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Betatron radiation diagnostics for AWAKE Run 2

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Abstract

AWAKE Run 2 aims to preserve the transverse normalised emittance of an externally injected electron beam throughout acceleration, requiring a new diagnostic to measure the beam emittance after the accelerating plasma stage. Spectroscopy of the betatron emission from the electron beam could be suitable for this case. The method of trace-space reconstruction from a measured betatron spectrum is described via a simplified analytical model in order to assess its suitability for the AWAKE experiment. The expected betatron emission from witness electrons at AWAKE is characterised using 3D quasi-static PIC simulations, revealing a measurable quantity of UV to soft x-ray emission. Practical challenges for the measurement are discussed.

Keywords: Accelerators, Beam driven plasma accelerators, Beam diagnostics, Radiation sources

1. Introduction

The AWAKE experiment [1] is a GVm⁻¹-class plasma wakefield accelerator (PWFA) at CERN. Electrons are injected into a 10-metre-long laser-ionised Rubidium plasma column from an adjacent RF linac and accelerated in a wakefield driven by a 400 GeV proton beam from the CERN SPS. A high-momentum proton driver facilitates large energy gains in a single stage of acceleration [2] and therefore reduces the complexity of a future collider based on plasma acceleration; other PWFA-based schemes, such as those driven with lepton beams, necessitate

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synchronisation and alignment between multiple plasma stages to overcome driver energy depletion and achieve beam energies useful for high energy physics experiments. Proof-of-concept experiments in AWAKE Run 1 have demonstrated electron acceleration in a proton-driven wakefield [3]. AWAKE Run 2 aims to preserve transverse electron beam emittance throughout acceleration and non-intercepting diagnostic systems to measure aspects of beam quality, such as the emittance of the accelerated electron beam, will therefore be central to the development of proton-driven PWFA. Betatron radiation has been shown to be a valuable single-shot, non-invasive diagnostic tool for laser-driven plasma wakefield accelerators (LWFAs): the average transverse beam size can be deduced from a measured critical frequency of betatron radiation and can also, when paired with an independent measurement of beam divergence and energy, be used to infer an uncorrelated normalised emittance [4–6]. Alternatively, the correlated emittance inside the plasma wakefield may be reconstructed with betatron and electron energy spectra alone, by analysing a betatron spectrum as the result of a convolution of single-particle orbits [7]. A diagnostic using the latter technique could be implemented for the AWAKE experiment in Run 2, and here we set out the suitability of this method with some initial diagnostic considerations. In section 2 an approximate analytical model of the betatron radiation from a generic beam of electrons is introduced to demonstrate how the correlated beam emittance is recovered from a betatron spectrum. Section 3 presents particle-in-cell simulations that estimate the quantity and properties of the betatron radiation foreseen in AWAKE Run 2. The effect of a varying transverse beam size on the betatron emission is shown, and requirements for the use of the reconstruction technique are described. Practical considerations for implementing a diagnostic system based on betatron radiation spectroscopy are described in section 4, and the number of detectable photons is estimated using the simulated emission.

2. Betatron Radiation

Electrons in ion cavities execute transverse, or betatron, oscillations and emit radiation with spectral characteristics determined by the distribution of single-electron orbits within a beam envelope [8]. With constant energy, and in the absence of radiation reaction, transverse electron motion in a fully blown-out ion channel within a plasma of electron density n_p is governed by the focusing force from the resulting wakefield. It works against the transverse electron momentum to produce sinusoidal betatron oscillations with a frequency $\omega_\beta = \omega_p / \sqrt{2\gamma}$ and

49 a period $\lambda_\beta = 2\pi/k_\beta \simeq 2\pi\sqrt{2\gamma}/k_p$, where $\omega_p = (n_p e^2/m_e \epsilon_0)^{1/2}$ and k_p is the wavenumber
 50 associated with the plasma wavelength. The electron Lorentz factor is γ and m_e , e and ϵ_0 are the
 51 electron mass, electron charge and vacuum permittivity respectively.

Figure 1: The transverse motion of an electron trapped in an ion channel, both in the case of constant energy (a) and with a non-zero accelerating field E_z (b).

52 If the electron energy varies, e.g. in the case of a non-zero accelerating field E_z , the equation
 53 of motion can instead be modelled as a parametric oscillator of slowly varying frequency. This
 54 results in a decaying amplitude and frequency of betatron motion as an electron gains energy,
 55 known as adiabatic damping. To illustrate this in the case of an electron accelerated at AWAKE,
 56 figure 1 shows the transverse motion of a single electron (initially with $\gamma_0 = 426$), trapped in an
 57 ion channel with $n_p = 7 \times 10^{14} \text{ cm}^{-3}$, for both constant energy and with a non-zero accelerating
 58 field $E_z = 0.14 E_0$ where $E_0 = m_e c \omega_p / e$ is the wave-breaking field and c the speed of light in
 59 vacuum. These parameters correspond to the baseline case for AWAKE Run 2. The strength
 60 of transverse oscillation is characterised by the parameter $\alpha_\beta = \gamma k_\beta r_\beta$, where r_β is the betatron
 61 amplitude. The ensemble average $\bar{r}_\beta$ is equivalent to the Gaussian beam radius σ_r . For $\alpha_\beta \gg$
 62 1 the electron motion results in a broadband spectrum that is synchrotron-like, emitted within
 63 an angle $\theta \sim \alpha_\beta / \gamma$ from the acceleration axis, and has a shape given by the universal function
 64 $S(\omega/\omega_c)$:

$$S(x) = x \int_x^\infty K_{5/3}(\xi) d\xi, \quad (1)$$

where $K_{5/3}$ is a modified Bessel function of the second kind. Such a spectrum falls exponentially after a critical frequency ω_c , which divides the spectrum into two equal parts and, for a relativistic electron in a pure ion column, may be expressed [9]

$$\omega_c = \frac{3}{2} \gamma^3 c r_\beta k_\beta^2. \quad (2)$$

The power radiated by an electron over all angles, averaged over one betatron period, is [10]

$$\langle P_\gamma \rangle = \frac{\pi e^2 c}{3\epsilon_0} \frac{\gamma^2 \alpha_\beta^2}{\lambda_\beta} = \frac{\pi e^2 c}{3\epsilon_0} \frac{\gamma^4 k_\beta^2 r_\beta^2}{\lambda_\beta}. \quad (3)$$

Dividing equation (3) by the average photon energy in a synchrotron spectrum $\hbar\langle\omega\rangle = (8/15\sqrt{3})\hbar\omega_c$, the number of photons emitted per electron, per betatron period N_β , and integrated over all angles is [10]:

$$\langle N_\gamma \rangle = \frac{5\sqrt{3}\pi}{6} \alpha \alpha_\beta, \quad (4)$$

where α is the fine structure constant.

2.1. Emittance Reconstruction

Using equations (1-4) a simple radiation model can be created. We start with a plasma of density $n_p = 1 \times 10^{18} \text{ cm}^{-3}$, within which a radially symmetric Gaussian electron beam of $\sigma_r = 1.325 \mu\text{m}$ propagates for one betatron oscillation in a fully blown-out ion channel. The radiating beam has $\gamma = 2000$ and contains 1 pC of charge (6.25×10^6 electrons). We assume zero energy spread $\sigma_\gamma = 0$, that $\sigma_r \approx r_{\beta,RMS}$ and $\alpha_\beta \gg 1$ for most $r_{\beta,i}$. The ion channel is generated by a separate drive beam, itself with a density n_d such that $n_d/n_p \gg 1$. With equation (1) a series of synchrotron spectra $S(\omega_j/\omega_{c,i})$ are generated, covering a range of frequencies ω_j and betatron amplitudes $r_{\beta,i}$, which each have a corresponding critical frequency $\omega_{c,i}$ given by equation (2). The resulting spectral distribution is shown in figure 2(a). Each spectrum $S(\omega/\omega_{c,i})$ is then assigned a number photons, according to its critical energy $\omega_{c,i}$, using equation (4), as shown in figure 2(b). This spectral distribution $W_{i,j}$ represents the number of photons emitted per electron, in one betatron oscillation, over a range of i betatron amplitudes, and j photon energies ($\hbar\omega_j$). A Gaussian distribution of betatron amplitudes $R(r_{\beta,i})$ with $r_{\beta,RMS} = 1.325 \mu\text{m}$ is used to populate $W_{i,j}$ and is shown in 2(c). Finally, when summed over all betatron amplitudes $r_{\beta,i}$, a total spectrum W_j is found according to equation (5) as seen in 2(d).

Figure 2: (a) The spectral distribution $S(\omega_j/\omega_{c,i})$; (b) the spectral distribution emitted per electron $W_{i,j}$, where $\Delta\hbar\omega = 10^{-3}\hbar\omega$ and $\Delta r_\beta = 10^{-3}r_\beta$; (c) the original and recovered Gaussian beam distribution $R(r_{\beta,i})$; and (d) the total spectrum W_j , the result of populating $W_{i,j}$ and summing over all betatron amplitudes, which has a critical energy $\hbar\omega_c = 3.78$ keV (dash-dotted line).

$$W_j = \sum_i R(r_{\beta,i}) W_{i,j} \quad (5)$$

89 The critical energy of W_j is $\hbar\omega_c = \hbar\langle\omega\rangle/(8/15\sqrt{3}) = 3.78$ keV, and the spectrum gives the
 90 number of photons in 0.1% bandwidth ($\Delta\hbar\omega = 10^{-3}\hbar\omega$) per photon energy $\hbar\omega$. The recovery
 91 of a beam profile R from an independently measured betatron spectrum Σ_j and electron energy
 92 spectrum Γ (not required for this model with $\sigma_\gamma = 0$) was demonstrated by Curcio et al. (2017)
 93 [7], by imposing that the total theoretical spectrum W_j is equal to Σ_j . Inverting equation (5) as

$$R(r_{\beta,i}) = \sum_j \Sigma_j W_{i,j}^{-1}, \quad (6)$$

94 enables the reconstruction of a beam profile R from a measured spectrum Σ_j , given an inverse
 95 matrix $W_{i,j}^{-1}$, where $W_{i,j}^{-1}$ is the Moore-Penrose pseudoinversion of the radiation model $W_{i,j}$. By
 96 considering the modified Lorentz factor due to the relativistic transverse motion, it is possible to
 97 approximate the divergence θ due to the betatron motion itself [7]:

$$\theta = \sqrt{\frac{\sqrt{1 + \frac{1}{2}\gamma^2 r_\beta^2 k_\beta^2}}{4\gamma}} r_\beta k_p. \quad (7)$$

With a series of weighted positions $x_i = R(r_{\beta,i})$, and corresponding angles $x'_i = \theta(r_{\beta,i})$, the correlated geometric r.m.s. emittance may be calculated via equation (8), where angle brackets represent the average over i values.

$$\varepsilon_{r.m.s.} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}. \quad (8)$$

Testing this procedure by setting $\Sigma_j = W_j$ recovers a beam with a transverse profile (also shown in figure 2(c)) which, if it had emitted the spectrum shown in 2(d), would have a geometric emittance $\varepsilon_{r.m.s.} = 2.37 \times 10^{-3}$ mm mrad and normalised emittance $\varepsilon_n \simeq \gamma \varepsilon_{r.m.s.} = 4.74$ mm mrad. Such a model could be used to reconstruct the transverse normalised emittance of any independently measured betatron spectrum Σ_j so long as the assumptions of the model are fulfilled, i.e. electrons emit in wiggler motion over one betatron period.

3. Simulated Betatron Emission

The three-dimensional quasi-static particle-in-cell (PIC) code QV3D [11], built on the framework of VLPL [12], is used in the presented simulation study. PIC simulations do not explicitly model synchrotron-like emission, as the short wavelengths cannot be resolved by the simulation grid. In QV3D, the emission is calculated analytically per particle, per timestep. Each macroparticle emits a synchrotron spectrum with a shape given by equation (1) along its trajectory, from which the transverse momentum change of the macroparticle in one timestep is calculated. This change in momentum determines how many photons of that spectrum are emitted. The emission (critical frequency, photon count and direction in polar angle θ and azimuthal angle ϕ) is saved, and the macroparticle momentum updated, which also allows radiation reaction to be modelled [13]. The raw spectrum from the code is saved as a critical photon spectrum $N(\omega_c)$; the total betatron spectrum $N(\omega)$ is reconstructed by convolving equation (1) with $N(\omega_c)$:

$$N(\omega) = \int_0^\infty S\left(\frac{\omega}{\omega_c}\right) N(\omega_c) d\omega_c. \quad (9)$$

3.1. AWAKE Run 2

In AWAKE Run 2, the two-dimensional transverse normalised emittance ε_n^* will be preserved at its injected value by matching the electron beam to the plasma, while the final relative energy spread $\sigma_{\gamma_f}/\langle\gamma_f\rangle$ will be kept to the %-level by beamloading the proton-driven wakefield. Beam parameters to meet these conditions were previously determined by Olsen et al. (2018) [14]. Matching requires an initial equilibrium transverse beam size that, for a given beam energy, will result in a preserved normalised emittance [15]. The electron beam drives its own local non-linear plasma wave in addition to the proton-driven wakefield in order to facilitate matching. The transverse dynamics of the electron beam are then somewhat independent from the quasi-linear wakefield driven by the proton beam, whose principle role is to provide longitudinal acceleration. In this section we reproduce the Run 2 model of Olsen et al. [14] and include betatron radiation emission in order to provide estimates for the number of detectable photons for a betatron radiation diagnostic at AWAKE. Simulations in QV3D were performed with cell sizes of $\Delta x = \Delta y = 0.01 c/\omega_p$ and $\Delta z = 0.1 c/\omega_p$ in the transverse and longitudinal directions respectively. Four particles per cell were used for the bulk plasma, which had an electron density $n_p = 7 \times 10^{14} \text{ cm}^{-3}$. The simulation domain had dimensions of $10 \times 5 \times 5 c/\omega_p$. To avoid simulation of the full proton beam a single short bunch, with transverse size $\sigma_{x,y} = 200 \mu\text{m}$, $\sigma_z = 40 \mu\text{m}$ that contains 1.46×10^{10} protons ($n_{pb} = 0.83 n_p$), is used to produce the accelerating field E_z . The proton beam energy was set with $\gamma_{pb} = 426$, the typical value on extraction from the CERN SPS. In the simulation the proton mass is increased by a factor of 10^6 in order to suppress the transverse evolution of the proton beam, and therefore eliminate any betatron radiation it might produce. The witness electron beam has $\gamma_{eb} = 426$, $\sigma_{x,y} = 5.25 \mu\text{m}$, $\sigma_z = 60 \mu\text{m}$ and contains 100 pC (6.25×10^8 electrons). With $n_{eb} = 34.3 n_p$ the electron beam is well within the blow-out regime. The initial ε_n^* and energy spread $\sigma_{\gamma_i}/\langle\gamma_i\rangle$ are 2 mm mrad and 0.1% respectively. In Figure 3 (a) the simulated emission integrated over all angles is shown with $\hbar\omega_c = 357 \text{ eV}$, containing a total of $N_e = 9.83 \times 10^8$ photons. Figures 3 (b-d) show the evolving beam parameters: energy spread saturates at less than 6%, while the normalised emittance remains close to the matched value; some emittance growth is seen, due largely to the erosion of the bunch head by interaction with the plasma. The angular betatron spectrum for $\theta > 1 \text{ mrad}$ from the acceleration axis z is shown in figure 4. In this baseline AWAKE Run 2 case, with $\sigma_{x,y} = 5.25 \mu\text{m}$, most emitted photons (96.9 %) are confined to small angles less than 1 mrad and have energies in the

Figure 3: The angle-integrated ($0 < \theta < 5$ mrad) betatron spectrum $N(\omega)$ (a) and electron beam γ , relative energy spread $\sigma_\gamma / \langle \gamma \rangle$ and transverse normalised emittance ϵ_n^* over 10 metres (b-d) for the baseline AWAKE Run 2 case with $\sigma_{x,y} = 5.25 \mu\text{m}$. The spectrum gives the number of photons in 0.1% bandwidth ($\Delta\hbar\omega = 10^{-3}\hbar\omega$) per photon energy $\hbar\omega$.

range 1-1000 eV. Above 1 mrad 3.1 % remain which, although a small fraction of the total, still provides a reasonable number of photons per 0.1% BW in absolute terms. These photons have lower energies in the range 1-100 eV. To determine the effect of larger beam radii on the emitted betatron spectrum, the baseline simulation was scanned between $\sigma_{x,y} = 5.25 \mu\text{m}$ and $\sigma_{x,y} = 40 \mu\text{m}$. The resulting angle-integrated spectra are shown in figure 5(a). According to equation (2), ω_c should scale linearly with r_β and equivalently with $\sigma_{x,y}$. However, the simulated spectra show only small changes in the critical energy $\hbar\omega_c$ over a range of different beam sizes. Two mechanisms limit the energy of emitted photons to produce the simulated spectra: firstly, due to adiabatic damping of $\sigma_{x,y}$ discussed in section 1, the beam sizes fall to less than $5 \mu\text{m}$ in all cases. Secondly, an increasing electron beam size results in a falling beam density n_{eb} and ratio n_{eb}/n_p which, since the electron beam blows out its own ion channel, results in an increasingly linear wakefield with weaker transverse forces to drive the betatron motion. With $\sigma_{x,y} = 40 \mu\text{m}$ the ratio falls to $n_{eb}/n_p = 0.59$, and $\hbar\omega_c$ is lower than the emission from the smaller $\sigma_{x,y} = 20 \mu\text{m}$ beam. Figure 5(b) shows that neither the number nor divergence of photons emitted into angles $\theta > 1$ mrad have a strong scaling with transverse beam size, although they do increase.

Figure 4: The total spectrum $N(\omega, \theta)$ distributed in polar angle $\theta > 1$ mrad for the baseline AWAKE Run 2 case with $\sigma_{x,y} = 5.25 \mu\text{m}$. This angular spectrum gives the number of photons in 0.1% bandwidth ($\Delta\hbar\omega = 10^{-3}\hbar\omega$) per photon energy $\hbar\omega$ per polar observation angle θ

Figure 5: The angle-integrated ($0 < \theta < 5$ mrad) spectra $N(\omega)$ for the AWAKE Run 2 baseline case scanned over beam size, from $\sigma_{x,y} = 5.25 \mu\text{m}$ to $\sigma_{x,y} = 40 \mu\text{m}$ (a). The spectrum gives the number of photons in 0.1% bandwidth ($\Delta\hbar\omega = 10^{-3}\hbar\omega$) per photon energy $\hbar\omega$. The number of photons emitted for $\theta > 1$ mrad (solid line) and the radiation divergence $\theta_{95\%}$ as a function of beam size $\sigma_{x,y}$ (dashed line) is shown in (b). $\theta_{95\%}$ is defined as the angle that contains 95% of the photons remaining above 1 mrad.

165 The one-dimensional beam profile cannot be reconstructed from these simulated spectra us-
 166 ing the simple analytical model presented in section 2. This is because the model does not
 167 account for evolving beam properties, and therefore cannot simulate betatron emission when it
 168 is integrated over multiple betatron periods. At AWAKE the electron beam will complete ~ 100
 169 betatron periods along the accelerator and the beam size and energy will continually evolve dur-
 170 ing this time. The combination of a relatively modest accelerating gradient and many betatron

oscillations means that the radiating electron beam contributes a significant portion of the final betatron spectrum well before it has reached its final energy, despite the strong scaling of radiated power with γ seen in equation (3). Therefore for the experimental measurement itself, a radiation model that accounts for beam evolution must be employed. This is an ongoing area of study. Electron orbits that correspond to undulator motion, i.e. $\alpha_\beta \ll 1$, should also be considered; for an AWAKE-like beam with $\sigma_r = 5.25 \mu\text{m}$ in an ion channel with $n_p = 7 \times 10^{14} \text{ cm}^{-3}$, a large fraction of the electrons in the 1σ beam core oscillate with $\alpha_\beta < 1$. Consequently, a single-particle model [8] that reproduces the spectrum under all regimes of electron motion should be implemented in order to accurately reconstruct the beam profile [7]. Furthermore, the electron beam energy spread following acceleration will have an appreciable impact on the final betatron spectrum, and the beam profile and emittance reconstruction require an independent electron beam energy spectrum.

4. Diagnostic Considerations

Betatron emission from LWFAs has been measured using a number of techniques. It can be measured directly after its parent beam has been removed [4], but may also be extracted and measured indirectly via a highly oriented pyrolytic graphite (HOPG) crystal [16]. The spectrum can be reconstructed with an x-ray charge-coupled device (CCD) in single-photon counting mode, or with a set of Ross filters. Betatron radiation from a PWFA has also been measured previously [17]. The AWAKE experiment is a challenging environment for measuring betatron radiation as a substantial x-ray background exists from upstream interaction of SPS protons with the beamline, and because the relativistic proton beam itself is rigid and cannot be transversely kicked to isolate the betatron radiation. Core and defocused protons have a divergence of 1 mrad on exiting the plasma and their presence inhibits a direct, on-axis measurement. However, as shown in figure 4, a significant remnant of the emission falls outside 1 mrad. A measurement station 5 m downstream from the accelerating plasma stage with a hole radius corresponding to such an angle, e.g. 5 mm radius at 5 m, would allow the proton beam to pass and enable a partial measurement of the emission, i.e. those photons having energies between 1-100 eV and falling in angles above 1 mrad.

A conceptual layout is shown in figure 6. A combination of conventional Al and multilayered mirrors are readily available to measure in the 1-100 eV range. Multilayered optics could be

Figure 6: Layout for a potential betatron radiation diagnostic for AWAKE Run 2. Electrons e^- are kicked by the energy spectrometer dipole, leaving protons p^+ and betatron radiation γ_β to propagate. An array of on-axis UV-coated Al or multilayered mirrors may be used to pick off betatron photons in the 1-100 eV range, to be detected by a UV-VUV CCD camera.

used to select certain photon energies or, if rotated according to Bragg's law, be used to build a spectrum. Ultraviolet (UV) photons with energies ≥ 3.1 eV cannot propagate in air and require vacuum, further complicating the measurement. The measurement would profit from being split into two phases. First a UV measurement between 200-400 nm wavelengths (< 3.1 eV) can be used to extrapolate a spectrum and estimate the beam size σ_r , and would confirm the observation of a significant quantity of betatron radiation. Second, a vacuum UV (VUV) measurement (3.1-100 eV) that provides spectroscopy in a larger range, improving the precision of the fitted spectrum and enabling correlated emittance reconstruction. A local background of transition radiation from strongly defocused protons (> 1 mrad) striking the UV/VUV optics would reduce the signal to noise ratio on each shot. However this could be addressed by characterising the signal in the absence of accelerated electrons. Finally, due to the x-ray background in the beamline, sensitive cameras should be positioned at some minimum distance from the beamline, which could further reduce the number of detectable photons. Using the photon emission from the baseline simulation ($\sigma_{x,y} = 5.25 \mu\text{m}$) in the previous section it is possible to estimate the number of detectable photons per unit area of sensor N_d from the total emitted, N_e :

$$N_d = gR_m\delta_\theta N_e, \quad (10)$$

where $\delta_\theta = 0.031$ is the fraction of photons falling above 1 mrad, determined in the simulations, and R_m is the fraction reflected by the mirror, assumed to be 0.6. The geometrical factor

$g = \delta\Omega/4\pi$ accounts for the loss of photons over a distance r between the mirror surface and the
 CCD sensor plane. The sensor with area A occupies a solid angle $\delta\Omega = A/r^2$. A VUV CCD cam-
 era such as an Andor Newton SO has a sensor area of 178.89 mm^2 , and a $26 \mu\text{m} \times 26 \mu\text{m}$ pixel
 size. From the baseline simulation $N_e = 9.83 \times 10^8$ photons, this camera positioned 200 mm
 from the reflecting mirror should be subject to $6.62 \times 10^3 \text{ photons mm}^{-2}$. With the native pixel
 area of $6.76 \times 10^{-4} \text{ mm}^2$ this may be too few photons to register above the noise floor of the CCD
 sensor. However, with 8×8 pixel binning, the pixel area may be increased to $4.33 \times 10^{-2} \text{ mm}^2$
 and the photons per pixel increased from ~ 5 to ~ 286 . Finally, assuming the CCD QE ~ 0.6
 one could expect ~ 172 detected photons per binned pixel in the range 1-100 eV. Pixel binning
 would result in a poorer sensor resolution of $\sim 200 \mu\text{m}$, although at 5 m downstream from the
 accelerator, the radiation divergence of $\theta = 2 \text{ mrad}$ for the baseline case, shown in figure 5(b),
 would produce a relatively large 10 mm spot radius. Despite the weak scaling of radiation prop-
 erties with $\sigma_{x,y}$ seen in simulations, larger beams sizes would still produce a broader distribution
 containing more photons, which would increase the number of detectable photons. Only 28 % of
 the total detectable photons have an energy $< 3.1 \text{ eV}$ in the baseline case, so a larger sensor may
 be required for the initial lower energy measurement.

5. Conclusions

The next phase of the AWAKE experiment aims to demonstrate the preservation of trans-
 verse beam emittance throughout acceleration. This requires a new emittance measurement after
 the accelerating plasma stage, which could be facilitated by betatron radiation spectroscopy in
 conjunction with the electron beam energy spectrum. The reconstruction of the one-dimensional
 electron beam transverse profile and emittance is possible with a simple betatron radiation model
 in the limit of full wiggler motion. However, owing to both the adiabatic damping of the elec-
 tron beam size over many betatron periods and quasi-undulator motion of electrons accelerated
 at AWAKE, a diagnostic reconstructing the beam emittance will require a single-particle beta-
 tron radiation model that takes account of the evolving beam properties over the 10 m plasma
 column. This is currently under development. The angular acceptance of the measurement is
 limited by the diverging proton beam, however quasi-static three-dimensional simulations show
 that a reasonable quantity of soft x-ray to UV betatron photons will be produced in AWAKE Run
 2 that will be measurable above 1 mrad and away from the proton beam. The number and spatial

distribution of photons will be relatively insensitive to the electron beam transverse size as the beam drives its own wakefield, which becomes increasingly linear and provides weaker focusing as the beam density falls. Due to the restrictions of the experimental environment UV-VUV (1-100 eV) spectroscopy could be the most promising method to partially measure the betatron spectrum and reconstruct the emittance of accelerated electrons, however issues such as camera positioning and background radiation suppression must be addressed.

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- AWAKE Run 2 aims to demonstrate the preservation of beam quality throughout acceleration
- New diagnostics are required to measure transverse emittance of the accelerated electron beam
- This article proposes using UV-VUV spectroscopy to reconstruct the electron beam emittance, and the reconstruction technique is described
- Quasi-static three-dimensional simulations with the particle-in-cell code QV3D are used to provide estimates for the radiation spectrum and spatial distribution for expected electron beam properties in AWAKE Run 2
- The number of detectable photons is estimated in order to inform the measurement

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Declaration of Interest Statement*Declaration of interests**

☐ ☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

None