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Dual damage criteria in a crash stop

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ABSTRACT

This study proposes a method to apply dual criteria for the assessment of product damage based on the deceleration signal in a crash-stop incident. To evaluate the shock effects, a procedure is proposed to decompose the original crash signal into two components, i.e. an equivalent crash signal and a residual shock signal. The characteristics of crash signals are analysed in both time and frequency domains. It is found that the equivalent crash signal can represent the original crash signal if the dominant frequency (f_n) of the product structure is relatively lower than the effective frequency (f_s) of the crash signal, i.e. $\frac{f_n}{f_s} \leq \frac{1}{5}$. Otherwise, the original crash signal should be used in the assessment of product damage if $\frac{f_n}{f_s} > \frac{1}{5}$. The dual damage criteria method is proposed by combining the maximum-acceleration vs. velocity-change ($A_{\max}$ - ΔV) diagram and shock response spectrum (SRS), i.e. the former can offer a lower bound of damage boundary on $A_{\max}$ - ΔV diagram while the latter can define an upper bound of damage boundary on SRS graph. The method of dual damage criteria is illustrated numerically using two types of cantilever beams, as an example. The proposed method can be applied with the critical SRS (S_c) method recommended in ASTM D3332-99 for the packaging design and test, and may also be considered in the design and test of other impact energy absorbers in a crash-stop incident.

Keywords: Crash-stop incident; Damage boundary criteria; Acceleration-velocity change ($A_{\max}$ - ΔV) diagram; Shock response spectrum (SRS)

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1. Introduction

Crash-stop phenomena exist widely in various engineering applications, such as product fragility and packaging [1], ‘soft landing’ of re-entry capsule and spacecraft [2,3], impact protection of nuclear waste flask [4], design of crushable structures [5], or impact energy absorber (IEA) [6], in various transportation vehicles (e.g., automotive, rail, ship, aircraft). These applications concern the survivable crash-stop of the carrier container for the safety of its occupants (e.g., passenger, product or good). A typical container can be generically simplified into two main parts, i.e. the protector (e.g., packaging, IEA, cushion) and the protected component to host the occupant, as illustrated in Fig. 1. Usually, the protector is crushable to limit the impact force and absorb impact energy to maintain a relatively rigid envelope of survivable space and reduce the shock effects transmitted to the occupant [7]. The transmitted shock effects are usually characterised by the specified deceleration-time history applied to the occupant, which can be used to analyse occupant’s response [8].

Fig. 1. A generic container in a crash-stop incident, which can be simplified into two main parts, i.e. the protector (e.g., packaging, IEA, cushion) and the protected component (e.g., product, passenger, good).

The damage or injury of an occupant in an impact incident has been evaluated by the maximum acceleration against velocity change graph (i.e. $A_{\max}$ - ΔV diagram), as proposed in [9], where a rectangular acceleration-time history was used to derive a conservative $A_{\max}$ - ΔV diagram. The derivation of $A_{\max}$ - ΔV diagram has the same nature and procedure as the derivation of pressure-impulse (p - i) diagram, a widely applied method to assess the damage of structures subjected to blast loading [10-12]. The American Society for Testing and Materials (ASTM) incorporated the $A_{\max}$ - ΔV diagram into the Standard Test Methods for Mechanical Shock Fragility of Products (ASTM D3332-99) [13]. Mechanical shock in a structure is usually characterised by high frequency oscillations over short time duration. Because the high frequency oscillation cannot be completely represented by the velocity change, $A_{\max}$ - ΔV diagram alone cannot describe the crash effect.

Mechanical shock can be produced either by the stress wave propagations and reflections in the structural domain due to the application of an intensive transient loading, or directly by a transient loading with high frequency oscillations due to the structural behaviour of packaging protector (e.g., progressive crushing) [14-16]. Shock response spectrum (SRS), introduced initially by Biot [17], is an efficient approach to assess the severity of shock environment (e.g., [18,19]), which has different types of indicators, i.e. absolute acceleration (AA), pseudo acceleration (PA), pseudo velocity (PV), acceleration pseudo velocity (APV) and relative displacement (RD), as summarised in [20-22]. Although SRS was discussed in [9], it was only considered for the damage of products (or devices) when product's dominant natural frequency (f_n) is higher than the dominant frequency of the deceleration-time history. Later on, the concept of critical SRS (S_c) derived from trapezoidal acceleration-time history was used in the packaging design of product protection [23], which was adopted in ASTM D3332-99 [13], as summarised in Appendix. Qian et al. [24] experimentally and numerically conducted shock tests on a packaged washing machine based on ASTM D3332-99. Shock test data showed that the packaging design using the $A_{\max}$ - ΔV diagram is conservative, while the assessment method based on the S_c method may provide support for the development of more effective packaging system. In other words, it is possible to design the cushion to transmit higher accelerations than the critical maximum acceleration ($A_{\max,cr}$) without causing damage to the product. Although the ASTM D3332-99 has been taken as a testing standard in packaging industry, its valid application range did not seem to have been clearly stated because only several idealised crash signals were considered. In addition, the procedure to apply joint $A_{\max}$ - ΔV diagram and the S_c method to assess the product damage is not detailed in ASTM D3332-99 [13].

On the other hand, vehicle crashworthiness can be generally considered as the fragility problem of a packaged product where the 'packaging' is replaced by the passive IEA and the product represents the carried passenger or goods. Several crashworthiness indices have been proposed to evaluate the crashing performance of an IEA, e.g., energy absorption (EA), specific energy absorption (SEA), peak crushing force (PCF), mean crushing force (MCF), crushing force efficiency, stroke efficiency, total efficiency, and energy-absorbing effectiveness factor [25-29], which only focus on IEA's overall crashworthiness performance with little concern of shock oscillation. Consequently, product damage and human injury criteria in a crash incident are usually assessed using the parameters determined by the overall crashworthiness responses of the IEA, which is irrelevant to the oscillation of acceleration-time history.

The shock environments of vehicles during normal transportation are characterised in [30-32]. Nevertheless, limited works were reported on the shock analysis in vehicle crashworthiness. Foley and Gens [33] applied SRS method to describe the mechanical vibration and shock environment in

normal rail and truck transportation (e.g., run in, run out and crossing rail switches, bridges and highway underpasses). Later, Magnuson [34] found that the time durations of the mechanical shock in normal rail transportation are similar to those in rail coupling or shunting; but the magnitude of the coupling shock is severer than the magnitudes of the mechanical shocks in normal rail transportation because the coupling event is a designated low-velocity impact. When two acceleration-time curves are similar in time domain, they may have large difference in frequency-domain, as shown in [35] for two acceleration-time curves in crash incidents even if they both meet the impact condition specified in relevant standard (ECE R17) [36]. Therefore, it was concluded that the SRS method is a useful tool for distinguishing the potential damage or injury caused by similar crash signals in time domain [35], which demonstrates the importance of shock analysis in a crashworthiness design. It is beneficial to consider product/occupant safety as a general crash-stop problem in both packaging and vehicle crashworthiness due to their similarity.

The objective of the present work is to develop a method for the assessment of structural damage based on the crash signal in a general crash-stop incident. Without causing confusion and for the simplification, the term of ‘damage’ may generally include structural damage and human injury, the term ‘structure’ may include product, device, structural component and human body or part, and the term of ‘acceleration’ may generally include deceleration, in the rest of this paper if no particular notation is given. Section 2 discusses the characteristics of a crash signal, the procedure of signal decomposition and the calculation of characteristic frequency ratios. Section 3 focuses on the structural damage assessment criteria using $A_{\max}$ - ΔV diagram and SRS method. Section 4 validates the proposed damage criteria by two types of cantilever beams with relatively low and high natural frequencies.

2. Crash signal characteristics

2.1 Signal decomposition

Generally, the crush load-time history of a package/IEA can be related to an acceleration-time history at a representative location of the protected object. Without losing generality, we consider the original crash signal (OCS) in the form of acceleration-time history ($A(t)$). In order to assess the shock effects, we herein decompose the OCS into two components, $A_1(t)$ and $A_2(t)$, i.e.

$$A(t) = A_1(t) + A_2(t) \quad (1)$$

where $A_1(t)$ is the equivalent crash signal (ECS) whose velocity-change ΔV is identical to that of $A(t)$; $A_2(t) = A(t) - A_1(t)$ is the residual shock signal (RSS) whose integral of acceleration over time equals zero. It is noted that the term of ‘equivalent crash wave’ used in some publications [37] has the same meaning of ECS.

$A_1(t)$ can be determined by different ways. For the OCS with an overall ideal pulse shape, ECS satisfying the above requirements can be easily identified. As an example, the OCS of an object attached to a thin-walled tube subjected to an axial collision can be divided into an equivalent square signal (ESS) and a RSS in Fig. 2. The ΔV of ESS equals that of OCS, while the integration of RSS is zero. The OCS and ESS are shown in Fig. 2(a) while the RSS is shown in Fig. 2(b). t_d denotes the overall time duration of the shock signal.

Fig. 2. Typical load curve associated with an axially crushed thin-walled cylinder tube: (a) OCS and ESS; (b) RSS ($t_d = 0.030$ s).

For more complex OCSs shown in Fig. 3(a), the following general procedure is recommended. Firstly, a low-pass filter with a given frequency (f_L) is used to remove high frequency oscillations in the OCS. Then, the filtered signal is compensated by a vertical translation to meet the requirement of same velocity-change, i.e.

$$A_1(t) = A(t)|_{\text{low-pass filtered by } f_L} + A_3 \quad (2a)$$

$$A_3 = \frac{1}{t_d} \left\{ \int_0^{t_d} A(t) dt - \int_0^{t_d} [A(t)|_{\text{low-pass filtered by } f_L}] dt \right\} \quad (2b)$$

where the ΔV of $A_3(t)$ equals the difference between the ΔV of $A(t)$ and the ΔV of the low-pass filtered curve; f_L is determined by

$$f_L = \frac{f_1 + f_2}{2} \quad (3)$$

where f_1 and f_2 are the nonzero frequencies corresponding to the highest and the next highest amplitudes from the FFT of OCS with satisfying the condition of $f_1 < f_2$. For example, the typical acceleration curve associated with a frontal barrier crash test of a vehicle is shown in Fig. 3(b) [38] and its corresponding FFT graph is plotted in Fig. 3(d), where $f_1 = 11$ Hz, $f_2 = 22$ Hz. Therefore, $f_L = 16.5$ Hz according to Eq. (3).

Fig. 3. (a) Typical acceleration curves associated with a vehicle crash where crash pulse-1 was extracted from a small four door sedan in [38] (the location of accelerometer was unspecified); crash pulse-2 was extracted from the unbelted occupant in [37]; crash pulse-3 was extracted from the B-pillar of a truck in [37]. (b)~(c) Typical acceleration curve associated with frontal barrier crash test of a vehicle [38]: OCS, the equivalent half-sine signal (EHSS) and RSS. (d) The fast Fourier transform (FFT) of the OCS shown in Fig. 3(b).

2.2 Characteristic frequency ratios

To compare the structural responses to OCS and ECS and to explore the effect of RSS, a single-degree-of-freedom (SDOF) model is employed. The product is idealised as a lumped mass (m) with elastic coefficient (k), damping coefficient (c) and natural frequency ($f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$), which is loaded by the acceleration-time history acting on the base, i.e. $\ddot{x}(t) = A(t)$. The responses of the SDOF, e.g., the absolute acceleration ($\ddot{y}$) and relative displacement ($z = y - x$), can be described by

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{x} \quad (4)$$

Solutions of Eq. (4) can be easily obtained by Duhamel integration, which will not be repeated here. According to structural dynamics [39], the transmissibility (TR) for a damped SDOF system subjected to a harmonic excitation is given by

$$TR = \left\{ \frac{1 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2} \right\}^{\frac{1}{2}} \quad (5)$$

where $\omega_n = 2\pi f_n = \sqrt{\frac{k}{m}}$; ω is the angular frequency of harmonic excitation; ξ is damping ratio ($c = 2\xi\omega_n$). The displacement transmissibility is identical to the acceleration transmissibility, which is plotted in Fig. 4 for $\xi = 0$ and 0.05 [39].

Fig. 4. Transmissibility of a SDOF system subjected to a harmonic base excitation.

As shown in Fig. 4, the transmissibility is zero as $\frac{\omega_n}{\omega} \rightarrow 0$, peaks at $\frac{\omega_n}{\omega} \approx 1$, and approaches unity as $\frac{\omega_n}{\omega} \rightarrow \infty$. Therefore, two characteristic frequency ratios, i.e. $\lambda_1 = \frac{\omega_n}{\omega_1}$ and $\lambda_2 = \frac{\omega_n}{\omega_2}$ can be determined by $\lambda_1 = \max\{\lambda \text{ with } \|TR-0\| < \varepsilon_1\}$ and $\lambda_2 = \min\{\lambda \text{ with } \|TR-1\| < \varepsilon_2\}$, respectively. Normally, a 5% relative accuracy is sufficient for structural design [3], and thus, $\lambda_1 \approx 1/5$ and $\lambda_2 \approx 5$ are obtained respectively from Fig. 4 when $\varepsilon_1 = \varepsilon_2 = 0.05$ are taken.

Considering an array of SDOF models subjected to the same base excitation, a SRS curve can be obtained. Fig. 5 presents the normalised absolute acceleration SRS (AASRS) curves driven from six typical ideal crash pulses, i.e. ESS, equivalent positive triangular signal (EPTS), equivalent negative triangular signal (ENTS), equivalent isosceles triangular signal (EITS), EHSS and equivalent haversine signal (EHVS) [37]. These six ideal crash signals have identical ΔV and t_d . The signal's effective frequency is defined as the lowest dominant frequency in this study, i.e. $f_s = f_1$ in Fig. 3(d) as an example. Newton [9] found that the AASRS for a given shape of the crash signal is approximately a straight line through the origin when $\frac{f_n}{f_s} < \frac{1}{6} \approx 0.167$. $0.167f_s$ is the lower

176 knee frequency on the SRS of OCS, which is close to the characteristic ratio of $\frac{1}{5} = 0.2$ obtained
 177 from Fig. 4 using 5% relative accuracy (i.e. the characteristic ratio value may change when the
 178 relative accuracy is changed). Signal's effective frequency and effective time duration are defined
 179 as $\frac{A_{max}}{\Delta V}$ and $\frac{\Delta V}{A_{max}}$, respectively, in [9,40] for given A_{max} and ΔV .

180
 181 **Fig. 5.** The normalised AASRS curves for an undamped SDOF system subjected to various crash signals.

182 To validate the characteristic ratios (λ_1 and λ_2), three types of SDOF models with relatively
 183 low, medium and high natural frequencies are discussed. The typical acceleration curve associated
 184 with the frontal barrier crash test of a vehicle in Fig. 3(b) is taken as the base excitation, whose
 185 effective frequency equals the reciprocal of time duration, i.e. $f_s = f_1 = \frac{1}{t_d}$ in this study (Fig. 3(d)).
 186 The corresponding results are tabulated in Table 1.

187 **Table 1.** Results of the SDOF models with relatively low, medium and high natural frequencies when subjected to
 188 base excitations ($\ddot{x}(t)$ =OCS, ECS and RSS, respectively) in Fig. 3(b)~(c).

Natural frequency, f_n (Hz)	$\frac{f_n}{f_s}$		Structural response			Relative difference between OCS and ECS
			from OCS	from ECS	from RSS	
2.146	$\frac{1}{5}$	$ \dot{y} _{max}$ [g]	13.8	13.8	0.215	0%
		$ z _{max}$ [mm]	549	548	8.08	0%
10.730	1	$ \dot{y} _{max}$ [g]	31.7	36.5	5.06	15.14%
		$ z _{max}$ [mm]	68.2	78.7	10.5	15.40%
53.650	5	$ \dot{y} _{max}$ [g]	42.7	23.5	26.1	-44.96%

$ z _{max}$ [mm]	3.67	2.0200	2.24	-44.96%
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From Table 1, it is obvious that the structural responses rely on the natural frequency of the structure, i.e. (i) when f_n is relatively low ($\frac{f_n}{f_s} \leq \frac{1}{5}$), negligible differences are observed between the results derived from OCS and ECS. For example, if $f_n = 2.146$ Hz or $\frac{f_n}{f_s} = \frac{1}{5}$, differences of $|\ddot{y}|_{max}$ and $|z|_{max}$ are both close to 0; (ii) when f_n equals f_s (or they are generally in the same order), obvious differences can be found between the results from OCS and ECS, e.g., if $f_n = 10.73$ Hz or $\frac{f_n}{f_s} = 1$, differences of $|\ddot{y}|_{max}$ and $|z|_{max}$ are 15.14% and 15.40%, respectively; (iii) when f_n is relatively high ($\frac{f_n}{f_s} \geq 5$), significant differences are captured between the responses resulted from OCS and ECS, e.g., if $f_n = 53.65$ Hz or $\frac{f_n}{f_s} = 5$, differences of $|\ddot{y}|_{max}$ and $|z|_{max}$ are both -44.96%.

It shows that OCS and ECS result in very similar responses in a SDOF model with relatively low frequency, no matter what the shape of crash signal is. It implies that ECS is able to represent the OCS for assessing the damage of structures with relatively low natural frequencies while the RSS has great effect on the responses of structures having relatively high natural frequencies, which demonstrates the necessities of the signal decomposition discussed in Section 2.1 and the determination of the characteristic frequency ratios. More accurate criteria for structural damage assessment can be established by employing the A_{max} - ΔV diagram and SRS graph with the consideration of characteristic frequency ratios, as shown in next section.

3. Structural damage boundary criteria

3.1 $f_n/f_s \leq 1/5$

As discussed in Section 2, ECS is able to represent the OCS well when $\frac{f_n}{f_s} \leq \frac{1}{5}$. The ESS-based A_{max} - ΔV diagram is introduced in this case to assess the structural damage. In general, structural damage is usually defined by an overall displacement failure criterion in p - i diagram [11], thus, the isodamage boundary in an A_{max} - ΔV diagram can also be determined by the maximum relative displacement (z_{max}). However, this does not prevent the use of other failure criteria if they are more representative and available, e.g., the damage boundary based on yield stress (σ_y) is presented in Section 4. There are three main advantages for the use of A_{max} - ΔV diagram with signal decomposition, i.e.

(i) $A_{\max}$ - ΔV diagram can be employed for quick assessment because it only requires the information of the crash signal (i.e. no structural information is needed). If an isodamage boundary in the form of $A_{\max}$ - ΔV diagram is obtained, the crash signal is represented by a point ($A_{\max}$, ΔV) on the diagram. Then, it is easy to judge whether the point falls in the damage region or non-damage region.

(ii) It is usually hard to replicate OCS in laboratory. However, it is relatively easier to generate the filtered ECS by available equipment (e.g., shaker table), which can produce the same damage level as that produced by OCS.

(iii) Although SRS can also be used to assess the structural damage [20], and a particular relationship between SRS and $A_{\max}$ - ΔV diagram can be derived for a given signal (e.g., rectangular signal in [9]) based on SDOF, $A_{\max}$ - ΔV diagram is not restricted to SDOF. Like a p - i diagram, $A_{\max}$ - ΔV diagram derived using SDOF model can be applied to multi-degree-of-freedom (MDOF) models and real structures in simulations, experiments and practical loading scenarios. The $A_{\max}$ - ΔV diagram based on MDOF and simulation is discussed in Section 4.

According to Eq. (4) with damping neglected, $z_{\max}$ depends on parameters (m , k , $A_{\max}$, and t_d) for a given crash signal (i.e. acceleration-time history herein) when $z_{\max} < z_{cr}$. z_{cr} denotes the critical relative displacement of a structure associated with a given damage level. By following the same procedure for the derivation of p - i diagram [11], two non-dimensional quantities are defined to determine the normalised maximum relative displacement, $z_{\max}/z_{cr}$.

$$a = \frac{mA_{\max}}{z_{cr}k} \quad (6a)$$

$$v = \frac{\Delta V}{z_{cr}\sqrt{k/m}} \quad (6b)$$

For a given z_{cr} associated with a prescribed structural damage level, there is

$$g(a, v) = \frac{z_{\max}}{z_{cr}} = 1 \quad (7)$$

which defines all points, (a , v), on an isodamage curve for the given damage level prescribed by the critical relative displacement. The ideal isodamage boundary derived from a monotonically-decreasing crash signal is presented in Fig. 6 to illustrate the general characteristics of the a - v diagram.

Fig. 6. The ideal isodamage boundary on the a - v diagram.

The isodamage boundary has two asymptotes (a_{cr} and v_{cr} , as normalised critical maximum acceleration and critical velocity change), and can be divided into three regimes, i.e.

- Regime I: $a > a_1$, $v \approx v_{cr} = \lim_{a \rightarrow \infty} [g(a, v) = 1]$;
- Regime II: $a \leq a_1$ and $v \leq v_1$, structural damage is controlled by both a and v , and
- Regime III: $v > v_1$, $a \approx a_{cr} = \lim_{v \rightarrow \infty} [g(a, v) = 1]$.

a_1 and v_1 are the threshold values to distinguish Regimes I, II, and III on the a - v diagram, which are determined by accuracy requirements of $v_1 = \min \{v \text{ with } ||v - v_{cr}|| < \varepsilon_1\}$ and $a_1 = \min \{a \text{ with } ||a - a_{cr}|| < \varepsilon_2\}$, respectively. Again, we take 5% relative accuracy, i.e. $\varepsilon_1 = \varepsilon_2 = 0.05$. For an undamped SDOF system subjected to ESS, the ratio between a_1 and a_{cr} equals 2 [40] and the ratio between v_1 and v_{cr} equals $\frac{\pi}{2}$ [9]. Regimes I, II and III represent impulsive, dynamic and quasi-static regimes, respectively [11].

Furthermore, Eq. (7) for ESS leads to $(\Delta V)_{cr} = \frac{1}{\pi} \frac{A_{max,cr}}{f_n}$ [9], or

$$v_{cr} = \frac{A_{max,cr}}{2\pi^2(f_n)^2 z_{cr}} \quad (8)$$

when Eq. (6b) is applied where $A_{max,cr}$ and $(\Delta V)_{cr}$ are respectively the critical maximum acceleration and critical velocity change of the A_{max} - ΔV diagram. If the average acceleration is introduced as

$$A_{avg} = \frac{\Delta V}{t_d} = \frac{\int_0^{t_d} A(t) dt}{t_d} \quad (9)$$

260 Then the safe region in Regime I, i.e. $\frac{v}{v_{cr}} \leq 1$, is equivalent to

$$261 \quad A_{avg} \leq \left(\frac{1}{\pi}\right) \cdot A_{max,cr} \cdot \left(\frac{f_n}{f_s}\right)^{-1} \quad (10)$$

262 Since the asymptote $v = v_{cr}$ of the a - v diagram in Regime I applies to different crash signals
263 [39,40], the ideal ESS can be used to determine v_{cr} . In addition, A_{avg} is the same as A_{max} for
264 ESS. Therefore, under the condition of $\frac{f_n}{f_s} \leq \frac{1}{5}$, no damage assessment is required if the A_{avg} of

$$265 \quad \text{OCS is lower than } \left(\frac{1}{\pi}\right) \cdot A_{max,cr} \cdot \left(\frac{f_n}{f_s}\right)^{-1}.$$

266 The A_{max} - ΔV diagram or the normalised a - v diagram introduced here can still be adopted in
267 the following section, but its effectiveness is questionable as shown in Section 4. Besides, the SRS
268 can also be used when $\frac{f_n}{f_s} \leq \frac{1}{5}$ to construct the upper bound of damage boundary, which will be
269 introduced in the following section where the dual damage criteria are proposed.

270 **3.2 $f_n/f_s > 1/5$**

271 As shown in Table 1, the high frequency oscillations in OCS have significant influence on the
272 responses of structures having relatively high natural frequencies. Obvious differences could be
273 observed between the structural responses derived from OCS and ECS, i.e. ECS is unable to
274 represent OCS effectively. SRS is a widely adopted method for the evaluation of the severity of
275 shock environment. SRS is defined as the maximum responses of a series of SDOF systems having
276 the same damping, but different natural frequencies, to a given transient shock signal. When $\frac{f_n}{f_s} > \frac{1}{5}$,
277 A_{max} - ΔV diagram can still be used for the assessment of damage under certain conditions while SRS
278 should be introduced to supplement the assessment of damage caused by high frequency oscillatory
279 shock effect. Therefore, dual damage criteria by combining A_{max} - ΔV diagram and SRS graphs
280 should be considered

281 The a - v diagram with z_{cr} damage level derived from six ideal crash signals is presented in Fig.
282 7. It can be found that the rectangular shaped signal, i.e. ESS, forms the most conservative boundary
283 in Regimes II and III, because ESS defines the lowest A_{max} for given ΔV on a - v diagram comparing
284 with crash signals having other shapes. In addition, the A_{max} of a measured base excitation (i.e. the
285 crash signal) may be influenced by other factors, e.g., noise, sampling frequency and signal filtering.
286 Therefore, A_{max} - ΔV diagram (or a - v diagram) based on ESS was recommended as a conservative
287 damage diagram in packaging industry [13]. It should be noted that the maximum responsive
288 acceleration of the SDOF system is approximately the maximum acceleration of the base excitation

for high-frequency structures [20], which reduces the conservative extent of the ESS-based $A_{\max}$ - ΔV diagram. The advantage of ESS-based $A_{\max}$ - ΔV diagram is that it can be adopted for the initial quick assessment. If the base excitation, represented by a point $(A_{\max}, \Delta V)$ or (a, v) , falls into the non-damage region, then the structure is safe. Otherwise, further damage assessment with the aid of SRS method is necessary.

Fig. 7. The normalised a - v diagrams of six ECSs with various shapes.

SRS is currently the standard method for quantifying the severity of mechanical shock events. It uses the responses of standardised structures (SDOF oscillators) to the base excitation for the assessment of shock severity rather than using the base excitation signal directly. As it is mentioned in Section 3.1, it is usually hard to reproduce the complex OCS (see Fig. 3(a)) in a laboratory. Thus, establishing the damage boundary in SRS graph based on the exact OCS is nearly impossible. But the decomposed smooth ECS whose ΔV is identical to that of OCS, and the oscillating RSS whose ΔV is zero, can be fairly easily reproduced without losing their shock features and be repeated in laboratory. According to the distributive law of convolution, the SRS damage boundary based on the critical loadings associated with ECS and RSS signals can give an upper bound of the SRS damage boundary for OCS, i.e.

$$\text{SRS}(\text{OCS}) \leq \text{SRS}(\text{ECS}) + \text{SRS}(\text{RSS}) \quad (11)$$

The upper bound of SRS damage boundary will be shown later by examples in Section 4 (Fig. 16). Three types of SRS curves, i.e. pseudo-acceleration SRS (PASRS), pseudo-velocity SRS (PVSRs), and relative displacement SRS (RDSRS) (see [20]), for an undamped SDOF model where PASRS=AASRS have following relationships [20]

$$\lg(\text{PASRS}) - \lg(2\pi f_n) = \lg(\text{PVSRs}) = \lg(2\pi f_n) + \lg(\text{RDSRS}) \quad (12)$$

PVSRS can be plotted on a 4-coordinate graph according to Eq. (12) where the iso-RDSRS, iso-PVSRS and iso-PASRS lines have 45° , 0° , and -45° slopes, respectively. The upper bound of PVSRS damage boundary can be constructed as the summation of PVSRS curves derived from critical ECS and RSS signals (Eq. (11)), which are respectively obtained by varying the peak accelerations of ECS and RSS signals while keeping the time duration unchanged until structural damage occurs. The PVSRS can also be adopted in the situation when $\frac{f_n}{f_s} \leq \frac{1}{5}$ to construct the upper bound of damage boundary, although ECS is able to represent OCS well in this situation (see Section 4). The ESS-based S_c method in ASTM D3332-99 [13] may also be adopted here to construct the lower bound in PVSRS graph, which may reduce the conservativeness of ESS-based $A_{\max}-\Delta V$ diagram. This procedure will be followed in Section 4 when the damage boundaries are constructed for a fully clamped beam as an example. The PVSRS curves of six typical ideal ECSs on 4-coordinate graph are shown in Fig. 8. Both Figs. 5 and 8 show that ESS predicts the largest response, and therefore, gives the most conservative damage prediction when compared with other idealised ECS. It should be noted that other types of SRS may be selected for the S_c method, as indicated in Appendix.

Fig. 8. Normalised PVSRS of various crash signals on a 4-coordinate graph.

In brief, ECS is able to represent the OCS well when $\frac{f_n}{f_s} \leq \frac{1}{5}$. Particularly, no damage will occur if following two conditions are satisfied, i.e. (i) $\frac{f_n}{f_s} \leq \frac{1}{5}$, and (ii) $A_{avg} \leq \left(\frac{1}{\pi}\right) \cdot A_{max,cr} \cdot \left(\frac{f_n}{f_s}\right)^{-1}$. Otherwise, for $\frac{f_n}{f_s} > \frac{1}{5}$, the ECS cannot represent the OCS, and the OCS should be assessed. In either

case, it is easier and cheaper to reproduce the smooth ECS and oscillatory RSS signals in laboratory when their shock features are kept in comparison with the reproduction of more complex OCS signals. It should be mentioned that the method proposed here is applicable to the crash signals in a crash stop characterised by a finite velocity change and $A(t) \geq 0$, which covers most cases of vehicle crash and packaging requirements. In other words, the ΔV of the crash signal is a monotonically increasing function of time. Pyroshock, ballistic shock and navy shock signals are not dominated by velocity change, and therefore, are out of the scope of this study.

4. Validation of the damage boundary criteria

The method of dual damage criteria is proposed for various crash-stop incidents in package dropping and vehicle crash etc., as discussed in Section 1. A frontal barrier crash test of a vehicle [38] is taken to illustrate and demonstrate the signal decomposition and damage assessment. A cantilever beam with multiple natural frequencies is taken as a generic representative of the onboard product or occupant, which subjects base excitations ($\ddot{x}(t)$) from the vehicle (see Fig. 9(a)). The IEA and crushable structure, similar to the cushioning material in a package dropping senary, determine the OCS to the cantilever beam. The highly oscillatory signal in Fig. 3(b)~(c) is used as an example of base excitations, which has been decomposed into the smooth $A_1(t)$ (i.e. the EHSS with $\Delta V \neq 0$) and the oscillatory $A_2(t)$ (i.e. the RSS with $\Delta V = 0$) by following the procedure of signal decomposition in Section 2. The RSS illustrated in Fig. 3(c) inherits the high oscillation from OCS, because it is obtained by subtracting EHSS from OCS. The scaled crash signals (SCSs) are shown in Fig. 9(b)~(c), with their key parameters summarised in Table 2.

Two types of cantilever beams with relatively low ($\frac{f_n}{f_s} \approx \frac{1}{5}$) and high ($\frac{f_n}{f_s} \approx 5$) dominant natural frequencies are considered. The geometry and physical properties of these two cantilever beams are tabulated in Table 3 together with the lowest five natural frequencies for each beam. To judge the “significance” of a vibration mode, the modal participation factor (r) [41] is calculated and its values are tabulated in Table 3. Then, the f_n is taken as the structure’s dominant natural frequency having the highest value of modal participation factor. Therefore, $f_n = f_{n1}$ for both beam I and beam II in this study. To evaluate the potential damage of the beam due to the base excitation, the dual criteria are adopted to assess the damage of structures with different values of $\frac{f_n}{f_s}$. The $A_{\max}$ - ΔV diagram is employed to build the lower bound of damage boundary, while the PVSRS graph is chosen to construct the upper bound of damage boundary.

Fig. 9. (a) Illustration of a cantilever beam subjected to base excitation; (b)~(c) Scaled crash signals (SCSs) with the same shape as OCS in Fig. 3(b): (b) SCS-1 to SCS-4 that beam I is subjected to; (c) SCS-5 to SCS-8 that beam II is subjected to.

Table 2. Key parameters of SCSs in Fig. 9(b)~(c) ($f_s = 10.73$ Hz) .

Base excitation, $\ddot{x}(t)$ [g]	Scaled factor, α [-]	$A_{\max}$ [g]	t_d [s]	ΔV [9.8 m/s]
SCS-1	0.323	11.202	0.09324	0.431
SCS-2	1.4	48.615	0.09324	1.872
SCS-3	0.9	31.253	0.09324	1.203
SCS-4	1	34.725	0.09324	1.337
SCS-5	2.5	86.812	0.09324	3.343
SCS-6	7.0	243.075	0.09324	9.360
SCS-7	3.5	121.537	0.09324	4.680
SCS-8	4.5	156.262	0.09324	6.017

Table 3. Geometry and physical properties of two types of cantilever beams with relatively low and high values of

$$\frac{f_n}{f_s}$$

Geometry and physical properties								
Width $\times$ Length $\times$	Area moment of inertia, I_m	Density, ρ [kg/m ³]	Elastic modulus,	Yield stress, σ_y	Poisson's ratio, ν [-]	Viscous damping	Natural frequencies	Modal participation

	Thickness [mm ³]	[mm ⁴]		E_0 [GPa]	[MPa]		ratio, ξ [-]	[Hz]	factor [-]
Beam I	100 × 1950 × 10	8333.333	2770	71	280	0.30	0	$f_{n1} = 2.154$; $f_{n2} = 13.465$; $f_{n3} = 37.762$; $f_{n4} = 74.055$; $f_{n5} = 122.406$;	$r_1 = 1.82$; $r_2 = -1.01$; $r_3 = 0.59$; $r_4 = -0.42$; $r_5 = 0.33$;
Beam II	100 × 390 × 10	8333.333	2770	71	280	0.30	0	$f_{n1} = 53.858$; $f_{n2} = 336.615$; $f_{n3} = 944.052$; $f_{n4} = 1851.38$; $f_{n5} = 3060.14$;	$r_1 = 0.81$; $r_2 = -0.45$; $r_3 = 0.27$; $r_4 = -0.19$; $r_5 = 0.15$;

4.1 Beam I with $f_n/f_s \approx 1/5$

The first natural frequency of beam I is 2.154 Hz, i.e. $\frac{f_n}{f_s} \approx \frac{1}{5}$. Responses of the cantilever beam are tabulated in Table 4, including maximum absolute acceleration, maximum relative displacement and relative differences between the results derived from OCS and ECS. From Table 4, it can be found that the $|z|_{max}$ values derived from OCS and ECS are close to each other, although relatively larger difference is observed for $|\ddot{y}|_{max}$ values. Such observations are slightly different from the results of SDOF in Table 1. Since the damage boundary is generally controlled by the maximum relative displacement for low-frequency structures [20], the results in Table 4 support the proposed characteristic frequency ratio in Section 2.

Table 4. Results of the cantilever beam I with $\frac{f_n}{f_s} \approx \frac{1}{5}$ when subjected to base excitations in Fig. 3(b)–(c).

Natural frequencies of		Structural response			Relative difference
Beam I [Hz]		from OCS	from ECS	from RSS	between OCS and ECS
$f_{n1} = 2.154$;	$ \ddot{y} _{max}$ [g]	32.41	25.09	33.30	-22.59%
$f_{n2} = 13.465$;					
$f_{n3} = 37.762$;					
$f_{n4} = 74.055$;	$ z _{max}$ [mm]	863.40	873.90	15.80	1.22%
$f_{n5} = 122.406$;					

Structural damage (including failure) is usually determined by a local damage criterion based on critical stress (strain), or an overall damage criterion based on critical acceleration or displacement at a selected reference point of a structure. It should be noted that the material damage

associated with microscale mechanisms is not considered. To build the damage boundary, the yield stress (σ_y) of cantilever beam is defined as the critical stress, i.e. $\sigma_{cr} = \sigma_y = 280$ MPa. When anywhere of the beam becomes yielded, the loading is defined as the critical loading (i.e. critical crash signal). To construct the conservative isodamage boundary, the base excitation in the form of rectangular shaped pulse is varied by changing its time duration (t_d) and magnitude (A_{max}).

The finite element analysis is carried out in commercial FE code ANSYS®. The number of solid elements of beam I is 600, as shown in Fig. 10(a). One side of the beam is fixed and subjects to base excitation while the other side is free (see Fig. 9). A bilinear curve is used to model the elastic-plastic behaviour of metallic material (see Table 3). The mesh sensitivity convergence is examined using different meshes, and the results show that the displacement of the free end has only 0.9% relative difference when the element number is doubled. Therefore, the current mesh scheme is sufficient for the analysis. The FE model of this cantilever beam can be validated by theoretical analysis with Matlab code [42,43]. As shown in Fig. 10(b), high consistency is observed in the relative displacement derived from FE modelling and theoretical calculation, respectively, when cantilever beam I subjects to the OCS of Fig. 3(b).

Fig. 10. (a) Finite element model of cantilever beam I; (b) the relative displacement derived from FE modelling and theoretical calculation, respectively, when cantilever beam I subjects to the OCS of Fig. 3(b).

(a) The lower bound of damage boundary based on A_{max} - ΔV diagram

The isodamage boundary associated with the critical stress is obtained by numerical simulation and plotted in Fig. 11 for rectangular crash signal as a lower bound of the A_{max} - ΔV diagram. This isodamage boundary can be divided into following three regimes,

- Regime I: $A_{max} > 9.0g$, structural damage is controlled by both A_{max} and ΔV , but $\Delta V = (\Delta V)_{cr} \approx 1.0 \times (9.8 \text{ m/s})$ obtained from Eq. (8) offers a lower bound, which is quite close to the iso-damage boundary in this regime;
- Regime II: $A_{max} \leq 9.0g$ and $\Delta V \leq 1.57 \times (9.8 \text{ m/s})$, structural damage is controlled by both A_{max} and ΔV , and
- Regime III: $\Delta V > 1.57 \times (9.8 \text{ m/s})$, $A_{max} = A_{max, cr} \approx 4.5g$.

The OCS, EHSS (EHSS is used as ECS shown in Fig. 3(b)) and RSS of SCS-1 in Table 2 are shown by three points in Fig. 11, which fall into the non-damage region. In other words, the crash pulse, whose average acceleration is lower than $\left(\frac{1}{\pi}\right) \cdot A_{max, cr} \cdot \left(\frac{f_n}{f_s}\right)^{-1}$, causes no damage to a cantilever beam with $\frac{f_n}{f_s} \approx \frac{1}{5}$. The OCS and ECS of SCS-2 to SCS-4 in Table 2 are also marked in Fig. 11, which fall into the damage region.

Fig. 11. The A_{max} - ΔV diagram derived from rectangular crash signal for beam I with $\sigma_{cr} = 280 \text{ MPa}$,

where the scaled crash signals are marked as points.

(b) The upper bound of damage boundary based on the PVSRS graph

The upper bound of damage boundary is constructed by employing the PVSRS graph. The critical crash signals with the same shapes as ECS and RSS causing structural failure can be obtained by varying the coefficients of magnitudes until the σ_{max} equals failure stress of 280 MPa. It should be mentioned that the time duration remains unchanged to keep $\frac{f_n}{f_s} = 5$. The upper bound of damage boundary is calculated as the summation of PVSRS curves derived from EHSS-like and RSS-like critical loadings, as shown in Fig. 12.

Fig. 12. PVSRS curves of the upper bound of damage boundary of beam I and various SCSs on a 4-coordinate graph.

The critical PVSRS curve derived from ESS-shape critical loading, i.e. S_c , is also plotted in Fig. 12. But, it should be noted that the overall lower bound of damage boundary is still defined by the ESS-based A_{max} - ΔV diagram (shown in Fig. 11) for two main reasons, i.e. (i) the A_{max} - ΔV diagram is not restricted to SDOF model, but can be established from simulations or experiments; (ii) the A_{max} - ΔV diagram directly based on crash signals is easier for quick assessment. Therefore, the procedure of damage assessment is: (i) if the crash signal represented by a point (ΔV , A_{max}) falls in the no damage area of ESS-based A_{max} - ΔV diagram, then the structure is safe; (ii) if the crash signal falls in the damage area of A_{max} - ΔV diagram, then the PVSRS graph is needed for further assessment. Under this condition, the structure is still safe if the PVSRS of crash signal is lower than S_c [13], while the structure is damaged if the PVSRS of crash signal is higher than the upper bound of damage boundary. The crash signal, whose PVSRS is above S_c while below the upper bound of

damage boundary (see the inset of Fig. 12), may or may not cause damage to the structure. The critical PVSRS curve derived from OCS-shape critical loading is also plotted in Fig. 12, which is located between the upper bound of damage boundary and S_c , as expected. According to the S_c method in [13] (see Appendix), structural damage occurs when the PVSRS of a given crash signal at $f_n/f_s = 0.2$ or in the range of 0.5-2 times the $f_n/f_s = 0.2$ is beyond the upper bound of damage boundary on PVSRS graph. The critical PVSRS at $f_n/f_s = 0.2$, i.e. the PVSRS value of the upper bound damage boundary at $f_n/f_s = 0.2$, is $PV_{cr} = 0.857$ m/s, as shown in Fig. 12.

The time histories of the maximum equivalent stress, i.e. von-Mises stress, for beam I under excitations of SCS-1 and SCS-4 are plotted in Fig. 13(b). The damage conditions evaluated by A_{max} - ΔV diagram, PVSRS graph and FE simulation are summarised in Table 5. For example, the maximum stress under SCS-1 is 95.97 MPa (< 280 MPa), which is consistent with the assessment based on both A_{max} - ΔV diagram and PVSRS graph.

Fig. 13. The maximum equivalent stress for beam I: (a) contour plot for SCS-1; (b) time histories for SCS-1 to SCS-4.

Table 5. Results of the cantilever beam I with $\frac{f_n}{f_s} \approx \frac{1}{5}$ subjected to different SCSs.

Base excitation, $\ddot{x}(t)$	Damage boundary		FE simulation
	Lower boundary	Upper boundary	
	($A_{\max}-\Delta V$)	(PVSRS)	
SCS-1	No damage	No damage	No damage
SCS-2	Damage	Damage	Damage
SCS-3	Damage	No damage	No damage
SCS-4	Damage	No damage	damage

4.2 Beam II with $f_n/f_s \approx 5$

The first natural frequency of beam II is 53.858 Hz, i.e. $\frac{f_n}{f_s} \approx 5$. Responses of the cantilever beam are tabulated in Table 6, which reveals that the $|\ddot{y}|_{\max}$ and $|z|_{\max}$ derived from OCS and ECS are significantly different, although their relative difference values are slightly different from the results of SDOF in Table 1.

Table 6. Results of the cantilever beam II with $\frac{f_n}{f_s} \approx 5$ when subjected to base excitations in Fig. 3(b)~(c).

Natural frequencies of		Structural response			Relative difference between OCS and ECS
Beam II [Hz]		from OCS	from ECS	from RSS	
$f_{n1}=53.858$;	$ \ddot{y} _{\max}$ [g]	67.71	24.11	55.77	-64.39%
$f_{n2}=336.615$;					
$f_{n3}=944.052$;	$ z _{\max}$ [mm]	5.72	3.15	3.53	-44.98%
$f_{n4}=1851.383$;					
$f_{n5}=3060.138$;					

(a) The lower bound of damage boundary based on $A_{\max}-\Delta V$ diagram

The yield stress (σ_y) of the cantilever beam is also defined as the critical stress, i.e. $\sigma_{cr} = \sigma_y$, to determine the lower bound of damage boundary based on $A_{\max}-\Delta V$ diagram. To construct the conservative isodamage boundary, the base excitation of rectangular crash signal is varied by changing its time duration (t_d) and magnitude ($A_{\max}$). As shown in Fig. 14, the isodamage curve with critical stress obtained by numerical simulation is plotted in the $A_{\max}-\Delta V$ diagram, where an isodamage boundary is divided into following three regimes,

- Regime I: $A_{\max} > 226g$, structural damage is controlled by both $A_{\max}$ and ΔV , but $\Delta V = (\Delta V)_{cr} \approx$

1.0×(9.8 m/s) obtained from Eq. (8) offers a quite close lower bound of the iso-damage boundary in this regime;

- Regime II: $A_{max} \leq 226g$ and $\Delta V \leq 1.57 \times (9.8 \text{ m/s})$, structural damage is controlled by both A_{max} and ΔV , and

- Regime III: $\Delta V > 1.57 \times (9.8 \text{ m/s})$, $A_{max} = A_{max, cr} \approx 113g$.

The positions of crash signals (i.e. SCS-5 to SCS-8) in A_{max} - ΔV diagram are shown in Fig. 14.

The damage conditions are summarised in Table 7 and will be analysed later.

Fig. 14. The A_{max} - ΔV diagram derived from rectangular crash signal for beam II with $\sigma_{cr} = 280 \text{ MPa}$.

(b) The upper bound of damage boundary based on PVSRS graph

Similarly to the procedure in Section 4.1(b), the critical crash pulses with the same shapes as ECS and RSS causing structural failure can be obtained by varying the coefficients of magnitudes until the σ_{max} equals 280 MPa, as shown in Fig. 15. Then, the upper bound of damage boundary and S_c on PVSRS graph are constructed, which, together with the PVSRS curves of various crash signals, is shown in Fig.16. Same assessment procedure as that in Section 4.1(b) can be applied. The critical PVSRS at $f_n/f_s = 5$ is $PV_{cr} = 1.235 \text{ m/s}$.

Fig. 15. The critical loadings with the same shapes as EHSS and RSS, respectively, causing structural failure.

Fig. 16. PVSRS curves of the upper bound of damage boundary of beam II and various SCSs on a 4-coordinate graph.

(c) Analysis of scaled signals

Scaled signals (SCS-5 ~ SCS-8) in Fig. 9(c) and Table 2 are discussed to have a better understanding of the dual damage criteria. These SCSs are shown in $A_{\max}$ - ΔV diagram in Fig. 14, while their corresponding PVSRS curves are plotted in Fig. 16. In addition, the time histories of the maximum equivalent stresses in beam II derived from different SCSs are illustrated in Fig. 17. Results of assessment are summarised in Table 7.

Fig. 17. The time histories of the maximum equivalent stresses in beam II derived from SCS-5 to SCS-8.

Table 7. Results of the cantilever beam II with $\frac{f_n}{f_s} \approx 5$ subjected to different SCSs.

Base excitation, $\ddot{x}(t)$	Damage boundary		FE simulation
	Lower boundary	Upper boundary	
	($A_{\max}-\Delta V$)	(PVSRS)	
SCS-5	No damage	No damage	No damage
SCS-6	Damage	Damage	Damage
SCS-7	Damage	No damage	No damage
SCS-8	Damage	No damage	damage

From Tables 5 and 7, we can obtain following conclusions:

- (a) The structure is safe if the crash signal parameters locates under the lower bound of damage boundary on $A_{\max}-\Delta V$ diagram. For $\frac{f_n}{f_s} \leq \frac{1}{5}$, the base excitation can be represented by its ECS.
- (b) When the crash signal paramters locate above the lower bound of damage boundary in $A_{\max}-\Delta V$ diagram, the structure is still safe if the PVSRS of the crash signal is lower than S_c while the structure is damaged if the PVSRS of the crash signal is above the upper bound of damage boundary.
- (c) The senarios outside senarios (a) and (b) are uncertain.

Overall, in addition to the existing standard testing methods in Appendix, the following

additional test procedure for complex crash signals is proposed, i.e. (i) the lower bound of damage boundary in $A_{\max}$ - ΔV diagram is constructed by following the same procedure in standard [13]; (ii) the OCS is decomposed to ECS and RSS by following the method of signal decomposition in Section 2.1; (iii) the magnitudes of ECS and RSS are scaled without changing the signal shape until the critical loadings that cause damage to the structure are obtained; (iv) PVSRS curves of critical ECS and RSS signals are calculated; (v) the upper bound of damage boundary in PVSRS graph as the summation of PVSRS curves of critical ECS and RSS signals can be derived; (vi) conduct the the assessment using dual damage criteria with $A_{\max}$ - ΔV diagram and PVSRS graph.

5. Conclusions

The assessment of structural damage in a crash-stop incident is investigated based on the decomposition of crash signal, in which the OCS is separated into a smooth ECS and an oscillatory RSS. The shock effects on the responses of structures with various frequency ratios ($\frac{f_n}{f_s}$) are analysed. The method of dual damage criteria is proposed by combining characteristics of the $A_{\max}$ - ΔV diagram and PVSRS graph, in which a lower bound of damage boundary is constructed in $A_{\max}$ - ΔV diagram while the PVSRS graph is used to assess the structural damage when the parameters of the crash signal is above the $A_{\max}$ - ΔV diagram. In addition to the S_c method recommended by ASTM D3332-99, an upper bound of damage boundary is proposed on PVSRS graph to further reduce the uncertainty of the damage assessment. It is found that the ECS is able to represent the OCS if the structure's natural frequency is relatively low compared with the effective frequency of the crash signal, i.e. $\frac{f_n}{f_s} \leq \frac{1}{5}$. Under this condition, no damage will occur if $A_{avg} \leq \left(\frac{1}{\pi}\right) \cdot A_{max,cr} \cdot \left(\frac{f_n}{f_s}\right)^{-1}$. Otherwise, for $\frac{f_n}{f_s} > \frac{1}{5}$, the ECS cannot represent the OCS, and the OCS should be assessed. In this case, the dual damage criteria on $A_{\max}$ - ΔV diagram and PVSRS graph can be applied to provide more accurate damage assessment for crash signals. This study underpins the mechanics principles to support further improvement of the relevant standards for the damage assessment of occupied structures subjected to crash loads.

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Appendix: Summary of ASTM D3332-99 (2016) [13]

This appendix summarises the test methods and damage boundaries in ASTM D3332-99 (2016) [13]. Two test methods (i.e. Test Method A and Test Method B) are introduced for the determination of the shock fragility of products. A damage boundary (i.e. the $A_{\max}$ - ΔV diagram) and a “critical” SRS plot (S_c) are adopted to assess the damage condition of products.

Test Method A is used firstly to determine the critical velocity change, $(\Delta V)_{cr}$, of the product in Regime I of the $A_{\max}$ - ΔV diagram. The procedure is described by: (a) Set the shock test machine so that the shock signal produced has a velocity change below the anticipated $(\Delta V)_{cr}$ of the product; (b) Perform one shock test and determine whether damage due to shock has occurred; (c) If no damage has occurred, set the shock test machine for a higher velocity change and repeat the shock test until product damage occurs. Thus, the $(\Delta V)_{cr}$ is obtained. Half-sine shock signals are normally used to perform this test since they are relatively easy to control.

Test Method B is used secondly to determine the critical acceleration, $A_{\max,cr}$, of the product in Regime III of the $A_{\max}$ - ΔV diagram. The procedure is described by: (a) Set the shock test machine so that it will produce a shock signal having a velocity change of at least $1.57 \cdot (\Delta V)_{cr}$ determined in Test Method A (see Fig. 7 for the position of $v = 1.57 \cdot v_{cr}$), while the maximum acceleration ($A_{\max}$) level of the first shock signal should be below the anticipated $A_{\max,cr}$; (b) Perform one shock test and determine whether damage due to shock has occurred; (c) If no damage has occurred, set the shock test machine for a higher $A_{\max}$ and repeat the shock test until product damage occurs. Thus, the critical shock loading and its corresponding $A_{\max,cr}$ are obtained. Trapezoidal shock signals with very short rise and fall times are normally used to perform this test since it can be easily generated in a laboratory to approximate an ideal ESS. Therefore, the $A_{\max}$ - ΔV diagram bounded by the critical velocity change and critical maximum acceleration for a given damage level can be established by following the above Test Methods A and B. The critical SRS curve (S_c) can be derived from the critical shock loading determined by Test Method B. Although AASRS is used in D3332-99, it realises ‘different types of SRS analyse, and therefore, other types of SRS are acceptable.

Product damage will not occur if (i) the shock signal represented by a point ($A_{\max}$, ΔV) falls in the no damage region in the critical $A_{\max}$ - ΔV diagram; or (ii) the SRS of the shock signal (or base excitation) lies below the S_c in a frequency range of approximately $\frac{1}{2}$ to 2 times the product’s (dominant) natural frequency (f_n).

Abbreviations

Abbreviation	Full name
AA	Absolute acceleration

AASRS	Absolute acceleration SRS
APV	Acceleration pseudo velocity
ASTM	American Society for Testing and Materials
ASTM D3332-99	Standard Test Methods for Mechanical Shock Fragility of Products
EA	Energy absorption
ECS	Equivalent crash signal
EHSS	Equivalent half-sine signal
EHVS	Equivalent haversine signal
EITS	Equivalent isosceles triangular signal
ENTS	Equivalent negative triangular signal
EPTS	Equivalent positive triangular signal
ESS	Equivalent square signal
FFT	Fast Fourier transform
HIC	Head injury criterion
IEA	Impact energy absorbers
MCF	Mean crushing force
MDOF	Multi-degree-of-freedom
OCS	Original crash signal
PA	Pseudo acceleration
PASRS	Pseudo-acceleration SRS
PCF	Peak crushing force
PV	Pseudo velocity
PVSRS	Pseudo-velocity SRS
RD	Relative displacement
RDSRS	Relative displacement SRS
RSS	Residual shock signal
SCSs	Scaled crash signals
SDOF	Single-degree-of-freedom
SEA	Specific energy absorption
SRS	Shock response spectrum
<i>TR</i>	Transmissibility

Notation	Description
$A(t)$	Acceleration-time history (or deceleration-time history)
$A_1(t)$	Equivalent crash signal
$A_2(t)$	Residual shock signal
a	Normalised maximum acceleration
A_{avg}	Average (or mean) acceleration
a_{avg}	Normalised average acceleration
a_{cr}	Normalised critical maximum acceleration
A_{max}	Maximum acceleration
$A_{max}\text{-}\Delta V$ diagram	Maximum acceleration-velocity change diagram
$A_{max,cr}$	Critical maximum acceleration
c	Damping coefficient
f_1	The nonzero frequency corresponding to the highest amplitude from the FFT of OCS
f_2	The nonzero frequency corresponding to the next highest amplitude from the FFT of OCS
f_L	Low-pass filter frequency
f_n	Natural frequency
f_s	Effective frequency
g	Standard gravity acceleration
I_m	Area moment of inertia
k	Stiffness
m	Mass
$p\text{-}i$ diagram	Pressure-impulse diagram
PV_{cr}	Critical pseudo velocity
r	Modal participation factor
S_c	Critical SRS
t	Time
t_d	Overall time duration of the crash signal
v	Normalised velocity change
v_{cr}	Normalised critical velocity change
$\ddot{x}(t)$	Acceleration-time history acting on the base of SDOF model
$\ddot{y}$	The output absolute acceleration of SDOF model
$z = y - x$	The relative displacement

z_{cr}	Critical relative displacement
z_{max}	Maximum relative displacement
ΔV	Velocity change
$(\Delta V)_{cr}$	Critical velocity change
λ_1, λ_2	Characteristic frequency ratios
ν	Poisson's ratio
ξ	Damping ratio
ρ	Density
σ_{cr}	Critical stress
σ_y	Yield stress
ω	The angular frequency of harmonic excitation
ω_n	The angular frequency of structure

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