td>
</tr>
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<td>( p_{ref} )</td>
<td>Reference pressure = ( 2 \times 10^{-5} )</td>
<td>[N/m²]</td>
</tr>
<tr>
<td>( p' )</td>
<td>Acoustic pressure</td>
<td>[N/m²]</td>
</tr>
<tr>
<td>( P )</td>
<td>Rotor power</td>
<td>[W]</td>
</tr>
<tr>
<td>( q )</td>
<td>Heat flux</td>
<td>[W/m²]</td>
</tr>
<tr>
<td>( Q )</td>
<td>Rotor torque</td>
<td>[N·m]</td>
</tr>
<tr>
<td>( r )</td>
<td>Rotor radius</td>
<td>[m]</td>
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<tr>
<td>( r_x, r_y, r_z )</td>
<td>Observer location in x, y, and z cartesian system</td>
<td>[m]</td>
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<tr>
<td>( R )</td>
<td>Rotor tip radius</td>
<td>[m]</td>
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<tr>
<td>( \mathcal{R} )</td>
<td>Magnitude of radiation vector</td>
<td>[m]</td>
</tr>
<tr>
<td>( \mathbf{R} )</td>
<td>Vector of flux residuals</td>
<td>[-]</td>
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<tr>
<td>( R_c )</td>
<td>Reynolds number = ( \rho V_{ref} l_{ref} )</td>
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<tr>
<td>( S )</td>
<td>Shape function</td>
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<td>( S )</td>
<td>Source term</td>
<td>[-]</td>
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<tr>
<td>( S_{ij} )</td>
<td>Strain rate tensor</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( S_{pp} )</td>
<td>Far-field broadband noise spectrum</td>
<td>[Pa²/Hz]</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
<td>[s]</td>
</tr>
<tr>
<td>( t_c )</td>
<td>Thickness to chord ratio</td>
<td>[-]</td>
</tr>
</tbody>
</table>
\( T \) Rotor thrust, Fluid temperature [N],[K]

\( \mathcal{T} \) Thickness acoustic source term [-]

\( u, v, w \) Cartesian velocity component [m/s]

\( U_n, V_n, W_n \) Normalised cartesian velocity component = \((U, V, W) / V_{ref}\) [-]

\( V_{\infty} \) Free stream velocity [m/s]

\( \psi \) Volume of fluid domain \([m^3]\)

\( \psi \) Cell volume \([m^3]\)

\( V_{ref} \) Reference velocity [m/s]

\( \mathbf{W} \) Vector of conserved variables [-]

\( y^+ \) Non-dimensional wall-distance [-]

\( z \) Normalised rotor radius = \( r/R \) [-]

**Greek Symbols**

\( \alpha \) Local angle of attack [rad]

\( \beta \) Blade setting angle [rad]

\( \gamma^+, \gamma^- \) Pitch gains [-]

\( \delta \) Relative error [-]

\( \delta_{ij} \) Kronecker delta [-]

\( \varepsilon \) Blade lean angle [rad]

\( \zeta, \zeta_i \) Computed thrust/torque and required value [N],[N \cdot m]

\( \eta \) Propulsive efficiency [-]

\( \Theta \) Angle between unit radiation and normal vectors. [rad]

\( \theta \) Observer polar angle [rad]

\( \kappa \) Interference coefficient [-]

\( \lambda \) Blade sweep angle [rad]

\( \mu \) Dynamic viscosity [Ns/m²]
\( v \) Induced velocity component \([\text{m/s}]\)

\( \rho \) Working fluid density \([\text{kg/m}^3]\)

\( \sigma \) Rotor solidity [-]

\( \sigma_{ij} \) Viscous stress tensor \([\text{N/m}]\)

\( \varphi \) Blade azimuth angle \([\text{rad}]\)

\( \phi \) Local inflow angle, observer azimuth angle \([\text{rad, rad}]\)

\( \phi_e \) Phase angle due to blade lean [-, [rad]]

\( \phi_\lambda \) Phase angle due to blade sweep [-, [rad]]

\( \Psi_D \) Fourier transformed chordwise distribution of drag [-]

\( \Psi_L \) Fourier transformed chordwise distribution of lift [-]

\( \Psi_T \) Fourier transformed chordwise distribution of thickness [-]

\( \Omega \) Rotor rotational speed \([\text{rad/s}]\)

Subscripts

\([-\cdot]_h\) Hub component

\([-\cdot]_i\) Self-induced component

\([-\cdot]_{\text{max}}\) Maximum value

\([-\cdot]_{mi}\) Mutually-induced component

\([-\cdot]_r\) Relative component

\([-\cdot]_t\) Tip component

\([-\cdot]_x\) Axial component

\([-\cdot]_\theta\) Tangential component

\([-\cdot]_\infty\) Free-stream component

\([-\cdot]_{0.75}\) Component at 75% rotor span

\([-\cdot]_{1,2}\) Fore/aft rotor
"Wha kens perhaps yet but the world shall see
Thae glorius day when folk shall learn to flee:
When, by the powers of steam, to anywhere,
Ships will be biggit that can sail i' the air
Wi' a as great ease as on the waters now
They sail, an' carry heavy burdens too."

Andrew Scott, 1826
Chapter 1

Introduction

The first chapter of this thesis introduces the Counter Rotating Open Rotor (CROR) concept, its benefits and current challenges. A literature survey has been carried out to identify the state-of-the-art in tackling these challenges from which the basis of the present work has been formed. A description of the motivations follows before a summary of the work carried out is presented.

1.1 Description of CROR

With increasing environmental pressure on the aviation industry, there has been a renewed interest in the CROR concept of late. The CROR comprises two blade rows rotating in opposite directions around a common shaft. The renewed interest comes as a consequence of the increased efficiency relative to turbofan technologies [1, 2, 3, 4]. The improved efficiency results from the counter-rotation of the aft rotor as it removes the residual swirl imparted onto the flow by the fore rotor. Additionally, without the drag of a duct and a higher by-pass ratio, the CROR performance is further improved.

The CROR concept and its benefits have been studied since the early days of aviation (see, e.g. Ref. [5, 6]). However, the oil crisis of the 1980s perhaps saw the most significant developments in CROR technology with a period of intense research aimed at delivering a commercial engine [7]. CROR development in this period resulted in two concepts to the flight test stage, Figure 1.1, see Ref. [8, 9]. However, with a number of technical issues still to overcome and a drop in oil prices, CROR development was subsequently scaled back, and a commercial engine was never produced.

With an increasing awareness of aviation’s environmental impact, the CROR concept has once again come to light. The CROR concept has been the focus of intense research over the past decade, with current efforts focussed on addressing a number
of challenges to make the CROR commercially realisable. Figure 1.2 shows state-of-the-art concepts currently under testing.

![Figure 1.1: CROR concepts of the 1980s under flight testing.](image)

Whilst general aviation aircraft (light propeller-driven aircraft with a maximum take-off mass $< 8,618 \text{ kg}$) may not have such a significant environmental impact as civil aircraft, there is increasing focus on reducing their emissions [12, 13]. Furthermore, while the endeavours of most current CROR research remains focussed on civil aviation aircraft, the use of CROR in general aviation is receiving attention from a number of general aviation engine manufacturers [14]. Despite the difference in aircraft class, there is a great deal of commonality between the underlying physics of the CROR in each case. Furthermore, there will be overlap in the challenges and their solutions between applications to both classes of aircraft.

The increasing environmental pressures have also inspired further propulsion concepts. In particular, there is a great deal of interest in electrifying flight, either fully or intermediately, in a hybrid configuration. The use of two combined powerplants lends itself naturally to CROR applications. Hybridisation may be more promising for general aviation class aircraft where the power requirements are perhaps more realisable with the current state-of-the-art or near-future hybrid technologies. Not only does hybridisation offer even greater reductions in emissions, but the independent control also offers greater safety during the take-off segment.

Despite the promise of increased efficiency, there remains a number of technical issues to overcome before the adoption of the CROR as a conventional propulsion system. For example, installation and aircraft integration, certification and noise all present challenges for the modern CROR [15, 16]. Furthermore, these challenges are not necessarily commensurate with increases in efficiency. The work presented in this thesis is concerned with the challenges surrounding CROR noise. In particular,
the focus is on developing tools to evaluate CROR designs and novel concepts that offer reduced CROR noise without significantly affecting performance. The current work primarily deals with general aviation. However, the methods used and the conclusions drawn are not exclusively limited to this class of aircraft.

![Modern CROR concepts under testing as part of the EU CleanSky SAGE project [17].](image)

1.2 Literature Survey

The previous section has identified that noise remains a pertinent issue for CROR. In this section, the open literature is reviewed, bringing together current knowledge and concepts related to the reduction of CROR noise. The aim of this is, therefore, to identify the most significant noise source and hence the area to explore to significantly reduce CROR noise. The literature survey is divided into several sub-sections, grouping the various noise-reducing concepts or technologies.

A number of studies have identified several general principles for the design of low-noise CROR [15, 18, 19, 20]. These design philosophies include increased axial spacing, aft rotor clipping and blade sweep, to name but a few. However, the main focus is on reducing the interaction source. To proceed, a number of these concepts are further explored in addition to discussing some more advanced concepts to reduce CROR noise.

1.2.1 Blade Sweep

For rotors with transonic tip speeds, introducing blade sweep is a well-known method for reducing rotor noise [21]. For lower tip speeds, the effect of sweep is less significant. Noise reductions due to sweep comprise two sources. Firstly, sweep has the effect of reducing the relative Mach number along the span (analogous to
CHAPTER 1. INTRODUCTION

swept wings), and hence reducing the strength of the quadrupole source. Secondly, sweeping back successive blade elements results in a de-phasing effect between blade sections [18].

Experimental studies have demonstrated significant noise reduction by the use of blade sweep for high tip speeds. For example, Dittmar, Jeracki & Blaha [22] compared the unswept SR2 with the swept SR1-M and SR3 rotor blades up to supersonic tip speeds in the NASA Lewis 8 × 6 ft wind tunnel. Figure 1.3 shows the three rotors with progressively increasing levels of sweep. The authors demonstrated that both swept rotors offered significant noise reductions compared to the SR2 rotor, with the SR3, the most highly swept, offering the most substantial reductions in noise. The same conclusions were made by Dittmar [23] using the numerical acoustic model of Farassat [24, 25, 26].

Figure 1.3: Three rotors tested in the NASA Lewis tunnel to investigate sweep, from Ref. [23].

With regards to CROR, blade sweep will have the same effect for the self-noise of each rotor. However, sweep may also affect the interaction between blade rows, in particular, the wake and tip vortex development. Simonich, McCormick & Lavrich [27] carried out an experimental investigation on the interaction of a swept stationary vane upstream of a rotating blade. For the case of the aft-swept vane, the leading-edge vortex\* moved outboard towards the tip, coalescing with the tip vortex and resulting in a greater interaction component. On the other hand, for the forward-swept vane, the leading-edge vortex moved inboard and remained distinct from the tip vortex, resulting in a reduced interaction component compared to the aft-swept vane.

\*The high loading at take-off conditions results in separation at some spanwise location along the leading-edge. This separated flow subsequently reattaches along the chord creating an area of high recirculation and is known as a leading-edge vortex.
1.2. LITERATURE SURVEY

While blade sweep may be used to reduce rotor noise, it must be carefully considered when applied to CROR for its effect on the interaction noise component. Furthermore, the benefits of sweep are only realised for high tip speeds. Therefore, in the case of general aviation CROR, sweep perhaps may not be of great importance with regards to noise reduction.

1.2.2 Rotor Trim

It is possible to trim a CROR to meet the thrust or torque requirements of a given flight condition with a combination of fore and aft rotor speed and setting angles. Therefore, there is more than one combination of rotor speed and setting angle to deliver the required thrust. In addition to delivering the aircraft thrust or torque, the given combination must also satisfy low-noise requirements.

Zachariadis & Hall [28] investigated changes to setting angles and rotor speed to improve CROR efficiency and noise at take-off conditions. For the given configuration, the authors found that the baseline trim settings resulted in a highly loaded fore rotor with flow separation in some regions. The setting angles were reduced to decrease the high incidence, while the rotor speed was increased to maintain the required thrust. The new trim settings resulted in improved aerodynamic performance. While the increased rotational speed increased self-noise, the improved aerodynamic performance reduced the interaction noise and subsequently, the total CROR noise.

In addition to controlling the self-noise, the rotor speed has a significant effect on the interaction noise. Aside from the aerodynamic effects on the interaction noise, the rotor speed can affect the acoustic interaction [29, 30]. The acoustic interaction is the constructive and destructive addition of the acoustic pressure signals from fore and aft rotors. Therefore, careful control of rotor speed may result in greater destructive addition and hence lower total noise. However, this has to be balanced with the resulting aerodynamic effects.

CROR can be trimmed to meet the aircraft requirements by a number of configurations: equal torque ratio, $\frac{T_2}{T_1} = 1$, equal thrust ratio, $\frac{T_2}{T_1} = 1$, or unequal thrust and unequal torque ratios. Delattre & Falissard [31] investigated the aerodynamic and aeroacoustic effects of fore and aft rotor torque ratio. A total thrust was maintained by adjusting blade setting angles. Delattre & Falissard found that an increase in torque ratio resulted in a reduction in the emitted SPL across polar directivities. The increase in torque ratio resulted in a reduction in the fore rotor loading and hence its corresponding wake deficit. Therefore, the interaction with the rear row was reduced and resulted in a lower interaction noise component. The total noise was reduced due to the significance of the interaction component at take-off. An overall increase in propulsive efficiency also accompanied the reduction in noise.
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The method of trimming the CROR rotors to deliver the required aircraft thrust or torque has been found to have a significant effect on the resulting noise. Therefore, optimisation of setting angles and rotational speeds can ensure reduced noise whilst satisfying the aircraft loading requirements.

1.2.3 Geometry Modifications

Aft Rotor Clipping

As the tip vortex from the fore rotor is trailed backwards, it impinges on the aft rotor, Figure 1.4. This interaction results in an unsteady loading component and hence unsteady noise source on the aft row. Clipping, the reduction of the aft rotor radius, can reduce CROR noise by minimising the tip vortex interaction. The vena-contracta of the tip vortex can be estimated to determine the level of clipping required. However, the behaviour of the tip vortex will change as a result of loading and non-axial flight. Therefore, across all operating points, the properties of the tip vortex must be well understood. Clipping the aft rotor results in a reduced blade area. Increasing the chord or rotational speed can be used to offset the increased loading.

Peters & Spakovszky [32] attempted to isolate the various interaction noise sources numerically. By considering the noise from the aft rotor in the 90-100% span region, the tip vortex interaction component was isolated. Peters & Spakovszky found that the tip vortex interaction component could be as important as the viscous wake component. It was also subsequently shown that a redesigned CROR with a reduced aft rotor (amongst other design changes) resulted in reduced noise and a significant reduction in the tip vortex interaction component.
Using the NASA Lewis 9 × 15 ft anechoic wind tunnel, Woodward [33] investigated the acoustic performance of an advanced CROR design (F7/A7) at take-off conditions. In a subsequent analysis of a similar design with a 25% reduced aft rotor radius, Woodward & Gordon [34] investigated the effects of clipping. The authors found a considerable reduction in noise with the clipped aft rotor. Dittmar & Stang [35] conducted a similar experiment at cruise conditions. Dittmar & Stang also found that clipping was successful in reducing CROR noise at cruise. Noise reductions resulted from a reduction of aft tip speed in addition to a reduction in the interaction tones as the aft rotor was below the vena-contracta of the fore rotor tip vortex.

Majjigi, Uenishi & Gliebe [36], observing the significant noise reduction achieved when clipping the aft rotor, Figure 1.5, noted the importance of accounting for the tip vortex interaction component when numerically modelling CROR interaction noise. From experimental and empirical results, Majjigi, Uenishi & Gliebe developed a tip vortex interaction model that was able to capture the effects of clipping and tip vortex interaction. This problem has been tackled by a number of authors, including more recently by Kingan & Self [37] and Quaglia et al. [38].

![Figure 1.5: Comparison of clipped (F7/A3) and unclipped (F7/A7) CROR directivities, adapted from Ref. [36]](image)

Clipping of the aft rotor has been identified as a method for reducing the interaction of the fore tip vortex and the aft rotor, and hence the resulting interaction noise. However, the properties of the tip vortex at all flight conditions must be understood to ensure the rotor has sufficient clipping to realise its benefits.


Axial Spacing

Both potential and wake interactions decay with increasing distance. Therefore, the axial spacing between blade rows is an important design consideration. As the axial distance increases, the viscous wake from the fore rotor has a greater time to mix, resulting in a weaker wake with a lower velocity deficit. The result of this weaker wake is a reduction in the interaction noise source [19].

The effect of axial spacing was studied experimentally by Woodward [33] at take-off conditions. Woodward found that increasing the axial spacing had a positive impact, resulting in a reduction of the noise. Woodward & Gordon [34] performed a similar study with a clipped aft rotor. Comparing their study with that of Woodward [33], the authors found that the clipped rotor was more sensitive to changes in axial distance. Figure 1.6 compares the results from both studies to demonstrate this. The unclipped rotor is affected by both the tip vortices and viscous wake of the fore rotor. On the other hand, the clipped rotor is only affected by the viscous wake. The increased sensitivity of the clipped rotor results from the fact that the viscous wake decays more rapidly than the tip vortex.

![Figure 1.6: Effect of axial spacing on rotor alone and interaction tones. Adapted from Ref. [34].](image)

Dittmar [39] studied the effects of spacing in the NASA Lewis 8 × 6 ft tunnel for a CROR blade pair at cruise conditions and observed some interesting results. Whilst the expected trends were observed for a wind tunnel speed of Mach 0.8, the expected trends were not observed for the reduced tunnel speeds of Mach 0.76 and Mach 0.72. For each wind tunnel speed, the setting angle was fixed. Therefore, the reduced speed resulted in an increased inflow angle and hence a greater loading on
each blade. As a result, the wake contraction is greater, and the tip vortex has a greater impact area on the aft row. Despite the weakening due to increased axial spacing, the increased effect of the tip vortex interaction resulted in increased noise levels. This mechanism is demonstrated in Figure 1.7. The variation in conclusions found with increasing spacing demonstrates the complex interaction found in CROR. Furthermore, it highlights the coupling of clipping and axial spacing.

![Images of blade configurations at different Mach numbers](image)

*(a) M = 0.8  
(b) M = 0.76  
(c) M = 0.72*

Figure 1.7: *Cause of increased noise with increased spacing at cruise Mach numbers.*  
*Adapted from Ref. [39].*

While the axial spacing has a significant effect on the wake and tip vortex interaction, it may also be important for the potential interactions. Parry [40] and Parry & Crighton [41] found that potential interactions were the dominant interaction source when predicting the noise of the Fairey Gannet aircraft. The low blade count of the Fairey Gannet CROR blade pair suggests the potential interactions will be significant for general aviation class aircraft. Similarly to the wake components, the potential interactions will decay with increasing axial spacing. Therefore, increased spacing should also reduce the potential interaction noise source.

In the development of modern CROR designs, there has been a focus on increased spacing to reduce CROR noise. For example, Peters & Spakovsky [32] described the use of increased spacing (amongst other measures) to reduce both wake and potential interaction noise sources of an advanced CROR design. In describing the
collaborative design of a modern CROR, Van Zante [4] described the noise-reducing
benefit of increased axial spacing. However, Van Zante also discussed the need to
optimise this spacing. Increasing the spacing results in the tip vortex being allowed
to contract more before meeting the aft rotor, and may result in impingement with
the aft rotor. The optimisation of spacing, taking into account the aft clipping, was
also discussed by Khalid et al. [42].

It has been established that increasing the axial spacing may be beneficial in re-
ducing both potential and wake interaction noise sources. However, the increased
spacing may result in a greater tip vortex interaction. Furthermore, aerodynamic
performance or installation restrictions may limit the axial spacing.

Blade Shape Modifications

In an attempt to reduce the wake interaction noise, Delattre & Falissard [43] and
Delattre et al. [44] investigated the effect of placing a small protuberance on the
leading-edge of the fore rotor of the Onera HTC5 CROR [45]. The concept aimed
to modify the wake structure from the fore rotor and reduce its impact on the aft
rotor. Using the Onera elsA CFD solver [46, 47, 48], Delattre & Falissard and Delattre
et al. performed a numerical analysis of a CROR with a fore blade modified with
a small protuberance at the 80% span location, Figure 1.8. The authors found that
the protuberance had a minimal impact on the CROR performance at take-off and
practically no effect at cruise conditions.

The noise produced by each configuration at take-off conditions was studied
based on the aerodynamic computations. The authors found that the modified CROR
had reduced noise levels for upstream and downstream (from the overhead position)
polar directivities. Additionally, the protuberance also caused a reduction in the
dominant interaction harmonics.

The observed noise reduction due to the protuberance arises due to a weakening
of the tip vortex. Considering the unmodified configuration, the high loading of the
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The formation of a leading-edge vortex [28, 49]. The small protuberance acts to prevent the merging of the leading-edge vortex with the tip vortex, resulting in two weaker co-rotating vortices leaving the fore row. The interaction of the two weaker vortices results in a reduced interaction with the aft rotor and hence, a reduction in noise.

While Delattre & Falissard [43] and Delattre et al. [44] did not investigate the acoustic effects of the protuberance at cruise, due to the leading-edge vortex being weaker under cruise conditions, it is hypothesised that this concept would have little impact on the noise during cruise conditions.

Recall that control of the leading-edge vortex has also been discussed by Simonic, McCormick & Lavrich [27], however, in terms of blade sweep. Forward or backward sweep was found to move the leading-edge vortex inboard or outboard respectively. Additionally, Khalid et al. [42] discussed additional camber and modified thickness to prevent the roll-up of the leading-edge vortex.

In an effort to reduce wake interaction noise, Weckmüller & Guérin [51] investigated the use of serrations on the trailing-edge of the fore rotor of the Airbus AI-PX7 CROR [52], as illustrated in Figure 1.9. This concept proposes to increase the mixing of the wake behind the fore rotor, similar to chevrons found on modern turbofan nozzles, and hence reduce the wake deficit and the resulting interaction with the aft rotor. Using high-order numerical tools, the authors investigated three serrated designs using a steady RANS simulation. The CROR aerodynamic performance was not significantly affected by any of the trailing-edge serration designs. Following the steady simulations, an unsteady RANS was performed on the most promising design. This was then coupled to an acoustic solver. Weckmüller & Guérin found that the serrations had little effect on the self-noise. On the other hand, the serrations had

Figure 1.9: Airbus AI-PX7 CROR with trailing-edge serrations on the fore rotor, from Ref [50].
a greater benefit with increasing frequency. An OASPL reduction of 0.2 dB resulted. The authors argued that the poor overall reduction was a result of the weaker than expected mixing process and the dominance of the tip vortex interaction clouding any noise reductions resulting from the serrations [3].

Jaron et al. [50] later revisited the serrated blade concept. To identify a design with greater noise reductions in terms of serration count and depth, Jaron et al. utilised an evolutionary optimisation routine. Using a steady RANS approach coupled to an acoustic solver, Jaron et al. found a design offering up to 1 dB gains over the baseline Airbus AI-PX7 CROR with little impact on the aerodynamic performance. However, the design was on the border of the optimiser limits of serration depth, suggesting a design with lower noise may exist.

While the use of serrations on the fore rotor trailing-edge was shown to offer reduced noise compared to a baseline, the manufacturing complexity of such a design must be further considered against the acoustic benefit. Furthermore, the concept should be experimentally studied to verify its performance.

Barakos & Johnson [53] studied a number of blade and hub concepts. In particular, three concepts were studied: (i) a staggered blade concept, where every second blade was axially displaced aft-wards; (ii) an unequally spaced concept, where blades are azimuthally unequally spaced; (iii) blades with an off-loaded tip, increased pitch with a reduced twist at the tip. These concepts are illustrated in Figure 1.10.

Barakos & Johnson used the HMB CFD solver to compare the three concepts to a baseline design. From the CFD computations, the near-field acoustic impact of each configuration was compared. The authors found that there was no significant
difference in the aerodynamic performance between each configuration. The off-loaded tip configuration reduced noise levels considerably. On the other hand, the staggered and unequally spaced concepts were shown to introduce sub-harmonics, spreading the tones over a greater range of frequencies and thus changing the sound quality. Nonetheless, the overall sound energy increased compared to the baseline.

This study was later extended by Chirico, Barakos & Bown [54], considering the effects of these configurations on interior cabin noise. The authors observed similar trends to those reported by Barakos & Johnson [53]. However, the differences between the three configurations reduced with respect to the cabin noise predictions.

The studies performed on the blade and hub configurations have shown that the off-loaded tip can reduce the resulting noise. On the other hand, the alternative hub configurations showed higher sound energy. The authors studied these concepts for near-field and cabin noise. However, it would be interesting to explore these further in the far-field to investigate their community noise impacts. Furthermore, the predictions were made for a Single Rotation Propeller (SRP) and would, therefore, require further analysis for CROR applications.

### 1.2.4 CROR Optimisation

Increases in computational power have allowed for the development of optimisation routines employing high-fidelity analysis tools. To this end, a number of works have utilised optimisation tools to target designs towards low-noise while satisfying, amongst others, aircraft thrust and propulsive efficiency requirements.

Lepot et al. [55] and Schnell et al. [56] described the CROR optimisation investigated by a number of agencies in the framework of the European DREAM project. The works described the optimisation of a modern CROR based on maximising efficiency at the top of climb whilst simultaneously minimising take-off noise. In addition to this, geometrical and structural constraints had to be satisfied. The optimisation utilised an evolutionary algorithm coupled to a steady RANS solver to identify promising designs. Pseudo-acoustic criteria, namely mean-flow properties of the fore rotor wake, were used to evaluate the fitness in terms of noise. The most promising designs were then further studied with unsteady RANS simulations coupled to an acoustic solver. The optimisation identified a design that could reduce noise over most of the polar directivity at take-off while also meeting the design constraints. The authors noted that the structural constraints were the most limiting to the design. Additionally, the staggering, leading- and trailing-edge profiles and spanwise loading were reported to have the most significant impact on the optimisation.

Villar, Lindblad & Andersson [57] used an Evolutionary Algorithm to optimise the efficiency of a CROR at the top of climb. The performance of each design was
evaluated from CFD simulations. However, to reduce the number of CFD evaluations, a meta-modelling approach was used. The optimisation, based on aerofoil parametrisation, identified a design with improved performance relative to the baseline. Whilst this work does not consider the aeroacoustic impact of the CROR, the meta-model approach presents an interesting methodology to reduce the computational cost of optimisation studies utilising high-order tools.

Grasso et al. [58] performed a multi-disciplinary, multi-objective optimisation of a CROR. An Evolutionary Algorithm, coupled to a meta-model to decrease computational cost by reducing the number of executions of the numerical models, was used to identify high performing designs that maximised isentropic efficiency while reducing sound power level. Optimisations were carried out on blade profile shape as well as spanwise sweep and lean. The optimisation was also subject to structural constraints. RANS CFD, coupled to an analytical acoustic code, was used to compute the aerodynamic performance and tonal and broadband noise sources. The authors identified a number of promising designs along the Pareto front. However, aerodynamic and aeroacoustic objectives were found to be competing. Figure 1.11 compares the designs giving the highest efficiency and lowest noise. Low-noise designs resulted from increased spacing and reduced chord to reduce the wake deficit. Low-noise designs also featured sweep and lean to de-phase radial contributions. Sweep and lean were also used on the high-efficiency designs bringing the blade tips together.

![Figure 1.11](image.png)

(a) Design identified with highest efficiency  
(b) Design identified with lowest noise

Figure 1.11: Optimised CROR designs identified by Grasso et al., blue shows optimised geometry against the grey baseline. Adapted from Ref. [58].

The use of optimisation routines has shown the ability to identify CROR designs with reduced noise. However, the objective of low-noise is not commensurate with the objective of high-efficiency. The studies found in the open-literature used a combination of high-order methods. However, restraints on computational cost required
simplifications of the physics (the use of steady simulations for an inherently unsteady problem), or limitations on the design space. On the other hand, the use of meta-models seems a promising method for reducing the computational cost. Despite all this, optimisation of the CROR design shows the potential to design a CROR with low-noise whilst simultaneously meeting further design constraints.

1.2.5 Fore Rotor Trailing Edge Blowing

Further to the use of passive concepts to reduce CROR noise, active concepts have also demonstrated potential noise reductions. Akkermans, Stuermer & Delfs [59, 60, 61] performed a numerical investigation on the ability of trailing-edge blowing to reduce CROR noise at take-off. The concept aims at reducing the wake deficit from the fore rotor, and hence the interaction with the aft row, by the injection of air along the trailing-edge across the blade span. Whilst the concept was shown to make a significant impact in reducing the wake deficit while maintaining aerodynamic performance, it was unsuccessful in reducing CROR noise. Insufficient clipping of the aft rotor resulted in tip vortex interaction dominating the noise emissions. Therefore, any gains from the trailing-edge blowing, which does not impact the tip vortex, were not realised.

A later numerical study by Akkermens, Stuermer & Delfs [62, 63] realised the noise benefits of the trailing-edge blowing concept using an aft rotor with increased clipping. Again, the trailing-edge blowing significantly reduced the wake deficit at minimal aerodynamic cost (in fact an efficiency gain was observed for the fore rotor). The use of trailing-edge blowing resulted in reduced noise levels across most polar directivities. The noise reductions resulted from a more pronounced effect on the interaction tones, in contrast to the minimal impact it had on the self-tones. Compared to the study by Akkermans, Stuermer & Delfs, the removal of the tip vortex interaction component allowed the noise reduction to be more easily observed. A reduction of 1 dB was demonstrated at the overhead position, whilst up- and downstream directivities showed a maximum of 5 dB reduction relative to the baseline.

To date, only numerical studies have investigated the trailing-edge blowing concept. Therefore, the predicted noise reductions need to be experimentally verified. Furthermore, as the mass flow of air will likely be taken as bleed from the engine, the cost in terms of additional fuel consumption must be well understood.

1.2.6 Installation and Shielding

Installation of a rotor on an aircraft can present a vastly distorted flowfield in comparison to the uniform inflow of the idealised isolated case. For example, installation at incidence and boundary layer ingestion both introduce some distortion
to the flow. In particular, pylon installation has received significant attention with regards to CROR installation. In pylon mounted rotors, where the pylon is upstream of the rotor (which is the typical configuration of modern CROR), the wake from the pylon results in periodic changes in incidence and hence a periodic loading on the rotor. This unsteady loading then results in an unsteady noise source. The resulting noise is analogous of stator-rotor interactions, with the upstream stator having a single blade. The resulting noise will be of mode \( v = k_2 - k_1 N_1 \) and of frequency \( k_1 N_1 n_1 \), where \( k_1 \) and \( k_2 \) are all integers, \( n_1 \) is the rotor rotational frequency and \( N_1 \) is the fore rotor blade count. The noise will, therefore, generate tones at the front rotor Blade Passage Frequency (BPF)\(^\dagger\).

The installation effects of pylon mounted CROR has been studied by a number of authors. Stuermer & Yin [64], using the DLR TAU CFD code [65] and APSIM acoustic code [66], studied the installation effects of a pylon mounted \( 10 \times 8 \) CROR\(^\ddagger\) at both take-off and cruise conditions. Stuermer & Yin observed that at cruise, the pylon had a significant impact on the rotor alone tones of both fore and aft rotors, resulting in increased noise levels in the up- and downstream directions. At take-off, the effect was less pronounced on the aft rotor. The unsteady loading due to the pylon resulted in greater noise levels for the fore rotor tones (equivalent to the pylon interaction tones) in the upstream direction. At both conditions, the pylon was observed to have a minimal impact on the rotor interaction tones.

\[\Omega_2 = 0 \text{ and } N_2 = 1 \text{ in Equation (3.12).}\]

\[\text{This notation specifies fore and aft blade counts, i.e. } N_1 \times N_2 \text{ and is used throughout the thesis.}\]

\[\dagger\]This result can be achieved with reference to the acoustic modelling section below (§3.3), in particular by setting \( \Omega_2 = 0 \) and \( N_2 = 1 \) in Equation (3.12).

\[\ddagger\]This notation specifies fore and aft blade counts, i.e. \( N_1 \times N_2 \) and is used throughout the thesis.

Figure 1.12: Full aircraft installation configuration studied by Stuermer, from Ref [67].

In a later study, Stuermer [67] compared experimental and numerical studies...
of isolated, semi-isolated (pylon only) and full-installation (full aircraft) CROR, Figure 1.12. Stuermer found that the DLR TAU CFD solver was able to predict the aero-
dynamic performance and the resulting noise directivity trends relatively well. How-
ever, for the full aircraft simulations, there were notable over-predictions of OASPL in
the up- and downstream directions. Stuermer attributed the discrepancy to the fail-
ure to account for the scattering effect of the fuselage. The impact of full installation
was additional unsteady noise sources. These unsteady sources were more promi-
nent for the fore rotor, resulting in increased noise levels in the upstream direction.

Kleinert et al. [68] presented a numerical study of installation effects of a pylon
mounted CROR. In particular, the authors investigated a pusher configuration at two
pylon spacings and a tractor configuration. Kleinert et al. again observed the acous-
tically detrimental impact of the CROR installation. Comparing the pusher config-
urations of differing spacing, the authors noted a reduced impact on the increased
spacing configuration. For both tractor and pusher configurations, the presence of
the pylon had a negligible effect on the CROR interaction tones. In the pusher con-
figuration, the pylon wake had a more significant impact on the fore rotor, increasing
fore rotor tone levels, particularly in the upstream direction.

On the other hand, for the tractor configuration, the downstream pylon had a
greater impact on the aft row, increasing the aft rotor tones in the downstream di-
rection. Stagnation of the flow at the pylon leading-edge resulted in unsteady effects
on the aft row due to the proximity of the downstream pylon. These unsteady com-
ponents increased the unsteady noise at the aft rotor tones. However, increasing the
pylon spacing can reduce its effects.

The installation of the CROR has shown to have a significant impact on noise
emissions. Trailing-edge blowing has been proposed to address the installation effects
of pylon mounted configurations. In particular, slats in the trailing-edge of the pylon
inject a mass flow of air to reduce the velocity deficit caused by the pylon wake.

In an experimental investigation, Gentry Jr., Booth Jr. & Takallu [69] compared
the performance of an SRP mounted behind a pylon with and without pylon blow-
ing. The authors found that pylon blowing was successful in reducing the velocity
deficit in the pylon wake. Furthermore, only a minor impact on the aerodynamic
performance of the SRP resulted from the blowing.

Shivashankara, Johnson & Cuthberston [70] experimentally investigated the acous-
tic effects of pylon installation of a CROR. The authors observed that the pylon had a
negligible impact on the interaction tones, but was more significant for the self-tones.
The presence of the pylon resulted in as much as a 12 dB increase in the fore rotor
tones. The authors further studied the impact of pylon blowing. The decrease of the
wake deficit due to pylon blowing resulted in a reduction in the rotor tones to levels
CHAPTER 1. INTRODUCTION

comparable with the isolated CROR case.

Ricourd et al. [71] likewise performed an experimental investigation to determine the potential acoustic benefit of pylon blowing on installed CROR. Ricourd et al. again observed that the presence of the pylon had little impact on the interaction tones, but increased the fore rotor tones. Studying the spectral content of the noise, the authors observed that in addition to reducing the pylon interaction tones to a level comparable to the isolated case, pylon blowing could also lower in the broadband component.

Stuermer, Yin & Akkermans [72] discussed DLR’s efforts in investigating pylon trailing-edge blowing. The authors set out to investigate the potential benefits of pylon blowing applied to a modern CROR at take-off conditions. The numerical study demonstrated the ability of pylon blowing to reduce the wake deficit of the pylon. Additionally, the injection of the additional mass flow of air was found to have no significant impact on the thrust or propulsive efficiency. The numerical study again showed the primary effect of pylon blowing was a reduction in the fore rotor tones. Stuermer, Yin & Akkermans demonstrated that the use of pylon blowing resulted in a minor benefit in OASPL, with the most significant reductions observed in the upstream direction.

Whilst pylon blowing has shown the potential to alleviate the impacts of installation at the source, the use of aircraft shielding has been proposed by a number of authors to reduce noise at the observer.

Kingan, Powles & Self [73] and Kingan & Sureshkumar [74] presented theoretical models to account for centrebody scattering when computing the noise of CROR, with the centrebody assumed to be an infinitely long cylinder. In both works, the authors found that scattering from the hub contribution had a significant effect on the resulting noise emissions. Furthermore, Kingan & Sureshkumar found that this scattering could be reduced by acoustically lining the CROR centrebody.

Lummer et al. [75] presented a Boundary Element Method (BEM) coupled with a CROR noise model to compute the effect of shielding and reflection from a CROR. Lummer et al. demonstrated that the aircraft tail was effective in shielding the noise from the CROR. However, due to the reflection from the tailplane, the unshielded side of the CROR saw an amplification of the acoustic pressure. While the CROR model generally matched experimental data, the reflection model was not validated.

In a similar study, Sanders et al. [76] presented a BEM to compute the installation effects where the noise was calculated based on uRANS CFD simulations. Comparing the numerical and experimental data of the isolated CROR, Sanders et al. noted significant discrepancies. Sanders et al. attributed the differences to the acoustic reflections within the wind tunnel, which were not accounted for in the free-field
1.2. LITERATURE SURVEY

conditions of the simulation. A further study by Sanders et al. [77] supported this hypothesis with BEM calculations to account for the effects of the wind tunnel walls.

Guo & Thomas [78] presented a parametric experimental study on the impact of shielding of CROR noise from a hybrid-wing-body with various vertical tail and acoustic liner configurations. The authors found that the blended-wing-body was effective in shielding the CROR noise in the flyover plane but not so effective in the sideline plane. However, the introduction of a vertical tail improved the shielding in the sideline plane. The authors also noted the difference in shielding behaviours of the fore rotor, aft rotor and interaction tones, resulting from differences in their directivity and coherence.

In the frame of the Clean Sky WENEMOR project [79], extensive numerical and experimental investigations were carried out investigating installation effects of modern CROR. The project tested a large number of CROR configurations and aircraft configurations, including U-, L- and T-tail configurations, the angle of the CROR mounting pylon and engine-wing distance, to name but a few. Figure 1.13 illustrates the experimental facilities and the range of installation configurations considered within the project.

Sanders et al. [77] detailed some of the numerical contributions to the WENEMOR project, whilst Kennedy, Eret & Bennet [80] reported on some of the experimental work within the project. In both experimental and numerical studies, the L- and U-tail configurations (i.e. where there is a horizontal surface below the CROR) generally resulted in greater reductions in ground noise. Furthermore, the angled tailplane offered more significant noise reductions than the horizontal counterpart. However, due to the different shielding behaviours of the alone- and interaction-tones, no single configuration offered noise reductions across all frequencies.

Dürrwächter, Keßler & Krämer [81] presented a numerical tool-chain for assessing the installation effects of a CROR. Using uRANS CFD data to inform an acoustic solver coupled to a BEM to compute the acoustic shielding of aircraft surfaces, the authors investigated a number of aircraft configurations, namely T-, U- and L-tail. The authors used the EPNL metric, computed from simulated certification flyover points, to compare each configuration. Therefore, the results can be more realistically used to gauge the relative benefit of each case. The shielding in the lateral direction showed marginal benefits due to the symmetry of the configuration (i.e. two CROR either side of the tail-plane). In the flyover case, the L- and U-tail configurations showed the most significant reduction in noise.

Modern CROR are typically proposed to be pylon mounted to the airframe. However, the presence of the pylon upstream of the fore rotor results in an unsteady inflow and hence a further unsteady noise source occurring at harmonics of fore BPF. Pylon
trailing-edge blowing has been demonstrated, both experimentally and numerically, to reduce CROR noise at the source. On the other hand, shielding from the aircraft empennage has also proven, both numerically and experimentally, effective at reducing noise at the observer. Whilst pylon blowing is seen as a promising solution to negate the acoustic impact of the pylon, the efficiency penalty on the engine (as the air will likely be bled from the engine) must be considered. Additionally, the aircraft empennage must also serve its aerodynamic and aeromechanic roles in addition to providing shielding for CROR noise.

Figure 1.13: Clean Sky WENEMOR project scale aircraft and some of the configurations studied, from Ref [80].
1.3. OBJECTIVES OF THE THESIS

1.2.7 Summary

A survey of the open literature has been undertaken to bring together current CROR noise-reducing concepts and the methodologies used for their analysis. The literature survey has identified that targeting the interaction component offers the greatest potential for reducing CROR noise. Novel configurations have shown the ability to reduce the interaction component. Additionally, optimisation methods have also demonstrated the ability to reduce the interaction source. However, the literature review has shown that the use of high-order models within optimisation tools are still currently limited by computational resources, e.g. the use of steady models for an inherently unsteady problem and a limited design space. As a result, there is presently scope to identify further noise-reducing concepts using a range of low- and high-order methodologies.

1.3 Objectives of the Thesis

The main aim of this work is to develop a low-order tool-chain for the analysis of CROR and to use this tool, supported by high-fidelity analysis, to identify potential noise reductions for CROR. To this end, the work presented in the thesis comprises the following objectives:

1. Develop and validate a low-order, multi-disciplinary CROR design tool.
2. Demonstrate the use of the design tool to identify high-performing CROR designs for general aviation aircraft.
3. Use the low-order CROR design tool to investigate a number of novel configurations for reducing CROR noise in the terminal area.
4. Perform high-order aerodynamic analysis to investigate the noise sources of these novel concepts.
5. Use coupled high-order aerodynamic-aeroacoustic tools to substantiate the capabilities of the novel concepts.

1.4 Thesis Structure

The thesis is divided into eight chapters and proceeds as follows: Chapter 2 is divided into two parts, presenting the low- and high-order aerodynamic models used in the analyses. Validation and a sensitivity analysis is presented for both.
Chapter 3 is divided into three parts. First, CROR aeroacoustics is described with an introduction to the main noise sources. Following this, both low- and high-order aeroacoustic models are described with an accompanying validation and sensitivity analysis for each.

Chapter 4 focuses on the low-order structural model. Following the development of the model, a sensitivity analysis is presented.

Chapter 5 presents a parametric study of CROR noise, investigating the effect of blade count and tip speed. A design study based on optimisation is then described.

Chapter 6 investigates a number of novel noise-reducing CROR concepts using the presented low-order design tools.

Chapter 7 further investigates the novel noise-reducing CROR concepts. However, the described high-order tools are utilised to confirm the conclusions made in the previous chapter and provide greater insight into the noise generating mechanisms.

Chapter 8 provides the main conclusions of the work and a number of recommendations for future work.
Chapter 2

Aerodynamic Modelling

Aerodynamics, in the current context of CROR, is the interaction between air (the working fluid), and the rotor blades. By studying this interaction, it is possible to estimate the performance of the CROR. In particular, how efficient the rotor blades convert their supplied power (from the engine), into useful power, i.e. thrust to propel the aircraft (multiplied by the aircraft forward speed). Furthermore, the analysis of all other aspects of the CROR analysis, e.g. noise or structural, relies on the CROR aerodynamics.

This chapter presents two approaches to studying the aerodynamics of CROR. Firstly, a low-order approach is described, whereby Blade Element Momentum Theory (BEMT) is extended for CROR applications. The low-order method, while limited in the detail of physics, is suitable for preliminary design studies due to its low computational cost. On the other hand, using a CFD solver, a high-order approach is also taken. Whilst more computationally expensive, the high-order analysis allows for a more in-depth analysis of the aerodynamics of the CROR.

2.1 Low-Order Aerodynamic Modelling

Low-order models are those that provide reduced physical insight for low computational cost. Whilst low-order models may not be appropriate for a detailed design or for understanding the underlying physics of a problem; they are sufficient for parametric and preliminary design studies. To this end, a low-order model to compute the aerodynamic performance of CROR has been developed. This model aims to provide sufficient details of the performance such that changes to the CROR configuration may be assessed.

In this section, BEMT is extended to CROR. The theory of the model is presented, followed by a validation and sensitivity study to verify the abilities and limitations of the given model.
2.1.1 Blade Element Momentum Theory

Blade Element Momentum Theory (BEMT) combines the simplicity of Momentum Theory and the ability of Blade Element Theory to represent a greater geometric detail to give a robust and reliable solution methodology for propeller and rotor analysis.

Momentum Theory

The momentum theory of Froude [82] and Rankine [83] has been used extensively for the analysis of rotary wings, including marine and aircraft propellers, helicopters and wind turbines. Due to its low computational cost, its simplicity and the fact that little input data is required, momentum theory has been a popular method for obtaining first-order approximations of rotary-wing performance since its initial development.

Momentum theory, also known as actuator disc theory, models the propeller/rotor as a planar rotating disc with surrounding stream tube, Figure 2.1. Momentum theory assumes that there is no flux through the stream tube and that the flow is incompressible and inviscid. Despite the assumption of constant velocity through the disc, there exists a pressure discontinuity through the disc. This discontinuity results in a flow acceleration with accompanying contraction of the streamtube aft of the disc.

The momentum theory formulation is well documented throughout the literature (see, e.g. [84, 85]), and, therefore, its development is unwarranted here. To account for variable inflow and wake rotation, the elemental form of momentum theory is used. The thrust and torque produced over a differential area, \( dA \), of the disc are given by:

\[
\begin{align*}
\text{d}T &= 2\rho (V_\infty + v_{i})v_i \text{d}A,
\end{align*}
\]  

(2.1)
and

\[ dQ = \rho r v_{i+i} (V_{\infty} + v_{i+i}) \, dA. \] (2.2)

Where \( v_{i+i} \) and \( v_{i+i} \) are the self induced axial and tangential velocities respectively. The elemental area, \( dA \), is given by:

\[ dA = r \, dr \, d\psi. \] (2.3)

Considering axisymmetric loading and using the annular formulation where \( dA = 2\pi r dr \) the thrust and torque are:

\[ dT = 4\pi \rho r (V_{\infty} + v_{i+i}) v_{i+i} \, dr, \] (2.4)

\[ dQ = 2\pi \rho r^2 v_{i+i} (V_{\infty} + v_{i+i}) \, dr. \] (2.5)

Blade Element Theory

Blade Element Theory (BET) is based on the discretisation of the rotor blade into a number of discrete radial elements. The two-dimensional sectional aerodynamics of each section is computed in isolation and are then integrated along the span to give the total rotor loads. BET requires more input data and can, therefore, provide greater detail than momentum theory for negligible additional computational cost.

Considering the velocity components at the blade element, Figure 2.2, the thrust and torque at each element are given by:

\[ dT = \frac{1}{2} F_T N \rho V_{rel}^2 C_a c \, dr, \] (2.6)

\*The total area would be given by: \( A = \int_0^{2\pi} \int_{r_i}^r r \, dr \, d\psi. \)
\[ \text{d}Q = \frac{1}{2} F_T N \rho V_{rel}^2 C_n c r \text{d}r. \]  

(2.7)

Where \( V_{rel} \) is the relative velocity component to the blade element and from Figure 2.2, is given by:

\[ V_{rel} = \sqrt{V_x^2 + V_\theta^2} = \sqrt{(V_\infty + v_{ix})^2 + (\Omega r - v_{iy})^2}. \]  

(2.8)

The axial and normal force coefficient, \( C_a \) and \( C_n \), are related to the local 2D aerodynamic coefficients by:

\[
\begin{bmatrix} C_a \\ C_n \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} c_l \\ c_d \end{bmatrix}.
\]  

(2.9)

The aerodynamic coefficients, \( c_l \) and \( c_d \) are approximated from simple aerodynamic models for rotary wings [86] or taken from look-up tables for a given aerofoil section and angle of attack. The local angle of attack is related to the inflow angle and the local setting angle:

\[ \alpha = \beta - \phi, \]  

(2.10)

with the inflow angle \( \phi \):

\[ \phi = \tan^{-1} \left( \frac{V_x}{V_\theta} \right) = \tan^{-1} \left( \frac{V_\infty + v_{ix}}{\Omega r - v_{iy}} \right). \]  

(2.11)

Finally, \( F_T \) is the Prandtl correction factor [87, 88] (though Glauert’s modified version of the tip correction is more commonly used [89]). This correction accounts for the zero loading at the blade hub and tip due to 3D effects. This simple correction is computed from the local blade radius and inflow angle:

\[
\begin{align*}
F_h &= \frac{N (r - r_h)}{2 r \sin \phi}, \\
F_t &= \frac{2 \cos^{-1} \left( e^{-f_h} \right)}{\pi}, \\
f_h &= \frac{N (R - r)}{2 r \sin \phi}, \\
F_t &= \frac{2 \cos^{-1} \left( e^{-f_t} \right)}{\pi}, \\
F_T &= F_h F_t.
\end{align*}
\]  

(2.12)

**Combined Blade Element Momentum Theory**

Both momentum and blade element theories are combined to give a unified set of equations that provide a robust and reliable numerical scheme for computing rotor performance. From an initial guess of the induced velocities, BET is used to compute
the initial rotor performance. Combining the momentum and blade element theory equations, and recasting in terms of the induced velocities:

\[
\nu_i = \frac{dT|_{BET}}{4\pi \rho r (V_\infty + \nu_i)}, \quad (2.13)
\]

\[
\nu_\theta = \frac{dQ|_{BET}}{2\pi \rho r^2 (V_\infty + \nu_i)}, \quad (2.14)
\]

an iterative solution methodology can be used to obtain the induced velocities and the blade loading.

### 2.1.2 Extension to Counter Rotating Open Rotors

The analysis of CROR presents a complex problem. Due to the mutual interaction between both blade rows, in its current form, the BEMT is unsuitable to perform analysis of CROR. Therefore, to account for the interaction between blade rows, both momentum and blade element theories have been extended.

#### Momentum Theory

The extension of momentum theory to the CROR case requires the consideration of a number of additional velocity components. In addition to the self-induced components, there now exists ‘mutually-induced’ components. These mutually-induced components are introduced to account for the interaction each rotor has on the other. To proceed, Figure 2.3 shows a schematic of the updated momentum theory problem.

![Momentum theory schematic for CROR.](image)

The velocity components at each location in the domain comprise both self- and mutually-induced components. Mutually induced components are the product of the
self-induced component and an ‘interference’ coefficient. Thus, for each rotor:

\[ v_{\text{mix}1} = \kappa_{x21} v_i x_2; \]
\[ v_{\text{mix}2} = \kappa_{x12} v_i x_1; \]
\[ v_{\text{mix}1} = \kappa_{\theta21} v_i \theta_2; \]
\[ v_{\text{mix}2} = \kappa_{\theta12} v_i \theta_1. \]  

(2.15)

Where, e.g. \( v_{\text{mix}1} \) characterises the effect of the axial velocity of the aft rotor acting on the fore rotor. With analogous definitions for the other terms.

The interference coefficients characterise the propagation of the self-induced terms in the direction of the opposing rotor. Beaumier [90] gives the following description and values of these interference coefficients:

- \( \kappa_{x12} \in [1:2] \); the induced velocity approaches two far downstream and is equal to one at the rotor disc;
- \( \kappa_{x21} \in [0:1] \); the induced velocity is zero far upstream and equal to one at the disc;
- \( \kappa_{\theta12} \approx 2 \); the induced swirl quickly approaches 2 behind the disc;
- \( \kappa_{\theta21} \approx 0 \); the swirl from the aft rotor does not propagate upstream.

To characterise the effect of spacing between the two blade rows, the following interference coefficients are introduced:

\[ \kappa_{x12} = \left( \frac{x}{D} + 1 \right), \]
\[ \kappa_{x21} = \left( 1 - \frac{x}{D} \right), \]
\[ \kappa_{\theta12} = \left( \frac{x}{D} \right)^{1/4} + 1, \]
\[ \kappa_{\theta21} = 0. \]  

(2.16)

where \( x \) is the axial location of interest, e.g. for the velocity component at location 3, this would take a value of \( g/2 \), with \( g \) the spacing between blade rows. These terms have been developed from results from classical momentum theory and the description given by Beaumier. From the interference coefficients, the velocity components at each location in the domain, Figure 2.3, can be computed, as summarised in Table 2.1. Note that the far downstream position has been assumed to correspond to one diameter downstream (From classical momentum theory, this is the location where the induced velocity is approximately twice that at the disc [91]).
Table 2.1: CROR velocity components.

<table>
<thead>
<tr>
<th>Component</th>
<th>Axial ($U$)</th>
<th>Tangential ($V$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$V_\infty$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$V_\infty + v_{i_1} + v_{i_2} \kappa_{x_{12}} (g)$</td>
<td>$v_{i_1} - v_{i_2} \kappa_{\theta_{12}} (g)$</td>
</tr>
<tr>
<td>3</td>
<td>$V_\infty + v_{i_1} \kappa_{x_{12}} \left( \frac{\theta}{2} \right) + v_{i_2} \kappa_{x_{21}} \left( \frac{\theta}{2} \right)$</td>
<td>$v_{i_1} \kappa_{\theta_{12}} \left( \frac{\theta}{2} \right) - v_{i_2} \kappa_{\theta_{21}} \left( \frac{\theta}{2} \right)$</td>
</tr>
<tr>
<td>4</td>
<td>$V_\infty + v_{i_1} \kappa_{x_{12}} (g) + v_{i_2}$</td>
<td>$v_{i_1} \kappa_{\theta_{12}} (g) - v_{i_2}$</td>
</tr>
<tr>
<td>5</td>
<td>$V_\infty + v_{i_1} \kappa_{x_{12}} (2R) + v_{i_2} \kappa_{x_{21}} (2R)$</td>
<td>$v_{i_1} \kappa_{\theta_{12}} (2R) - v_{i_2} \kappa_{\theta_{21}} (2R)$</td>
</tr>
</tbody>
</table>

From the velocity components, the thrust and torque from each rotor can be computed. The total thrust is given by the change in axial momentum within the streamtube. Hence,

$$T = \dot{m} (U_5 - U_1)$$

(2.17)

The total thrust is also given by the sum of the thrust from each rotor:

$$T = T_1 + T_2,$$

(2.18)

The individual components can be approximated as:

$$T_1 = \dot{m}_1 (U_3 - U_1),$$

(2.19)

and

$$T_2 = \dot{m}_2 (U_5 - U_3).$$

(2.20)

However, as $U_3$ does not represent the far downstream and far upstream locations for the fore and aft rotors respectively, the above equations are not strictly true. Nevertheless, these expressions are required in order to approximate the contributions from each rotor. Note that the expression for the total thrust remains true.

The thrust from each rotor can also be computed by considering the pressure jump over rotor, $T = A \Delta p$. This requires a similar assumption that the pressure in the midplane between the rotors is equal to the ambient value. Again, whilst this is not an accurate representation it is sufficient for approximating the individual rotor contributions.

Returning to the momentum theory derivation and expanding the velocity terms for the fore rotor and considering the thrust of an annular element:

$$dT_1 = \left( V_\infty + v_{i_1} \kappa_{x_{12}} \left( \frac{\theta}{2} \right) + v_{i_2} \kappa_{x_{21}} \left( \frac{\theta}{2} \right) - V_\infty \right) d\dot{m}_1.$$  

(2.21)
The mass flow rate through the fore rotor is expressed as:

\[ \dot{m}_1 = \rho U_1 dA_1 \]
\[ = 2\pi \rho r_1 \left( V_\infty + v_{i_1} + v_{i_2} \kappa_{x_{21}} (g) \right) dr_1. \] (2.22)

Hence, the elemental thrust produced by the fore rotor is expressed as:

\[ dT_1 = 2\pi \rho r_1 \left( V_\infty + v_{i_1} + v_{i_2} \kappa_{x_{21}} (g) \right) \left( v_{i_1} \kappa_{x_{12}} \left( \frac{g}{2} \right) + v_{i_2} \kappa_{x_{21}} \left( \frac{g}{2} \right) \right) dr_1. \] (2.23)

Similarly, expanding the velocity terms for the aft rotor:

\[ dT_2 = \left[ \left( V_\infty + v_{i_1} \kappa_{x_{12}} (2R) + v_{i_2} \kappa_{x_{21}} (2R) \right) - \left( V_\infty + v_{i_1} \kappa_{x_{12}} \left( \frac{g}{2} \right) + v_{i_2} \kappa_{x_{21}} \left( \frac{g}{2} \right) \right) \right] \dot{m}_2. \] (2.24)

The mass flow rate through the aft rotor is given by:

\[ \dot{m}_2 = \rho U_4 dA_2 \]
\[ = 2\pi \rho r_2 \left( V_\infty + v_{i_1} \kappa_{x_{12}} (g) + v_{i_2} \right) dr_2. \] (2.25)

The elemental thrust from the aft rotor is then:

\[ dT_2 = 2\pi \rho r_2 \left( V_\infty + v_{i_1} \kappa_{x_{12}} (g) + v_{i_2} \right) \left[ v_{i_1} \left( \kappa_{x_{12}} (2R) - \kappa_{x_{12}} \left( \frac{g}{2} \right) \right) + v_{i_2} \left( \kappa_{x_{12}} (2R) - \kappa_{x_{21}} \left( \frac{g}{2} \right) \right) \right] dr_2. \] (2.26)

The torque produced by the CROR will be given by the change in angular momentum through the streamtube. For the fore rotor, the torque is:

\[ dQ_1 = V_2 r_1 \dot{m}_1. \] (2.27)

Substituting the mass flow rate and expanding the velocity term:

\[ dQ_1 = 2\pi \rho r_1^2 \left( V_\infty + v_{i_1} + v_{i_2} \kappa_{x_{21}} (g) \right) \left( v_{i_1} - v_{i_2} \kappa_{x_{21}} (g) \right) dr_1. \] (2.28)

Similarly, for the aft rotor, the elemental torque is given by:

\[ dQ_2 = V_4 r_2 \dot{m}_2. \] (2.29)

Again, expanding the velocity term and substituting the mass flow term:

\[ dQ_2 = 2\pi \rho r_2^2 \left( V_\infty + v_{i_1} \kappa_{x_{12}} (g) + v_{i_2} \right) \left[ v_{i_1} \kappa_{x_{12}} (g) - v_{i_2} \right] dr_2 \] (2.30)
Note, the torque from each rotor will be in opposite directions and will, therefore, have a cancelling effect.

**Blade Element Theory**

The extension of BET to application to CROR only requires that the velocity triangles are updated. The velocity triangles for a CROR configuration are shown in Figure 2.4. The relative velocity of each radial element then becomes:

\[
V_{rel} = \sqrt{V_x^2 + V_\theta^2} = \sqrt{(V_\infty + v_{ix} + v_{mix})^2 + (\Omega r - v_{i\theta} + v_{mitr})^2}. \tag{2.31}
\]

The inflow angle is now given by:

\[
\phi = \tan^{-1} \left( \frac{V_\infty + v_{ix} + v_{mix}}{\Omega r - v_{i\theta} + v_{mitr}} \right). \tag{2.32}
\]

These are then used to compute the sectional aerodynamics and thus the thrust and torque of each rotor using the equations above for the isolated rotor.

**Combined Blade Element Momentum Theory**

Similarly to the case of the isolated rotor, the momentum and blade element theories are combined. However, as the performance of both rotors is now strongly coupled, a more robust solution methodology is required to solve for the rotor performance.

Similarly to the case of the isolated rotor, an initial guess of the induced velocities
is first made, i.e.:

\[ \nu^{(0)} = (\nu_{i_1}^{(0)}, \nu_{i_2}^{(0)}, \nu_{i_3}^{(0)}, \nu_{i_4}^{(0)}) \]  

(2.33)

The loading of each rotor is then computed using this initial guess. Following this, the blade element and momentum theory equations are combined and results in a system of non-linear equations of the form \( [F(x)] = 0 \):

\[
\begin{align*}
\text{d}T_1|_{\text{BET}} - \text{d}T_1|_{\text{MT}}(\nu_{i_1}, \nu_{i_2}, \nu_{i_3}, \nu_{i_4}) &= 0, \\
\text{d}T_2|_{\text{BET}} - \text{d}T_2|_{\text{MT}}(\nu_{i_1}, \nu_{i_2}, \nu_{i_3}, \nu_{i_4}) &= 0, \\
\text{d}P_1|_{\text{BET}} - \text{d}P_1|_{\text{MT}}(\nu_{i_1}, \nu_{i_2}, \nu_{i_3}, \nu_{i_4}) &= 0, \\
\text{d}P_2|_{\text{BET}} - \text{d}P_2|_{\text{MT}}(\nu_{i_1}, \nu_{i_2}, \nu_{i_3}, \nu_{i_4}) &= 0. 
\end{align*}
\]

(2.34)

To solve this system of equations, Newton’s method is used. Newton’s method can be easily programmed and can provide quadratic convergence (when close to the solution). Extended to a system of non-linear equations, it is generally given by [92]:

\[ \nu_i^{(k)} = \nu_i^{(k-1)} - J(\nu_i^{(k-1)})^{-1}F(\nu_i^{(k-1)}). \]

(2.35)

Where \( F \) is a vector containing the set non-linear equations; \( k \) is the iteration counter; and \( J \) is the Jacobian matrix. The Jacobian matrix is the matrix of first-order partial derivatives, i.e. \( \frac{\partial F_i}{\partial \nu_j} \). In this case, the Jacobian can be solved analytically. However, in practice, it was solved numerically using a finite differencing scheme. In particular, forward differencing was used:

\[
\left( \frac{\partial F(\nu)}{\partial x} \right)^{(k)} = \frac{F(\nu^{(k+1)}) - F(\nu^{(k)})}{\Delta x} + O(x) 
\]

(2.36)

The use of higher-order differencing, e.g. central differences, in addition to Richardson extrapolation [93], was also investigated. However, these higher-order approximations showed no significant effect on the computed Jacobian or induced velocities. Therefore, given the additional computational cost, they were not used.

For cases where the initial guess is far from the solution, Newton’s method will converge very slowly. For generic CROR geometries and operating point, a reasonable initial guess may not be first obvious. To overcome this, and to improve the robustness of the numerical method, the Levenberg-Marquardt algorithm [94, 95] was implemented to solve the BEMT system of equations.

Newton’s method is fundamentally based on the evaluation of the search direction, \( d_k \), which from Equations (2.35) is \( J(\nu_i^{(k)})^{-1}F(\nu_i^{(k)}) \). The Levenberg-Marquardt algorithm provides a more robust evaluation of the search direction, not failing when
far from the solution or when the Jacobian is singular. The hybrid algorithm combines Newton’s method and the method of steepest descent. Hence, the Levenberg-Marquardt algorithm uses the following evaluation of the step direction [96]:

\[
d_k = - \left( J(v_i^{(k)}) J(v_i^{(k)})^T + \lambda_k I \right)^{-1} J(v_i^{(k)}) F(v_i^{(k)}),
\]

where \( I \) is the identity matrix and \( \lambda_k \) is a scalar constant (that can change for each iteration). \( \lambda_k \) controls the direction and magnitude of the search direction. As \( \lambda_k \) approaches zero, the search direction uses the Newton method, whereas when it tends to infinity, the steepest descent method is used.

Expanding the equations for the fore and aft rotor thrust, Equations (2.23) and (2.26), shows the equations contain square terms in fore and aft rotor axially induced velocity, respectively. Therefore, when solving the system of equations there are two possible solutions. Further examining the equations shows these are both concave functions with two real roots. Considering the most common case of a positive thrust, the physical solution is the positive root. Obtaining this physical result relies on a sufficiently accurate initial guess of the induced velocity. In the present case, an initial guess which is on the right hand side of the function vertex would be required to ensure a search direction that moves towards the physical solution. In the present analysis, it has been found that a small positive value for the initial guess of the induced velocity is sufficient to guarantee the physical solution to the system of equations.

The Levenberg-Marquardt algorithm is used to successively find the roots to the system of non-linear equations, i.e. solve for the induced velocities. Once successive results converge, the induced velocities are used to compute the rotor loading. The process is repeated until suitable convergence is met of the induced velocities. Hence, we have two iteration loops, an inner, solving the induced velocities, and an outer, successive-substitution [97] method to solve the loading. The solution methodology is summarised in Figure 2.5. From the described solution methodology, the performance of a generic CROR under a given operating point can be rapidly estimated.

2.1.3 Rotor-Rotor Interactions

So far, the interaction between blade rows has only been considered as a steady problem, with a change in apparent velocity caused by the opposing rotor. However, the interaction between blade rows is naturally unsteady and complex. The interaction between blade rows can be broadly characterised as: (i) Wake interactions; (ii) Tip vortex interactions; (iii) Potential interactions.
Wake (i) and tip vortex (ii) interactions only propagate downstream, affecting only the aft rotor. On the other hand, potential interactions (iii) project both up and downstream, affecting both rotors. These interactions are characteristically harmonic, related to both the fore and aft blade passage frequencies. These interactions are most significant when considering CROR noise as they have a direct impact on the behaviour of the noise emissions.

Models have been previously developed to account for these interactions. To support integration into low-order acoustic codes, these are typically given in terms of a harmonic lift component. Due to the length and complexity of the expressions, the models will not be presented here. Instead, the reader is directed towards the work of Parry [40, 98], Parry & Crighton [41] and Ekoule et al. [99] for a description of wake (i) and potential (iii) interactions. For a description of tip vortex interactions (ii), the reader is directed towards the work of Kingan & Self [37].

Figure 2.5: Workflow for solving CROR BEMT equations.
2.1.4 2D Aerodynamics

To compute the total rotor loads, the BEMT method relies on the aerodynamic properties of each radial element along the span. The aerodynamic properties must be known for a range of angles of attack for each aerofoil at each radial element.

In the present work, the aerodynamic data comes from a number of sources. For example, a look-up table has been generated for the NACA-65 aerofoil family from the experimental data of Abbot & von Doenhoff [100]. Collation of the experimental data results in a database of aerodynamic properties for NACA-65 aerofoils accounting for thickness, camber and angle of attack. A database for the NACA-16 family was also constructed, however, using a 2D aerofoil code. The aerodynamic properties at each radial element are then interpolated from the look-up table.

The small changes in thickness and camber, combined with the changes in the angle of attack along the span, when interpolated from the look-up tables, typically results in a non-continuous distribution along the span. This discontinuity tends to make the convergence of the BEMT more difficult. However, this difficulty may be overcome by smoothing the solution along the span.

Another data source is to use some aerodynamic approximations to the aerofoil behaviour. From thin aerofoil theory, the lift coefficient is approximated. The drag coefficient, on the other hand, is computed from the method presented by Bailey [101] for calculating the aerodynamics of a lifting rotor. Whilst this data has a greater number of assumptions about the flow, it does not suffer the same numerical problems of the look-up tables. Furthermore, they are less computationally expensive. As will be demonstrated later, these approximations also provide sufficient accuracy in predicting the rotor performance.

2.1.5 Model Corrections

To improve the capabilities and minimise the limitations of the CROR aerodynamic analysis, a number of corrections can be added. For example, the Karmen-Tsien compressibility correction and a drag divergence model [102] can be used to account for the effects of higher Mach numbers. Modern civil aviation CROR typically feature high blade count designs; therefore, a cascade correction can be used to account for the interaction that will occur between blades towards the blade root [103]. The Prandtl tip correction was used included to account for 3D effects. The simple model presented by Stepniewski & Keys [104] was used to compute the radius of contraction to account for the effects of streamtube contraction. Outside this radius, no effects of mutual interaction were accounted for. The aerofoil data, both approximations and look-up tables are appropriate only in a range of angles of attack. Leishmann [105] suggest a sinusoidal profile to lift and drag coefficients
to extend the 2D aerodynamics to ±180°, thus extending the range of the aerofoil database.

### 2.1.6 Rotor Trim

The analysis of rotor blades typically requires the matching of thrust or torque to compare rotor performance. Similarly, the operation of rotor blades requires a given thrust or torque to be delivered to meet the aircraft requirements. The rotor blade can be trimmed to provide the required thrust by either varying the blade setting angle or the rotor rotational speed. Figure 2.6 defines the setting angles and direction of rotation of each rotor used in the analysis.

![Figure 2.6: Definition of CROR fore and aft rotor setting angles and rotation senses.](image)

To trim the rotor blades, either the blade setting angle or rotor rotational speed was successively altered until the thrust or torque produced was within a given tolerance of the required value. To aid convergence to the required value, the change in setting angle or rotor rotational speed was computed according to:

\[
\Delta = \Delta + \gamma^+ |\zeta - \zeta_T|, \quad \zeta < \zeta_T \\
\Delta = \Delta - \gamma^- |\zeta - \zeta_T|, \quad \zeta > \zeta_T
\]

(2.38)

\(\Delta\) represents either blade setting angle (\(\beta\)) or rotor speed (\(\Omega\)); \(\zeta\) is the computed thrust or torque, and \(\zeta_T\) the corresponding required value; \(\gamma^+\) and \(\gamma^-\) are numerical gains. For trimming by setting angle, these gains are of the order \(10^{-5}\), whilst when trimming by rotational speed, these are of the order \(10^{-2}\). To avoid a solution oscillating about a given value, it is recommended to have different values of \(\gamma^+\) and \(\gamma^-\).

The CROR trimming procedure comprises two iterative loops. The first loop adjusts either rotational speed or setting angle of the the aft rotor (depending on
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input specification). Once the aft rotor has met the trim requirements, the second iterative loop adjusts the fore rotor. After each adjustment of fore rotor rotational speed or setting angle, the aft rotor trim is then re-evaluated. The procedure is demonstrated in Figure 2.7. The procedure implemented only allows for adjustments in either setting angle or rotational speed. Whilst one is varied, the other is held constant. This avoids issues surrounding multiple solutions to the trim variable.

![Diagram](image)

Figure 2.7: Implemented rotor trimming routine.

2.1.7 Model Validation

To verify the BEMT predictions, the CROR aerodynamic model has been compared against a number of available experimental and numerical data. Whilst CROR data is scarce, a number of performance data have been obtained to perform the validation. These data sets cover a range of configurations and operating points.

SR2 CROR

The BEMT model is first compared against the experimental work presented by Dunham et al. [106]. Both flowfield and thrust coefficient data were obtained for a
0.409 m diameter CROR of SR2 design [107]. The SR2 is a straight-bladed propeller from the NASA Advanced Turboprop Project [7], used as a baseline for investigations into the effect of sweep. The spanwise geometry of the SR2 propeller is presented in Figure 2.8.

![SR2 spanwise geometry](image)

Figure 2.8: SR2 spanwise geometry.

Figure 2.9 compares the thrust coefficient computed using the BEMT model and the experimental data obtained by Dunham et al. [106] for 4 × 4 and 8 × 8 CROR configurations. Results are compared over a range of advance ratios at a fore and aft blade setting angle of $\beta_{75} = 40.3^\circ$.

![Comparison of SR2 thrust coefficient](image)

Figure 2.9: Comparison of SR2 thrust coefficient.

Figure 2.9 shows that for both configurations, the thrust coefficient is generally well predicted. For the 8 × 8 configuration, the thrust coefficient is predicted well over the entire range of advance ratios. On the other hand, for the 4 × 4 configuration,
the thrust coefficient is predicted well for \( J \leq 1.4 \); beyond this, there is a significant discrepancy between the reported and computed data. This region of advance ratio results in a reduced angle of attack. At the extremes, this results in a negative angle of attack at some areas along the span. The discrepancy, therefore, may result from the poor aerodynamic resolution at low angles of attack. However, as the data presented by Dunham et al. did not include an analysis of errors, it is difficult to isolate the sources of the discrepancy.

The flowfield of a \( 4 \times 4 \) CROR operating at an advance ratio of \( J = 1.21 \) was computed by Dunham et al. using laser velocimetry. The rotational speed of both rotors were assumed equal and their value computed from an assumed free-stream velocity (within a reported range of speeds). Three components of velocity were taken at a number of axial planes, Figure 2.10, and averaged over a minute.

![Figure 2.10: Location of probes and CROR reference frame.](image)

Figures 2.11 and 2.12 compare the axial and tangential velocity components from the BEMT method and those reported by Dunham et al. The BEMT model neglects the radial flow and is, therefore, not compared with the published data. Figure 2.11 shows that the BEMT model has generally predicted the axial velocity profile well as it travels downstream. Whilst the downstream aft rotor location is generally well predicted, the upstream, downstream fore rotor and far downstream locations show a discrepancy with the experimental data. The maximum discrepancy between the BEMT model and experimental data was found at the far downstream location and corresponded to \( \sim 10\% \). Given the agreement with experimental data for the thrust coefficient at the current advance ratio, a better agreement would be expected for the flowfield. Therefore, the observed discrepancy may be due to the combination
of mutual interference coefficients used to compute the propagation of the velocity components. Additionally, the uncertainty of the rotor speed and inflow may result in the discrepancies in the upstream prediction and hence making it more difficult to predict the profiles downstream.

![Graphs showing BEMT and EXP comparisons for different locations.](image)

(a) Upstream fore rotor  (b) Downstream fore rotor  (c) Downstream aft Rotor  (d) Far downstream

Figure 2.11: SR2 Axial comparison.

However, the accuracy of the experimental data must be questioned here as it is unusual that the reported axial velocity does not accelerate further between the downstream aft rotor and far downstream locations. It would be expected that the axial velocity would show a higher value far downstream than that just downstream the aft rotor. However, this will be investigated further when comparing the high-order model with the presented data.

Figure 2.12 shows that for most locations, the BEMT model has predicted the tangential component reasonably well. Comparing the downstream fore and aft rotor location, the effect of the aft rotor is shown, i.e. the change in sign of the tangential component. The kink in the profile at \( \sim 0.95R \) results from the model accounting for the wake contraction. Outside the area of the contracted wake, the mutually-induced component is neglected, leaving only the self-induced component. This can also be observed in the axial velocity predictions. The far downstream location shows the greatest discrepancy compared with the experimental data. This discrepancy shows an overprediction of the aft rotor tangential component as it is propagated downstream, whereas the experimental data shows an almost net cancellation of the tangential component.
The observed overprediction may again result from the terms describing the mutual interaction too heavily influencing the development of each velocity component. Further experimental works should be compared in order to 'calibrate' the interference coefficients to capture the axial velocity development more accurately.

**DLR CROR**

Stuermer et al. [72] presented data for the performance of a $12 \times 10$ CROR at take-off conditions. Although the data presented is from numerical analysis and considers
only a single point, it useful to test the model against as it represents a modern and advanced CROR design. Figure 2.13 shows the CROR configurations studied by Stuermer et al. The data presented is also useful as it presents individual blade loads in addition to setting angles. To compare the results, the BEMT model was trimmed to match the reported thrust coefficient with both rotors rotating at the reported 1100 rev/min.

Table 2.2 compares the computed loading coefficients and blade setting angles calculated using the BEMT model and the DLR-TAU CFD solver. Table 2.2 illustrates that when trimmed to the same thrust coefficient, the BEMT model is able to predict relatively well fore and aft power coefficients. The BEMT model is shown to match well the corresponding setting angle required to trim the rotor.

The aft rotor shows an ~4% disagreement in resulting power coefficient. This disagreement may arise due to an overprediction of the mutually induced component. An increased axial component will result in an increased inflow angle, requiring an increased setting angle to deliver the same angle of attack to provide the required thrust. Table 2.2 also shows the increased setting angle required.

Table 2.2: DLR CROR performance computed using CFD and BEMT model.

<table>
<thead>
<tr>
<th></th>
<th>( C_{T1} [-] )</th>
<th>( C_{T2} [-] )</th>
<th>( C_{P1} [-] )</th>
<th>( C_{P2} [-] )</th>
<th>( \beta_{1s} [^\circ] )</th>
<th>( \beta_{2s} [^\circ] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEMT</td>
<td>0.3915</td>
<td>0.2882</td>
<td>0.5804</td>
<td>0.4835</td>
<td>35.08</td>
<td>45.62</td>
</tr>
<tr>
<td>DLR</td>
<td>0.3915</td>
<td>0.2883</td>
<td>0.5793</td>
<td>0.4647</td>
<td>34.85</td>
<td>41.40</td>
</tr>
<tr>
<td>( \Delta ) [%]</td>
<td>0</td>
<td>0.035</td>
<td>0.190</td>
<td>3.97</td>
<td>0.657</td>
<td>9.70</td>
</tr>
</tbody>
</table>

Rig-140 CROR

Zachariadis & Hall [108] presented both numerical and experimental data for the Rolls-Royce Rig-140 CROR. Rolls-Royce developed the Rig-140 CROR for investigations into the viability of CROR in the 1980s. This case is considered highly useful as the CROR has a number of design features of modern CROR whilst also having experimental data available to compare with. Rig-140 is shown in Figure 2.14.

The Rig-140 model is a \( 7 \times 7 \) CROR with straight blades. Whilst the detailed blade geometry remains proprietary, Zachariadis and Hall provided sufficient detail, Figure 2.14, to compare with the BEMT model. Whilst the detail is sufficient for BEMT analysis, there is insufficient detail to compare using high-order methods.

As reported by Zachariadis & Hall [108], thrust and power coefficient changes resulted from changes in the rotational speed at a constant setting angle. Therefore, this was replicated in order to compare the BEMT model. The range of advance ratios and hence rotor speeds were worked back from the combination of thrust and power...
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Figure 2.14: Details of the Rolls-Royce Rig-140 CROR (adapted from Ref. [108]).

coefficient and power coefficient and efficiency plots, i.e. from the expression: \( \eta = JC_T/C_P \). Whilst this does not necessarily replicate the test conditions, in particular unequal rotor speeds, it is acceptable here in order to make a comparison given the lack of available data.

Figure 2.15 compares the total thrust and power coefficients for the range of advance ratios computed using the BEMT model and the CFD and experimental results reported by Zachariadis & Hall at cruise conditions.

The BEMT model is seen to compare very well with both experimental and CFD results up to \( \sim C_P = 3 \), beyond this range, the BEMT diverges from the experimental results but agrees well with the CFD results up to \( \sim C_P = 4 \). The source of disagreement is not immediately apparent as thrust and power results are not isolated. Isolating the disagreement is further complicated by the likely difference in rotor speeds. The disagreement between CFD and experimental data is reported to be with deflections of the physical blades under operation not captured using CFD. These deflections are also not captured in the BEMT model and likely one of the reasons for the disagreement. Furthermore, no analysis of the experimental errors was reported, making it yet more difficult to identify the sources of error with the BEMT model.

2.1.8 Sensitivity Analysis

To validate the predictions of the BEMT model, it has been compared against a number of numerical and experimental data sets. The validation showed that while
the model generally performed well, there were areas showing discrepancy with reported data. These discrepancies may come from a number of sources. However, the data sets with which the BEMT model was compared did not provide any account for errors. Therefore, identifying the sources of discrepancy becomes difficult. To this end, a sensitivity analysis has been carried out to identify the effects of the accuracy of the most pertinent parameters of the CROR model.

Radial Discretisation

The BEMT model discretises each CROR blade into a number of 2D sections. The aerodynamic loading of each section is computed as if in isolation. The 2D loading from all elements are then integrated along the span to give the total blade forces. Therefore, the number of points used to describe the geometry of the blade will impact the final integrated loads. To minimise computational time, the current sensitivity analysis aims to find the minimum number of points to obtain an independent solution.

For the sensitivity analysis on the radial discretisation, the Rig-140 CROR was simulated at a constant tip speed and blade setting angle. Figure 2.16 presents the effect of the radial discretisation on both thrust and power coefficients as well as the computational time\(^\dagger\). The number of iterations required for convergence was not significantly affected by the radial discretisation. Figure 2.16 demonstrates that the radial discretisation quickly converges for both thrust and power coefficients.

\(^\dagger\)Computations carried out on a single Intel\textsuperscript{®} Core\textsuperscript{TM} i7-6700 CPU
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On the other hand, both the wall-time and CPU time exponentially increases for increasing radial discretisation. Whilst the computation time is not exhaustive for even 192 radial elements (compared to higher-order analyses), the use of the low-order model for optimisation or preliminary design demands minimal computational time. As a result, it is recommended that 32 radial points be sufficient for CROR analyses\textsuperscript{2}. Using 32 radial elements, the thrust coefficient differs by less than 2% of that obtained using 192 radial points, whilst the computational time was 862% lower.

Sensitivity to Geometry

When comparing the BEMT model with published experimental data, the CROR spanwise geometry is typically extracted from scanned images. Therefore, this may result in errors as the data is extracted and interpolated. Furthermore, it is important to understand the effects of the CROR parameters on its performance. The sensitivity of the rotor chord has been investigated to understand these effects.

For the sensitivity analysis, the chord was changed by a maximum $\pm 10\%$ and the resulting thrust and power coefficient for the Rig-140 CROR at a fixed blade setting over a range of advance ratios were computed. Figure 2.17 demonstrates the effect of changes in chord on the computed thrust and power coefficients, where the error bars represent the maximum increase and decrease in chord. Figure 2.17 shows that the effect of a 10% change in the chord is significant for both thrust and power coefficients. However, the effect is seen to diminish for increasing advance ratio. At low advance ratio, the angle of attack is higher and therefore

\textsuperscript{2}In order to ensure the 75% span index results in an integer, 32 elements are recommended as opposed to 30.
more critical to design changes.

To investigate further, Figure 2.18a shows the effect of changes in the chord at a constant advance ratio. Figure 2.18a shows that the effect of chord changes are slightly more important for the power coefficient. It is also observed that both thrust and power coefficient respond linearly to the changes in the chord. This result is expected given the blade element equations describing the loading terms. A 10% change in the chord is shown to have $\sim 5\%$ and $\sim 6\%$ change in thrust and power coefficients, respectively — not insignificant. However, in terms of capturing the reported geometry, chord changes are unlikely to be this large.

Figure 2.17: Effect of chord data on CROR performance.

Figure 2.18: Detailed performance effect of changes in rotor chord.
Figure 2.18b presents the effect of a constant chord change over a range of advance ratios. Figure 2.18b highlights the diminishing effect of the chord at higher advance ratios. It is shown that the thrust coefficient diminishes much more quickly for increasing advance ratio.

With respect to the SR2 validation, which showed the most considerable discrepancy at the higher advance ratios, it is unlikely that the inaccuracies of the captured geometry are to blame for the disagreement.

Sensitivity to Aerodynamic Data

The CROR BEMT model requires 2D sectional aerodynamic data to compute the rotor loads. The developed model can take this sectional data from either look-up tables for a given aerofoil section or from a linear approximation (§2.1.4).

Data taken from look-up tables populated with experimental data will have two primary sources of limitations. Firstly, the data will be for isolated aerofoils, not accounting for 3D effects and interaction with other radial sections. Secondly, the data is linearly interpolated from the table. The relationship with the given parameter may not be linearly dependent, and hence the interpolation will result in incorrect values.

For data taken from aerofoil aerodynamic codes or the linear approximations, the inherent approximations will result in inaccuracies in the resulting aerodynamics. To this end, a sensitivity analysis has been performed to investigate the effects of small changes in 2D aerodynamic data. For the study, the Rig-140 CROR was studied at a fixed setting angle over a range of advance ratios with linear aerodynamics used to represent the sectional loads. In each case, the lift and drag coefficients were independently altered by ±10%.

Figure 2.19 demonstrates the effect of lift and drag coefficients on the resulting total CROR thrust and power coefficients over a range of advance ratios. Figure 2.19 shows that, compared to the drag coefficient, the changes in the lift coefficient has a greater impact on both thrust and power coefficients over the entire advance ratio range. However, in both cases, their effect diminishes with increasing advance ratio. As described previously, this is due to the increased angle of attack at low advance ratios being more critical to design changes.

To investigate the effects of aerodynamic coefficients further, Figure 2.20 presents the change in thrust and power coefficient at a single advanced ratio for ±10% change in the aerodynamic coefficient.

Figure 2.20a shows that the effect of the lift coefficient is similar for both thrust and power coefficients. For increasing lift coefficient, both thrust and power coefficients linearly increase. A maximum ±5% change is observed for the maximum
Figure 2.19: Effect of aerodynamic data on CROR performance.

Figure 2.20: Effect of aerodynamic data at constant advance ratio.

changes in lift coefficient. For the drag coefficient, Figure 2.20b shows that it has a greater effect on the thrust coefficient, with a negligible effect on the power coefficient at this advance ratio. However, the maximum change now observed is $< 1\%$ for the maximum changes in the drag coefficient.
Figure 2.21 shows the effect of a 6% change in the aerodynamic coefficient over a range of advance ratios. The sensitivity of the loading coefficients to lift coefficient is shown to have a diminishing effect with increasing advance ratio, with a rapid decrease at higher advance ratios. Nonetheless, the lift coefficient is shown to have a greater effect for the range of advance ratios shown. There is only \( \sim 0.6\% \) change in power coefficient, with an even smaller change observed for the thrust coefficient when considering the drag coefficient.

The effect of advance ratio is seen to have opposing effects on thrust and power coefficients with regards to sensitivity to drag coefficient. However, the changes are < 1% over the entire range for both.

The aerodynamic data is used to compute sectional loading on the rotor blades. The sensitivity analysis showed that the lift coefficient had a greater impact than the drag coefficient on the thrust and power coefficients. Despite the diminishing effect at higher advance ratios, the sensitivity to lift coefficient was still significant. The sensitivity analysis carried out has highlighted the importance of accurately predicting the 2D aerodynamic data. As a result, the accuracy of the 2D data may be used to explain the discrepancies observed for both the SR2 and Rig-140 validation data.

This section has introduced a low-order aerodynamic model that has been extended to allow for predictions of CROR performance. In order to verify the prediction capabilities, the model was compared against both numerical and experimental CROR cases. Generally, the model predicted well both flowfield and loading coefficient data. However, there was greater discrepancy at higher advance ratios. With a lack of error analysis in the compared data, a sensitivity analysis was undertaken to explore the observed differences. The sensitivity study identified the accuracy of
aerodynamic data to be the most significant factor in explaining the discrepancies.

2.2 High-Order Aerodynamic Modelling

To gain a greater understanding of the complex aerodynamics of CROR, and in particular, better understand the aerodynamic noise generating mechanisms, a high-fidelity analysis is required. To this end, this section presents a high-order CFD methodology for studying CROR aerodynamics. The governing equations are first briefly presented before the CFD solver, mesh requirements and limitations are discussed. Flowfield predictions are compared against experimental data in addition to a mesh refinement and turbulence study are presented to verify the performance of the CFD solver and the methodology.

2.2.1 The Governing Equations

Rotor flows are characterised as being viscous, compressible, highly turbulent and are often unsteady in nature. Therefore, a detailed analysis of the rotor flowfield presents a complex problem. The flow is governed by the Navier-Stokes equations. These equations define the conservation of mass, momentum and energy within the fluid domain, and to close the system, the two thermodynamic equations of state.

Continuity describes the conservation of mass within the flow. In conservative form, the Navier-Stokes equations are written in a Cartesian system as:

**Continuity:**

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0. \tag{2.39}
\]

**Momentum:**

\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = \rho f_i - \frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{2.40}
\]

where the left-hand side represents the convective terms. The first term of the right-hand side describes the body forces. The remaining two terms represent the pressure gradient and \(\sigma_{ij}\), the viscous stress tensor. Applying Stoke’s hypothesis and assuming a Newtonian fluid:

\[
\sigma_{ij} = 2\mu \left[ S_{ij} - \frac{1}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right]. \tag{2.41}
\]

With, \(\mu\) the molecular viscosity and \(\delta_{ij}\), the Kronecker delta function. \(S_{ij}\) is the strain-rate tensor, given by:

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{2.42}
\]
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Energy:

\[
\frac{\partial (\rho E)}{\partial t} + \frac{\partial (\rho u_j E)}{\partial x_j} = \frac{\partial}{\partial x_j} [u_i (-p \delta_{ij} + \sigma_{ij}) - q_j] + \rho u_i f_i. \tag{2.43}
\]

\(E\) is the total energy of the fluid per unit mass and is defined as:

\[
E = \left[ e + \frac{1}{2} u_i u_i \right], \tag{2.44}
\]

\(e\) is the specific internal energy and \(\frac{1}{2} u_i u_i\) is the unit mass kinetic energy. \(q_i\) is the heat flux vector, and using Fourier’s law is given by:

\[
q_j = -k_T \frac{\partial T}{\partial x_j}. \tag{2.45}
\]

Here, \(k_T\) is the thermal conductivity and \(T\) is the fluid temperature. Viscosity is calculated using Sutherland’s law:

\[
\mu = \mu_{ref} \left( \frac{T}{T_{ref}} \right)^{3/2} \frac{T_{ref} + T_{Suth}}{T + T_{Suth}}. \tag{2.46}
\]

The system of equations is closed assuming an ideal gas to relate the pressure and density:

\[
p = \rho RT, \tag{2.47}
\]

where \(R\) is the specific gas constant of dry air, \(R = 287 \text{ J/kgK}\).

2.2.2 The HMB3 Solver

The HMB3 solver [109, 110] was used to carry out the CFD simulations in the present work. In order to begin solving the Navier-Stokes equations, some work is first required in order to make them more suitable for programming.

The HMB3 solver uses the conservative form of the governing equations. The Navier-Stokes equations are written in conservative form, allowing for a more convenient form where continuity, momentum and energy equations to be combined into a single expression:

\[
\frac{\partial W}{\partial t} + \frac{\partial (\mathbf{F} + \mathbf{F}^c)}{\partial x} + \frac{\partial (\mathbf{G} + \mathbf{G}^c)}{\partial y} + \frac{\partial (\mathbf{H} + \mathbf{H}^c)}{\partial z} = \mathbf{S} \tag{2.48}
\]

where, \(W\) is the vector of conservative variables:

\[
W = (\rho, \rho u, \rho v, \rho w, \rho E)^T \tag{2.49}
\]
ρ is the fluid density and \( u, v \) and \( w \) are the three components of velocity. \( F, G \) and \( H \) are the \( x-, y-, \) and \( z-\)components of the flux vectors. The inviscid components, denoted by \([\cdot]^i\) are:

\[
F^i = (\rho, \rho u^2 + p, \rho uv, \rho uw, u(\rho E + p))^T, \\
G^i = (\rho, \rho uv, \rho v^2 + p, \rho vw, v(\rho E + p))^T, \\
H^i = (\rho, \rho uw, \rho w^2 + p, w(\rho E + p))^T. 
\]

(2.50)

The viscous components, denoted by \([\cdot]^v\) are:

\[
F^v = (0, \sigma_{xx}, \sigma_{xy}, \sigma_{xz}, u\sigma_{xx} + v\sigma_{xy} + w\sigma_{xz} + q_x)^T, \\
G^v = (0, \sigma_{sx}, \sigma_{sy}, \sigma_{sz}, u\sigma_{sx} + v\sigma_{sy} + w\sigma_{sz} + q_u)^T, \\
H^v = (0, \sigma_{xz}, \sigma_{yz}, \sigma_{zz}, u\sigma_{xz} + v\sigma_{yz} + w\sigma_{zz} + q_z)^T. 
\]

(2.51)

\( S \) is a source term. For simulations assuming a rotating frame of reference (e.g. an isolated rotor in forward flight), the grid is fixed in the frame of reference of the rotor, and the source term represents the inertial effects due to the rotation of the reference frame. In particular, \( S = [0, -\rho \omega \times u_h, 0]^T \). \( \omega \) and \( u_h \) are respectively the rotational vector, and local velocity field in rotor reference frame. For simulations not assuming a rotating reference frame, this source term is set to zero, i.e. \( S = 0 \).

The governing equations can be non-dimensionalised to further improve programming simplicity. The above equations are non-dimensionalised with the fundamental units of measure: length, mass, time, and temperature. To obtain the non-dimensional form of the equations, the HMB solver requires only four reference variables: length, density, velocity and temperature. Non-dimensional variables are obtained through:

\[
x_i = \frac{x^*_i}{L_{ref}}, \quad \rho = \frac{\rho^*}{\rho_{ref}}, \quad u_i = \frac{u^*_i}{U_{ref}}, \quad T = \frac{T^*}{T_{ref}}. 
\]

(2.52)

Where the reference variables are problem dependant. The remaining problem variables can be obtained from the four reference variables:

\[
t = \frac{U_{ref} t^*}{L_{ref}}, \quad p = \frac{p^*}{\rho_{ref} R T_{ref}}, \quad \mu = \frac{\mu^*}{\mu(T_{ref})}, \quad e = \frac{e^*}{U_{ref}^2}, \quad k = \frac{\mu(T_{ref})}{\rho_{ref} L_{ref} U_{ref}^3 k^*}. 
\]

(2.53)

where, \( t \) is time; \( p \) is pressure; \( \mu \) is the viscosity; \( e \) the internal energy; and \( k \) the kinetic energy. \( R \) is the ideal gas constant.
2.2. HIGH-ORDER AERODYNAMIC MODELLING

The HMB3 solver solves the Unsteady Reynolds Averaged form of the Navier-Stokes equations in dimensionless, integral form using the arbitrary Lagrangian Eulerian [111] formulation for time dependant domains with moving boundaries:

\[
\frac{d}{dt} \int_{V(t)} \mathbf{W} dV + \int_{\partial V(t)} (\mathbf{F}_i(\mathbf{W}) - \mathbf{F}_v(\mathbf{W})) \cdot \mathbf{n} \, dS = \mathbf{S}. \tag{2.54}
\]

\( V(t) \) is the time dependant control volume, with \( \partial V(t) \) its boundary. The HMB solver discretises the Navier-Stokes equations using a cell-centred finite volume approach on a multi-block grid. This is done on a curvilinear coordinate system, and leads to the following set of equations to solve:

\[
\frac{d}{dt} (\mathbf{W}_{i,j,k} \mathbf{V}_{i,j,k}) + \mathbf{R}_{i,j,k} = 0. \tag{2.55}
\]

\( \mathbf{W}_{i,j,k} \) is the vector of conserved variables of each cell, with \( \mathbf{V}_{i,j,k} \) the cell volume. \( \mathbf{R}_{i,j,k} \) is the vector of flux residuals.

Osher’s upwind scheme [112] is used to evaluate convective fluxes due to its stability, robustness and accuracy. Second-order finite differencing is employed to discretise the viscous fluxes. The MUSCL variable extrapolation method [113] is used in order to provide second-order accuracy in space. A van Albada limiter [114] is used in areas of high-pressure gradients to remove spurious oscillations. An implicit dual time-stepping method is employed to perform time integration in order to achieve fast convergence [115]. The resulting system of linear equations are solved using the generalised conjugate gradient method with a BILU factorisation [116] as a preconditioner. Boundary conditions are applied using ghost cells on the exterior of the computational domain. In the far-field, ghost cells take the value of the free-stream. For solid bodies, the no-slip condition is set for viscous flows, whilst for Euler flows, the value is extrapolated from the interior (i.e. zero normal velocity).

2.2.3 Turbulence Closure

In order to compute the flow for most practical engineering applications, i.e. turbulent and of high Reynolds number, a large number of length and time scales must be resolved. A very fine, and hence large grid is therefore required to solve the flow field. This approach of completely resolving all scales is known as Direct Numerical Simulation (DNS). However, resolving the entire flowfield is computationally demanding. The computational requirements of DNS make it unfeasible for most applications and more so for rotor flows. Therefore, to proceed, some of the fluctuating turbulent flow must be modelled in order to obtain a flow solution for reasonable computational cost. A large number of turbulence models have been developed with
the Reynolds Averaged form of the Navier-Stokes equations to make obtaining a solution more practicable.

The HMB3 solver offers a number of RANS turbulence models to close the viscous stress tensor. In particular, the $k-\omega$ [117], $k-\omega$ SST [118] turbulence models have been used in this work.

The $k-\omega$ model, first devised by Wilcox in 1988 [117], is a Reynolds averaging model that has seen wide application in aerospace due to its superior performance in the near-wall region, adverse pressure gradients and strongly separated flows. However, a disadvantage of the model is its sensitivity to the free-stream boundary conditions. The $k-\varepsilon$ model [119] on the other hand, does not suffer the same free stream dependence and as a result, the $k-\omega$ and $k-\varepsilon$ models have been blended by Menter [118] to give the $k-\omega$ SST model. The blending is done such that the $k-\omega$ model is used in the near-wall region, whilst the $k-\varepsilon$ is used in the outer region. Hence, the $k-\omega$ SST combines the advantages of both $k-\omega$ and $k-\varepsilon$ models.

By modelling all turbulent scales, the RANS approach offers an attractive solution for solving turbulent problems. However, RANS formulations have a number of disadvantages, e.g. they can be too diffusive on coarse grids and are unable to provide unsteady data. As discussed previously, resolving all turbulent scales is too costly. A step between the two extremes of modelling and resolving is Large Eddy Simulations (LES). LES resolves those scales larger than the grid cells, with those smaller with a sub-grid model, resulting in reduced modelling uncertainties. However, the LES itself can still be computationally expensive, requiring a fine spatial and temporal resolution. Notwithstanding, LES can become affordable when coupled to RANS by using LES where required and RANS where it is reliable and accurate.

The Scale Adaptive Simulation (SAS) [120] is an improved uRANS approach that can provide LES like behaviour. The SAS model is a scale resolving approach based on introducing a second mechanical length scale (the von-kármán length scale). By adjusting the turbulence length scale depending on the flow conditions, the SAS model balances the modelled and resolved regions, resulting in an improved uRANS approach. The SAS formulation has been implemented in the HMB3 solver by coupling with the SST model [121]. The SAS model differs from the SST model only by the introduction of an additional source term, $Q_{\text{SAS}}$. The reader is directed towards Ref. [122] for more details on the SAS implementation within the HMB solver.

The SAS model has been used within this work and compared to the RANS turbulence models. Both aerodynamic and aeroacoustic predictions have been compared in order to verify its performance based on the additional computational cost for the specific work presented within this thesis.
2.2.4 Meshing Strategy

In order to solve the system of equations describing the CROR flowfield, the domain has to be discretised into a suitable computational grid. For the HMB3 solver, this consists of multi-block structured grids. Furthermore, solving unsteady rotor problems requires a method to account for the relative motion of the blades. HMB3 supports two methods, the sliding plane method [123], and the Chimera method [124]. Due to the simplicity offered for mesh generation and blade setting changes, the Chimera method was used throughout this work.

Chimera Method

The Chimera method works with overlapping and non-matching grids that are independently generated. The overlapping grids exchange field variables by interpolating between grid layers based on a defined hierarchy of grid levels. In the HMB3 solver, an interpolation search is performed. This search or localisation identifies grid blocks that potentially overlap. It then identifies cells that need to be interpolated and their interpolates based on a defined hierarchy, as well as calculating the interpolation weights. For overlapping regions, holes are cut in the grid so that field variables are only computed on one of the grid layers. The result of this localisation is illustrated in Figure 2.22. Green cells are computational cells; dark blue represents the solid holes or solid bodies, here the rotor blade; light blue represents overlapping cells; yellow highlights cells flagged for interpolation; and red the fringe cells - the last layer of computational cells. Further details of the Chimera implementation within HMB3 can be found in Ref. [124].

The Chimera method offers a number of benefits besides the ability to account for the relative motion of the rotor blades. Firstly, it allows for the rotor grids to be generated independently, simplifying the mesh generation process. Additionally, where blade setting angles are required to be changed, the Chimera components can be rotated without regenerating the mesh. Furthermore, for CROR geometry with large blade counts, only a single blade need to be meshed, the others are obtained by copying the first grid.

Whilst the Chimera technique offers a number of benefits, it also has a number of limitations in its current implementation. For example, at least two cells are required for interpolation, limiting the maximum rotor time step. Also, it is not currently possible to overlap the Chimera grid with solid bodies. Therefore, a small gap will exist between the rotor root and the spinner surface. However, if the hub region is unimportant, this can be acceptable.

The Chimera method has been successfully applied to CROR flows by a number of authors [48, 125, 126, 127, 128, 129]. Table 2.3 describes the Chimera grids used
in a number of these studies. Table 2.3 highlights the large cell counts required to capture the CROR flow physics. Note the first two studies performed aeroacoustic simulations, and hence the cell counts are considerably larger.

Table 2.3: Comparison of works employing Chimera technique for CROR analysis

<table>
<thead>
<tr>
<th>Work</th>
<th>Cells per Blade ($10^6$)</th>
<th>Background Cells ($10^6$)</th>
<th>CROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boisard et al. [48]</td>
<td>4.5</td>
<td>80</td>
<td>12x10</td>
</tr>
<tr>
<td>Busch et al. [127]</td>
<td>5.6</td>
<td>25</td>
<td>9x7</td>
</tr>
<tr>
<td>Stürmer et al. [128]</td>
<td>2.0</td>
<td>19</td>
<td>10x8</td>
</tr>
<tr>
<td>Stuermer and Yin [126]</td>
<td>1.5</td>
<td>N/A</td>
<td>8x8</td>
</tr>
</tbody>
</table>

**Spatial and Temporal Discretisation**

The use of the Chimera technique has the benefit that background and Chimera grid components are generated independently. However, when generating both grids, to reduce numerical errors in the area of the rotors, it is desired to have a similar cell size in both grids.
Implementation of the Chimera technique requires at least two cells for interpolation. This requirement, therefore, constrains both the azimuthal step size and the background azimuthal grid resolution to ensure there are enough interpolation cells as the rotor grids rotate. This is further complicated by the constraint on the grid and step size from acoustic criteria.

For near-field noise simulations, the frequency resolution is dictated by the grid spacing, with fine grids required for a high-frequency resolution. This constraint is removed as we only consider far-field noise. Considering a rotor with angular frequency $n \ [\text{rev/s}]$, the sampling frequency of the CFD simulation is governed by the azimuthal step size $\Delta \psi$:

$$f_{\text{samp}} = \frac{360n}{\Delta \psi}$$  \hspace{1cm} (2.56)

from Nyquist’s theorem \cite{130}, the maximum resolved frequency is computed according to:

$$f_{\text{max}} = \frac{180n}{\Delta \psi}$$  \hspace{1cm} (2.57)

Therefore, when generating a grid for aeroacoustic simulations, the Chimera grids must satisfy both grid and acoustic constraints. For the simulations carried out in this work, an azimuthal step size of $\Delta \psi = 0.5^\circ$ was used. This step size resulted in a maximum frequency resolution of around 6 kHz for a rotor rotating at a 1,000 [rev/min] as was the baseline case. This step size also satisfied the interpolation constraints.

**Mesh Generation**

The computational grid for the CROR simulations comprised a stationary background grid and $N_1 + N_2$ Chimera blade grids. The CROR simulations carried out in this work all employed the SR2 \cite{107} blade grid with a simplified spinner assembly. Therefore, a single grid has been generated which is scaled depending on the computational case.

The background grid consists of the spinner and free-field. A simplified spinner has been used to reduce the complexity of the mesh. The spinner is modelled on the SR2 spinner used in the work of Dunham et al. \cite{106}, however, it is extended far downstream to simplify the grid.

Only a quarter of the background grid is generated, exploiting the symmetry of the model. It is then copied around the azimuth to give the full grid. For the background topology, an O-grid is employed in the azimuthal direction and an H-grid used in the meridional plane. The topology of the quarter background grid is shown in Figure 2.23.
The grid is extended 6.5R upstream, 15R downstream, and 6R in the radial direction (Figure 2.23), far enough to ensure no recirculation in the computational domain. These locations were chosen based on previous rotor simulations using the HMB solver [122] and from CROR simulations found in the literature [48, 127]. The grid was refined in the rotor and near wake regions and coarsened towards the far-field in the axial and radial directions. Equally spaced cells were used around the azimuth. Refinement of the grid in the near-wall region of the spinner was used to ensure $y^+ \leq 1$. The background mesh is shown in frontal and meridional views in Figure 2.24. The quarter background grid was constructed of 352 blocks.

For the CROR analysis, both fore and aft rotors were of SR2 design, Figure 2.8. Therefore, using the Chimera technique, the grid for only a single blade is constructed. For the fore row, it is rotated to the required setting angle and copied
to give the full blade count. For the aft row, it is first mirrored about the y-axis before re-pitching and copying.

The rotor blade grid was created using a C-H topology. The blade topology is shown in Figure 2.25. The grid was refined towards the blade surface, and the hub and tip gap regions to ensure $y^+ \leq 1$. Each Chimera grid comprised 86 blocks.

The independent grid components are combined using a HMB grid utility to construct the full grid. Figure 2.26 shows the assembled surface grids. Orange represents the spinner surface; red and blue represent the fore and aft blade grids respectively; the black outlines show the Chimera boundaries for each blade grid.

Figure 2.24: Background grid topology.
Recall that in the current implementation of the Chimera technique, a small gap will exist between the rotor root and the spinner. The whole grid is scaled to give a tip chord equal to one. The scaling required between the different cases studied is then carried out in the post-processing. Blade setting angles for each case are estimated using the BEMT model described.

Grid Refinement Study

In order to ensure the CFD simulations were grid-independent, a grid refinement study has been performed. The grid refinement study is composed of two parts. Firstly, a study of the background grid and secondly, a study of the Chimera grid. Due to the large grid sizes required for CROR simulations, the grid study has been conducted for a single rotor. However, the results should be applicable to the full grid of the CROR case.

For the background grid study, three grids have been generated. These have been designated ‘coarse’, ‘medium’ and ‘fine’, representing their relative cell counts. The grid study has been concentrated on the area of the rotor blade and the near wake region. The area of refinement for the background grid study was from hub to 1.25R in the radial direction, and from -0.85R to 2R in the axial direction. The azimuth was equally refined due to the equal spacing. The refinements for each area for each grid are presented in Table 2.4. The cell count is equal in all other areas of each grid.

Using the medium Chimera grid (detailed below) steady single rotor simulations were performed for each grid. Table 2.5 presents the computed pressure and viscous
components of the normalised axial forces (note the change of reference frame for the steady simulation), and total thrust coefficient. Also shown in Table 2.5 is the difference in total thrust coefficient relative to the fine grid. Note: following the steady rotor implementation in HMB3, the pressure and viscous force coefficients are first converted to the helicopter thrust coefficient according to:

\[ C_{T_H} = C_{T_z} + C_{\tau_z} \frac{\sigma}{\pi R/c} = \frac{T}{\frac{1}{2} \rho (\Omega R)^2 A}, \] (2.58)

before being converted to the propeller definition which is used throughout this work:

\[ C_{T_{prop}} = \frac{T}{\rho n^2 D^4} = C_{T_H} \frac{\pi^3}{8}. \] (2.59)

Table 2.4: Background grid refinement parameters.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Axial</th>
<th>Azimuth</th>
<th>Radial</th>
<th>Total cells (10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>271</td>
<td>182</td>
<td>76</td>
<td>6.57</td>
</tr>
<tr>
<td>Medium</td>
<td>339</td>
<td>218</td>
<td>116</td>
<td>13.4</td>
</tr>
<tr>
<td>Fine</td>
<td>409</td>
<td>250</td>
<td>140</td>
<td>20.6</td>
</tr>
</tbody>
</table>
Table 2.5: Computed loads for each background grid.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>$C_{T_{x}}$</th>
<th>$C_{T_{z}}$</th>
<th>$C_{T}$</th>
<th>$\Delta C_{T_{fine}}%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>$-0.1073$</td>
<td>$0.2464$</td>
<td>$0.4024$</td>
<td>$1.452$</td>
</tr>
<tr>
<td>Medium</td>
<td>$-0.1067$</td>
<td>$0.2560$</td>
<td>$0.3994$</td>
<td>$0.721$</td>
</tr>
<tr>
<td>Fine</td>
<td>$-0.1059$</td>
<td>$0.2522$</td>
<td>$0.3966$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 2.5 shows that the refinement of the background grid has had little effect on the total thrust coefficient computed. The increase in cell count from 7 to 21 million cells has resulted in a $<1.5\%$ difference in thrust coefficient relative to the finest grid.

The refinement of the background grid is shown to have only a small impact on the total loading. The flowfield downstream is next studied for each grid refinement in order to better assess the grid independence.

Figures 2.27 and 2.28 respectively present the three components of normalised velocity\(^8\), 1R and 2R downstream of the rotor pitch change axis. Figure 2.27 shows that for both axial and tangential components the profiles of the medium and fine grids practically overlay one another. However, for the radial component, there is a discrepancy between the two at the locations of peak deficit. Whilst for the axial and tangential components, the coarse grid generally follows the fine and medium grids, there is an even greater discrepancy for the radial component.

\(^8\)The velocity components presented are normalised with respect to the steady rotor formulation.
2.2. HIGH-ORDER AERODYNAMIC MODELLING

Figure 2.27: Velocity profile comparison for three mesh densities 1R downstream of the rotor.

Figure 2.28: Velocity profile comparison for three mesh densities 2R downstream of the rotor.

Considering 2R downstream, Figure 2.28, for all three components, the medium and fine grids agree very well. Again the coarse grid, whilst generally showing a
similar profile, departs in magnitude with the two other grids.

Given the small change in computed thrust coefficient, in addition to the small changes in flowfield velocity as the grid was refined, the finest grid was considered to be mesh independent, i.e. further refinement would not yield differing results. The use of the coarse background grid was ruled out on two parts, it differed too greatly relative to the fine grid, and when used for CROR did not meet the interpolation criteria. While the medium grid showed discrepancy when comparing radial component, it matched well with the finest grid in both axial and tangential components. Given the additional computational time of the finer grid, which will only escalate when considering a full CROR grid, the medium grid was used for the subsequent CROR simulations. Despite the fine grid being mesh independent, the use of the medium grid is justified from the point of view that the reduction in computational cost is more beneficial than the penalty of a slight loss of accuracy. Additionally, the cell count of the medium grid is itself quite high when compared with other authors, Table 2.3.

The second element of the grid study is concerned with the Chimera component of the grid. The Chimera grid studies were carried out on the medium background grid. Due to the constraints on the Chimera grid component, two Chimera grids were generated, designated medium and fine to represent their relative cell counts.

Refinement for the Chimera grid component was simultaneously carried out normal to the blade, along the blade span and around the aerofoil section. The refinements for each area for each grid are presented in Table 2.6.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Aerofoil</th>
<th>Normal</th>
<th>Spanwise</th>
<th>Total cells ($10^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
<td>360</td>
<td>61</td>
<td>229</td>
<td>6.6</td>
</tr>
<tr>
<td>Fine</td>
<td>408</td>
<td>75</td>
<td>245</td>
<td>9.6</td>
</tr>
</tbody>
</table>

The constraints with the current implementation of the Chimera technique presents some difficulty when performing a grid study. A coarser grid than the medium is difficult to achieve whilst retaining the required number of interpolee cells. Similarly, a greater grid refinement would be wasted effort as the interpolation is still done on the same background grid, i.e. interpolation would be onto a coarser grid.

The loading computed for simulation using each grid is presented in Table 2.7. The refinement of the Chimera grid has shown a negligible effect on the computed total thrust coefficient. The viscous component shows a greater difference. However, the difference was found to be less than 1%.
To further evaluate the two grids, Figure 2.29 compares the velocity profile at the mid-chord position located at the 75% span position in the normal direction on both suction and pressure surfaces of the two grids. Note that the distance from the blade surface is normalised by the tip chord. Comparing the two profiles again shows very good agreement between the two cases, where the profiles for both grids practically overlap. Note the small kink in the suction surface profile at approximately 0.6 units along the normal results from the transition from Chimera to background grid.

The Chimera grid refinement will have the greatest effect in the vicinity or on the rotor blade. To evaluate the Chimera grids without concerning the previous comments regarding the interpolation onto a coarser grid, the computed blade skin friction coefficient is compared. Figure 2.30 shows the computed skin friction coefficient for each grid at three radial blade locations.

Comparing the skin friction coefficient computed for each grid at all three radial locations shows that the medium grid profile closely follows that of the fine grid. Over most of the profile, the two profiles practically overlap. However, there are a
number of locations where the two cases differ. In particular, there are differences in the regions of peak skin friction. However, these differences are relatively small.

![Comparison of skin friction coefficient for medium and fine Chimera grids.](image)

Figure 2.30: Comparison of skin friction coefficient for medium and fine Chimera grids.

The Chimera refinement study has shown that there is generally good agreement in a number of variables comparing the two grids. Firstly, a negligible change in total loading for both grids was found. Secondly, comparing the axial velocity in the normal direction from the blade surface showed very good agreement between both grids. Finally, comparing skin friction coefficient around the blade surface at a number of spanwise locations showed very small differences in skin friction coefficient as the grid has been refined from medium to fine. The greatest difference was found in the locations of peak skin friction, however the differences were found to be relatively small. Therefore, due to the close agreement between the two grids, the proceeding
analysis will utilise the medium Chimera grid. The use of the medium grid, despite the small differences in compared variables is justified from the point of view of computational cost. In particular, the savings in computational cost (which will significantly increase for the CROR case, i.e. eight blades as opposed to one here), are considered more important than the small differences observed in the grid study. Furthermore, the medium grid itself has a comparatively high cell count, Table 2.3.

A grid study has been performed on both background and Chimera grid components. In both studies, the medium grid was chosen for the proceeding CROR simulations. This decision came as a result of loading and flowfield predictions (relative to the finest grid) and considerations of computational cost.

2.2.5 Model Validation

The HMB3 solver has been validated for a wide range of complex flow cases (e.g. see Refs. [109, 131, 132]). However, to verify the ability of the solver to predict the performance of CROR, it has been compared against available experimental data. In particular, it has been compared with the work of Dunham et al. [106]. The data presented by Dunham et al. has been described previously for the validation of the BEMT model. While the quality of the reported data is not known and no errors have been discussed by Dunham et al., it remains the only source of open data (to the author’s knowledge) that provides sufficient detail for CFD analysis. Therefore, unlike the low-order models, the Rig-140 or DLR data cannot be compared with as detailed section geometry is not provided.

The CROR flowfield computed by the HMB3 solver is compared with the three-component laser velocimetry measurements presented by Dunham et al. at an advance ratio of $f = 1.21$. Figure 2.10 shows the axial planes corresponding to the locations where three velocity components have been recorded. For the CFD simulation, numerical probes have been placed in the grid cell-centres nearest to those of the experiment. Whilst the data presented by Dunham et al. were time-averaged over 1-minute, the CFD data have been time-averaged over a single rotor revolution. Furthermore, for the comparison of the velocity components, the CROR CFD simulation has been trimmed to within $\sim 3.5\%$ of the reported thrust coefficient. As Dunham et al. only provided a range of tunnel speeds, i.e. no precise value of inflow velocity was given, a value was assumed within this range. This was required in order to set the rotational speed, computed from the reported advance ratio. As a result, the operating conditions chosen represent an uncertainty in the validation. Figures 2.31, 2.32 and 2.33 respectively compare the axial, tangential and radial velocity components obtained experimentally with those predicted by the HMB3 solver.
Figure 2.31: CFD and experimental normalised axial velocity.

Figure 2.31 shows that the solver has generally overpredicted the axial velocity component, although the component downstream the aft rotor is relatively well resolved. It is expected that matching the reported thrust coefficient more closely may reduce this over-prediction. Additionally, the high rotational rate will result in a deformation of the rotor. However, the CFD model used the *cold* shapes and therefore will not be an accurate representations of the blade at the operating point. The disagreement at the upstream location may result from the uncertainty surrounding the operating conditions, which will further affect the downstream conditions. Additionally, the present analysis used a simplified spinner shape compared to that by Dunham *et al.* and this will also affect the inflow to the rotors and will contribute to the disagreement with the reported data.

The trends observed show a resemblance to those predicted by the BEMT model, i.e. an over-prediction of the downstream fore rotor and far downstream locations and a good match downstream the aft rotor. Similar trends closely matching those predicted by the HMB solver have also been found for the present case by researchers at the University of Rome using a Boundary Element Method [133]. The close match of the different methods brings into question the quality of the reported data. Nonetheless, the maximum discrepancy was found at the far downstream location and was $\sim 10\%$.

Considering the tangential component, Figure 2.32, the HMB3 solver compares well with the experimental data. In particular, the solver has identified the location of peak deficit for all locations. The greatest discrepancy is shown far downstream,
where the solver has predicted an almost cancellation of the tangential component, which is not demonstrated in the reported data. This disagreement may be attributed to the current matching to the reported loading and the difference in hot and cold rotor shapes.

Finally, the radial component, Figure 2.33, is considered. Note that no upstream radial component was presented by Dunham et al.. The HMB3 solver has again been able to identify the location of peak deficit. However, there is a difference in the compared magnitude of this deficit. Furthermore, the experimental data shows a reduced radial component inboard of the tip vortex in comparison to the predicted values. Far downstream the computed results show a deficit in the near-hub region, likely a result from the hub gap. This deficit is not a physical solution and is known a priori due to the requirements of the Chimera implementation. This area is, therefore, neglected.

A validation study has been undertaken to verify the prediction capability of the HMB3 CFD solver. Upon comparison, the solver is generally capable of predicting the velocity components and their development through the rotors and downstream. However, a number of discrepancies were observed. These discrepancies may be attributed to the unmatched loading and the fact that the cold rotor shape was modelled and uncertainties surrounding the inflow. Furthermore, the discrepancies observed may be further attributed to the unknown accuracy of the reported data. The close agreement between the HMB and a number of additional methods further support this. Nevertheless, the HMB solver has been able to identify key features of the
CROR flowfield not possible with the low-order analysis.

**Figure 2.33:** CFD and experimental normalised radial velocity.

**Turbulence Performance**

CROR flow is naturally highly turbulent, with high levels of vorticity in the rotor wake. The HMB3 solver offers a range of turbulence closures when solving the flowfield §2.2.3. Therefore, in order to assess the effect of turbulent model choice on the CROR flowfield predictions, a turbulence study has been performed.

For the turbulence study, the $k-\omega$, $k-\omega$-SST and SST-SAS models have been used in the prediction of the CROR flowfield. In particular, the turbulence models are compared for the validation test case. The experimental data presented no information on the turbulent properties of the setup. In particular, it is unclear whether a trip device was used to trigger a fully turbulent boundary layer across the blade. Therefore, given the complexity of predicting transition, and the lack of information on the transition behaviour of this complex case, no transition turbulence models have been used in this work. However, this may be an important feature for future analysis where more details of transition behaviour is available.

To compare the turbulence models, the flowfield upstream, downstream the fore rotor, downstream the aft rotor and far downstream have been studied. Figures 2.34, 2.35 and 2.36 respectively show the radial variation in the three components of velocity for each turbulence model.

Comparing the axial component, Figure 2.34 shows that, with the exception of
far downstream, all three turbulence models predict very similar velocity profiles at all axial locations. For the far downstream location, the SST and $k$-ω models closely match in predicted profile. However, the SAS model predicts a higher peak velocity and a greater wake contraction. However, this increases the discrepancy with the experimental data.

The tangential velocity comparisons, Figure 2.35, shows very close agreement between all models. All three models closely match in magnitude and location of the peak deficit.

Comparison of the radial component, Figure 2.36, shows the most significant difference between the three models. All models agree in the upstream direction, predicting no influence of the rotors on the upstream direction. In the downstream fore and downstream aft locations, the models agree very well in the region of the peak deficit. However, the magnitude of the peak deficit differs between the RANS models and the SAS. While the $k$ – $\omega$ and SST models predict similar deficits, the SAS model shows a reduced deficit. Further investigation showed an increased eddy viscosity ratio and modelled turbulent kinetic energy for the RANS models. In the far downstream region, the SAS model varies more significantly from the two RANS based models.

A turbulence study has been carried out to investigate the performance of three turbulence models implemented within the HMB solver. The results indicate that the RANS approaches followed the experimental trends more closely. Nonetheless, the differences predicted by the SAS approach were only small. The small differences suggest that the SAS model mostly behaved like the RANS models and only had small areas where the additional source term $Q_{SAS}$ was activated. The acoustic impact of the three turbulence models will be further assessed in a later chapter in order to inform the most suitable choice for the CROR analysis.
Figure 2.34: Normalised axial velocity for each turbulence model.

Figure 2.35: Normalised tangential velocity for each turbulence model.
2.2. HIGH-ORDER AERODYNAMIC MODELLING

2.2.6 HMB execution

With the large grid sizes required to compute the CROR flowfield, High-Performance Computing (HPC) resources were required to execute the HMB3 solver. Throughout the project, a number of HPC resources have been used. In particular, the Manchester HPC pool [134], the Tier 2 Cirrus HPC [135] and the former N8 Polaris HPC facility [136] were all utilised for running CROR simulations. Simulations were typically carried out on 500-540 cores. Due to the grid configuration (cell count per block), further cores would achieve no reduction in computing time.

To avoid high gradients in the initial stages of the unsteady simulations, the rotational speed is gradually increased from zero to the required speed over a single rotation. Furthermore, a coarser temporal resolution is used in the first two revolutions before being reduced to the required value. At least 4 rotor rotations were required to obtain statistical time-invariance of the flow variables, after which flowfield data were recorded over a further complete rotor revolution.

This sub-section has introduced a high-order approach for computing CROR aerodynamic performance. The governing equations of fluid flow have been introduced in terms of the HMB3 CFD solver. Furthermore, pre-processing requirements for the particular case of CROR has also been introduced. The solver has been shown to agree relatively well with reported data. Both mesh independence and turbulence modelling have been studied to verify the solvers performance further. The

Figure 2.36: Normalised radial velocity for each turbulence model.
high-order approach offers greater detail of the flow-physics in comparison to the low-order models, thus allowing for a greater understanding of the underlying noise generating mechanisms.

2.3 Summary

This section has introduced both low- and high-order modelling techniques for the analysis of CROR aerodynamic performance. Both methodologies were compared against available experimental data, showing sufficient accuracy for the work carried out in the thesis. Furthermore, a sensitivity analysis was carried out for each approach to verify their respective predictions further and understand the pertinent preprocessing procedures and input data.

The low-order models, while not suitable for a detailed understanding of the flow-physics, are extremely useful as a design tool due to their low computational cost. On the other hand, the high-order models are not yet suitable as design tools due to their onerous computational cost. However, they remain important to better understand the flow features and improve existing models.
Chapter 3

Aeroacoustic Modelling

The most significant component of aircraft noise is that due to the propulsion system, and this is especially true for propeller-driven aircraft. In an effort to investigate the aeroacoustic emissions of propellers, this chapter presents both low- and high-order strategies for computing propeller and CROR noise. A description of the various noise sources which contribute to rotor noise is first introduced. Following this, both low- and high-order modelling of CROR noise is discussed. Both methodologies are accompanied by a validation and sensitivity study.

3.1 Rotor Noise Sources

The theory of propeller noise was first investigated by Lynam and Webb [137]. However, the advancements of Gutin [138] are perhaps of the greatest significance. Gutin presented a theoretical development of the noise due to the rotating forces produced by the propeller and was shown to agree reasonably well with experimental predictions [139]. Significant advancements in the theory and understanding of the noise sources have since been made. It is the work of Lighthill [140] however, for which most modern aeroacoustic models are based [141]. The work of Lighthill has been developed further by Ffowcs Williams and Hawkings [142] to allow for easier numerical implementation of the noise sources, particularly for the case here of propellers. The Ffowcs Williams Hawkings equations form the basis of most modern rotor noise codes for both low- and high-order methods.

CROR noise poses a complex problem; it is highly tonal with significant noise in all directions over a wide frequency range. CROR noise comprises a number of sources. These sources are summarised in Figure 3.1.
CHAPTER 3. AEROACOUSTIC MODELLING

Figure 3.1: CROR noise sources.

**Tonal Noise**

Tonal noise is harmonic in nature, i.e. it is periodic. In the time domain, it can be represented by a constant rate pulse. For a rotor with $N$ blades and rotational frequency $n$, the tonal noise will appear as a peak at the fundamental Blade Passage Frequency, $BPF = nN$, in the frequency domain. The noise will then linearly decrease at integers of the BPF. Characteristic time and frequency responses of steady noise are shown in Figure 3.2.

![Figure 3.2: Rotor steady noise characteristics (adapted from [21]).](image)

Steady noise sources are typically grouped as thickness noise and loading noise. Thickness noise results from the volume displacement of the fluid as the rotor rotates. The resulting noise is proportional to the blade volume, with the frequency response characterised by the aerofoil shape [21] and can be represented as a monopole source.

Loading noise results from the pressure field caused by the aerodynamic loading as the rotor rotates. The loading source can be represented by a dipole. The loading contribution is proportional to the lift and drag (or thrust and torque) produced by the blade. For low-speed conditions, the loading terms typically dominate e.g. take-off. For increasing speeds, the thickness terms become more important, e.g. during cruise.
3.1. Rotor Noise Sources

Quadrupole Noise

For low to moderate speeds, the steady sources are linear. However, as the flow over the rotor becomes transonic, the noise becomes non-linear, and instead of acting on the blade surface, the volume around the blade needs to be considered. This non-linear noise can be characterised as a quadrupole source. Modelling of the quadrupole source requires properties of the transonic flow. However, for general aviation propellers, where the tip speed remains low, the quadrupole source is unimportant.

Broadband Noise

Broadband noise results from the random turbulent fluctuations throughout the flow. For a rotor, there are two sources of broadband noise [21]. Firstly, leading-edge broadband noise results from the ingestion of turbulent flow by the rotor. The magnitude of the resulting noise is proportional to the level of turbulence in the flow. This may result from natural turbulence of the flow or for example, from an upstream pylon. This source of broadband noise is also important in the case of CROR, where the aft rotor interacts with the turbulent flow created by the fore rotor [143].

Trailing-edge broadband noise is the second source of rotor broadband noise. The turbulent boundary layer present for most rotors will result in a fluctuation in loads that terminate at the blade trailing edge. The resulting noise is dependant on the properties of the rotor boundary layer. Figure 3.3 presents typical broadband noise responses in both time and frequency domains.

Unsteady Loading Noise

Non-axial flight, interaction with an upstream disturbance and rotor installation all give rise to unsteady loading noise. The unsteady loading on the rotor contributes to noise at the rotor harmonics. Furthermore, the circumferential directivity is no
longer axisymmetric and now appears as a number of lobes. The number of lobes is proportional to the order or frequency of the unsteady loading.

For CROR, the interaction between fore and aft rotors results in periodic loading on both rotors and, therefore, unsteady loading noise. Figure 3.4 shows the CROR interaction mechanisms that result in unsteady loading noise. Unsteady loading noise can be a significant source for the CROR, typically dominating for low-speed operations (e.g. take-off and climb-out) [144].

![Figure 3.4: CROR interactions resulting in unsteady loading.](image)

The unsteady aerodynamic loading on each rotor will be periodic with respect to the opposing rotor. As a result of the periodic loading, the CROR noise spectrum will consist of the rotor alone tones occurring at $BPF_1$ and $BPF_2$ and the so-called interaction tones: $k_1n_1N_1 \pm k_2n_2N_2$ ($\forall k_1,k_2,\in \mathbb{Z}$), i.e. the sum and difference tones. The CROR frequency spectrum, unlike the rapid decay seen for isolated rotors, is characterised by high amplitudes across the spectrum. Furthermore, during flyover scenarios, the CROR is shown to produce high noise levels near the axis of rotation, unlike the isolated rotor, which rapidly decays away from the rotor plane. The unsteady loading source is, therefore, an important source that cannot be neglected when considering CROR noise.

### 3.2 Noise Metrics

Whilst sound represents the physical pressure perturbations in the flow, noise describes the perception of this sound by the human ear [145] (although both are typically used interchangeably). Therefore, due to the subjective nature of noise, it can be difficult to quantify. As a result of this, a number of metrics have been developed to account for various properties of the sound. In this section, a number
of common metrics which have been used throughout this work are introduced.

The human ear is able to perceive a large dynamic range. It can generally detect sound pressures in the range of $20 \mu Pa \rightarrow 200 Pa$ [146]. Given the large range in acoustic pressure, it is typical to work on a logarithmic scale. As a result, the Sound Pressure Level (SPL) is defined:

$$SPL = 20 \log_{10}(p/p_{ref}),$$  \hspace{1cm} (3.1)

where $p_{ref}$ is taken as $20\mu Pa$ and represents the threshold of human hearing. The ear is capable of discerning $\pm 1$ dB, whilst $\pm 10$dB results in a noticeable difference in the sound level. As the scale is logarithmic, a doubling of the sound pressure results in a $\sim 6$ dB difference in sound level [147].

Noise is typically recorded and analysed as a signal (over a period of time), as opposed to a single event. Therefore, the sound pressure level of the signal is given by:

$$SPL = 20 \log_{10}(p_{rms}/p_{ref}),$$  \hspace{1cm} (3.2)

$p_{rms}$ is the root-mean-squared of the pressure signal.

The human ear can also perceive a large frequency range from around $20 \ Hz \rightarrow 20 \ kHz$. However, it is not equally sensitive to all frequencies [148]. As a result, it is typical to employ some weighting to a noise signal. The A-weighting is commonly used for environmental measurements. However, it is also used in the certification of general aviation Aircraft (light propeller-driven aircraft with a maximum take-off mass $< 8,618 \ kg$) [149]. The A-weighting is applied to account for the relative perception across frequencies of the ear (i.e. it is less sensitive for low-frequencies). A-weighting is graphically shown in Figure 3.5. The A-weighted SPL can be computed according to [150]:

$$L_A(f) = SPL(f) + 20 \log_{10}(G_A(f)) + 2 \ dB(A),$$  \hspace{1cm} (3.3)

where $G_A$ is the filter gain:

$$G_A(f) = \frac{12194^2 f^4}{(20.6^2 + f^2)(107.7^2 + f^2)^2(737.9^2 + f^2)^2(12194^2 + f^2)}$$  \hspace{1cm} (3.4)

For larger commercial aircraft, the EPNL metric is used for noise certification [151]. The EPNL is a weighted, integral metric. It is corrected for the tonal content of the signal and can account for the duration and loudness of the sound. The computation of the EPNL is a cumbersome exercise. Therefore, the reader is directed towards Refs. [145, 147] which detail the required process. The EPNL is a single number.
representing the noise over a single aircraft operation, it is calculated from flyover measurement and cannot be directly measured.

![Frequency Gain](image)

**Figure 3.5:** *A-weighting frequency gain.*

### 3.3 Low-Order Noise Modelling

There is a wide range of theories for describing rotor noise. To be used as a suitable design tool, the theory must provide sufficient physical detail at a low computational cost. To this end, this section introduces the low-order noise model which has been used throughout this work.

**Steady Noise**

To unify the various noise sources and unsteady loading terms, the analysis throughout this work utilises a frequency domain approach. Many authors have previously reported the expression for the steady noise of an isolated rotor [152, 153, 40, 21]. The acoustic pressure at observer location \( \mathbf{x} \) and time \( t \) due to the rotor is
given by:

\[
p'(\mathbf{x}, t) = \frac{-\rho c_0^2 N R}{4\pi r_z (1 - M_x \cos \theta)} \sum_{k_1=-\infty}^{\infty} \exp \left\{ j \left( \frac{k_1 N \Omega}{1 - M_x \cos \theta} \left( t - \frac{r_i}{c_0} \right) + k_1 N \left( \frac{\pi}{2} - \phi_l \right) \right) \right\} \times \int_{z_h}^{z_t} M_r^2 e^{-j(\phi_x + \phi_\nu)} J_{(Nk_1)} \left( \frac{k_1 N z_i \sin \theta}{1 - M_x \cos \theta} \right) \left\{ \begin{array}{l} T \\ D \\ L \end{array} \right\} \, dz \tag{3.5}\]

The observer distances \( \mathbf{x} = (r_x, r_y, r_z) \) and magnitude \( r_i \) and corresponding angles \( \theta_i \) and \( \phi_l \) are defined in Figure 3.6. \( z_h \) and \( z_t \) are respectively, the normalised hub and tip rotor radius; \( k_1 \) is the acoustic harmonic; \( \phi_x \) and \( \phi_\nu \) are phase terms due to blade lean and sweep; \( J_n(Z) \) is a Bessel function of the first kind, of order \( n \) and argument \( Z \); \( M_x, M_t \) and \( M_r \) are the free-stream, tip and relative Mach numbers respectively. The term \( (1 - M_x \cos \theta) \) is the Doppler frequency shift, with \( \theta \) the retarded observer angle; \( T, D, \) and \( L \) are the source terms due to thickness and drag and lift forces respectively. These are given by:

\[
\begin{bmatrix} T \\ D \\ L \end{bmatrix} = \begin{bmatrix} k_x t_c \Psi_T \\ jk_x c_d \Psi_D \\ jk_y c_l \Psi_L \end{bmatrix}, \tag{3.6}\]

where \( t_c, c_d, \) and \( c_l \) are the thickness-chord ratio, and lift and drag coefficients respectively. The chordwise wave numbers, \( k_x \) and \( k_y \), which represent non-compactness factors, are given by:

\[
k_x = \frac{k_1 N c M_t}{R M_r (1 - M_x \cos \theta)}
\]

\[
k_y = \frac{k_1 N c}{M_r} \left( M_r^2 \cos \theta - M_x \right) \tag{3.7}\]

Finally, the terms \( \Psi_T, \Psi_L, \) and \( \Psi_D \) are the Fourier transforms of the thickness, lift and drag chordwise distributions. These are given by [21]:

\[
\begin{bmatrix} \Psi_T \\ \Psi_D \\ \Psi_L \end{bmatrix} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \begin{bmatrix} f_T(x) \\ f_D(x) \\ f_L(x) \end{bmatrix} \exp(jk_x x) \, dx \tag{3.8}\]
where, \( f_T(x), f_D(x), \) and \( f_L(x) \), describe the thickness and loading distributions along the blade section chord. As the chordwise loading is not available from the BEMT approach, an analytical expression assuming a uniform loading distribution can be used [21]:

\[
\begin{align*}
  f_D(x) &= 1 & -\frac{1}{2} \leq x \leq \frac{1}{2} \\
  f_L(x) &= 1 \\
\end{align*}
\]  

(3.9)

which gives:

\[
\Psi_L = \Psi_D = \begin{cases} 
1 & (k_x = 0) \\
\frac{2}{k_x} \sin \left( \frac{k_x}{2} \right) & (k_x \neq 0)
\end{cases}
\]  

(3.10)

The assumption of uniform loading should be sufficient for the rotors considered throughout this work [18].

Studying the above expression gives some preliminary insight into the behaviour of rotor noise. The first exponential term in Equation (3.5) describes the properties of the radiating sound field. The term \( \frac{r_l}{c_0} \) indicates the acoustic waves will propagate radially outwards at sound speed, \( c_0 \) [21]. The term \( k_1 N \varphi_l \) illustrates that peak noise will be heard at frequencies of \( k_1 N \frac{\Omega}{2\pi} \), i.e. at multiples of BPF. It also shows that circumferential directivity, which will rotate with the propeller at its angular velocity, will have lobes proportional to \( k_1 N \). The terms to the right of the integral consist of the source terms, \( T, D, \) and \( L \), and the Bessel function. This Bessel function characterises the radiation efficiency of the acoustic pressure [29]. Where the Bessel function decays more rapidly for increasing order, \( \nu \). The source terms affect the magnitude of the acoustic pressure.

Further insight can be gained by carrying out a dimensional analysis of the above expression for the rotor noise dependence on the local blade velocity. Firstly the exponential terms are neglected Following this, the Bessel function term can also be neglected as for subsonic flows (as considered here) the argument will be less than the order. This leaves the following expression:

\[
p' \propto -\frac{\rho c_0^2 N R}{4\pi r_z (1 - M_x \cos \theta)} M_r^2 \left\{ \begin{array}{c} T \\ D \\ L \end{array} \right\}
\]  

(3.11)

Expanding the source terms for all components, including the wave numbers reveals that the thickness and drag components have a dependence of \( M_r^2 \) whilst the lift component has the dependence of \( M_r^3 \) (due to the additional Mach number term in \( k_y \)). For the thickness and loading terms these are the classical velocity relations for monopole and dipole sources respectively [154, 155].
3.3. LOW-ORDER NOISE MODELLING

Unsteady Noise

The unsteady loading component can form a significant contribution to CROR noise. Unsteady loading arises due to the aft rotor cutting through the wake and tip vortices of the fore rotor in addition to the potential interactions between both fore and aft rotors. The noise due to unsteady loading on a CROR has been derived by Hanson [156] and Parry [40]. The acoustic pressure due to unsteady loading on a CROR blade row is read as:

\[
p_{i}(x,t) = \frac{\alpha j \rho c_0^2 N_i R_i}{4 \pi R_z (1 - M_x \cos \theta)} \sum_{k_1 = -\infty}^{+\infty} \sum_{k_2 = -\infty}^{+\infty} \exp \left\{ j \left[ \mu \left( t - \frac{r_{1}}{c_0} \right) + \nu \left( q_{0} + \frac{\pi}{2} \right) \right] \right\} \times \int_{z_{hi}}^{z_{lt}} M_{r_i}^2 e^{i(\phi_i + \phi)} J_{\nu} \left[ \frac{\xi_{i} \sin \theta}{1 - M_x \cos \theta} \right] \left\{ k_{y}, c_{l_i}^{(k_i)} \Psi_{L_i}(k_j) \right\} dz_i, \] (3.12)

where,

\[
\mu = k_1 N_1 \Omega_1 + k_2 N_2 \Omega_2, \]
(3.13)

\[
\nu = k_1 N_1 - k_2 N_2, \]
(3.14)

\[
\xi = k_1 N_1 M_{t_1} + k_2 N_2 M_{t_2}. \] (3.15)

The indices \( i \) and \( j \) represent the currently considered and the opposite rotors respectively. For the fore rotor: \( i = 1, \ j = 2, \ \alpha = -1 \); for the aft rotor: \( i = 2, \ j = 1, \ \alpha = +1 \). Again, the observer location definitions are defined in Figure 3.6, note the reference to the aft rotor. Note: there are now two harmonics. For the fore rotor, \( k_1 \) would
be the acoustic harmonic, whilst $k_2$ would be the loading harmonic. With the opposite being true when considering the aft rotor. The non-dimensional wave numbers (again characterising non-compactness) are, in this case, given by:

$$k_x^i = \frac{c_i}{R_i M_{r_i}} \left[ \frac{\bar{z}}{1 - M_{x} \cos \theta} - k_j N_j (M_{t_1} + M_{t_2}) \right], \quad (3.16)$$

$$k_y^i = -\alpha c_i \left[ \frac{\bar{z} M_i z_i \cos \theta}{1 - M_{x} \cos \theta} + a \frac{v M_{x}}{z_i} \right]. \quad (3.17)$$

**Quadrupole Noise**

When the relative velocity of the rotor blade approaches transonic speeds, the linear loading terms become non-linear. This can be represented as a quadrupole source. To model this source requires details of transonic flow perturbations. Obtaining these details can be computationally expensive and as a result, would be out of the scope of a low-order analysis. Furthermore, in this work, the focus is on general aviation aircraft, where even during cruise, the rotors do not reach transonic speeds. As a result of this, the quadrupole term is not modelled in the present work. However, for a more in-depth review of the quadrupole source as well as modelling strategies, the reader is directed towards the works of Hanson & Fink [157] and Parry [158].

**Broadband Noise**

Broadband noise is typically characterised as being lower in amplitude than the tonal component. However, whilst the tonal component is significant at a few distinct harmonics, the broadband component is distributed across the whole frequency spectrum. It has also been shown that in some cases the broadband component can be as important as the tonal contribution for CROR [159].

Due to the nature of broadband noise, i.e. it is based on turbulent, and hence, random fluctuations, it can be difficult to compute. As discussed previously for CROR, there are two components of the broadband noise, the leading-edge component (due to rotor-rotor interaction) and the trailing-edge component. The equations describing the broadband component are typically exhaustive, containing many terms, in particular those describing the aerodynamic behaviour of the turbulent flow. As a result, the equations are fully described in Appendix A. Only the leading equations are shown here.

There are a number of models to compute the trailing-edge broadband component. The model described by Magliozzi et al. [21] is simple to use and is relatively accurate at predicting the broadband noise due to scattering of the turbulent boundary layer [147]. The model described by Magliozzi et al. is based on the work of
3.3. LOW-ORDER NOISE MODELLING

Amiet [160] on broadband noise on a flat plate. From Magliozzi et al. the far-field noise spectrum is read as:

$$S_{pp}(x,\omega) = \left(\frac{\omega b z}{2\pi c_0 \sigma^2}\right)^2 I_y(\omega) s |L|^2 S_{qq}(\omega, 0),$$ (3.18)

The second component of broadband noise from a CROR arises due to the turbulent interaction as the aft row interacts with the wake of the fore row. This is described as rotor-rotor broadband interaction noise. For the analysis, the model presented by Kingan [161] is used. This model is an updated model based on the original work of Blandeau [162].

From Kingan, the time-averaged rotor-rotor broadband pressure spectrum is given by:

$$\bar{S}_{pprr}(x,\omega) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} \frac{2\pi N_2}{(4\pi R_e)^2 (1 - M \cos \theta)^2}$$

$$\int_{R_h}^{R_l} \left(\frac{\pi V_r}{\ln(2)}\right) \left(\frac{\rho N_1 b_1/2}{r \cos \alpha_1}\right)^2 G(k_b)^2 S^2 J^2_{\phi} \left(\frac{\nu r \sin \theta}{1 - M \cos \theta}\right) \times$$

$$\Phi_{ww}(\tilde{K}_{X},\tilde{e}_{X})|\Psi_L(K_{X},0,\omega - n\Omega_2,k_c)|^2 dr.$$ (3.19)

Recall, the above equations are fully presented in Appendix A. The equations for the computation of the broadband source are given in a strip formulation. In particular, the equations are computed for each observer location for a given frequency and blade span discretisation. Therefore, it is convenient to code them in a matrix form to reduce the computational cost.

3.3.1 Model Validation

In order to verify the prediction of the CROR noise model, it should be compared against a number of experimental data sets. Despite the scarcity of data in the open literature, a small number of sources have been found with which to compare the CROR code. The code is first compared against the case of an isolated rotor before considering a CROR cases.

SR2 SRP

Block [163] investigated the noise from an isolated 4 bladed SRP at two operating points. The first case considers the rotor at a setting angle of 20.6° rotating at 120 rev/s. In the second case, the rotor is at a setting angle of 12.7°, rotating at 168 rev/s. In both cases, the freestream velocity is $\sim 30$ m/s. Figure 3.7 compares the reported
and computed Overall Sound Pressure Level (OASPL) for the two SRP operating cases.

![Graph](image1.png)

Figure 3.7: SRP noise prediction.

Figure 3.7 demonstrates that the acoustic model predicts well the OASPL of the SRP in both cases. There are some discrepancies towards the extremes of the observer angles. However, this was also found in numerical comparisons by Block [163] and is described as possibly being caused by nacelle diffraction as the polar angle approaches the rotor axis.

**SR2 CROR**

Directivity trends for a 0.409 m 4 × 4 scale CROR have been presented by Block [164]. With both rotors rotating at \( n \sim 192 \text{ rev/s} \), Block presented trends up to the third harmonic of BPF. The CROR noise was measured at a number of azimuthal and polar angles, Figure 3.8. However, the data was presented as polar directivities, having been averaged over the azimuthal component. To match the reported data, this averaging was done according to:

\[
\overline{SPL}(\theta, f) = 10\log \left( \frac{1}{m} \sum_{i=1}^{m} \frac{SPL(\theta_i, f)}{10} \right)
\]

where \( m \) is the number of azimuthal microphone positions, here 22.

Figure 3.9 compares the OASPL and SPL of the first three harmonics reported by Block [164] and those obtained using CROR noise model.
3.3. LOW-ORDER NOISE MODELLING

Figure 3.8: Microphone locations for CROR noise validation, adapted from Ref. [164].

Figure 3.9: Comparison of computed and reported SPL values for SR2 CROR.

Figure 3.9 shows that the OASPL has been very well captured by the CROR noise model over the entire directivity. The first harmonic of BPF is relatively well captured over the directivity. However, there is a slight discrepancy in the upstream direction. The second and third harmonics are not captured so well. There is good agreement
in the overhead position, owing to the steady component. However, moving up or downstream, there is a significant difference between the predicted and reported data. The reduced levels in the up- and downstream directions predicted by the noise model suggest the unsteady interaction has not been captured so well. This suggests a deficiency in the aerodynamic interaction model. As the polar directivities are averaged over the azimuthal component, the discrepancies are more difficult to attribute. Nonetheless, the OASPL has been predicted very well and given the fact that the total noise is the useful component for the present analysis, the model has predicted the CROR noise to a sufficient level of accuracy for the current work.

3.3.2 Sensitivity Analysis

In order to validate the CROR noise model, it has been compared against available experimental data. A sensitivity analysis has been undertaken to investigate potential sources of the discrepancy. Furthermore, the sensitivity analysis will demonstrate the effect of small changes in various parameters on various noise sources. The sensitivity analysis has been carried out on 3m diameter SRP and CROR rotating at 1,000 rev/min, representing the characteristics of the rotors used in the analysis throughout this work.

Rotor Thickness

The rotor blade thickness, or more precisely the blade volume, is the source of rotor thickness noise. The thickness of the blade has been varied by ±10% to investigate the sensitivity of this noise source. This should also indicate if small differences in thickness, e.g. from extracting the geometry will propagate to a large difference in thickness noise.

Figure 3.10 demonstrates the effect of rotor thickness on the resulting peak noise level of the thickness noise source. The relative velocity of the rotor throughout the sensitivity analysis is below transonic speeds and constant. The peak thickness noise is shown to vary by a maximum of ~ ±0.9dB over the blade thickness range tested. The characteristic parabolic arc representing a flyover was found to translate vertically for variations in blade thickness.

Extraction of the spanwise geometry may result in errors when representing the geometry. However, these are likely to be small. Figure 3.10 demonstrates that the resulting change in SPL is therefore small and accuracy of the thickness should not be a concern so long as the thickness of the section is accurate to a few per cent.
3.3. LOW-ORDER NOISE MODELLING

Figure 3.10: *Effect of blade thickness on thickness noise.*

**Rotor Speed**

The rotational speed of the rotor is known to have a significant effect on the resulting noise emission of the rotor. To quantify this effect, Figure 3.11 presents the change in SPL of thickness and loading sources as well as the EPNL due to differences in rotational speed. The investigation is carried out with the rotor delivering the same thrust for each rotational speed.

Figure 3.11: *Effect of Rotational speed.*

Figure 3.11 shows that the rotational speed has a greater effect on the thickness
component than the loading component. For the loading component, the change in rotational speed will also affect the sectional loading (to keep the total blade thrust constant) and hence will also affect the resulting noise source. Due to the dominance of the loading component for the current case, the large change in the thickness contribution is not seen in the total noise changes. Nonetheless, the total noise is seen to change by more than 2 dB for a 10% change in rotational speed. Figure 3.11 also shows that using an integral metric, here, EPNL, can change by as much as 5 dB for a 10% change in rotational speed. Comparing the trends of thickness and loading noise, it is shown that they do not follow the relations: \( SPL_T \propto U^4 \) and \( SPL_L \propto U^6 \). However, these relations cannot be easily seen as the relative velocity varies along the blade span, and the effect of rotational velocity is compared here. For the loading component, it is further complicated as the source strength of each spanwise location also varies.

The analysis of the sensitivity of the rotor rotational speed has shown that the resulting noise is highly sensitive to the rotor rotational speed. The effect of rotational speed will be more important when the thickness component will dominate, e.g. during cruise. Therefore, for accurate noise prediction, the rotational speed must be precisely replicated.

**Accuracy of Aerodynamic data**

The aerodynamic data used to trim the rotor is taken from 2D look-up tables or from linear approximations. To this end, it is important to understand the effects of the aerodynamic data on the resulting noise emissions. Given that the expression for the loading noise is of the same form as the thickness component, it is expected to have a similar response to changes in thickness. This response is shown in Figure 3.12, where the changes in peak SPL are shown for changes in the rotor lift coefficient for an isolated SRP. As the loading noise is dominant here, it is also possible to show the effect on the integral metric, EPNL.

The response shows that the noise is directly proportional to the source strength, here \( c_l \). The same response was found for the drag coefficient. The EPNL also shows a dependency on the rotor aerodynamics. For both metrics, the maximum difference is \( \sim 1\text{dB} \) for the maximum changes in lift coefficient. While this does not seem like a large change, a 1 dB change is equivalent to a \( \sim 15\% \) change in the acoustic pressure. Therefore, the accuracy of the aerodynamic predictions may have a significant impact on the resulting noise predictions.

CROR noise introduces an additional loading source — the noise due to unsteady loading. Therefore, the sensitivity of CROR noise to this source is investigated. The sensitivity analysis is applied by increasing/decreasing the unsteady component
equally across all harmonics. Figure 3.13 presents the effect of the harmonic loading component on the resulting noise of fore and aft rotors. The error bars represent a maximum change in the unsteady component of ±10%. The steady component is also presented for reference.

Figure 3.12: Effect of steady aerodynamic component on peak and EPNL noise levels.

For both fore and aft rotors, Figure 3.13 shows that the importance of including the unsteady component as it has a significant effect on the resulting flyover profile. Figure 3.13 also demonstrates that the overhead position is dominated by the steady
component of the noise, with both fore and aft unsteady components demonstrating little deviation from the steady component at this position.

It is shown that the greatest deviation in SPL is found near the axis of rotation. For the fore rotor, the greatest deviation is on the upstream side, whilst for the aft rotor, this is on the downstream side. The high SPL near the axis of rotation, which is demonstrated for the aft rotor, is characteristic of CROR noise. Figure 3.13 suggest a maximum deviation of around 1dB for a ±10% change in the unsteady loading. Even with the lower bound, the profile still retains its characteristic shape.

To investigate the unsteady loading further, Figure 3.14 compares the effect of unsteady loading changes for increasing harmonics of BPF at the overhead position ($\theta = 90^\circ$). Figure 3.14 shows that the steady component rapidly decays, for the first three harmonics, this is also true for the unsteady component. However, for both fore and aft rotors, the SPL then increases for increasing harmonics. As this is for the overhead position, the steady component is dominant, as demonstrated by equal values of the steady and unsteady cases at the first harmonic. For positions away from the rotor plane, the unsteady component is found to increase further for higher harmonics.

![Graph](image1.png)

![Graph](image2.png)

Figure 3.14: Harmonic components for steady and unsteady aerodynamics at overhead position, $\theta = 90^\circ$.

The maximum deviation for the unsteady component is around 1dB for the maximum ±10% change in the unsteady component. This difference does not change the overall trends observed but may become significant when integrating across the spectrum. Furthermore, the results have demonstrated that the accuracy of the unsteady
3.3. LOW-ORDER NOISE MODELLING

component has a significant impact on the resulting noise.

Overall, the sensitivity analysis of the unsteady loading component has demonstrated that and equal change in the harmonics resulted in a significant change in SPL along the flyover. Therefore, this may be a cause for an underprediction observed for the second and third harmonics in the validation cases.

Time Discretisation

The equations presented above require a discretisation of time for computation of the rotor noise. This discretisation results in a complete rotor revolution being divided into a number of time or azimuth steps. The resulting time step dictates how many points are used to describe the time signal or the sampling frequency of the noise and the maximum frequency that can be resolved. Therefore, the choice of time discretisation is problem dependant.

Ideally, the greatest time discretisation should be used to resolve the highest frequencies, or at least up to the upper range of audible frequencies. However, this is often not possible whilst minimising computational cost. As a result, a sensitivity analysis of the time discretisation has been undertaken. The case of a SRP at take-off conditions \( (M = 0.2, \Omega = 1,000 \text{ rev/min}) \) has been simulated as this is representative of the analysis throughout this thesis. A SRP has been used to reduce the number of resulting variables (i.e. the noise of a single rotor as opposed to two).

Figure 3.15 demonstrates the effect of increasing time discretisation on the resulting EPNL and maximum A-weighted SPL in a simulated constant-altitude flyover. These metrics have been used as they operate on amplitudes within frequency bands and are, therefore, much more greatly affected by the frequency resolution. Figure 3.15 shows that for an increasing number of time points, the resulting EPNL and \( LA_{\text{max}} \) rapidly converge to a final value. While the noise metrics rapidly converge, the resulting computational time exponentially increases. Therefore, a compromise on acceptable accuracy and computational time must be made. From the sensitivity analysis, a time discretisation of 1,000 points was chosen. The use of 1,000 points results in a difference of only 0.28 dB from the converged value obtained using 5,000 time points whilst reducing the computational time by 123%\(^*\). Whilst using 5,000 points does not represent a significant computational time itself, the objective of the low-order models is to minimise the computational cost. Furthermore, the use of 1,000 points results in \( \sim 8,000 \text{Hz} \) to be resolved, \( > 100 \) times the BPF, which is much more than is interesting.

\(^*\)Computations carried out on a single Intel® Core™ i7-6700 CPU
3.4 High-Order Noise Modelling

A high-order aeroacoustic analysis is performed based on detailed flow-field data from high-order aerodynamic tools. Therefore, the modelling should be able to capture a greater detail of the flow physics and the resulting noise emissions.

Two approaches are available to utilise CFD data for CROR noise predictions. Either a porous surface or a solid-surface formulation can be used. A comparison of the two approaches is described in Ref. [165]. A solid-surface formulation is used throughout this work.

3.4.1 Farassat 1A Code

The Farassat 1A formulation [153] of the original Ffowcs Williams-Hawking equation [142] was implemented to calculate rotor noise based on blade surface pressures computed form uRANS CFD simulations using the HMB solver. The Farassat formulation is given by a summation of thickness and loading contributions:

\[
p'(x,t) = p'_L(x,t) + p'_T(x,t)
\]  

(3.21)
where,

$$4\pi p_T'(x, t) = \rho c_0 \int_{f=0}^{f} \left[ \frac{M_n}{R (1 - M_r)^2} + \frac{M_n M_r}{R (1 - M_r)^3} \right] dS +$$

$$\rho c_0 \int_{f=0}^{f} \left[ \frac{c_0 M_n (M_r - M^2)}{R^2 (1 - M_r)^3} \right] dS \quad (3.22)$$

and

$$4\pi p_L'(x, t) = \int_{f=0}^{f} \left[ \frac{p_S \cos \Theta}{c_0 R (1 - M_r)^2} + \frac{M_r p_S \cos \Theta}{R (1 - M_r)^3} \right] dS +$$

$$\int_{f=0}^{f} \left[ \frac{p_S (\cos \Theta - M_n)}{R^2 (1 - M_r)^2} + \frac{p_S \cos \Theta (M_r - M^2)}{R^2 (1 - M_r)^3} \right] dS \quad (3.23)$$

Note that the quadrupole source has been neglected in the present work. The $\frac{1}{R}$ terms in both thickness and loading terms represent the far-field component, while the $\frac{1}{R^2}$ terms describe the near-field components.

$M$ is the Mach number, with subscripts $r$ and $n$ representing the Mach number in radiation and normal direction respectively, with $\hat{r}$ and $\hat{n}$ the unit radiation and normal vectors respectively. $\Theta$ is the angle between the normal and radiation vectors. $R$ is the magnitude of the radiation vector, $r$. Finally, $p_S$ is the pressure on the blade surface, $S$.

Blade surface pressure is extracted at each time step to compute the noise. Each blade is discretised into a number of panels from which the surface pressure is extracted and appended to an input file. From the local panel velocities and surface pressures, the acoustic pressure can be calculated for each radial and chordwise position on the blade. The source-dominant implementation of the Farassat formulation is used in the present work. In the source-dominant approach, the acoustic pressure is solved at a selected source time. The sound will then reach the observer at a time, $t = \tau + r/c_0$. This approach will result in unequally spaced observer time for each source. Therefore, an observer time is chosen and each source is interpolated onto this time. In the present work the upper and lower limits of the observer time were computed according to:

$$t_1 = 0.5(\min(R) + \max(R)) / c_0$$

$$t_2 = t_1 + \frac{2\pi}{\Omega}$$

(3.24)

The lower value was chosen to account for as many of the sources within the observer time array. However, it was necessary to extend the source time at the lower and upper extremes in order to ensure all sources could be interpolated onto the observer
time. Whilst the source-dominant formulation requires the need for interpolation and additional source time points, it avoids the need to solve backwards for the source time required in the observer dominant formulation. Following interpolation onto the observer time, sources can be integrated to give the total acoustic pressure of the rotor blade.

The Farassat implementation only calculates the tonal component and does not account for the broadband component. However, to consider the broadband component would require significant additional computational resources. Furthermore, the low-order analysis suggested the tonal component was more significant in the present case.

Figure 3.16 presents a typical blade discretisation used for the noise analysis. The blade discretisation is biased towards the blade tip where the greatest loading and hence noise source is located.

![Blade discretisation for aeroacoustic analysis.](image)

The implementation of the Chimera technique in the blade mesh generation does not allow for overlap between grids. Therefore, the resulting hub gap will produce a hub vortex which will impinge on the aft rotor and introduce a further noise source. This may be addressed by starting the blade dissection further along the span. This can be justified as the hub source is likely weak in comparison with the rest of the blade.

### 3.4.2 Model Validation

To verify the ability of the Farassat noise code to predict CROR noise, it has also been compared against the noise measurements of Block [164]. The HMB3 solver,
with the k-ω SST closure, was used to simulate the 4 × 4 SR2 CROR. A temporal resolution of 0.5° per step was used in the computation. Blade surface pressures were then extracted and the resulting acoustic pressure calculated using the noise code. The CFD simulation was carried out using the medium Chimera and background grids previously discussed.

Comparisons are made for azimuthally averaged SPL up to the third harmonic of BPF, with the averaging done according to Equation (3.20). The noise was computed at observer locations corresponding to the physical microphones shown in Figure 3.8.

Figure 3.17 compares the values obtained using the Farrassat noise code and those reported by Block [164].

![Graphs showing comparisons of OASPL, SPL for different harmonics.](image)

Figure 3.17: Validation of the high-order noise model for azimuthally averaged polar directivities.

The Farassat implementation is shown to agree very well with the reported data for the OASPL over the entire directivity range. The first harmonic also shows very good agreement. However, there is a notable discrepancy in the downstream region where the numerical model has over predicted the SPL. This suggests an overprediction of the aft rotor interactions (from the fore row) which will propagate in the downstream direction. The second harmonic is predicted well over most of the directivity with a slight disagreement near the rotor plane. Finally, the third harmonic is generally underpredicted across the entire directivity range. The predicted noise
levels result from the computed aerodynamic loads. Therefore, the discrepancies shown may arise to the resolution of the unsteady interaction between fore and aft rotors. However, as the polar directivities have been averaged over the azimuth, the discrepancies are harder to isolate, i.e. a single significant underprediction at a given azimuth angle may skew the averaged results.

Also overlaid on Figure 3.17 is the BEMT computations for the validation case. Comparing the high- and low-order approaches shows that the high-order analysis has been better able to capture the SPL over directivity range. In particular, the high-order method is able to predict the high levels near the axis of rotation for the higher harmonics. These results show that it is necessary to use a high-order approach, despite the significant additional computational cost, to capture some of the specific features of CROR noise.

Overall, the Farassat noise code has shown the ability to predict CROR noise to a suitable accuracy for this work. In particular, throughout this work, the interest is on community noise, which will typically consider the integrated noise level, which is shown to be predicted well here.

### 3.4.3 Effect of Turbulence Model

The CROR noise can be dominated by the unsteady interactions between the blade rows. Therefore, the choice of turbulence closure has been studied on the resulting flyover noise. The $k-\omega$-SST model has been used as a baseline and is shown to compare well to reported data. The validation case is further studied using the $k-\omega$ and SAS turbulence closures. The turbulence study aims to identify if the turbulence closure used has implications on the CROR tonal noise. Therefore, can the validation data be improved? Furthermore, as the three chosen models vary in computational cost, it is essential to weigh the cost of any improvements in noise prediction if they exist.

The turbulence study was performed for the validation case previously discussed for all three turbulence closures. Figure 3.18 compares the choice of turbulence closure on the flyover noise for the OASPL and first three harmonics of BPF. The comparison of the three turbulence models firstly suggests no great difference in the predicted peak values, with overlapping of all three models near the overhead position. As the polar angle moves away from the peak value, there becomes a significant difference between the turbulence models.

Throughout the polar angle range, there is no greatly discernable difference between the SST and SAS models. The close matching between SST and SAS models suggest only small areas where the additional source term $Q_{SAS}$ has been activated.
within the SAS model. The k-ω model shows the greatest difference. The most significant difference is observed near the axis of rotation. This area is dominated by the interaction between fore and aft rotors and suggests an increased prediction in the unsteady loading components using the k-ω model.

Comparing the trends of the k-ω with the reported data suggests an even greater discrepancy than the SST for the OASPL and first two harmonics of BPF. The k-ω model does, however, show improvements for the third harmonic. As a result of the improved prediction of the OASPL using the SST model and the reduced cost in comparison to the SAS model, the SST turbulence closure is used throughout this work in the analysis of CROR noise. In this work, the emphasis is placed on community noise and, therefore, if the interest should be shifted towards a specific noise component, the turbulence should again be studied to understand its effects on the resulting source of interest.

Figure 3.18: Effect of turbulence closure on CROR flyover noise for the noise validation case.
3.5 Summary

In this chapter, both low- and high-order methods for the computation of CROR noise have been presented. Both models were compared against experimental data for CROR noise and showed sufficient agreement to use them for future analysis. Whilst the low-order model is perhaps only suitable for comparing overall sound levels, its low computation time makes it ideal as a preliminary design tool. On the other hand, the high-order tool allows for a more in-depth analysis of the noise sources. However, this comes at the cost of a considerably higher computational cost.
Chapter 4

A Parametric Study of CROR Noise

The renewed interest in CROR for next-generation aircraft, in addition to the rapid expanse of eVTOL concepts utilising CROR configurations, demands design tools that can accommodate new configurations and large design spaces. In order to meet this demand, an objective of the present work is to develop a multi-disciplinary CROR design tool. To accommodate the large design space and the need to evaluate many design configurations, low-order models have been utilised. In this chapter, the low-order models that have been previously discussed are coupled to develop a preliminary CROR design and analysis tool.

One of the concerns surrounding the integration of CROR configurations is their noise emissions. Therefore, design choices for future configurations must focus on noise reductions and the parameters that affect this. Furthermore, efficiencies in a number of areas may have to be sacrificed to accommodate reductions in noise. To this end, this chapter will first present a parametric study of the two major design features controlling the noise emissions — the blade count and the rotational speed. Following this, the low-order models are coupled to an optimisation routine to develop the design tool. Whilst the parametric study has been carried out for a civil aviation sized aircraft, designs are studied for both general and civil aviation class aircraft.

4.1 Blade Count Investigation

The rotor blade count is a significant design choice. Whilst from a performance and structural perspective, it determines the loading required by each blade to deliver the total thrust, the choice of blade count also has a significant impact on the resulting noise emissions. Relative to the SRP the additional blade row of the CROR further complicates the design process as fore and aft blade count directly affect the
CHAPTER 4. A PARAMETRIC STUDY OF CROR NOISE

aerodynamic and acoustic interactions. Due to these further complications, the effect of blade count is studied on the resulting CROR noise emissions for a simulated flyover.

Changes in rotor blade count will affect both the thickness and loading sources as both rotor volume and blade loading change. Therefore, to isolate the effects of differing blade counts, the solidity, \( \sigma = N \bar{c} / 2\pi R \), is held constant. The solidity is maintained by scaling the blade chord and maintaining the spacing-chord ratio. This constraint ensures that the required loading per blade unit area is constant (i.e. \( \frac{Q}{NA} = const. \)) for all cases and the changes in noise due to blade count result from acoustic criteria only.

Table 4.1 shows the rotor design parameters used throughout the analysis. The SR2 geometry [107] was used for all cases. The analysis is carried for civil aviation class aircraft, which typically utilise large blade count designs [7]. As a result, the design values were chosen to be representative of this size of aircraft [52, 72].

Table 4.1: CROR design parameters for parametric noise study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \sigma )</th>
<th>( R_1 = R_2 ) [m]</th>
<th>( g/D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.37</td>
<td>2.13</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The analysis was carried out at take-off, climb-out and approach conditions. These are the most critical phases of flight for aircraft noise evaluation. Table 4.2 shows the open rotor operating characteristics at each of these flight conditions. The tip speed of both rotors are equal at \( M_t = 0.6 \), and for each case, the rotors were trimmed to deliver the same torque. For each case, there is, therefore, a difference in delivered thrust. However, designing for a given torque ratio allows for an appropriate design of the gearbox or power turbine to deliver the required torque ratio.

Table 4.2: Operating point parameters.

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Take-off</th>
<th>Climb-out</th>
<th>Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude [feet]</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Mach Number</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>( C_Q / \sigma )</td>
<td>1.02</td>
<td>1.08</td>
<td>0.467</td>
</tr>
</tbody>
</table>

Due to the highly directive nature of CROR noise and in line with civil aviation noise certification [151], the EPNL metric was used to characterise the CROR noise for the analysis. A flyover was simulated by placing an array of microphones 1,000 ft (304.8 m) below the CROR to compute the EPNL. The CROR was considered in axial flight throughout the analysis.
4.1. **BLADE COUNT INVESTIGATION**

For the investigation of blade count, a number of fore and aft blade count combinations are considered. In particular, fore blade counts from eight to fourteen, i.e. \( \{ N_1 \in \mathbb{Z} \mid 8 \leq N_1 \leq 14 \} \) and aft blade counts of \( N_2 = N_1 \pm 3 \) are studied. In the analysis, cases with equal blade counts are not considered as these are known to generate high noise levels due to the simultaneous blade passages [2]. Hence, for the blade count study, 42 blade count combinations for each operating point are studied.

Due to the large number of blade count configurations studied, only those producing the lowest and highest EPNL values are presented. Figure 4.1 shows the ten combinations producing the lowest EPNL for all three operating conditions, whilst Figure 4.2 shows those producing the ten highest EPNL values.

![Figure 4.1: Blade counts giving lowest EPNL for all operating points.](image)

Figure 4.1 shows that combinations with the highest blade counts produce the lowest computed noise. Conversely, Figure 4.2 shows that the greatest noise is produced by combinations with the lowest blade counts. This trend is in line with the trends observed by Parry & Vianello [2]. At take-off conditions, there is a difference of \( \sim 35 \) EPNLdB between the lowest and highest computed blade count combinations, illustrating the significant impact blade count has. Recall, the present analysis is from acoustic criteria only and other performance measures would be considered in the design process.

Comparing the effects of operating point in both Figures 4.1 and 4.2, shows that the trend are generally comparable across all operating points. This similarity results
from the importance of the loading source across all three operating points.

Figures 4.1 and 4.2 also show there is not a consistent increase or decrease in noise with blade count, i.e. at take-off while the $8 \times 5$ produces the highest EPNL, the $10 \times 7$ produces the second-highest, not the $8 \times 6$ as would be expected. This results from the complex interactions between the blade row, i.e. both potential and wake interactions. This illustrates the significance of accounting for the interaction component. Furthermore, it highlights the importance of studying the effect of blade count for CROR. For SRP, the results would be obvious, with CROR, the interaction adds an additional layer of complexity.

Whilst high blade counts result in low noise, the use of a large number of blades has to be weighed against practical issues such as installation as well as ensuring suitable aerodynamic performance.

![Graphs of EPNL vs Blade Count for Take-off, Climb, and Approach](image)

**Figure 4.2:** Blade counts giving highest EPNL for all operating points.

### 4.2 Tip Speed Investigation

Similarly to the blade count, the rotor tip speed is a critical rotor design choice. The tip speed has a significant effect on a number of CROR parameters, including the rotor loading, rotor stress, and importantly the resulting noise. Again, the use
of two rotors further complicates the design choice. As a result, fore and aft tip speed combinations are investigated. Tip speed combinations are first studied for the combination producing the greatest EPNL, namely the $8 \times 5$ configuration.

Tip Mach number combinations between 0.5-0.7 are investigated for each rotor, for take-off, climb-out and approach conditions (Table 4.2). The CROR parameters in Table 4.1 are again used throughout the analysis.

![Figure 4.3: Effect of tip speed on the EPNL of the $8 \times 5$ combinations at all operating points.](image)

Fig. 4.3 shows the computed EPNL over the range of tip speeds for each operating point for the $8 \times 5$ combination. The results demonstrate that the computed EPNL generally increases for increasing fore and aft tip speed. The largest EPNL was computed when $M_{t1} = M_{t2} = 0.7$ across all operating points. On the other hand, the lowest EPNL was computed for the tip speed combination, $M_{t1} = 0.6$, $M_{t2} = 0.5$, which is contrary to the expected case of $M_{t1} = M_{t2} = 0.5$. However, the difference between the two is not significant and results from small differences in the blade loading*. Whilst reductions in tip speed are shown to reduce the computed EPNL, this lower limit is constrained by the ability to trim at the given rotational speed.

Comparing the operating point, the trends are comparable across take-off, climb-out and approach conditions. In each configuration, the general trend of increased

*Small differences in loading are expected between cases as the aerodynamic code trims to the required loading within a given tolerance.
noise for increased rotational speed is observed. However, the increase in tip speed appears more sensitive for the aft rotor. This will result from the dominance of the aft rotor due to the interaction component for this case.

4.3 Combined Blade Count and Tip Speed Investigation

The previous two sections have demonstrated that both tip speed and blade count have a significant effect on the computed CROR noise and, therefore, are important design parameters. As a result, the combined effect of blade count and tip speed on the resulting EPNL is studied. For the analysis, tip speed combinations between Mach 0.5-0.7 are investigated for various blade count combinations. Due to the large parameter range, the analysis is carried out for take-off conditions (Table 4.2) only.

Figure 4.4 shows the effect of tip speed with increasing aft blade count. The results demonstrate the strong coupling between blade count, tip speed and the computed CROR noise. The lowest computed EPNL is found for the lowest tip speed combination and highest blade count combination.

\[
\begin{align*}
\text{(a)} & \quad N_1 = 8, \quad N_2 = 9 \\
\text{(b)} & \quad N_1 = 8, \quad N_2 = 10 \\
\text{(c)} & \quad N_1 = 8, \quad N_2 = 11
\end{align*}
\]

Figure 4.4: Effect of tip speed with increasing aft blade count.

Figure 4.5 shows the effect of tip speed with increasing fore blade count. Again, the trend of increased EPNL for increasing tip speed is observed. The lowest noise is
achieved with the lowest tip speeds and greatest blade counts.

![Graphs showing EPNL (dB) vs. tip speed for different blade counts.](image)

Figure 4.5: Effect of tip speed with increasing fore blade count.

Comparing the two cases presented (increasing aft blade count and increasing fore blade count), shows generally that the computed EPNL levels are similar for opposing pairs (e.g. $8 \times 9, 0.5 \times 0.6$ against $9 \times 8, 0.6 \times 0.5$). These comparable levels show the importance of both fore and aft rotors to the total noise emissions. For comparable blade pairs, the greatest noise is achieved when the rotor with the lowest blade count has the higher tip speed. For example, comparing $8 \times 9$ and $9 \times 8$ configurations at $M_{t1} = 0.5, M_{t2} = 0.6$, the $9 \times 8$ combination has the highest computed EPNL due to the combination of reduced blade count and increased tip speed. The results again show that when the tip speed reaches $M_t = 0.7$, there is a sharp rise in EPNL for all cases. Whilst the present study has highlighted the complexity of simultaneously considering the effect of both tip speed and blade count on the resulting CROR noise, the general approach of increasing blade count and reducing tip speed can be used to reduce noise.

The present study has so far considered only tip speed and blade count. The study highlighted the complex relationship between these two design choices and the difficulty of comparing designs. Therefore, introducing further design choices will only serve to increase the complexity. As a result, a more sophisticated approach
to considering a greater design space is required.

4.4 Open Rotor Optimisation

4.4.1 Optimisation Routine

Optimisation is the process of finding the set of input parameters (constrained or unconstrained), that give the best global performance for a given objective (or objectives). As a design tool, optimisation should allow for an efficient search of the design space to locate those parameters that give the greatest performance relative to the design goal(s) while simultaneously meeting the constraints of the design.

In the context of this work, optimisation has been used to identify CROR geometries and configurations that maximise the CROR performance based on a number of metrics.

Mathematically, optimisation is the minimisation of a given function. As a result, the earliest optimisation methodologies were gradient-based, e.g. secant method, method of steepest descent, etc. These methodologies are easy to code and computationally inexpensive. However, for large problems, or where the input function is not easily differentiable, as is the case here, these methods are unsuitable.

Increasing computational power allowed for the development of non-gradient based methods. For example, *Monte Carlo* methods. These methods get around the issues of large and complex problems. However, they randomly search the design space in search of the minima, and can, therefore, be computationally expensive [166].

Population-based optimisation routines are another breed of non-gradient based method. These again can handle large and complex design problems. However, they use a defined set of ‘rules’ to drive towards the minima. This approach results in an efficient search for the best performing design. For example, the Particle Swarm Optimisation (PSO) [167] mimics the social behaviour of flocking birds. The particles fly through the design space while their performance is recorded and updated such that all particles follow the paths of the high performing particles. However, if the PSO discovers a local minima, it is typically unable to move from the local to the global minima — a disadvantage of this method.

Evolutionary Algorithms form another branch of population-based methods. In particular, the Genetic Algorithm mimics the behaviour of natural selection [168, 169]. High performing individuals are bred together to successively improve individuals performance. Due to a number of the GA operations, the global minima may not be found, or it may take some time for it to be bred back into the population. However, the Genetic Algorithm is a robust method that has the ability to handle complex problems, considering a large design space efficiently. To this end, the Genetic Algorithm
was used in the development of the CROR design tool.

The operation of the Genetic Algorithm is presented in Figure 4.6. An initial population of Chromosomes, a data set containing the free parameters (known as Phenotypes), are first generated. The performance (or fitness) of each Chromosome is evaluated relative to the design goals. Upon evaluation, using processes borrowed from evolutionary theory (crossover, mutation, re-insertion), chromosomes are mated to produce a new generation of off-spring. The fittest individuals make it through successive generations, whilst the least-fit die-off (survival of the fittest). The process is repeated until stopping criteria is met, i.e. the global minimum is found.

![Figure 4.6: Operation of a Genetic Algorithm.](image)

**Constraint Handling**

Every design is subject to constraints. For example, in the present work, the design aims to minimise noise. However, this is subject to providing sufficient thrust
to the aircraft. Also of particular concern is the structural performance of the rotor blade due to design changes. In the present work, a simple beam bending model is employed to enforce structural constraints [170, 171, 172, 173]. In particular, the computed root stress is weighed against material yield stress.

In order to enforce design constraints, penalty functions were utilised. For example, if a set of input parameters resulted in a design that was particularly close to the yield stress of the material, the design was penalised. This penalty was in the form of setting a very low fitness value to ensure it did not progress to the next generation.

Penalty functions were also used to ensure the design stayed within the abilities of the models used. For example, designs were not considered when tip Mach numbers exceeded $M = 0.75$ as no quadrupole noise sources were considered in the noise analysis.

**Extension to Multiple Objectives**

Almost all engineering problems are required to satisfy multiple design goals, e.g. high aerodynamic performance whilst minimising noise. Furthermore, the competing objectives are not necessarily commensurable. Often improvements in one area lead to losses in another. Extending the optimisation to handle multiple objectives allows for finding a set of solutions that satisfy all design goals without degrading performance in a single objective.

To handle multiple objectives, the weighted sum approach [174] was used throughout this work. In this approach, objectives are defined within a single objective function and their corresponding performance metrics weighted together to give a single performance metric.

The weighted sum approach is easy to implement and computationally efficient [175]. The use of the weighted sum approach requires careful consideration of the problem to ensure variables are weighted appropriately. However, this may benefit a design where one objective may be valued more than another, so long as the other objective gives an adequate performance. Additionally, one has to ensure that all objectives are suitably normalised to ensure slight deviations in a single objective does not dominate the overall weighted fitness.

**Rotor Geometry Parametrisation**

Parametrisation is required to ensure that the spanwise geometry (e.g. chord, twist, etc.), is physically possible, realistic, and to some extent, manufacturable. Parametrisation ensures erratic variations in geometry are not generated and limits the number of points of inflexion (changes in directions) of the geometrical property along the span. From an optimisation context, parametrisation is also useful in that it
can significantly reduce the number of free parameters describing the geometry over the blade span.

The method of Kulfan [176] and Kulfan & Bussoletti [177] was used for parametrisation of the spanwise varying geometry components. The method proposed uses a class function/shape function transformation based on Bernstein polynomials to ensure smooth variations in parametrised geometry. The component shape function takes the following form [176]:

\[ S_i(\psi) = K_{i,n} \psi^i (1 - \psi)^{n-i} \]  

(4.1)

Where \( n \) is the order of the Bernstein polynomial, and \( i \), a counter of this. \( \psi \) represents the area in which the geometrical feature varies (e.g. here, blade span). \( K_{i,n} \) is the binomial coefficient, given by:

\[ K_{i,n} = \binom{n}{i} = \frac{n!}{i!(n-i)!} \]  

(4.2)

The individual terms of the Bernstein polynomial are analogous to Pascal’s triangle, as shown in Figure 4.7. For each \( n^{th} \) order of Bernstein polynomial, there is \( n+1 \) terms required to describe the polynomial. The overall shape function used to represent the geometry component, \( \zeta \), at each blade element \( j \), using an \( n^{th} \) order Bernstein polynomial is then:

\[ S_j = \zeta_{\text{max}} \sum_{i=0}^{n} \zeta_i S_i \]  

(4.3)

Where \( \zeta_i \in [0:1] \) represents the polynomial values from the optimiser, and \( \zeta_{\text{max}} \) is the maximum value that the geometrical feature can have. Using the chord distribution to illustrate, the blade element chord at each \( j^{th} \) blade element using a third-order Bernstein polynomial is then:

\[ c_j = c_{\text{max}} \sum_{i=0}^{n} \bar{c}_i S_i \]  

(4.4)

Which upon expansions is:

\[ c_j = c_{\text{max}} \left[ c_0 \left( \frac{r}{R} \right)^3 + 3c_1 \left( 1 - \frac{r}{R} \right) \left( \frac{r}{R} \right)^2 + 3c_2 \left( 1 - \frac{r}{R} \right)^2 \left( \frac{r}{R} \right) + c_3 \left( 1 - \frac{r}{R} \right) \right] \]  

(4.5)

With \( c_{\text{max}} \) set by the user and \( c_0 - c_3 \) given by the optimiser, the full spanwise variation of chord can be calculated. This approach reduces the number of free variables from \( m \) spanwise points to only four plus one for the maximum value.

The class/shape function approach allows for a smooth and well-behaved representation of the spanwise geometry to be quickly and easily generated using only
a small number of control points. However, using Bernstein polynomials, the control points are evenly spaced along the span. Therefore, this method is not suitable where more control would be required in certain areas, e.g. near the location of peak loading. This can be overcome using another parametrisation technique, e.g. B-splines. Nonetheless, the present parametrisation is suitable for the present purposes of demonstrating the capability of the optimisation routine.

Figure 4.7: Bernstein polynomial components represented analogously to Pascal’s triangle.

4.4.2 CROR design - Civil Aviation

A Genetic Algorithm optimisation routine [178, 179] was used to perform a preliminary design study for a civil aviation class aircraft. Three designs cases were considered, as summarised in Table 4.3. Case I builds on the previous parametric study, aiming to find the combination of blade counts and tip speeds that result in minimum noise. Case II also seeks to find the optimal combination of fore and aft blade count and tips speeds. However, the analysis is extended by considering the multiple objectives of minimum take-off noise and maximum cruise efficiency. Case III builds on case I by expanding the design space. The noise was characterised using the EPNL metric.

Table 4.3: Optimisation design objectives and free variables

<table>
<thead>
<tr>
<th>Case</th>
<th>Free-variables</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$N_1$, $N_2$, $M_{t_1}$, $M_{t_2}$</td>
<td>Take-off Noise</td>
</tr>
<tr>
<td>II</td>
<td>$N_1$, $N_2$, $M_{t_1}$, $M_{t_2}$</td>
<td>Take-off Noise, cruise efficiency</td>
</tr>
<tr>
<td>III</td>
<td>All</td>
<td>Take-off Noise</td>
</tr>
</tbody>
</table>

In the analysis of cases I and II, a CROR of 2.125 m fore and aft radius described by the SR2 geometry [107], Figure 2.8, was used. The upper and lower limits of blade
count and tip speed are presented in Table 4.4.

Case III considers a greater design space for the minimisation of take-off noise. In particular, the design space is opened to include tip radius, chord and twist distribution, and axial spacing. The upper and lower limits of the design variables are listed in Table 4.5. Third-order Bernstein polynomials describe chord and twist distributions. The values presented indicate their minimum and maximum values.

Table 4.4: Upper and lower limits of free variables for optimisation cases I and II.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1, N_2,$ [-]</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>$M_{t1}, M_{t2}, [-]$</td>
<td>0.3</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 4.5: Upper and lower limits of free variables for optimisation case III.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1, N_2,$ [-]</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>$M_{t1}, M_{t2}, [-]$</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>$R_1 = R_2$ [m]</td>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>$c_1/R_1, c_2/R_2$ [-]</td>
<td>0.25</td>
<td>1.25</td>
</tr>
<tr>
<td>$\beta_1, \beta_2$ [°]</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>$g/R_1$ [-]</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

The designs are evaluated at take-off (cases I and III), and take-off and cruise conditions (case II). Table 4.6 presents the operating parameters describing the conditions for each.

Table 4.6: Take-off and Cruise operating parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Take-off</th>
<th>Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude [ft]</td>
<td>1,000</td>
<td>36,000</td>
</tr>
<tr>
<td>$M_{\infty}$ [-]</td>
<td>0.2</td>
<td>0.7</td>
</tr>
<tr>
<td>$T_{req}$ [kN]</td>
<td>80</td>
<td>40</td>
</tr>
</tbody>
</table>

If the population size of the Genetic Algorithm is too small, the design space may not be fully explored. To this end, the effect of population size on the minimum objective value was studied for case I. This study should identify the minimum population size to obtain an independent solution. A range of population sizes from 10 to 500 individuals was studied. Figure 4.8 compares the minimum objective value obtained across the range of population sizes. Figure 4.8 shows that minimum objective value tends towards a final value as the population size increases. As the
population size approaches 100 individuals, the minimum objective value shows a
more settled value. The increase in population sizes between 100 and 500, there was
a 2% reduction in the minimum objective value. However, this came at the cost of
a significant increase in computational cost. As a result, subsequent optimisations
used population sizes of 100 individuals.

![Figure 4.8: Effect of population size on minimum object value.](image)

Table 4.7 presents the optimisation results for all cases of the civil aviation op-
timisation. Case I considered the reduction of take-off noise only. Table 4.7 shows
that the optimisation has identified a blade count and tip speed combination offer-
ing significant noise reductions. In fact, the computed noise is comparable to the
lowest levels found in the previous analysis (12 × 15 combination in the blade count
study). The optimal configuration shows reduced tip speed with a high blade count
combination. Increased blade counts may have offered a further reduction in noise.
However, this would breach the structural constraints placed on the optimisation.

<table>
<thead>
<tr>
<th>Case</th>
<th>$N_1$ [-]</th>
<th>$N_2$ [-]</th>
<th>$M_{t_1}$ [-]</th>
<th>$M_{t_2}$ [-]</th>
<th>$EPNL$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>12</td>
<td>13</td>
<td>0.61</td>
<td>0.54</td>
<td>53</td>
</tr>
<tr>
<td>II</td>
<td>12</td>
<td>11</td>
<td>0.59</td>
<td>0.63</td>
<td>55</td>
</tr>
<tr>
<td>III</td>
<td>12</td>
<td>13</td>
<td>0.58</td>
<td>0.64</td>
<td>52</td>
</tr>
</tbody>
</table>

Case II considered a simultaneous optimisation of both take-off speed and cruise
efficiency. Table 4.7 demonstrates that low noise can still be achieved. The computed
cruise efficiency was $\sim 0.82$. The reduced blade count comes as a result of the goal of
increased efficiency. However, this limits the available noise reduction. Nonetheless,
4.4. OPEN ROTOR OPTIMISATION

this case demonstrates the potential for CROR design for multiple design goals and constraints. No sweep was included in the optimisation. However, its inclusion would be expected to improve the efficiency in cruise through a reduction of the relative Mach number along the span.

Finally, case III builds on case I by expanding the design space, including variation in radius and spanwise geometry. Table 4.7 shows that the increased design space has allowed for a more significant reduction in noise. It is interesting to note, however, that the same blade count as case I has been retained. On the other hand, there is a large variation in tip speed, particularly for the aft rotor. Furthermore, the optimised design showed a reduced radius of $R_1 = R_2 = 1.72\ m$. Again, the optimisation was limited by the structural constraints imposed.

To explore case III further, Figure 4.9 compares the optimised twist and chord distributions with the SR2 baseline. Both fore and aft optimised rotors show a reducing chord for increasing radius, as opposed to the almost constant chord (before a sudden reduction near the tip) for the baseline SR2 case. The inner rotor section of the optimised aft rotor has a reduced chord relative to the fore. However, near the tip area, where loading is greatest, the aft rotor has an increased chord.

Comparing the twist distributions, both optimised and baseline distributions show similar trends. However, the optimised designs show a reduced twist over most of the span. As a result, an increased setting angle is required to maintain loading, with the chord and twist distributions giving a more optimal loading distribution.
To illustrate the designs obtained for each case, Figure 4.10 presents CAD representations for each optimisation case.

(a) Case I

(b) Case II

(c) Case III

Figure 4.10: CAD representations of the optimised civil aviation designs.

4.4.3 CROR design - General Aviation

In line with the current work, a design study has also been carried out for a general aviation class aircraft. Following a similar route to the civil aviation design, three cases are again considered, Table 4.3. However, the design variables have been adjusted for the general aviation case. Furthermore, the $L_{A_{\text{max}}}$ metric is now used to characterise the noise of each design. Table 4.8 details the upper and lower limits of
blade count and tip speed for cases I and II. Meanwhile, Table 4.9 details the lower and upper limits for design variable for case III.

Table 4.8: Upper and lower limits of free variables for optimisation cases I and II.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$, $N_2$, [-]</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>$M_{t1}$, $M_{t2}$, [-]</td>
<td>0.3</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 4.9: Upper and lower limits of free variables for optimisation case III.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$, $N_2$, [-]</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>$M_{t1}$, $M_{t2}$, [-]</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>$R_1 = R_2$ [m]</td>
<td>0.75</td>
<td>2</td>
</tr>
<tr>
<td>$c_1/R_1$, $c_2/R_2$ [-]</td>
<td>0.25</td>
<td>1.25</td>
</tr>
<tr>
<td>$\beta_1$, $\beta_2$ [°]</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>$g/R_r$ [-]</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

The operating parameters for take-off and cruise also differ for the general aviation case. Table 4.10 details the operating parameters for the general aviation analysis.

Table 4.10: Take-off and cruise operating parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Take-off</th>
<th>Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude [ft]</td>
<td>1,000</td>
<td>25,000</td>
</tr>
<tr>
<td>$M_\infty$</td>
<td>0.2</td>
<td>0.45</td>
</tr>
<tr>
<td>$T_{req}$ [kN]</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

In order to better quantify the optimised designs they are compared against a baseline design. A $4 \times 4$ CROR of 1.5 m radius with $M_{t1} = M_{t2} = 0.46$ of SR2 design is used here as the baseline design. The baseline design was computed to have a take-off noise of $L_{A_{max}} = 56.4$ dB(A) and cruise efficiency of $\eta_{cruise} = 0.84$ [-]. Table 4.11 presents the resulting designs for the three optimisation cases considered.

All three optimisation cases demonstrate reduced take-off noise levels relative to the baseline configuration. Furthermore, the cruise efficiency of case II shows improved levels relative to the baseline. Relative to the civil aviation case noise reductions are more substantial. The high thrust loading of the civil aviation case proved to be quite demanding and is thought to be the reason for the difference in noise reduction relative to the general aviation case.
For case I, a noise reduction of $\sim 10\text{dB}(A)$ has been achieved with high fore and aft blade counts and reduced tip speeds, as demonstrated by the parametric study. The fore rotor has an increased blade count relative to the aft rotor. The increased blade count serves to reduce blade loading and reduce interaction and hence noise source on the aft rotor. Further noise reduction may be achieved with increased blade counts and reduced tip speeds. However, this is constrained by the structural requirements imposed on the design. Whilst the additional weight and mechanical complexity of high blade counts were not considered here, they may play an important role in resulting designs.

For case II, the optimisation routine has identified a design that has simultaneously reduced noise at take-off and improved cruise efficiency. Efficiency gains of $\sim 1.6\%$ were observed compared to the baseline. This result was achieved with a reduced blade count. However, this reduced blade count has limited noise reduction in comparison with a design for noise only. Along the Pareto front, designs with reduced noise or increased efficiency were observed. However, the identified design had the lowest weighted objective value. This optimisation case highlights the competing objectives of CROR noise and efficiency.

A noise reduction at take-off is also achieved for case III with increased blade count and reduced tip speed. The increase in design space has allowed for an increased fore blade count and further reduced tip speed to be achieved before meeting the design constraints. Opening up the design space has offered a $\sim 13.6\text{ dB}$ noise reduction relative to the baseline configuration and offers the greatest reduction of all configurations.

Figure 4.11 compares the spanwise geometry of the baseline SR2 and optimised case III design. Immediately obvious is the large chord on the optimised aft rotor, resulting from the reduced blade count. Otherwise, the general shape is comparable to the SR2 baseline. Similarly, the twist distribution for the optimised case is fairly similar to the SR2 for both fore and aft rotor, with the exception of an increased twist near the root.

To illustrate the results from the optimisation design study, Figure 4.12 presents a CAD representation of the computed designs for each optimisation case.
4.5. SUMMARY

Low-order design tools have been coupled to an optimisation routine to investigate low-noise CROR designs for both civil and general aviation class aircraft. The design tool successfully identified low-noise designs. A structural model was used to apply design constraints and was found to limit the available noise reductions. A multiobjective design for low noise and high efficiency identified the competing nature of both objectives. However, designs must be further evaluated over the range of aircraft flight envelope to ensure it can deliver the required thrust and satisfactory performance for the range of operating conditions. These additional constraints can be added into the optimisation routine or tested separately. Overall, optimisation is a useful design tool able to account for various design constraints and objectives. Furthermore, optimisation has met the objective of reducing CROR noise.

This chapter has presented a parametric study of CROR noise. The parametric study has identified that low-noise can be achieved through high blade counts and low tip speeds. However, the study also identified a more complex relation when simultaneously considering both tip speed and blade count. This complexity is compounded for CROR due to the additional blade row. As a result of this complex relation, an optimisation routine has been coupled to the low-order models to develop a design tool to perform a design study of low-noise CROR. Carrying out
design studies of both civil and general aviation class aircraft resulted in reduced noise designs. These designs featured large blade counts and reduced tip speeds. However, further noise reductions were constrained by the structural requirements. A multi-objective optimisation highlighted that low-noise and high cruise efficiency as competing objectives.

Figure 4.12: CAD representations of optimised general aviation CROR.
Chapter 5

CROR Noise Reduction — Low-Order Analysis*

Whilst CROR can offer significant efficiency savings over conventional propulsion systems, there are concerns surrounding their noise emissions. These concerns may be addressed by studying novel concepts aimed at reducing CROR noise. However, to retain viability, these noise reductions must not significantly impact the cruise efficiency of the CROR. In this chapter, the low-order design tools developed within this work are used to investigate a number of novel CROR configurations aimed at reducing noise. Whilst low-order models may not be suitable for detailed and high accuracy design, they provide a suitable platform for investigating the effects of different designs and configurations. Furthermore, the use of low-order models allows for a large design space to be considered over a wide operating range. This chapter investigates a large number of blade count configurations to demonstrate preliminary viability of a number of concepts.

5.1 Description of Proposed Configurations

The novel configurations that have been proposed focus on a variable configuration CROR where a single blade row is locked, with the operative rotor providing the required thrust for the given aircraft manoeuvre. It is proposed to operate with a locked blade row during take-off, climb-out and approach conditions to reduce CROR community noise. Both rotors are operated at their design point during cruise to retain the high CROR efficiency.

As detailed in Figure 5.1, the analysis considers the following configurations:

• Locked fore rotor;
• Locked aft rotor;
• Folded aft rotor;
• Locked aft rotor with increased fore diameter.

Each configuration is compared to the baseline CROR, Figure 5.1a.

![Figure 5.1: Multi-configuration CROR concepts.](image)

The unsteady interaction noise from a CROR, due to the unsteady aerodynamic interaction, can be dominant at low-speed conditions [21]. Consequently, and as found from the literature survey, efforts should focus on the interaction component to reduce CROR noise. Locking a blade row (Figures 5.1b and 5.1c) is first considered to reduce the interaction component.

When locking a blade row, the operative rotor is required to provide the total thrust. As a result, it can be expected that the self-noise of this configuration will
increase. When locking a blade row, there will still exist an unsteady interaction component of the noise. However, this will be weaker than the fully operative CROR configuration. Therefore, to prove the viability of the locked row configurations, the reduction in the interaction component must reduce sufficiently to negate the increase in self-noise.

In an attempt to further reduce interaction noise, the locked-aft rotor is folded backwards, Figure 5.1d. Whilst there will again be an increase in the self-noise, the interaction component should now be eliminated. During cruise, the aft rotor is unfolded and operated at design conditions. The folding is controlled by a joint at the blade root. The folding rotor is only considered from an acoustic point of view, and detailed mechanical design is not undertaken. This is justified as there are a number of works considering a folding rotor for distributed propulsion concepts, e.g. see Refs. [180, 181]. The folded rotor concept will have a behaviour similar to that of the SRP. As a result, it is modelled as such throughout the analysis.

The noise gains of the locked rotor concepts are limited by the increase in self-noise as the operative rotor takes on the full thrust requirements. The effect of increasing fore radius with a locked aft rotor is studied to investigate the possible noise reduction of unloading the operative rotor. For increasing fore radius, tip speed is maintained by reducing the rotational speed.

In order to investigate the noise-reducing capabilities of the outlined concepts, a thorough analysis considering a large design and operating space must be considered. The use of low-order models affords such an exhaustive analysis which would be otherwise unaffordable with other numerical tools. The analysis is considered for the terminal areas, namely take-off, climb-out and approach conditions. Table 5.1 presents the details of these flight conditions. The values shown were computed for the Beechcraft 350i aircraft using the FLIGHT aircraft performance software [147, 182] to represent a general aviation class aircraft for which the analysis was performed.

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Take-off</th>
<th>Climb</th>
<th>Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude [ft]</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>$M_x$ [-]</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$C_T$ [-]</td>
<td>0.30</td>
<td>0.19</td>
<td>0.092</td>
</tr>
</tbody>
</table>

The aerodynamic performance and the resulting noise was computed for a range of blade count configurations from 4 to 6 on the fore rotor and 3 to 7 on the aft rotor. These blade count combinations were chosen as representative of modern propeller-driven general aviation aircraft. To isolate the effects of increasing blade count and to ensure a physical geometry as the blade count increases, the blade chord is scaled to
ensure constant solidity. Therefore, the differences in computed noise level are from acoustic criteria only.

The viability of each concept is determined based on comparisons of computed noise against the baseline configuration. In line with general aviation certification, the maximum A-weighted SPL \( L_{A_{\text{max}}} \) noise metric is used throughout the analysis [149]. Certification procedures require a single microphone measurement at the second segment of take-off. The aircraft altitude at which the measurement is taken is dependent on the aircraft performance. Therefore, to generalise and to allow for comparisons with climb-out and approach segments, a constant 1,000 ft (304.8m) flyover is simulated. Furthermore, a constant flyover allows for the CROR to be considered in steady level flight.

During the simulated flyover, only CROR sources are considered, i.e. the CROR is in isolation with no other aircraft sources accounted for. Considering the CROR in isolation is required so that the gains or losses of the current concepts can be compared. Furthermore, it is not envisaged that aircraft sources are likely to differ between installations of the proposed CROR concepts.

Following the operating requirements outlined in Table 5.1, a generic tractor CROR has been designed for the analysis. The CROR utilises the SR2 geometry [107], Figure 2.8. Further rotor design parameters are presented in Table 5.2. Clipping of the aft rotor is typically employed in CROR to minimise tip vortex interaction noise [32, 37]. However, in order to compare both fore-locked and aft-locked rotors, no clipping has been employed. For example, a clipped aft rotor may have an increased loading and therefore, would be disadvantaged when comparing to the fore rotor with reduced loading. Future designs using a locked-rotor configuration may be optimised to reduce tip vortex interaction noise through clipping.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \sigma )</th>
<th>( R_1 = R_2 ) [m]</th>
<th>( r_h / R )</th>
<th>( g / R )</th>
<th>( M_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.36</td>
<td>1.5</td>
<td>0.24</td>
<td>0.4</td>
<td>0.46</td>
</tr>
</tbody>
</table>

### 5.2 Locked-Fore Rotor

The locked-fore configuration is first considered. In the locked-fore configuration, the fore rotor is locked and trimmed to minimise torque, while the aft rotor is trimmed to deliver the thrust required by the aircraft plus that to overcome the computed drag of the locked row. The primary noise source from this configuration will be the self-noise of the baseline rotor and the interaction noise of the aft rotor operating in the wake and potential field of the locked-fore rotor.
5.2. LOCKED-FORE ROTOR

Figure 5.2 compares the computed $L_{A_{\text{max}}}$ for each blade count combination at each operating point calculated for the operative baseline and locked-fore configurations. Despite differences in absolute level, Figure 5.2 shows that the computed trends for each blade count combination are consistent across all three operating conditions. These comparable trends result from the dominance of the loading source at these particular conditions. It is understood during approach conditions that other aircraft noise sources may dominate the emitted sound field. However, this is not considered here as the interest is purely in comparing the different CROR configurations.

![Graphs showing noise comparison](image)

**Figure 5.2:** Comparison of noise between baseline CROR and a CROR with locked-fore blade row. CROR is the baseline configuration and inop the locked-rotor configuration.

To proceed with a discussion of the results, it is worthwhile to briefly describe the noise sources and what is being compared. CROR noise is known to be highly directive; typical flyovers are characterised by a peak near the rotor plane due to the self-noise and high values near the axis of rotation due to the unsteady interactions. In the analysis, comparisons are made using the $L_{A_{\text{max}}}$ metric. Therefore, comparisons are likely to be between those values near the rotor plane, i.e. the self-noise.
From Equation (3.12), in particular the integral term, the acoustic pressure is proportional to the product of some amplitude (the part containing the thickness or loading terms, although the loading will dominate here), and a radiation efficiency (the part containing the Bessel function). For both baseline and inoperative configurations, the steady loading source (giving rise to self-noise) will radiate with the same efficiency (i.e. $k_1 = 0$ for the aft rotor, $k_2 = 0$ for the fore rotor). Furthermore, as the solidity is held constant for increasing blade counts, the loading per unit blade area will also remain constant. As a result, for a given configuration, differences in noise will be from acoustic criteria due to changes in blade count. Differences in noise level observed between the two configurations will also arise due to the differences in loading between the two configurations, i.e. delivered by a single or two blade rows. Differences will also arise due to the specific blade count combination of the baseline configuration due to acoustic interactions.

The reasoning for the observed differences is evidenced with reference to Figure 5.2. For example, computed noise levels are comparable for equal aft blade counts (the operative row). When considering the baseline configuration, computed noise levels are observed to be similar between opposite blade pairs, for example, the $5 \times 4$ and $4 \times 5$ combinations. This similarity is only valid here as the rotors are equally loaded and operating at the same rotational speed.

Considering the baseline operative configuration, Figure 5.2 demonstrates a noise reduction for increases in both fore and aft blade counts. The exception to this occurs for equal blade count combinations, where the acoustic interaction (the constructive addition of fore and aft signals), results in increased sound level. This agrees with the results of §4.1. For the locked-fore configuration, it is shown that the computed noise decreases for increasing aft blade count (i.e. the operative row). Throughout the analysis, it was observed that the locked rotor produced a negligible contribution to the total noise. The result of equal blade counts is an increased noise of the baseline configuration over the locked-aft configuration. However, this may result in unsuitably high levels during cruise and must, therefore, be carefully assessed over the whole flight envelope.

With the exception of the $6 \times 7$ blade count combination, Figure 5.2 shows that the locked-fore combination is generally computed to produce less noise than the baseline configuration when $N_2 > N_1$. This reduction is seen to increase as the difference in blade count increases. Investigating the noise sources further, it was found that both fore and aft rotors of the baseline configuration produce less noise than the locked-fore configuration. However, the addition of their pressure signals results in a greater noise level.

With a reduction of around 3 dB(A), the lock-fore configuration with a $4 \times 6$ blade
count showed the greatest reductions against the baseline configuration. However, the 6 × 7 baseline configuration showed the lowest computed noise levels across all operating points. Nonetheless, the fore-locked configuration has demonstrated noise reductions relative to the baseline CROR whilst maintaining the required aircraft thrust. The current case used an unoptimised CROR that has been scaled to ensure equal loading across the blade count investigation. Therefore, an optimised design sized appropriately may offer more significant reductions in noise compared to those demonstrated here.

5.3 Locked-Aft Rotor

The locked-aft configuration comprises an operative fore rotor trimmed to deliver the required thrust and a locked aft rotor trimmed to minimise torque. In this configuration, the noise will be dominated by the self-noise of the operative fore rotor. The unsteady potential interactions, resulting in unsteady noise, will also contribute to the total noise. The interaction of the fore wake with the stationary aft row will result in unsteady noise being produced by the locked row.

Figure 5.3 compares the computed $LA_{\text{max}}$ for baseline and locked-aft CROR configurations for the range of blade count combinations over the three operating points. Similarly to the lock-fore analysis, Figure 5.3 demonstrates that the computed trends are consistent over the three operating conditions. Again, this is due to the dominance of the loading source at these particular operating points. Similarly to the locked-fore study, the approach conditions show the lowest noise levels.

The sources of difference in computed noise level, in this case, will be as described previously for the locked-fore analysis. Figure 5.3 shows that the computed $LA_{\text{max}}$ is comparable across fore blade counts. These comparable levels result from the requirement of constant solidity to ensure differences in noise level arise from acoustic criteria only. In addition to the effects of blade count, recall that differences in noise is also due to differences in loading, i.e. load split between one or two blade rows. To this end, the increased noise of the locked-aft configuration when $N_1 = 4$ results in greater computed noise than the operative configuration. As the fore blade count increases, the locked-aft configuration begins to demonstrate noise reductions. Therefore, the locked-aft configuration shows reduced noise compared to the baseline configuration when $N_2 \leq N_1$. In each case, the noise of the individual rotors of the operative configuration is computed to produce less noise than the locked-aft configuration. It is again, the acoustic interaction that results in increased noise levels. As the aft blade count increases and $N_2 > N_1$ the reduction in noise of the CROR configuration is such that the addition of the acoustic pressure of fore and aft rows
drops below that of the locked-aft configuration. The analysis found that the locked rotor of the locked-aft configuration produced a negligible contribution to the total noise.

The lowest predicted noise level was again observed for the $6 \times 7$ case of the baseline configuration. However, the locked-aft presented a number of blade counts offering noise reductions. The $5 \times 3$ combination demonstrated the most significant reductions relative to the operative configuration of $\sim 3$ dB(A) at take-off. These reductions can be further widened by increasing the fore blade count relative to the aft. This is the opposite conclusion found for the locked-fore configuration.

Comparing the locked-fore and locked-aft configurations show that the computed noise of the locked-aft configuration is marginally higher than the locked-fore configuration. Note, comparisons are made for opposing blade pairs, i.e. $4 \times 6$ compared with the $6 \times 4$ blade count. Under further inspection, this result arises from the computed aerodynamic performance. Locking the fore rotor was found to have an
aerodynamic benefit, with the computed induced velocity component lowering the inflow and hence required blade setting angle to deliver the required thrust. On the other hand, the locked-aft rotor did not benefit from the operative rotor, and therefore the locked-aft configuration had a poorer aerodynamic performance.

In line with general aviation certification [149], the LA_{max} noise metric has so far been used throughout the analysis. However, due to the highly directive nature of CROR noise, it is perhaps not the most suitable metric in which to compare the various configurations. Due to the properties of the EPNL metric (§3.2), it is perhaps more suited for the CROR analysis. To this end, using the same constant 1,000 ft (304.8 m) flyover, the EPNL has been computed for operative and locked-row configurations at the take-off conditions. Figure 5.4 compares the computed EPNL of the baseline and locked-rotor configurations for the range of blade counts.

![Figure 5.4: EPNL of baseline operative CROR and a CROR with a locked blade row. CROR is the baseline configuration and inop the locked-rotor configuration.](image)

Using the EPNL metric will make isolating the noise differences between configurations more difficult. The steady components will again be important. However, the interaction components are now better represented with the noise metric. The steady tones will again radiate with the same efficiency across configurations. Additionally, there are a number of interaction harmonics that will radiate very efficiently along the rotor axis (ν = N_1 k_1 + N_2 k_2 = 0) for all configurations. However, the amplitudes will differ as the loading changes for each configuration.

Figure 5.4 presents similar trends for both locked-fore and locked-aft configurations as when the LA_{max} metric is used. However, the differences between operative and inoperative configurations now appear wider. Therefore, the use of the EPNL metric has not highlighted further combinations demonstrating noise reductions. However, its use, and hence the inclusion of interaction components (i.e. those not near the plane of rotation) result in greater differences between inoperative and
baseline configurations. Locking the fore rotor results in a maximum reduction of $\sim 4$ EPNLdB with a $4 \times 6$ blade count. When the aft row is locked, a maximum reduction of $\sim 6$ EPNLdB can be achieved with a $5 \times 3$ blade count. Again, the lowest noise is achieved with the operative configuration utilising $6 \times 7$ blade count.

Whilst locking the aft rotor was found to offer noise reductions for a number of blade counts, these reductions came at the cost of a $\sim 5\%$ reduction in efficiency. Nevertheless, the take-off, climb-out and approach conditions are comparatively short segments relative to the cruise segment and, therefore, this penalty may be justifiable as the cruise efficiency is retained when using the proposed concepts.

5.3.1 Folded Aft Rotor

In an effort to further reduce the interaction noise, the effect of folding the locked-aft rotor backwards is studied, Figure 5.1d. In the folded-aft configuration, the aft rotor is locked and folded backwards in the terminal area. In cruise conditions, it is unfolded and operated at the design point. The folded rotor should eliminate the interaction component of the CROR noise. Furthermore, compared to the locked-aft case, the inoperative rotor drag is eliminated and hence reduces the loading demands and the subsequent loading source of the operative fore rotor.

The folded-aft rotor is modelled as an isolated SRP. The SRP is trimmed to deliver the required thrust and the flyover simulated in order to compute the $LA_{\text{max}}$. In order to evaluate the performance of the folded-aft configuration, Figure 5.5 compares the computed $LA_{\text{max}}$ of the locked-aft and folded-aft configurations at take-off conditions. Figure 5.5 demonstrates that the folded aft rotor concept is computed to offer significant noise reductions in comparison to the locked-aft configuration across all blade counts. The reduction in noise results from a number of sources. Firstly, the contribution of the unsteady interaction between the locked and operative rotor is not present in the folded configuration. In the unfolded configuration, the presence of the locked-aft rotor increases the loading demands on the operative rotor. This increased loading arises from the increased drag due to the presence of the aft rotor. Furthermore, it was found that the presence of the locked-aft rotor induced a velocity on the fore rotor such that an increased setting angle was required to achieve the target thrust. These additional loading components are eliminated by folding the aft rotor. When $N_1 = 4$, a maximum reduction of $\sim 10$ dB(A) was computed when the locked-aft rotor was folded backwards.
5.3. LOCKED-AFT ROTOR

Across all blade counts, significant noise reductions are observed. Comparing the computed noise of the folded-rotor with the baseline also demonstrates noise reductions across all blade counts. Nonetheless, the additional complexity and mass of the folding mechanism must be weighed against the presented benefits. However, as the aft rotor will be used only for cruise, it can be designed independently of any take-off requirements.

5.3.2 Increased Fore Radius

When either fore or aft rotor is locked, noise reductions are limited by the increase in self-noise as the operative rotor now provides the total required thrust. Therefore, to reduce this loading component, the effect of increasing the fore radius of the locked-aft configuration is studied. For increasing radius, the tip speed is maintained by reducing the rotational speed. Noise reductions will primarily arise due to reduced blade element loading and the reduced rotation rate. Furthermore, the increased rotor radius will have a similar effect as clipping the aft rotor radius by reducing the tip vortex interaction with the aft rotor.

The fore rotor radius is increased by a maximum of 30% for the range of fore and aft blade count combinations at the take-off operating point. Figure 5.6 compares the computed $L_{A_{\text{max}}}$ for both baseline and aft-locked configurations with increased fore radius at the take-off operating point. Figure 5.6 demonstrates a computed noise reduction for both configurations relative to no increase in fore tip radius, see Figure 5.3a.

Considering the baseline configuration, for cases where the aft rotor noise dominates, in particular, $N_2 > N_1$, the impact of increasing fore tip radius has a negligible effect. Comparing the effects of increasing tip radius shows there is not a linear effect.

Figure 5.5: Comparison of total noise locked-aft and folded-aft rotors for take-off conditions.
on the computed noise for both configurations. This result is believed to arise from the trade-off between the reduction in rotational speed to maintain the tip speed and the resulting required increase in setting angle. Nonetheless, there are a number of blade counts that demonstrate increased reductions when the fore rotor tip radius is increased.

As the fore rotor radius increases, there are a number of additional parameters that further complicate the changes in noise. Firstly, as the tip radius increases, the torque ratio of the operative configuration will reduce, and this is known to have an important role on the resulting noise [31]. Additionally, the effect of unloading is expected to be more significant for the locked-aft configuration. As the fore row of the operative configuration produces only half of the required loading, in comparison to the total loading produced by the operative row of the locked-aft configuration, the operative configuration will be less sensitive to changes in loading.

Figure 5.6: Effect of increasing radius on locked-aft and baseline CROR configurations.
The increase in fore radius, nonetheless, offers increased reductions for the locked-aft configuration over the baseline configuration. To further demonstrate this point, Figure 5.7 presents the computed noise reduction of the locked aft configuration with $6 \times 4$ blade count over the operative configuration for a range of increasing tip radii. Note that the negative value corresponds to reduced noise relative to the operative configuration. Figure 5.7 demonstrates that a noise reduction of up to $\sim 8$ dB(A) can be achieved with the maximum increased radius of 30%. Additional noise reductions will also result from a reduction in the tip vortex interaction noise as the radius increases.

![Figure 5.7](image)

**Figure 5.7: Difference in noise between locked-aft and operative CROR for increasing radius for the $6 \times 4$ combination.**

As the tip radius is increased, the resulting computed propulsive efficiency was found to reduce significantly. For a 10% increase in radius, a drop in efficiency of $\sim 10\%$ resulted. This efficiency drop results from a move away from the optimal lift-drag ratio as the radius increases and the rotational rate is reduced, the inflow angle to rotor will change. This is an important point, particularly as the aircraft transitions to cruise. The difference between take-off and cruise design points now widens as the fore rotor is designed for a much higher loading during take-off compared to its baseline configuration counterpart.

Increasing the fore radius was utilised in order to reduce the loading of the operative row of the locked-aft configuration. For a number of configurations, this demonstrated further reductions than the baseline locked-aft configuration. However, the reduction in noise was also accompanied by a significant drop in propulsive efficiency. However, it may be possible to alleviate the efficiency reduction by optimisation of the design. Furthermore, the lowest noise was achieved by an operative CROR configuration with increased tip radius.
5.4 Summary

In this section, low-order models have been used to investigate the noise-reducing potential of a number of novel CROR configurations. In particular, the concept of locking a blade row has been studied to reduce CROR interaction noise. The analysis showed that the concepts could successfully reduce noise for a number of CROR designs. A maximum reduction of $\sim 3\,\text{dB(A)}$ when either fore or aft rotor is locked was observed for take-off conditions. It was also found that further reductions could be achieved by folding the aft rotor or increasing the fore rotor radius of the locked-aft concept at selected blade counts. Nonetheless, the lowest noise was observed for a high blade count baseline CROR. However, the use of a high blade count must be weighed against the additional mass and the aerodynamic implications. Therefore, future work should consider an optimisation of the novel CROR configurations in a multidisciplinary approach. Furthermore, the low-order models can only provide indicative results and must be further verified.
Chapter 6

CROR Noise Reduction — High-Order Analysis*

The previous chapter (§5) indicated that for selected blade counts, CROR noise could be reduced in the terminal area by locking a blade row. The locked row is operated at design speed during cruise to retain the high propulsive efficiency of the CROR. The previous analysis highlighted the potential of the locked-rotor concept utilising low-order models. Therefore, these findings must be further investigated with models providing a higher fidelity representation of the physics. The aim of this section is then to study the noise-reducing potential of the locked-rotor concept utilising high-order models. Furthermore, the use of high-order models will provide a greater description and insight into the noise sources of the locked-rotor concept.

6.1 Description of Test Cases

Four configurations are studied: (i) operative CROR (baseline); (ii) locked-fore CROR; (iii) locked-aft CROR; (iv) an isolated single rotation rotor (SRP). These configurations are illustrated in Figure 6.1. The baseline and locked rotor configurations have $N_1 = N_2 = 4$ rotor blades, whilst the SRP has $N = 4$ rotor blades. The SRP may be likened to the folded aft rotor studied previously.

All configurations are of SR2 design, Figure 2.8. Pertinent design features are outlined in Table 6.1. Note: the design differs from the previous low-order analysis as the current rotors do not have to be scaled in order to compare a large variation in blade count. Despite the fact that clipping has been identified as a design solution to reduce CROR noise (§1.2), it is not utilised here in order to better compare the

configurations. In particular, when comparing locked-fore and locked-aft configurations, it is important that the rotors have similar loading such that one configuration is not acoustically biased. This is achieved by utilising the same design for all configurations. Nonetheless, future designs could utilise clipping to further reduce noise.

Table 6.1: Rotor design parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R_1 = R_2$ [m]</th>
<th>$r_h/R$</th>
<th>$g/R$</th>
<th>$M_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1.5</td>
<td>0.24</td>
<td>0.4</td>
<td>0.46</td>
</tr>
</tbody>
</table>

The analysis was carried out at take-off conditions. In particular, the free-stream Mach number was $M_\infty = 0.2$, take-off thrust, $T = 6$ kN. The configurations were simulated in steady level flight at an altitude of $h = 1,000$ ft (304.8 m). These values are used to be representative of a general aviation class aircraft.

![Figure 6.1: Baseline and inoperative configurations.](image-url)
6.2 Aerodynamic Analysis

The HMB solver was used to perform the high-order CFD analysis for each configuration. The medium background and Chimera grids described in § 2.2.4 were used for each computation. In addition, each simulation utilised the k-ω SST turbulence closure. The HMB3 solver adopts a non-dimensional approach. To return to physical flow properties, a Reynolds number, based on tip chord of \( R_e = 7.34 \times 10^5 \) was used to simulate the described take-off properties.

Unsteady simulations, i.e. those for the baseline and locked-rotor configurations, required at least four rotations to give time-invariance of flow variables. A final revolution was subsequently computed from which flow variables were extracted. The unsteady simulations used a time step corresponding to 0.5° for which blade surface pressure data were extracted. The SRP configuration utilised a steady simulation. Convergence was monitored using a number of metrics. These included tracking residuals at each time-step and monitoring surface loads and flow variables at each time step using numerical probes placed at a number of locations within the flowfield. Convergence was met when loads and flow variables showed a periodic solution. Figure 6.2 demonstrates a number of the convergence monitors for the baseline configuration. Namely axial pressure and viscous blade forces, and flowfield density at a location downstream of the fore rotor. The convergence monitors show that after some initial large transient fluctuations, the solution converges to a periodic variation in each variable. There is a slight fluctuation located \( \sim 4500 \) iterations. This change results from varying fore and aft rotor setting angles to adjust the thrust closer to the target value. Following this sudden fluctuation, the variables quickly returned to a periodic value.

The flowfield and surface data were extracted over the final rotation for post-processing. The data extracted is used to perform an aerodynamic analysis of each configuration. This analysis compares the aerodynamic performance of each configuration, allowing for any aerodynamic benefits or penalties to be weighed against any corresponding changes to the noise with reference to the baseline configuration. The aerodynamic analysis also allows for greater insight into the noise generating mechanisms that drive the differences in noise between each configuration.

Whilst the locked-rotor concept is concerned with reducing CROR noise, it must also provide the required thrust for the aircraft. For the take-off condition considered here, the required thrust was computed as \( T = 6 \) kN. The initial setting angles for each configuration were computed using the BEMT aerodynamic tool. The operative configuration was trimmed with an equal thrust split, whilst the SRP configuration and the operative rotor of the locked-rotor configurations were trimmed to deliver the full required thrust. Table 6.2 details the setting angles used for each configuration.
The locked-aft and baseline configurations required slight adjustments during the CFD computations. Table 6.3 presents the computed thrust for each configuration.

All configurations are within 5% of the target thrust. These could be trimmed closer to the required value with finer adjustment of the setting angle. However, this would require significant additional computational cost and withal the current values are sufficiently close to compare the aerodynamic and aeroacoustic performance of each configuration. For example considering the SRP configuration, using the BEMT tool, a 5% change in thrust resulted in < 0.5 [dB] change in peak noise level.
Table 6.2: Fore and aft rotor setting angles for each configuration.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\beta_{175}$ [$^\circ$]</th>
<th>$\beta_{275}$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>33.0</td>
<td>32.7</td>
</tr>
<tr>
<td>Locked-fore</td>
<td>83.2</td>
<td>37.2</td>
</tr>
<tr>
<td>Locked-aft</td>
<td>36.8</td>
<td>83.4</td>
</tr>
<tr>
<td>SRP</td>
<td>37.8</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.3: Computed thrust for each configuration. Target thrust = $6\text{ kN}$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_1$ [kN]</th>
<th>$T_2$ [kN]</th>
<th>$T_1 + T_2$ [kN]</th>
<th>$\Delta_{\text{Target}}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>3.05</td>
<td>3.06</td>
<td>6.11</td>
<td>+1.89</td>
</tr>
<tr>
<td>Locked-fore</td>
<td>-0.145</td>
<td>6.34</td>
<td>6.14</td>
<td>+2.37</td>
</tr>
<tr>
<td>Locked-aft</td>
<td>5.94</td>
<td>-0.01</td>
<td>5.85</td>
<td>-2.56</td>
</tr>
<tr>
<td>SRP</td>
<td>6.30</td>
<td>-</td>
<td>6.30</td>
<td>+4.83</td>
</tr>
</tbody>
</table>

Table 6.3 shows that the baseline configuration has an almost equal thrust ratio, i.e. $\frac{T_2}{T_1} \sim 1$. Considering the locked-rotor configurations, the operative rotors have all shown the capability to provide the required thrust for the aircraft. Concurrently, the locked rotor for all configurations produces minimal levels of negative thrust.

Whereas the operative rotor of the locked-rotor configurations was trimmed to deliver the total thrust, the locked rotor was trimmed to minimise torque. Table 6.4 presents the computed torque for each configuration. The baseline configuration shows a torque ratio of almost unity, i.e. $\frac{Q_2}{Q_1} \sim 1$. Although the design was for equal thrust, the equal torque will result from a high efficiency of the configuration. The locked rotor of the locked-fore configuration has been successfully trimmed to a sufficiently small value. On the other hand, the locked rotor of the locked-aft configuration produces a significant torque value. This torque results from the induced swirl from the fore rotor impacting on the aft rotor. Therefore, this indicates that the BEMT model has underpredicted this component. This torque may be reduced by adjusting the rotor setting angle. However, this torque reduction will have to be weighed against resulting negative thrust.

The operative rotors of all locked-row configurations, in addition to the SRP configuration, produce a torque of similar magnitude to the absolute sum of both baseline rotors. Despite this high torque reaction, the value is well within the limits of the engine\(^\dagger\).

\(^\dagger\)The torque reaction is within the limits of the engine used by the aircraft modelled to obtain the thrust requirements.
Table 6.4: Computed torque and efficiency of each configuration.

<table>
<thead>
<tr>
<th>Case</th>
<th>$Q_1$ [kNm]</th>
<th>$Q_2$ [kNm]</th>
<th>$\eta$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-2.50</td>
<td>2.49</td>
<td>0.794</td>
</tr>
<tr>
<td>Locked-fore</td>
<td>0.233</td>
<td>5.55</td>
<td>0.739</td>
</tr>
<tr>
<td>Locked-aft</td>
<td>-5.22</td>
<td>1.55</td>
<td>0.737</td>
</tr>
<tr>
<td>SRP</td>
<td>-5.44</td>
<td>-</td>
<td>0.750</td>
</tr>
</tbody>
</table>

Table 6.4 also shows the computed efficiency, where the efficiency was computed according to:

$$\eta = \frac{V_\infty (T_1 + T_2)}{\Omega_1 Q_1 + \Omega_2 Q_2}.$$  \hspace{1cm} (6.1)

The baseline configuration is shown to have the highest efficiency of all configurations. The locked-rotor configurations show a reduced efficiency relative to the baseline.

One source of improved efficiency of the CROR arises due to the counter-rotation of the aft rotor as the angular momentum from the fore rotor is transformed into useful axial momentum. In particular, the counter-rotation of the aft rotor induces a tangential velocity in the opposite sense from the fore rotor, resulting in a partial cancellation of the tangential velocity downstream. Figure 6.3 shows the computed time-averaged normalised tangential velocity far downstream of all configurations. The baseline configuration is shown to result in an almost complete cancellation of the downstream tangential component. This cancellation is in contrast to all other configurations which show significant downstream tangential components.

Locked-fore and locked-aft configurations have downstream components opposite in direction due to the opposing sense of their operative rotors. Considering the locked-aft configuration, the induced tangential component from the fore rotor will pass over the stationary aft row, and this interaction is seen to impact the resulting downstream profile. The reduction of the tangential component in the inboard sections demonstrates that the aft rotor may be adjusted to act as an outlet guide vane and reduce the tangential component further. Conversely, the fore rotor of the locked-fore configurations may be used as an inlet guide vane. However, these configurations would require some further study.

The mutual interactions between CROR blade rows result in unsteady loading and hence, an unsteady acoustic sources across both rotors. Figure 6.4 presents the computed axial force produced by each operative blade row over a single rotation for baseline and locked-row configurations.
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Figure 6.3: Azimuthal velocity profiles time-averaged over a single rotation. \( V_{ref} \) is the reference velocity, equivalent to \( M = 0.2 \).

Figure 6.4: Computed rotor axial force for each configuration over a single revolution.
The axial force of fore and aft rotors of the baseline configuration shows an 8-peak sinusoidal response about a mean value. This response results from the simultaneous blade passage of fore and aft blades. In the current case where $N_1 = N_2 = 4$ and $\Omega_1 = \Omega_2$, fore and aft blades are coincident every $45^\circ$. The aft rotor shows a phase shift of $\sim 15^\circ$ relative to the fore rotor. The interaction with the fore wake, which meets the aft rotor at a time shortly following the blade passage, leads to this phase shift. The similar amplitudes in the axial force response suggest that both upstream and downstream interactions are equally important in the present case.

The locked-fore and locked-aft configurations show a 4-peak response around a mean value. In these cases, with a locked rotor, simultaneous blade passages will occur only four times per revolution, i.e. every $90^\circ$. The locked-aft configuration shows a sinusoidal response. On the other hand, the locked-fore configuration has a more impulsive response. This response comes as a result of the interaction with the fore wake, as the fore rotor is locked, the wake will traverse in the axial direction only and, therefore, will only interact with the aft row intermittently as it cuts through the discrete wake.

The axial force responses shown for each configuration suggests that the interactions will be more significant for the equal blade count configurations. To investigate these interactions, the harmonic component of the axial force is studied. Figure 6.5 compares the harmonic content of the axial force for the operative rotor of each configuration. Note that axes have been scaled between sub-figures to highlight the magnitude for each configuration. The baseline configuration shows the highest force amplitude. The spectra for the baseline configuration peaks at the second fundamental harmonic, with the largest values shown for subsequent even values of the BPF. The baseline configuration shows comparable amplitudes between fore and aft rotors. This similarity highlights that both up- and downstream interactions are important. The importance of both components has previously been shown for the Fairey Gannet aircraft [40, 41] which may be representative of general aviation propeller design.

When either fore or aft rotor is locked, Figure 6.5 demonstrates that the amplitude of each harmonic is substantially reduced compared to the baseline configuration. The locked-rotor configurations display an almost continuous decay of amplitude with increasing harmonic. The interaction with the stationary upstream row of equal blade count results in interaction at each BPF. With the exception of the fundamental harmonic, the locked-fore configuration shows consistently higher amplitudes than the locked-aft counterpart. This increased amplitude suggests the downstream wake and potential interactions are stronger than those travelling upstream from the locked-aft rotor.
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Figure 6.5: Harmonic content of axial force of operative rotor for each configuration. Note the axis scale in each sub-figure.

Studying the harmonic content of the axial force from each configuration has indicated that the strongest interactions will occur for the fully operative configuration. The interactions are reduced in amplitude for the locked-rotor configuration.

The unsteady interactions between fore and aft rows may be further studied by considering the wake behaviour between the blade rows. Figure 6.6 compares the computed axial velocity variation around the azimuth at a location corresponding to the 75% blade span in the radial direction and the mid-plane between both rotors in the axial direction. The SRP configuration is also shown at a distance equivalent to the mid-plane between fore and aft rotors, i.e. \( \frac{d}{2} \). All configurations show a periodic wake pattern about a mean value. The locked-rotor configurations demonstrate the greatest velocity deficit. However, increasing the blade count of the operative rotor reduces the amplitude of this deficit.

It is interesting to note the periodic behaviour of the locked-fore configuration despite the measurement plane being upstream the aft rotor. These observations demonstrate the significance of the upstream influence of the potential interactions from the operative rotor.

To demonstrate the effect of the upstream interaction, Figure 6.7 compares the azimuthal profiles, a half-spacing (\( \frac{d}{2} \)) upstream of the fore rotor at the 75% span location. Firstly, the locked-fore configuration show minimal influence. However, the sinusoidal pattern corresponding to the aft blade count and the accelerated velocity
component relative to the free-stream shows the impact of the operative aft rotor even for this increased upstream location.

Despite the increased loading of the SRP configuration, the baseline configuration demonstrates an increased amplitude. This increased amplitude will result from the addition of contributions from both fore and aft rotors.

Analysing the locked-aft configuration, the amplitudes are higher than that of the SRP configuration, despite delivering the same loading. Therefore, this further suggests the locked blade row does contribute to the unsteady interactions.

Overall, analysis of the velocity profiles has highlighted the importance of the upstream interactions for all configurations.

Without clipping the aft rotor, the tip vortex of the fore rotor can impinge on the aft rotor, resulting in a significant increase in the radiated noise [37]. While clipping has been demonstrated to reduce CROR noise (see §1.2), it has not been used in the current analysis in order to compare fore and aft rows like-for-like. Nonetheless, it remains important to understand the tip vortex interaction source. To visualise the tip vortices, Figure 6.8 shows iso-surfaces of $Q$-criterion [183] for each configuration, where $Q = 0.2$ for each. The use of $Q$-criterion shows the vortical structures are resolved well for all configurations.

![Figure 6.6](image1.png)  
(a) Baseline and SRP  
(b) Locked-fore and locked-aft

**Figure 6.6:** Azimuthal variation of axial velocity $x = \frac{x}{2}$ downstream the fore rotor.

![Figure 6.7](image2.png)  
(a) Baseline and SRP  
(b) Locked-fore and locked-aft

**Figure 6.7:** Azimuthal variation of axial velocity $x = \frac{x}{2}$ upstream the fore rotor.
Figure 6.8: Contours of axial velocity with iso-surfaces of Q-criterion, \( Q = 0.2 \) for each configuration.
The current implementation of the Chimera technique requires a small gap between the rotor and spinner. This hub-gap results in unphysical hub-vortices. These hub-vortices are seen for the SRP and locked-rotor configurations where the blade loading is higher. These hub vortices may have an impact on the acoustic emissions. However, these are unphysical flow features and can be neglected by considering the noise sources from a more outboard position along the blade. This is justified as it is not anticipated that the hub section will be a significant noise source.

The tip vortices of the SRP and locked-rotor configurations show a greater width than the baseline configuration. Furthermore, as the vortices of the SRP and locked-rotor configuration are resolved to further downstream, indicates their greater strength. This increased strength is a result of their increased loading.

Considering the baseline configuration, the interaction of the fore tip vortices with the aft row is clearly visible. Following the interaction with the rear row, the tip vortex then collides with the tip vortex from the aft row. This secondary interaction may add a further contribution to the CROR noise.

Despite the blade row being stationary, a tip vortex emanates from the fore blades of the locked-fore configuration. However, the non-rotation results in the tip vortices travelling only axially aft wards. They then interact with the rear row in both cases. Additionally, the fore tip vortices are also seen to interact with the tip vortices of the aft row. However, the fore rotor is very weakly loaded. Therefore, the tip vortices will be of lower strength. Furthermore, the reduced loading will result in reduced wake contraction and will, therefore, interact with a more outboard portion of the aft row in comparison with the baseline configuration.

The tip vortex from the fore rotor of the locked-aft configurations is observed to impinge on the stationary aft rotor. However, as the rotor is not heavily loaded, it may not be such a significant noise source in comparison to the operative rotor loading and thickness sources.

The presented aerodynamic analysis has highlighted the performance of all configurations. In particular, the ability of the locked-rotor configurations to deliver the required thrust, albeit with an efficiency penalty relative to the baseline, has been shown. Furthermore, analysis of some aerodynamic features unique to CROR has demonstrated a number of unsteady interaction noise sources. Notably, the equal importance of both up- and downstream interactions have been demonstrated.
6.3 Aeroacoustic Analysis

Aeroacoustic simulations were carried out for all configurations. The Farassat-1A formulation was implemented to compute the tonal noise of each configuration at a range of observer locations. Broadband sources were not computed due to the additional grid and computational requirements. In addition, the low-order analysis indicated that the tonal component was most significant for the present configurations. Surface pressure, computed using the HMB solver, in addition to the blade geometry, were used as inputs into the Farassat-1A noise code to calculate the tonal noise.

A time resolution resulting from $0.5^\circ$ azimuthal steps was used in each simulation. This discretisation resulted in a maximum resolved frequency of $\sim 6$ kHz. For each configuration, both polar and azimuthal directivities are investigated. An array of 51 numerical microphones, evenly spaced between $\theta = 10^\circ$ and $\theta = 170^\circ$ located 1,000 ft (304.8 m) below the CROR are used to simulate a constant altitude flyover. A ring of 180 evenly spaced numerical microphones are placed around the CROR to study the azimuthal directivity. The microphones are located at the mid-point between both rotors, 10 diameters from the centre of rotation. The position of the numerical microphones and definition of polar angle are illustrated in Figure 6.9.

The objective of locking a blade row in the terminal area is to reduce CROR noise. Figure 6.10 compares the total computed A-weighted SPL for each configuration for the 1,000 ft simulated flyover. The computation reveals that the baseline CROR produces the highest noise levels over the entire directivity. Locking either fore or aft blade row, whilst maintaining the required thrust, is shown to reduce these noise levels. The SRP is computed to produce the overall lowest noise levels, with significant
reductions across the entire directivity.

![Graph of A-weighted SPL computed for each configuration](image)

**Figure 6.10:** A-weighted SPL computed for each configuration.

To characterise and compare the noise levels of each configuration across the directivity, Table 6.5 presents the computed EPNL and $L_{A_{\text{max}}}$ metrics for each case. Table 6.5 demonstrates that the baseline configuration produces the highest level in terms of both metrics. The noise of the baseline configuration can be reduced by $\sim 3.6$ dB(A) or $\sim 6.2$ EPNLdB by locking the fore rotor. Locking the aft rotor offers further reductions relative to the baseline of $\sim 7.9$ dB(A) and $\sim 11.4$ EPNLdB.

**Table 6.5: Noise levels for each configuration.**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>EPNL [dB]</th>
<th>$L_{A_{\text{max}}} \text{ [dB(A)]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>62.7</td>
<td>68.2</td>
</tr>
<tr>
<td>Locked-fore</td>
<td>56.5</td>
<td>64.6</td>
</tr>
<tr>
<td>Locked-aft</td>
<td>51.3</td>
<td>60.3</td>
</tr>
<tr>
<td>SRP</td>
<td>38.2</td>
<td>50.0</td>
</tr>
</tbody>
</table>

The SRP configuration demonstrates the greatest reductions, with $\sim 24.5$ EPNLdB and 21.5 dB(A) reductions relative to the baseline. Whilst showing the lowest noise, the SRP will not retain the high efficiency in cruise as offered by the CROR. However, if the SRP is considered as a locked-aft configuration with the aft rotor folded, this high cruise efficiency may be recovered and these noise reductions prove very promising.
The components of each configuration are further studied to investigate the sources of the different noise level between each configuration. Therefore, to proceed, Figure 6.11 presents the noise sources for the baseline configuration. From Figure 6.11a, it can firstly be observed that both fore and aft rotors retain high SPL values across the polar directivity, highlighting the significance of the interaction component on both rotors. The high contribution from both fore and aft rotors across the directivity further verifies the importance of both up and downstream interactions in the present case. Due to the equal rotational speed and blade count of fore and aft rotors, it is not possible to extract the interaction component. However, the pressure derivative term of the Farassat loading source identifies the contribution due to unsteady loading. Furthermore, the equal loading of both rotors would suggest a similar level of self-noise. Therefore, the observed differences will result from the interactions for each blade row.

Figure 6.11b compares the loading and thickness contributions for the baseline configuration. The thickness contribution is shown to have a negligible contribution to the total noise across the entire directivity. This was found to be the case for all configurations, with the loading contribution dominating over the whole directivity. The thickness component would be expected to be more important as the relative velocity of the rotors is increased.

Figure 6.12 compares the contributions from fore and aft rotors for both locked-rotor configurations. For the locked-fore configuration, Figure 6.12a, with the exception of the area near the rotor plane, the locked rotor makes a negligible contribution to the total noise. However, as the directivity approaches the rotor plane, the peak
levels of the locked rotor contribute significantly to the overall noise. The high noise levels of the aft rotor across the directivity is indicative of the importance of the interaction from the locked fore rotor. In particular, the wake and potential interactions from the stationary upstream blades.

Figure 6.12b compares the fore and aft contributions of the locked-aft configuration. It is interesting to note that in the locked-aft configurations, the locked rotor now plays a more significant part in the total noise over a greater range of directivities. These similar levels result from sources. Firstly, relative to the locked-fore configuration, the component of the locked rotor is increased. This increase arises from the downstream interactions from the operative fore rotor and is identified from the pressure derivative term in the Farassat formulation of the loading noise. Therefore, the interactions on the locked rotor are greater for the locked-aft configurations than the locked-fore configurations.

Secondly, relative to the locked-fore configuration, the operative rotor noise reduces. The cause of this again relates to the interaction. In this case, the upstream interactions of the locked rotor are weaker. Therefore, it can be said that concerning the locked rotor, the downstream interactions are more significant than the upstream interactions. These points highlight the importance of considering the design of the locked rotor, something that was not observed when utilising the low-order models.

![Figure 6.12: Rotor noise contributions for locked-rotor configurations.](image)

The significant levels of the locked rotors for both configurations can also be described from the use of the A-weighting. Without weighting the operative rotor is the dominant source over all observer angles. The locked rotor is less impacted by the
A-weighting due to higher levels at higher frequencies. The reductions in SPL of the operative rotor due to the weighting therefore reduces its noise to levels comparable to the locked rotor, as observed in the figures above.

The azimuthal directivity is studied to investigate the noise of each configuration further. Figure 6.13 presents the computed azimuthal directivities of the baseline and SRP configurations. The A-weighting masked a number of the interesting features present for the current configurations. Therefore, to retain these features, the unweighted SPL is presented. Additionally, the scales have been adapted for each configuration to highlight some of these features.

Figure 6.13a compares the contributions of fore and aft rotors for the baseline configuration. Both fore and aft rotors show an 8-peak response around the azimuth, owing to the simultaneous blade passage of this configuration. The peaks of the fore and aft directivities correspond to the peak loading, as shown in Figure 6.4a. Due to the higher noise levels of the fore rotor, the total noise shows an 8-peak directivity pattern with peaks corresponding to those of the fore rotor.

Figure 6.13b demonstrates the continuous and symmetrical directivity pattern of the SRP configuration. This continuous pattern results from the steady assumptions made during the aerodynamic analysis.

Figure 6.14 compares the computed azimuthal directivity for the locked-rotor configurations. Figure 6.14a shows that over most angles, the noise of the locked-fore configuration is dominated by the operative aft rotor. The locked rotor shows an interesting pattern around the azimuth formed of 4 distinctive peaks each composed...
of two sub-peaks. These features arise from dominance of the contributions from the second and third harmonic components. Whilst the first harmonic component has a 4 lobed pattern around the azimuth, the second and third harmonics show 8 lobes around the azimuth. Additionally, the second and third harmonics show peak values greater than that of the first harmonic. With the peaks of the first harmonic aligning with every second trough of the second and third harmonics and the troughs of the first harmonic aligning with the troughs of every other trough of the second and third harmonics, the resulting trend shows the two sub-peaks followed by a large trough.

Figure 6.14b presents the azimuthal directivities of the locked-aft configuration. The fore rotor of the locked-aft configuration dominates the total noise over the entire directivity, not previously observed using the A-weighting. Whilst the locked rotor shows a negligible contribution to the overall noise, it presents an interesting 8-peak pattern around the azimuth, with peaks occurring every $\sim 45^\circ$. This 8-peak pattern arises due to the interaction with the potential field and wake from the fore rotor.

The analysis of the azimuthal directivity of all configurations has highlighted the interactions between fore and aft rotors and their contributions to the total noise are now also more clearly shown. In addition to this, the importance of the locked rotor contribution is highlighted without the use of the A-weighting.

![Figure 6.14: Azimuthal directivity of rotor components for locked-rotor configurations.](image-url)
6.3. AEROACOUSTIC ANALYSIS

baseline and SRP configurations.

Figure 6.15a shows high total SPL values across the harmonics for both fore and aft rotors. The equal blade count and rotational rates of fore and aft rotors of the baseline configuration mean that it is not possible to extract the interaction tones. However, the high values observed at higher harmonics indicate the importance of the interactions. Fore and aft rotors produce similar levels at the fundamental harmonic, resulting from similar self-noise. As the harmonics increase, the fore rotor has the greatest contribution to the total noise for most harmonics. These increased levels at the higher harmonics further support the previous analysis showing the importance of the upstream interactions. Here, however, the increased values suggest that in the present case, the upstream component is more significant than those travelling downstream. The high levels observed for increasing harmonics is contrary to the almost continuous decay found for the isolated SRP configuration, Figure 6.15b.

![Figure 6.15: Harmonic components of baseline and SRP configurations at the overhead position.](image)

Figure 6.16 compares the harmonic content of the locked-rotor configurations at the overhead position. For the locked-fore configuration, Figure 6.16a shows that at the overhead position the total SPL at each harmonic is generally not dominated by either fore or aft (or operative or locked) rotor, but in fact, is comprised of significant contributions from both rotors. The high values for both fore and aft rotors for increasing harmonics is indicative of the importance of the interaction for both rotors. This fact also further highlights the importance of the contribution of the locked rotor.

Figure 6.16b compares the harmonic content of the locked-aft configurations at
the overhead position. For the locked-aft configuration, the fundamental and second harmonic of BPF is dominated by the contribution from the operative rotor. However, as the harmonic increases, the locked rotor makes the most significant contribution for the majority of harmonics. These high levels for the locked rotor result from the interaction with the wake and potential field from the operative upstream rotor.

Comparing locked-fore and locked-aft configurations further, it is generally observed that the locked rotor of the locked-aft configuration has higher noise levels due to being in the wake of the upstream operative rotor. On the other hand, the operative rotor of the locked-fore configuration generally shows higher levels due to being in the wake of the stationary upstream row. As a result, in the case of the locked-rotor configurations, the downstream interactions are more important than those travelling upstream.

An acoustic analysis of the locked-rotor concept has been carried out. To quantify the performance of the novel configurations, they have been compared with a baseline CROR (both rotors running). It was found that the lowest noise was achieved with the locked-aft configuration. However, it could be reduced further when folded back (as modelled by an isolated SRP). The analysis demonstrated the importance of the locked-rotor noise contribution, the significance of which was not generally observed in the low-order analysis and suggests a deficiency of the model. The locked rotor was found to have a significant contribution across a number of harmonics and directivities. The analysis was carried out on a design of equal fore and aft blade
count and therefore does not represent the most acoustically optimal configuration. As a result, future work should consider more acoustically optimal designs, e.g. unequal blade count and rotor speeds.

6.4 Summary

The previous chapter identified the potential of the locked-rotor concept to reduce CROR noise. In an attempt to further verify this potential, this chapter aimed at utilising high-order tools to investigate the novel concept. Firstly, an aerodynamic analysis was carried out to ensure that each configuration could provide the required thrust. Each configuration could successfully deliver the required thrust with a small penalty in efficiency. Additionally, the locked-aft configuration was found to produce a significant torque from the locked rotor. Further analysis is required to determine the impact on performance to reduce this. A detailed study of the rotor loads highlighted the equal importance of both up and downstream interactions.

An acoustic analysis further demonstrated the ability of the locked rotor concepts to reduce CROR noise, with locked-fore and locked-aft configurations offering gains of \(\sim 3.6\) and \(\sim 7.9\) dB(A) respectively. The lowest noise was, however, observed for the SRP, with gains of \(\sim 21.5\) dB(A). The SRP does not meet the objectives of the same cruise performance. However, if this is taken as the folded rotor concept, the results are extremely positive. The analysis highlighted the importance of modelling the locked rotor contribution, with further analysis required on reducing the contribution from the locked rotor. Additionally, the analysis should be extended to a more acoustically optimal design to investigate further noise reductions for existing high-performing designs using the proposed concept. Furthermore, the present investigation, whilst confirming the low-order predictions, is only numerical. Therefore, to fully verify the noise-reducing potential of the proposed concepts, an experimental study is required. Nonetheless, the present numerical analyses prove promising for the reduction of CROR noise.
Chapter 7

Conclusions and Future Work

Increasing environmental pressures have resulted in a renewed interest in the CROR concept. However, amongst others, noise remains a pertinent issue for the CROR. A literature review identified that optimisation may be a useful resource to incorporate low-noise within the design. Additionally, the literature review found that the interaction noise component offered the greatest potential for noise reduction. To this end, this thesis details the work carried out to develop and utilise numerical tools to investigate a number of novel configurations to reduce CROR noise for general aviation aircraft.

The chapter proceeds by outlining the main conclusions of the presented work. Following this, a number of recommendations for future work are discussed.

7.1 Conclusions

There are a number of conclusions from the work presented within this thesis. These conclusions reflect the objectives outlined in the opening chapter of the thesis:

1. Develop a low-order, multi-disciplinary CROR design tool

A number of low-order numerical tools for the analysis of CROR have been developed. Firstly, a BEMT model has been extended for the analysis of CROR. Secondly, an acoustic model to compute CROR noise has been implemented. The model is coupled with the aerodynamic model and is capable of computing the important unsteady loading source. Validation and sensitivity studies for both aerodynamic and aeroacoustic models proved their ability to match reported data relatively well.

The models are coupled together in the form of a single design and analysis tool. The low execution time and the ability of the model to predict the required performance indicates this to be a useful preliminary design tool for CROR.
2. Demonstrate the use of the design tool to identify high-performing CROR designs for general aviation aircraft.

The low-order models have been implemented within a Genetic Algorithm optimisation routine to incorporate low-noise into the design of a CROR for general aviation.

The optimisation routine identified a number of designs with reduced noise relative to a baseline configuration for three different design objectives. The optimisation study found that low noise was achieved with large blade counts and reduced tip speeds. A multi-objective design considering take-off noise and cruise efficiency identified that these two objectives were competing. Nonetheless, a design with reduced noise and increased efficiency was identified. Comparable results were also found when applying the design to civil aviation class aircraft.

Therefore, the coupling of the low-order models to an optimisation routine could successfully identify designs of low-noise. This result further highlights the ability of the preliminary design tools.

3. Use the low-order CROR design tool to investigate a number of novel configurations for reducing CROR noise in the terminal area.

The low-order design tools have been used to investigate the potential to reduce CROR noise using a novel multi-configuration CROR. In an effort to reduce the interaction component, it is proposed to lock either fore or aft rotor in the terminal area. During cruise, the locked rotor is operated at design conditions to retain the high CROR efficiency. Investigating a large number of blade counts, locking either fore or aft rotor resulted in a maximum noise reduction of \(\sim 3\) dB(A) relative to the baseline configuration at take-off conditions. However, these came at the cost of \(\sim 5\%\) penalty in efficiency. Furthermore, the lowest noise was obtained using a blade count of \(6 \times 7\) for the baseline configuration.

Further noise reductions were sought by folding the locked-aft rotor backwards and thus eliminating any interaction component. The folded-aft configuration offered further noise reductions of \(\sim 10\) dB(A). Additionally, offloading the operative rotor was considered by increasing the tip radius. The increased radius configuration demonstrated an \(\sim 8\) dB(A) reduction relative to a baseline with the same tip extension. Further gains would be envisioned from a reduction of the tip vortex component.

Overall, the novel configurations proposed demonstrated significant noise reductions relative to the baseline for a number of blade count combinations. These reductions prove promising for this preliminary analysis. However, these reductions must be further verified. Furthermore, the analysis considered these concepts from
an acoustic point of view only and did not account for such things as the mechanical complexity, aerodynamic performance, etc. which must inevitably be weighed against the noise reductions.

4. Perform high-order aerodynamic analysis to investigate the noise sources of these novel concepts.

uRANS CFD simulations, using the HMB3 solver, were carried out on the baseline CROR configuration in addition to a number of locked-rotor configurations. The analysis confirmed the ability of the locked-rotor configurations to deliver the required thrust. Furthermore, this was demonstrated to be delivered for a $\sim 5\%$ efficiency penalty. Additionally, the locked-aft configuration showed a large torque reaction for the locked rotor which must be further studied.

The aerodynamic analysis highlighted the importance of the interaction between blade rows. In particular, the equal importance of up- and downstream interactions for the operative rotors. The analysis suggested that the downstream interactions of a locked rotor were stronger than those travelling upstream. As a result, fluctuating loads on the operative rotor were higher on the locked-fore configuration. On the other hand, the fluctuating loads on the locked rotor were higher for the locked-aft configuration. The analysis also suggested that tip vortex interaction may be a significant source for all configurations.

5. Use coupled high-order aerodynamic-aeroacoustic tools to verify the capabilities of the novel concepts.

Unsteady blade surface pressure from the CFD computations was used as input to an acoustic solver based on the Farassat 1A formulation to calculate the tonal noise of each configuration. The acoustic solver firstly confirmed the noise-reducing capabilities of the locked-rotor configuration. The locked-fore configuration demonstrated noise reductions of $\sim 3.5$ dB(A), while the locked-aft showed reductions of $\sim 6.5$ dB(A) relative to the baseline configuration. Noise reductions were more significant for the SRP, $\sim 17$db(A). However, The SRP does not offer the required high cruise performance. Nonetheless, if this is representative of the folded-aft concept, it shows promising results. The acoustic analysis also highlighted an interesting observation that the locked rotor can in fact make a significant contribution to the total noise.

Whilst noise reductions have been observed, the analysis presented here is purely numerical, and an experimental study is required to verify the predictions. Furthermore, the present analysis was carried out for equal blade count and rotational speeds which perhaps does not represent the most acoustically optimal configuration. Therefore, to better quantify the potential of the proposed concept, additional CROR configurations should be considered.
To conclude, the aim of this thesis was to investigate a number of concepts to reduce CROR noise in the terminal area whilst retaining high efficiency. The concepts proposed demonstrated the ability to reduce CROR noise through a reduction in the interaction component. This resulted in an efficiency loss. However, this loss does not impact the cruise segment and is, therefore, deemed acceptable. Hence, the presented work has, through the development and use of a number of numerical tools, met the objectives set out at the beginning of the thesis.

7.2 Future Work

Whilst the present work has met the outlined objectives and investigated the ability of a number of concepts to reduce CROR noise, there are a number of limitations and areas of future work. In this section, recommendations for future work are described.

Firstly, while the low-order analysis identified a number of blade count combinations that offered noise reductions when either fore or aft row was locked. Ultimately though, the lowest noise was obtained for the baseline configuration. However, this was obtained from $6 \times 7$ blades and may not be feasible for a general aviation class aircraft. Furthermore, the analysis was from an acoustic point of view only. Therefore, it would be interesting to use the optimisation routine to investigate further noise gains of the locked-rotor concepts from a multi-disciplinary approach.

The high-order analysis presented a number of interesting results. For example, the importance of the noise of the locked rotor was identified. Furthermore, the configurations considered were not acoustically optimal, e.g. equal blade counts, no clipping, etc. Therefore, further configurations should be studied to further realise the potential of the proposed locked-rotor configurations. In particular, the effect of the blade count on the locked rotor should be further studied using the high-order analysis. This would give greater insight into the role of the locked rotor in the total noise generation and may support the development of the low-order models with high-order data.

Finally, the noise reductions found throughout the analysis have been from numerical analysis only. Therefore, these should be studied in experimentally to prove the noise reductions and the effect on CROR performance.
Appendix A

Appendix A - Broadband Noise

This supplementary section provides greater detail on the equations describing
the broadband noise sources. The equations presented here contain a large num-
ber of terms. Therefore, to avoid adding an exhaustive list of further terms to the
nomenclature, additional terms used in this section are described where used.

A.1 Trailing-Edge Source

The model described by Magliozi [21] was used to compute the trailing-edge
broadband noise. Recall, from §3.3, the trailing-edge broadband noise is given by:

\[ S_{pp}(x, \omega) = \left( \frac{\omega b z}{2 \pi c_0 \sigma^2} \right)^2 l_y(\omega)s |\mathbf{L}|^2 S_{qq}(\omega, 0), \tag{3.18} \]

where, \( \omega \) is the radian frequency, \( \omega = 2\pi f \); \( b \) and \( s \) are the semichord and semispan,
respectively. \( l_y \) is the spanwise correlation length, approximated as:

\[ l_y \approx \frac{2.1 U_c}{\omega}, \tag{A.1} \]

with \( U_c \) the convection velocity. Returning to Equation (3.18), \( \sigma = \sqrt{x^2 + \beta^2 z^2} \) is the
Prandtl-Glauert transformed distance to the trailing-edge, with \( \beta = 1 - M^2 \). \( \mathbf{L} \) is the
generalised lift, given as:

\[ |\mathbf{L}|^2 = \frac{(1 + M_c - M) D \sigma^4}{z^2 M_c r f^2 K_s^2}, \tag{A.2} \]

where the directivity factor, \( D \) is:

\[ D = \frac{2 \cos^2(\theta / 2)}{[1 - (M - M_c) \cos(\theta)]^2(1 - M \cos(\theta))} \tag{A.3} \]
\( r \) and \( \theta \) are the retarded observer locations; \( M_c \) is the convective Mach Number (= \( U_c / c_0 \)); \( K_x = \omega b / U_c \). The pressure spectrum \( S_{qq} \) is obtained from curve-fitting data for an aerofoil (as a posed to a flat-plate) and is expressed as:

\[
S'_{qq} = \frac{0.000666 \tilde{\omega}}{1 - 5.489 \tilde{\omega} + 36.74 \tilde{\omega}^2 + 0.1505 \tilde{\omega}^5}.
\] (A.4)

This spectrum is for a single aerofoil surface, i.e. upper and lower components must be added. \( \tilde{\omega} \) is a Strouhal number:

\[
\tilde{\omega} = \omega \delta^* / U
\] (A.5)

with \( \delta^* \) the displacement thickness of the turbulent layer which can be approximated as:

\[
\frac{\delta^*}{c} = 0.047 R_c^{-1/5}.
\] (A.6)

\( R_c \) is the Reynolds number based on chord.

### A.2 Rotor-Rotor Interaction Source

The interaction of the aft row with the turbulent wake of the fore rotor results in an additional broadband noise source. The model described by Kingan [161] has been used in the current work.

Recall, from §3.3, the time averaged rotor-rotor broadband pressure spectrum due to rotor-rotor interaction is given by Kingan [161] as:

\[
\tilde{S}_{pp}^{RR}(x, \omega) = \sum_{k_1 = -\infty}^{\infty} \sum_{k_2 = -\infty}^{\infty} \frac{2 \pi N_2}{(4 \pi r_1)^2 (1 - M \cos \theta)^2} \int_{R_h}^{R_i} \left( \frac{\pi V_r}{\ln(2)} \right)^2 \left( \frac{\rho N_1 b_{1/2}}{r \cos \alpha_1} \right)^2 G(k_b) S^2 \left( \frac{k \sin \theta}{1 - M \cos \theta} \right) \times \Phi_{ww}^{(2)}(\tilde{K}_{X_2}, \hat{e}_{X_2}) \left| \Psi_L(K_{X_2}, 0, \omega - n \Omega_2, k_c) \right|^2 dr. \quad (3.19)
\]

\( \Phi_{ww}^{(2)} \) is the two-wave number turbulent velocity spectrum:

\[
\Phi_{ww}^{(2)}(\tilde{K}_{X_2}, \hat{e}_{X_2}) = \frac{4}{9 \pi} \frac{w_{rms}^2}{k_c^2} \left( \frac{\tilde{K}_{X_2}}{k_c} \right)^2 \left( 1 + \left( \frac{\tilde{K}_{X_2}}{k_c} \right)^2 \right)^{7/3} \quad (A.7)
\]

where, \( K_{X_2} = \frac{\omega - \Omega_2}{V_2}, \) \( \hat{e}_{X_2} \) is the unit normal aligned to the chordwise direction of the aft rotor, and:

\[
k_c = \frac{\sqrt{\pi \Gamma(5/6)}}{\Lambda_1 \Gamma(1/3)}.
\] (A.8)
$\Lambda_1$ is the integral lengthscale, $\Lambda_1 \sim 0.42b_{\frac{1}{2}}$ and $\Gamma(\cdot)$ is the Gamma function (See p.255 of Ref. [97]). $b_{\frac{1}{2}}$ is the wake half width:

$$b_{\frac{1}{2}} = 0.25 \sqrt{c_{d_1}c_1 L_w} \quad (A.9)$$

with $c_{d_1}, c_1$ and $L_w$ the fore drag coefficient, fore chord, and $L_w = (g - \frac{G}{2} \cos \beta_2)/\cos \beta_1$ is the wake helical length, where $g$, $\beta_1$ and $\beta_2$ are the axial rotor spacing and fore and aft setting angles respectively.

As estimated from Blandeau [162], $w_{\text{rms}} \sim 0.404 u_{\text{def}}$; where, $u_{\text{def}}$ is the wake centreline velocity deficit:

$$u_{\text{def}} = 2V_1 \sqrt{\frac{c_{d_1}c_1 \ln(2)}{\pi L_w}} \quad (A.10)$$

Returning to Equation (3.19),

$$G(k_b) = \exp \left\{ -\frac{k_b^2}{4\ln(2)} \right\}, \quad (A.11)$$

is the normalised Fourier coefficient for the wavenumber $k_b$,

$$k_b = \frac{k_2 N_1 b_{\frac{1}{2}}}{r \cos \beta_1} \quad (A.12)$$

$S$ is defined as:

$$S = \frac{c_2}{2} \left[ \frac{\kappa \cos \theta_1 \sin \beta_2}{1 - M \cos \theta_1} - \frac{k_1 \cos \beta_2}{r} \right] \quad (A.13)$$

where $\kappa = \omega/c_0$, and $J$ is the Bessel function.

Finally, $\Psi_L$ is the acoustically weighted lift function. The lift function is computed according to the frequency of the gust, for low frequency gusts, i.e.:

$$\left| \frac{\kappa M_{r_2}}{\beta_{r_2}} \right| < \frac{\pi}{4},$$

$$\Psi_L(KX_2,0,\omega,k_c) = \frac{1}{\beta_{r_2}} S \left( \frac{\sigma}{\beta_{r_2}} \right) \exp \left\{ f \left( \frac{\sigma}{\beta_{r_2}} \right) \right\} \left[ J_0(\eta') - j J_1(\eta') \right]. \quad (A.14)$$

For high frequency gusts, i.e.: $\left| \frac{\kappa M_{r_2}}{\beta_{r_2}} \right| > \frac{\pi}{4}$

$$\Psi_L(KX_2,0,\omega,k_c) = \sqrt{\frac{2}{\mu' \pi (1 + M_{r_2})^2 (j \omega')^2}} \exp \left\{ j \frac{k_c + j \sigma}{\beta_{r_2}} \right\} E^* \left[ \sqrt{\frac{4\mu'}{\pi}} \right]. \quad (A.15)$$

where:

$$\beta_{r_2} = \sqrt{1 + M_{r_2}^2},$$

$$f(M_{r_2}) = (1 - \beta_{r_2}) \ln(M_{r_2}) + \beta_{r_2} \ln(1 + \beta_{r_2}) - \ln(2)$$
σ = 0.5K_2c_2

S is the Sears’ function:

\[ S = \frac{1}{jx[K_0(jx) + K_1 jx]} \]  \hspace{1cm} (A.16)

η’ and μ’ are given by:

\[ \eta' = \frac{\sigma M_{r_2}^2}{\beta_{r_2}} - k_c, \]
\[ \mu' = \frac{\sigma M_{r_2}}{1 + M_{r_2}} + k_c \]  \hspace{1cm} (A.17)

k_c is:

\[ k_c = \frac{c_2}{2} \left[ \frac{\kappa \cos \theta_1 \cos \phi_2}{1 - M \cos \theta_1} + \frac{n}{r_2} \sin \phi_2 \right]. \]  \hspace{1cm} (A.18)
Bibliography


