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An Investigation into Finite Similitude for High-Rate Loading Processes: Advantages in Comparison to Dimensional Analysis and its Practical Implementation

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Abstract

Exploration of the behaviour of large-scale structures under impact and explosive loading can in principle be achieved by means of scaled experimentation. In practice however, scaled experiments can be unrepresentative of high-rate phenomena due to non-scalable strain-rate effects and altered material behaviours. Critical is proper experimental design but this is presently impeded by the inability of dimensional analysis to cater for the absence of exact similarity in high-rate loading scenarios.

This paper re-examines the place of high-rate scaled experimentation in the light of a new similarity theory founded on the distortion of space (called finite similitude), which has recently appeared in the open literature. The advantage of finite similitude over dimensional-analysis based techniques is demonstrated in the paper, which is able cope (to some extent) with the breakdown in similarity with an approximate approach called inexact-finite similitude. It is shown to be independent of loading type (unlike competing methods), which is a feature demonstrated through the analysis of a scaled plate subject to a free-air explosion.

The work presented here provides the first attempt to test the practical importance of the new theory through detailed experimental trials. The experimental investigations reaffirm the theoretical studies and reveal that the developed inexact-finite similitude theory can practically be used for predicting full-scale behaviour by conducting experimental tests on trial models of different sizes and even different materials. For this purpose, circular tubes subjected to high

loading rate axial impact using a gas-gun apparatus are chosen as case studies. It is shown that in most cases, scaled experimentation provides superior predictions of buckling shapes, shortenings, temporal response of axial force and peak loads in comparison with numerical simulation. Analytical outputs for mean force and peak load are also shown to be significantly inferior to those returned through scaled experimentation.

Keywords: scaling; finite similitude; high loading rate processes; experimental results.

1. Introduction

Despite the advances made in theoretical and numerical techniques, full-scale experimentation of huge structures such as trains, ships and aeroplanes is inevitable [1]. However, full-scale experimentation can impose high costs and can also be highly time consuming. It is common practice to complement large-scale studies with the application of developed theoretical analysis and advanced computer programs. The application of proven simulation methods undoubtedly leads to a significant reduction in the number and cost of full-scale experiments. Scaled experimentation provides an alternative approach involving the trialling of small-scale experiments, which can also lead to a substantial reduction in the requirement for full-scale trials [1]. Scaled experiments and simulation have their own pros and cons with each limited in their ability to replicate full-scale behaviours. Scaled experiments are known to lose similarity in practical settings and modern simulation requires experimental data for its implementation. The approaches are not in competition however and are complementary and it is through the improvement of each that better predictions can be achieved.

The focus in this paper is on scaled experimentation which can provide an alternative and somewhat different source of experimental data for simulation. Attention is restricted to the analysis and understanding of the response of structures when subject to impact loadings, which is pertinent to the design of safety-critical and safety-protection equipment. Impact loaded structures are known to be problematic for scaling since replica scaling (i.e. traditional scaling) leads to distorted scaled models due to the non-scalability of strain rate effects [2-4]. Attempts have been made to address this issue with the aim of improving the representability of scaled experiments. In particular, an alternative form of similitude (non-direct similitude) has been developed using a new set of dimensionless numbers based on impact mass, initial impact velocity and dynamic yielding stress [1, 5-7]. Non-scalable strain-rate effects were corrected

61 using correction factors for impact velocity and impact mass, respectively in refs. [1] and [5].
62 Both approaches led to good predictions of full-scale behaviours for relatively simple perfectly-
63 plastic case studies. Also, by correcting the initial impact velocity of perfectly-plastic cylindrical
64 shells subjected to axial impact [6], the behaviour of full-scale model was predicted to good
65 accuracy using small-scale models. Knowledge about the strain rates involved is required for
66 determining the correction factors in refs. [1] and [5], which is an acute source of weakness in
67 the presented methods. Similarly, a mean value for strain rate was used in ref. [6], which suffers
68 in a similar respect since determination of a mean value for a complex structure is not
69 practicable. The approach was applied in ref. [7] with use of the Norton-Hoff constitutive law in
70 which the velocity correction factor was obtained as a function of the dimensional scaling factor
71 and the exponent in the Norton-Hoff equation. This provided an exact match for rigid, perfectly-
72 plastic case studies [7]. Also, a new technique was proposed by Wei and Hu [8] in which apart
73 from adjusting the impact conditions, additional mass was added to components of the model to
74 balance the strain rate effects.

75 For dimensional scaling based on the methods presented in refs. [1, 3-7], identical material
76 properties are employed in both full-scale and small-scale models, which can be restrictive. This
77 can be due to reasons such as different material properties arising out of the manufacturing
78 processes for full-scale and small-scale models [9], manufacturing problems [10], and high cost
79 and experimental restrictions [10]. It is often required and indeed beneficial to allow different
80 materials to be used for full-scale and small-scale experiments. However, only a few studies in
81 impact mechanics have been done trialling scaling methods with different materials for full-scale
82 and small-scale experiments [9-15]. In refs. [10] and [13] for example different material
83 properties are applied to full-scale and small-scale models, with density, yield stress, strain
84 hardening and strain rate compensated by correcting the initial conditions such as mass and
85 velocity of the impactor. However, absent from refs. [9-13] is a definitive procedure for
86 incorporating different material properties including strain hardening and strain rate effects. In
87 an effort to provide a systematic approach to high-rate loading processes, a non-geometric
88 method is presented in ref. [14] based on the recently developed finite similitude theory [16].
89 Inexact-finite similitude was introduced in which different material properties were compensated
90 by correcting the initial conditions such as mass and velocity of impactor [14]. In addition, by
91 building on the approach employed in ref. [14] a method is presented in ref. [15] to account for

material failure under for high-rate loading. Different material properties of full-scale and small-scale models including density, yield stress, strain hardening, strain rate, thermal softening and also failure strains were considered by correcting the initial conditions such as mass and velocity of the impactor and initial temperature.

In addition to the above mentioned deficiencies for developed scaling methods based on dimensional analysis [9-13], it must be emphasised that they are predominantly restricted to impact loadings controlled by velocity. It is shown here that the developed scaling methods based on finite similitude are independent of loading type, which is demonstrated explicitly for plates subjected to free air explosion. The plates are scaled numerically using the recently developed inexact-finite similitude theory [14].

A major criticism of the scaling method presented in refs. [14-15] is that it has to-date not been validated experimentally. This paper aims to correct this deficiency and validate experimentally the developed inexact-finite similitude theory for high-rate loading processes. Axially impacted tubes are selected as case studies being important load bearing and energy absorbing elements. In particular, thin-walled tubes due to their lightness, high energy absorption capacity, long crushing length and high energy absorption to weight ratio, are one of the most important energy absorption systems, especially in automotive industries [17-18]. Thin-walled tubes are used in automotive industries as a solution to minimize the effect of impact on vehicle occupants and for the purpose of increased car safety [17]. The special significance of thin-walled tubes in protection of transportation systems has for the past two decades made their impact behaviour an important research area [17-21]. Another reason for their consideration here is the involvement of various deformation mechanisms including highly nonlinear plasticity and strain rate effects associated with high loading rate impact [17-21]. These considerations make tubes a particularly good candidate for examining the practicality of the proposed scaling method.

A brief review of finite similitude theory is presented in Section 2 followed by a description of inexact finite similitude theory in Section 3. The fundamentals of dimensional analysis are also considered in this section to gauge the criticality of different Pi groups. It is revealed that for impact mechanics there is little scope and different choices have the sole effect of setting the rate at which a scaled experiment is performed. Section 4 examines the steps necessary for the practical implementation of inexact-finite similitude. In order to demonstrate the relative ease of application of the scaling methodology, analytical models are considered in Section 5. The poor

predictive capability of the analytical approaches considered is highlighted in this section. To provide more evidence for independency of inexact-finite similitude theory from loading type, numerical results obtained from LS-Dyna finite-element models of plates subjected to free air explosion are presented in Section 6. The numerical and experimental results for tubes subjected to axial impact under different loading rates are presented in Section 7. Particular focus is on the prediction of tube shortening, buckling shape, axial force versus time and peak load by means of small-scale tube experiments. Both identical and different materials are investigated for scaled models for comparison with full-scale behaviour. Full-scale tubes are made of steel in this study and small-scale tubes which are scaled to half the size of the full-scale tubes are made of steel and copper materials. The work is complemented by numerical simulation (Abaqus models) in an attempt to provide digital representations of the full and small-scale experiments. This complimentary approach is achieved by adoption of the Johnson-Cook constitutive equation, whose coefficients are experimentally determined for the tube materials used in the experiments and details on this are provided in Appendix A.

2. A brief background of finite similitude theory

Finite similitude is somewhat unusual in that it is founded on a metaphysical mechanism that cannot be achieved practically. It is defined in terms of space distortion, where an artefact or piece of apparatus is imagined to be scaled by the mechanism of space contraction. Although space contraction cannot be done the mathematics of such a process can be interrogated and contrasted against experimental reality. Although imagined, this process nonetheless makes the effects of scaling explicit in contrast with the alternative competing approach of dimensional analysis. Scaling therefore is represented in simple terms by differential identity $d\mathbf{x}_{ps} = F d\mathbf{x}_{ts}$, which in component form is $dx_{ps}^i = F_j^i dx_{ts}^j$, where $F_j^i = \partial x_{ps}^i / \partial x_{ts}^j$ is both spatially and temporally invariant. The coordinate functions $\mathbf{x}_{ps}$ and $\mathbf{x}_{ts}$ pertain to two orthonormal inertial frames, where the subscripts make reference to the physical space (full scale) and trial space (small scale), respectively. The identity $d\mathbf{x}_{ps} = F d\mathbf{x}_{ts}$ captures both anisotropic and isotropic forms of scaling with the latter of particular interest here and represented by $d\mathbf{x}_{ps} = \beta d\mathbf{x}_{ts}$, where β is linear-length scalar and reflects the extent of the scaling involved. Elemental volume and area identities are immediately obtainable from this expression via Nanson's identities, i.e.

153 $dV_{ps} = \beta^3 dV_{ts}$ and $\underline{n}_{ps} d\Gamma_{ps} = \beta^2 \underline{n}_{ts} d\Gamma_{ts}$, where dV and $d\Gamma$ are elemental volume and area,
 154 respectively, and $\underline{n}$ is a unit normal vector. Another aspect important to scaled experimentation
 155 is process times, where it can be anticipated that processes in the two spaces run at different
 156 rates. This is captured by the differential relationship $dt_{ps} = h dt_{ts}$, where h is a positive scalar,
 157 which measures the extent of this difference.
 158 With the establishment of the mathematics of space scaling attention now turns to the governing
 159 physics in the two spaces and it is advantageous to adopt a particularly generic representation at
 160 this point and to use equations incorporating explicit reference to geometric measures (i.e.
 161 volume, area). This ensures that scaling effects are explicitly identifiable and all pertinent
 162 physical behaviour is reflected. Transport equations in their integral form provide the pertinent
 163 description and take the form

$$\frac{D^*}{D^* t_{ps}} \int_{\Omega_{ps}} \rho_{ps} \psi_{ps} dV_{ps} + \int_{\Gamma_{ps}} \rho_{ps} \psi_{ps} (\underline{v}_{ps} - \underline{v}_{ps}^*) \cdot \underline{n}_{ps} d\Gamma_{ps} = - \int_{\Gamma_{ps}} \underline{J}_{ps}^{\Psi} \cdot \underline{n}_{ps} d\Gamma_{ps} + \int_{\Omega_{ps}} \rho_{ps} b_{ps}^{\Psi} dV_{ps} \quad (1)$$

$$\frac{D^*}{D^* t_{ts}} \int_{\Omega_{ts}} \rho_{ts} \psi_{ts} dV_{ts} + \int_{\Gamma_{ts}} \rho_{ts} \psi_{ts} (\underline{v}_{ts} - \underline{v}_{ts}^*) \cdot \underline{n}_{ts} d\Gamma_{ts} = - \int_{\Gamma_{ts}} \underline{J}_{ts}^{\Psi} \cdot \underline{n}_{ts} d\Gamma_{ts} + \int_{\Omega_{ts}} \rho_{ts} b_{ts}^{\Psi} dV_{ts} \quad (2)$$

164 where ρ , ψ , $\underline{v}$, $\underline{J}$, b and $\underline{n}$ are density, physical field, velocity field, flux, source and unit
 165 normal on the boundary Γ of the control volume Ω , and where the star-temporal derivative
 166 $D^*/D^* t_{ps} = \partial/\partial t_{ps}|_{X_{ps}^*}$ (and $D^*/D^* t_{ts} = \partial/\partial t_{ts}|_{X_{ts}^*}$) is related to control volume velocity by the
 167 identity $\underline{v}_{ps}^* = D^* \underline{x}_{ps}^*/D^* t_{ps}$ (and $\underline{v}_{ts}^* = D^* \underline{x}_{ts}^*/D^* t_{ts}$).

168 Control volume motion is described by the diffeomorphism map $\underline{x}_{ps}^*(X_{ps}^*, t_{ps}): \Omega_{ps}^* \rightarrow \Omega_{ps}$ (and
 169 $\underline{x}_{ts}^*(X_{ts}^*, t_{ts}): \Omega_{ts}^* \rightarrow \Omega_{ts}$), which relates the reference control volume Ω_{ps}^* (and Ω_{ts}^*) and the
 170 current control volume Ω_{ps} (and Ω_{ts}) [22-23], where $\underline{x}_{ps}^*$ (and $\underline{x}_{ts}^*$) are control volume points.
 171 Eqs. (1) and (2) are presently unrelated but substitution of Nanson's identities for isotropic
 172 scaling into Eq. (1) gives

$$\begin{aligned} \frac{D^*}{D^* t_{ts}} \int_{\Omega_{ts}} \rho_{ps} \psi_{ps} \beta^3 dV_{ts} + \int_{\Gamma_{ts}} \rho_{ps} \psi_{ps} \beta^3 \left[h \beta (\underline{v}_{ps} - \underline{v}_{ps}^*) \right] \cdot \underline{n}_{ts} d\Gamma_{ts} \\ = - \int_{\Gamma_{ts}} h \beta^2 \underline{J}_{ps}^{\Psi} \cdot \underline{n}_{ts} d\Gamma_{ts} + \int_{\Omega_{ts}} \rho_{ps} \beta^3 h b_{ps}^{\Psi} dV_{ts} \end{aligned} \quad (3)$$

173 which is identical to Eq. (2) (after multiplication with α^ψ) if and only if the common integrands
 174 match, which provides the following identities:

$$\rho_{ts} \psi_{ts} = \alpha^\psi \beta^3 \rho_{ps} \psi_{ps} \quad (4a)$$

$$\underline{v}_{ts}^* = h \beta^{-1} \underline{v}_{ps}^* \quad (4b)$$

$$\underline{v}_{ts} = h \beta^{-1} \underline{v}_{ps} \quad (4c)$$

$$\underline{J}_{ts}^\psi = \alpha^\psi h \beta^2 \underline{J}_{ps}^\psi \quad (4d)$$

$$\rho_{ts} b_{ts}^\psi = \alpha^\psi h \beta^3 \rho_{ps} b_{ps}^\psi \quad (4e)$$

175 where the scalar α^ψ provides added generality to reflect the fact that proportional physics is
 176 being sought.

177 To achieve finite similitude in high loading rate processes the identities in Eq. (4) must be
 178 satisfied for the transport equations for volume, mass, momentum and movement. The transport
 179 equations for volume, mass, momentum and movement can be achieved according to Table 1 by
 180 setting the physical field, flux and source in Eq. (4).

181

182 **3. Inexact-finite similitude theory**

183 According to presented information in the previous section, scaling factors for high-rate loading
 184 processes are: dimensional scaling factor (β), mass scaling factor (α^ρ), momentum scaling
 185 factor (α^v) and time scaling factor (h). Momentum scaling factor is a dependent parameter
 186 since $\alpha^v = h \beta^{-1} \alpha^\rho$. Thus, only three of the introduced scaling factors are independent
 187 parameters (i.e. β , α^ρ and h), which means there are only three degrees of freedom. By using
 188 these three degrees of freedom, three of the scaled-model properties can be fixed on the intended
 189 properties (more information is presented in ref. [16]). It is appreciated however that high-rate
 190 loading processes involve many parameters including density, yield stress, strain hardening and
 191 strain rate. Moreover one of the three degrees of freedom is used for dimensional scaling leaving
 192 only two parameters that can be fixed on the intended properties. It is evident that exact-finite
 193 similitude is not possible for high-rate loading processes and an alternative approach is required
 194 where similitude is lost in a measured manner. The approach employed here is called inexact-
 195 finite similitude, which involves the weak enforcement of a critical similitude identity. It was

demonstrated in ref. [14] that by means of careful experimental design good numerical results could be obtained.

The details of inexact-finite similitude will now be examined in brief. Using the first degree of freedom, dimensional scaling factor (i.e. β) can be set to be equal with any value. The second degree of freedom (i.e. α^ρ) allows inertia effects to be captured (i.e. density is set by $\rho_{ts} = \alpha^\rho \beta^3 \rho_{ps}$). With the last scaling factor, the mean value of yield stress, which is a function of plastic strain and strain rate can be fixed for capturing the effects of static yield stress, strain hardening and strain rate. The method is presented in detail in refs. [14-15] and is founded on a weak representation of $h = \sqrt{(1/\alpha^\rho \beta)(Y_{ts}/Y_{ps})}$. In brief, substitution of $(\alpha^\rho)^{-1} = \beta^3 \rho_{ps} (\rho_{ts})^{-1}$ into $h = \sqrt{(1/\alpha^\rho \beta)(Y_{ts}/Y_{ps})}$ and setting $Y_{ts} = \sigma_{dts}$ and $Y_{ps} = \sigma_{dps}$ gives

$$h\beta^{-1} = \sqrt{\frac{\rho_{ps} \sigma_{dts}}{\rho_{ts} \sigma_{dps}}} \quad (5)$$

where subscript d refers to dynamic.

Eq. (5) is a similitude condition that cannot be practically met in rate-sensitive mechanics but the weaker version

$$\beta_v = \sqrt{\frac{\rho_{ps} \frac{1}{\varepsilon_{\max}^p \dot{\varepsilon}_{\max}} \int_0^{\varepsilon_{\max}^p} \int_0^{\dot{\varepsilon}_{\max}} \sigma_{dts}(\varepsilon^p, \beta \beta_v \dot{\varepsilon}) d\varepsilon^p d\dot{\varepsilon}}{\rho_{ts} \frac{1}{\varepsilon_{\max}^p \dot{\varepsilon}_{\max}} \int_0^{\varepsilon_{\max}^p} \int_0^{\dot{\varepsilon}_{\max}} \sigma_{dps}(\varepsilon^p, \dot{\varepsilon}) d\varepsilon^p d\dot{\varepsilon}}} \quad (6)$$

can be satisfied, where for convenience and to comply with common usage the velocity scaling parameter $\beta_v = h\beta^{-1}$ is introduced and note that $\underline{v}_s = \beta_v \underline{v}_{ps}$.

Note also that ε^p and $\dot{\varepsilon}$ respectively represents plastic strain and strain rate and $\varepsilon_{\max}^p$ and $\dot{\varepsilon}_{\max}$ are respectively maximum plastic strain and maximum strain rate. The maximum plastic strain and maximum strain rate can be determined using numerical simulation. The dynamic yield stress σ_d is assumed here to be a function of plastic strain and strain rate. The Johnson-Cook

constitutive equation is used for determining the dynamic yield stress and in the absence of thermal effects [16] takes the form

$$\sigma_d = \left(A + B(\varepsilon^p)^n \right) \left(1 + C \ln \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right) \quad (7)$$

where A is initial yield stress, $\dot{\varepsilon}_0$ is a reference strain rate and B , n and C are coefficients determined by experimental means.

An iterative method must be employed for calculating of velocity scaling factor β_v since it is present on both sides of Eq. (6). Note however, its calculation is dependent only on the materials selected for the full and trial scale experiments and broad conditions of the processes under investigation.

3.1. Comparison with dimensional analysis

Dimensional analysis has had significant success in the field of continuum mechanics and is directly applicable to governing partial differential equations. In impact mechanics the momentum equation is critical and takes the form

$$\rho \frac{Dv}{Dt} = \text{div}(\underline{\underline{\sigma}}) \quad (8)$$

where ρ is density, v is material velocity and $\underline{\underline{\sigma}}$ is the 2nd-order Cauchy stress tensor.

The dimension of Eq. (8) is $[\tilde{M} \tilde{L}^{-2} \tilde{T}^{-2}]$, where $\tilde{t}$, $\tilde{L}$ and $\tilde{M}$ are the symbols representing the dimensions of time, length and mass. The means to a dimensionless form of Eq. (8) is achieved by identifying variables and introducing scalars, i.e. set $x_i = \chi X_i$ and $t = \tau \Upsilon$ along with $v_i = \gamma V_i$ and $\sigma_{ij} = \varsigma \Sigma_{ij}$, where the upper case symbols are dimensionless in these relationships. Substitution of the scaled relationships into Eq. (8) provides

$$\rho \frac{\gamma}{\tau} \frac{DV}{D\Upsilon} = \frac{\varsigma}{\chi} \text{Div}(\underline{\underline{\Sigma}}) \quad (9a)$$

where the initial upper case letter on the differential operators indicate that these are now dimensionless.

237 Transforming this equation into dimensionless form is simply a matter of dividing through to
 238 provide a unit multiplier for any one of the operators, i.e.

$$239 \quad \frac{DV}{DY} = \frac{1}{\rho} \frac{\varsigma \tau}{\gamma \chi} \text{Div}(\underline{\Sigma}) \quad (9b)$$

240 which is dimensionless and provides the Pi group $(\rho \gamma \chi)^{-1} \varsigma \tau$.

241 The validity of Eq. (9) for a scaled model is assured if $(\rho \gamma \chi)^{-1} \varsigma \tau$ is invariant with scale and
 242 assuming that $\chi_{ps} \neq \chi_{ts}$ and $\tau_{ps} \neq \tau_{ts}$, it follows that $(\rho_{ps} \gamma_{ps} \chi_{ps})^{-1} \varsigma_{ps} \tau_{ps} = (\rho_{ts} \gamma_{ts} \chi_{ts})^{-1} \varsigma_{ts} \tau_{ts}$ is
 243 required. Note however that χ , τ and γ must be related since $\tau \chi^{-1} \gamma$ provides a dimensionless
 244 group, which arises on consideration of the identity $D/Dt = \partial/\partial t + \underline{v} \cdot \nabla$. This has the
 245 dimensionless form

$$246 \quad \frac{D}{DY} = \frac{\partial}{\partial Y} + (\tau \chi^{-1} \gamma) \underline{V} \cdot \text{Grad} \quad (10)$$

247 where setting $\tau \chi^{-1} \gamma = 1$ ensures that the identity is identical in both the physical and trial spaces.

248 Note then that $(\rho \gamma \chi)^{-1} \varsigma \tau = (\rho \chi^2)^{-1} \varsigma \tau^2$ is a dimensionless number contrasting inertial and
 249 applied forces and it follows that $(\rho_{ts} \chi_{ts}^2)^{-1} \varsigma_{ts} \tau_{ts}^2 = (\rho_{ps} \chi_{ps}^2)^{-1} \varsigma_{ps} \tau_{ps}^2$ is required or equivalently

$$250 \quad \frac{\varsigma_{ts}}{\varsigma_{ps}} = \frac{\rho_{ts}}{\rho_{ps}} \left(\frac{\tau_{ps}}{\tau_{ts}} \frac{\chi_{ts}}{\chi_{ps}} \right)^2 \quad (11)$$

251 The invariance of $\underline{\Sigma}$ requires that $\underline{\sigma}_{ts} = (\varsigma_{ts}/\varsigma_{ps}) \underline{\sigma}_{ps}$, which can be contrasted against the finite
 252 similitude identity $\underline{\sigma}_{ts} = \alpha^\rho h^2 \beta \underline{\sigma}_{ps}$, which on substitution of $\rho_{ts} = \alpha^\rho \beta^3 \rho_{ps}$ yields the exact
 253 same identity. The critical observation to be made here is that in impact mechanics, dimensional
 254 analysis does not provide significantly different choices depending on the Pi group selected, as
 255 suggested in references [10] and [13]. Once densities and scale are set, then α^ρ is determined
 256 from $\rho_{ts} = \alpha^\rho \beta^3 \rho_{ps}$ and the only thing that can be changed in $\underline{\sigma}_{ts} = \alpha^\rho h^2 \beta \underline{\sigma}_{ps}$ is h , which
 257 dictates the timing of the scaled experiment. The following statements can be made:

- (i) In finite similitude theory, the properties of scaled model which predicts the prototype behaviour with zero error is completely clear and the scaling relations are obtained by minimizing the properties of the trial model against the scaled model. However, using dimensional analysis the error source is unclear and this leads to different choices of Pi group.
- (ii) The large number of possible Pi groups (see ref. [13]) gives the impression that some choices are better than others but these choices can in effect mask the underlying physical effect in a scaled model. This can be contrasted against the finite similitude theory where the number of degrees of freedom is known and their physical meaning is clear.
- (iii) Contrary to dimensional analysis, the finite-similitude theory involves the properties of scaled model (including loading) determined by specifying the independent scaled parameters (here α^ρ , β and h). Since all loading types can be described by these independent scaling factors, the obtained finite similitude based techniques including inexact-finite similitude are independent of loading types. To support the claim, the numerical scaling results of plates subjected to free air explosion is provided in Section 6.

4. The practical application of inexact-finite similitude

The finite similitude approach has the ability to project results from the trial to the physical space and vice versa. Four models arise with the approach; two physical (full and trial experiments) and two virtual (projections of the full and trial experiments). The scaled model is the projection of the full-scale experiment and can be contrasted against a trial experiment. Similarly the reverse is possible and the trial experiment can be projected to contrast against the full-scale experiment. This feature of the similitude approach is a consequence of it being underpinned by a space distortion map. Although at first sight the theory appears somewhat involved its application is relatively straightforward and can be expressed by the following six steps:

- (i) Specify geometrical and material properties of the structure and also mass and velocity of the projectile (mass and distance of charge in explosive process) for the full-scale model.
- (ii) Specify the dimensional scaling factor (i.e. β) and the material of structure for the trial model.

- (iii) Calculate the density scaling factor ($\alpha^\rho = \rho_{ts} (\beta^3 \rho_{ps})^{-1}$) and the velocity scaling factor β_v by solving Eq. (6).
- (iv) Calculate the velocity and mass of the projectile for the trial model respectively using the velocity and mass relationships $v_{ts} = \beta_v v_{ps}$ and $M_{ts} = \alpha^\rho M_{ps}$ (mass and distance of charge in explosive process).
- (v) Conduct experimental tests on the trial model.
- (vi) Reverse the experimental results of the trial model to obtain the full-scale model results.

Prior to testing the efficacy of presented method with detailed numerical and experimental trials it is of interest to explore first simple analytical examples to illustrate the relative ease of application.

5. Scaling with analytical models

The usefulness of available analytical relations is investigated in the section for the purpose of predicting experimental results of axially impacted tubes. Moreover, the procedure detailed above for application of the scaling method is demonstrated on relatively simple analytical relations. Due to the involvement of various deformation mechanisms, highly nonlinear plasticity and strain rate effects with tubes subjected to high loading rate impact [17-21] there exists no comprehensive analytical method for predicting the behaviour of these structures [2]. However, two different approaches founded on energy [2, 24-25] and stress waves [21, 26-28] are available for predicting limited tube behaviour under high-rate axial impact. Analytical expressions for mean force and peak load are of interest here. Accordingly, both the energy and stress-wave approaches are examined in the context of scaling the axially impacted tubes.

5.1. Energy based model

An approximate analytical relation for rigid, perfectly-plastic axisymmetric tubes was obtained by Alexander [2, 24-25]. Referring to Fig. 1, the external mean force P_m transfers the external work $P_m \times 2l$ to the tube and leads to a completely flattened wrinkle, and is assumed to equate with the dissipated plastic work at the hinges (i.e. at points a , b and c in Fig. 1). By minimizing the obtained mean force (i.e. P_m) versus wrinkle length (i.e. l) it is possible to obtain a least

upper bound expression for P_m . The relationship for the approximation of mean force for tubes with wrinkles on the outside is

$$\frac{P_m}{M_0} = 29.31 \sqrt{\frac{R}{H}} + 6.28 \quad (12a)$$

where the plastic collapse moment M_0 for the cross-section is

$$M_0 = \left(\frac{2\sigma_0}{\sqrt{3}} \right) \frac{H^2}{4} \quad (12b)$$

and where σ_0 is the static yield stress, R and H respectively represents the mean radius and thickness of the tube.

At this point, dynamic effects have not being considered and consequently Eq. (12a) represents the mean force of a tube subjected to static loading. However, corrections can be made to Eq. (12a) to enable it to be used for approximating the mean force of tubes subjected to low-rate axial impact. Utilisation of the dynamic yield stress (i.e. $\sigma_d = \sigma_0 \left(1 + C \ln \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) \right)$ in place of σ_0 in Eq. (12) provides the modified expression

$$P_m = \frac{H^2 \sigma_0}{2\sqrt{3}} \left(1 + C \ln \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) + 29.31 \sqrt{\frac{R}{H}} + 6.28 \right) \quad (13)$$

which provides improved accuracy despite the neglect of inertial influences.

The strain rate in this relationship is typically set to be equal to a mean value (i.e. $\dot{\epsilon} = \frac{v}{L}$).

The modified relationship can be used for low rate impact loading with for strain rates typically lower than 0.10/ms [29]. An initial comparison of the result of Eq. (13) is made against the force versus time curve of a full-scale experiment (termed F-S Model 1 (which experiences outer folds); which is trialled in Section 7) and provides the result presented in Fig. 2(a). The details of F-S Model 1 are presented in Section 7.1 but in short it consists of a tube with an inner diameter equal to 28.60 mm, thickness of 2 mm and length equal to 60 mm and is impacted axially by a 1 kg mass with velocity 40 mm/ms. It is clear from Fig. 2(a) that there is a significant difference between the analytical and measured results. Despite the large differences

involved it is still of interest to apply the scaling procedure outlined in Section 4 in this analytical study.

1. Consider then a rigid perfectly plastic steel tube with thickness, mean radius and length of respectively equal with 2 mm, 15.30 mm and 60 mm which is impacted axially by a 1 kg striking mass having velocities 1–6 mm/ms (the steel's material properties are presented in Appendix A). The mean force applied to the tube is approximated by

$$P_{mps} = \frac{H_{ps}^2 \sigma_{0ps}}{2\sqrt{3}} \left(1 + C_{ps} \ln \left(\frac{\dot{\epsilon}_{ps}}{\dot{\epsilon}_0} \right) + 29.31 \sqrt{\frac{R_{ps}}{H_{ps}}} + 6.28 \right) \quad (14)$$

which is Eq. (13) but applied to the full-scale model.

2. With $\beta = 2$, the thickness, mean radius and length of trial tube respectively are obtained and are equal to 1 mm, 7.65 mm and 30 mm, respectively. Also, the trial tube material is selected to be copper with properties found in Appendix A.
3. The density scaling factor (i.e. $\alpha^\rho = (1/\beta^3)(\rho_{Cu}/\rho_{St})$) and velocity scaling factor (Eq. (6)) are calculated and equate to 142.90×10^{-3} and 0.66, respectively.
4. The velocity (i.e. $v_{0ts} = \beta v_{0ps}$) and mass ($M_{ts} = \alpha^\rho M_{ps}$) of the striking mass for the trial model equates to $v_{0ts} = 0.66 \times (1 \text{ to } 6) = 0.66 - 3.96$ mm/ms and $M_{ts} = 0.14 \times 1 = 0.14$ kg, respectively.
5. The mean force of trial model as a function of strain rate is

$$P_{mts} = \frac{H_{ts}^2 \sigma_{0ts}}{2\sqrt{3}} \left(1 + C_{ts} \ln \left(\frac{\dot{\epsilon}_{ts}}{\dot{\epsilon}_0} \right) + 29.31 \sqrt{\frac{R_{ts}}{H_{ts}}} + 6.28 \right) \quad (15)$$

6. By reversing the trial model response (i.e. $P_m^{\text{rev}} = (\beta/\alpha^\rho h^2) P_{mts}$), the full-scale tube response can be obtained.

Projections of the constitutive response for the calculated scaling factors (i.e. $\beta = 2$, $\alpha^\rho = 142.90 \times 10^{-3}$ and $h = \beta \beta_v = 1.32$) are contrasted against the constitutive equations of steel

and copper materials for full and trial-scale models in Fig. 2(b). Note how mean values of stress for these curves match, which is a feature of the weak-similitude condition enforced by Eq. (6). Fig. 2(c) reveals how the mean force in the full-scale tube is predicted to good accuracy by a projection of the trial model result.

5.2. Stress wave based model

An analytical relation for peak loads is presented in refs. [26-27] based on biaxial-stress wave propagation in elastic-plastic strain rate insensitive tubes and takes the form

$$P_{Peak} = 2\pi RH \left(\frac{2\sigma_0}{\sqrt{3}} + 2v_0 \sqrt{\frac{\rho E_h}{3}} \right) \quad (16)$$

where R , H , σ_0 , v_0 , ρ and E_h represent the mean radius, thickness, static yield stress, initial impact velocity, density and linear strain hardening, respectively.

Eq. (16) is derived employing the von-Mises yield criterion and it should be appreciated that it is not strictly applicable to strain-rate sensitive tubes. However, in the absence of anything better, the peak loads from this relationship are compared with peak loads for strain rate sensitive tubes in Section 7. This relationship does however provide a reasonable order-of-magnitude estimation for peak loads of strain rate insensitive tubes. For example, consider a strain rate insensitive tube with R , H , σ_0 , v_0 , ρ and E_h equal to 15.12 mm, 2.10 mm, 0.14 GPa, 57 mm/ms, 2.70×10^{-6} kg/mm³ and 557.62×10^{-3} GPa [19], respectively. The peak load of this tube was measured experimentally and is 35.70 kN [19]. The peak load is predicted to be 48.36 kN using Eq. (16), so suffers a 35.46% error in comparison with the experimental peak load.

It is of interest to apply the procedure of Section 4 in this case to provide a second example of the scaling process involving a relatively simple analytical model.

1. Consider then a strain-rate insensitive tube made of Al 6061-T6 having R , H , σ_0 , v_0 , ρ and E_h equal to 15.12 mm, 2.10 mm, 0.31 GPa, 120 mm/ms, 2.70×10^{-6} kg/mm³ and 1.24 GPa [19], respectively. The peak load of the tube can be approximated as

$$P_{ps} = 2\pi R_{ps} H_{ps} \left(\frac{2\sigma_{0\ ps}}{\sqrt{3}} + 2v_{0\ ps} \sqrt{\frac{\rho_{ps} E_{h\ ps}}{3}} \right) \quad (17)$$

which is Eq. (16) but applied to the full-scale model.

2. With $\beta = 2$, R and H of trial tube are 7.56 mm and 1.05 mm, respectively, the trial tube material is selected to be Al 2024-T3, with σ_0 , ρ and E_h equal to 3.66×10^{-1} GPa, 2.78×10^{-6} kg/mm³ and 2.80 GPa [19], respectively.
3. The density scaling factor (i.e. $\alpha^\rho = (1/\beta^3)(\rho_{Al\ 2024}/\rho_{Al\ 6061})$) and velocity scaling factor (from Eq. (6)) are calculated and equate to 128.70×10^{-3} and 1.36, respectively.
4. The velocity (i.e. $v_{0\ ts} = \beta v_{0\ ps}$) of the striking mass ($M_{ts} = \alpha^\rho M_{ps}$) for the trial model is calculated to be 162.97 mm/ms.
5. The mean force of trial model can be obtained as

$$P_{ts} = 2\pi R_{ts} H_{ts} \left(\frac{2\sigma_{0\ ts}}{\sqrt{3}} + 2v_{0\ ts} \sqrt{\frac{\rho_{ts} E_{h\ ts}}{3}} \right) \quad (18)$$

which is Eq. (16) but applied in this instance to the trial-scale model.

6. By reversing the trial model response (i.e. $P_{rev} = (\beta/\alpha^\rho h^2)P_{ts}$), the full-scale tube response can be obtained.

Projections of the constitutive response for the calculated scaling factors (i.e. $\beta = 2$, $\alpha^\rho = 128.70 \times 10^{-3}$ and $h = \beta\beta_v = 2.72$), are contrasted against the constitutive equations of Al 6061 and Al 2024 materials in Fig. 3. Again note how mean values of stress for these curves match in accordance with Eq. (6). The full-scale tube peak load is 122 kN and is predicted to be equal to 100 kN (18% error) using a trial model made from a different aluminium alloy. The accuracy of the predicted-peak load although limited by the assumptions underpinning Eq. (16) does provide a reasonable order-of-magnitude result.

6. Scaling of plates subjected to free air explosion

As it mentioned above, the developed techniques underpinned by finite-similitude theory such as inexact-finite similitude (see Section 3) are independent of geometry and also loading type. Although this contention is shown theoretically in Sections 2 and 3 (and also in refs. [14-15]), it is worthwhile highlighting this feature here by means of numerical examples. Refs. [14-15] presents numerous numerical scaling results for different geometries and different types of impact loadings. But it should be emphasized however that impact loadings are invariably dictated by the impacting velocity and therefore in order to investigate loading-type dependency an alternative form of loading is required to be considered. This is achieved here by applying the inexact finite similitude to free-air explosion, where impact velocity is not a feature. It is show here that by means of correcting the mass of the explosive charge for the trial model, the results of full-scale model can be predicted to good accuracy. It should be emphasized that the vast majority of research conducted on the scaling of explosive processes are limited in the sense they are performed with replica scaling [2, 30-31] in which strain-rate effects and different material properties are not captured.

6.1. Simulation procedure

Plates with a clamped external boundary condition which are subjected to shock waves are simulated using the finite element software LS-Dyna [32]. In all performed simulations, 100 and 2 deformable solid elements have been used in radial and thickness directions, respectively. Also, 098-SIMPLIFIED_JOHNSON_COOK has been used as the material model; note that this material model accounts for strain hardening and strain rate effects according to the Johnson-Cook constitutive equation (Eq. 7).

The explosive loading applied to the plates is based on the ConWep method, which is a formulation derived for free explosions of different explosive materials by Randers-Person and Bannister in 1997 for investigating the response of vehicles to landmine blasts [33]. In this method, an equivalent weight of TNT is used, i.e. other kinds of explosive materials are converted to equivalent TNT by using a conversion factor. The simulations based on the ConWep method can be performed using the LS-Dyna finite element software in two different ways. One approach is the neglect of the negative phase of explosive wave or alternatively it can be included but accounted for by the Friedlander equation; the latter approach is adopted here.

6.2. Numerical results

In order to ensure that the LS-Dyna numerical model is sound its output is initially compared and validated against reported results in the open literature. Consider then a circular plate with radius and thickness, respectively equal to 1000 mm and 50 mm, which is clamped on its external boundary. Poisson ratio, density, elastic modulus, yield stress and hardening plastic modulus respectively are 0.28, 7.85×10^{-6} kg/mm³, 210 GPa, 1 GPa and 2 GPa. For simulating of the mentioned material properties, 003-PLASTIC_KINEMATIC material model with isotropic hardening is used. The distance between plate and centre of explosive charge is 500 mm and also the mass of the explosive charge is equivalent to 50 kg TNT. The dimensionless midpoint deflection has been compared with that reported by Neuberger et al. [30] according to Fig. 4 which shows good agreement. Also it is revealed that the maximum value of midpoint deflection in this paper (i.e. 1.93) is in good agreement with that reported in ref. [30] (i.e. 1.90).

For the full and scaled models, shock waves are applied in opposition to considering the details of the explosive charge and the connecting fluid medium (i.e. air). By means of correction to the mass of the explosive charge, trial-space scaling is achieved; the efficacy of which is investigated here numerically for plates subjected to explosive loading.

The numerical results of six different models including full-scale and trial models 1-5 are presented in this section. Tabulated in Table 2 are the properties of the full-scale, externally clamped circular plate made from expensive magnesium along with details of the explosive loading. The dimensional scaling factor of trial models 1-5 is set equal to 10; in the other words, the dimensions of trial plates are scaled to one tenth of full-scale plate. The trial models 1-5, respectively are made of magnesium, copper, aluminum 7039, aluminum 1100 and steel 1006, with properties tabulated according to Table 3. The geometrical and explosive charge properties of the trial models are presented in Table 2. The maximum values of plastic strain and strain rate required for Eq. (6) are set equal with 1 and 10/ms (see Section 7.3 for justification), respectively.

The pressure-time curves for the shock waves for the explosive charges are depicted in Fig. 5. These curves pertain to a common element at the centre of the circular plates and are of the expected exponential form for reported pressure-time curves for blast loading [34]. The results

show that by correcting the mass of the explosive charges for the trial models, that the reversed pressure-time curves match to a good approximate with the full-scale model.

The deflection of the plate on a specified path is depicted in Fig. 5, where it is revealed that the response of full-scale model is predicted using small-scale trial models, with errors less than 9%.

The plastic-strain distribution of full-scale and trial models is presented in Fig. 6; in this case also it is evident that the plastic strain distribution of full-scale model has been anticipated with a suitable accuracy using trial models. Note here that the depicted errors for figures have been

calculated based on the areas under the curves. It should be mentioned that the results of trial models have been reversed using the relationships: $t_{ts}^{rev} = h \times t_{ts}$, $\delta_{ts}^{rev} = \beta \times \delta_{ts}$ and

$Pr_{ts}^{rev} = (1/\alpha^\rho h^2 \beta) \times Pr_{ts}$, where t , δ and Pr denote time, deflection and pressure, respectively.

7. Design of scaled experiments

In this section, the developed inexact-finite similitude theory for high-rate loading processes [14-15] is validated experimentally for axially impacted circular tubes. The process of validation is achieved by the combined application of both experiments and simulation. An initial step is the creation of digital representations of the experimental processes to reveal both measured and unmeasured variables. Experimental simulation is achieved using the commercial finite element software package Abaqus version 2016 [37]. This initial step validates the numerical results and provides useful information for the scaled-experimental trials and reinforces the complementary nature of simulation and experimentation as mentioned above in the introductory section. In particular, plastic strain and strain rate ranges of axially impacted tubes are obtained as these are required in Eq. (6). The solution of Eq. (6) ensures that the dynamic stress versus plastic strain and strain rate of physical and trial models are minimized over determined ranges. The minimisation takes that form that the difference between projected and actual curves are minimised by alternating the areas between the curves as is visible in Figs. 2 and 3. Finally, the results of the full-scale tubes are predicted by conducting experimental tests on the trial models.

7.1. Experimental procedure

This section is in two parts with the first part detailing the equipment used for applying the axial impacts onto the circular tubes and associated measuring equipment such as load cells. This is

followed in the second part by details on the properties of the circular tubes, which have different geometrical dimensions and are manufactured from two different materials. The materials selected for this study are steel and copper materials, where the associated tests for the determination of their mechanical properties can be found in Appendix A.

7.1.1. Experimental equipment

The stationary circular tubes are axially impacted using a gas gun apparatus, which is depicted in Fig. 7. A schematic view is provided in to Fig. 8, where critical features are identified and labelled. By means of setting the input pressure, projectiles with different masses can be propelled axially at different velocities onto the awaiting specimens.

A load cell is used for measuring of impact load as detailed in Figs. 8 and 9. Strain gauges in the load cell measure the variation of voltage caused by the impact, which is amplified using an amplifier (see Fig. 8) and then a digital storage oscilloscope records it as a function of time. The recorded voltage-time response is converted to a force-time response by a scale factor of $1 \text{ mV} = 63.183 \times 10^{-3} \text{ kN}$.

For conducting experimental tests, the stationary tubes are placed on a rigid wall as depicted in Fig. 9 and are impacted axially by shot projectiles. Four different projectiles for impacting tubes are used having masses of 1.143 kg, 1 kg, 0.143 kg and 0.125 kg. The projectiles used are presented at Fig. 10 along with their fillers, which are Ertalons of masses 0.043 kg and 0.025 kg.

7.1.2. Preparation of samples

The geometrical dimensions of tubes made of steel and copper materials (with mechanical properties found in Appendix A) are tabulated in Table 4. Also provided in Table 4 are the mass and impact velocity of the projectiles used in the study. The properties of trial models depend on the scaling method presented in Sections 2, 3 and 4 but it is convenient to discuss all the materials used in the study in this section. An overall view of the prepared circular tubes is depicted in Fig. 11(a), where it is apparent that tubes of various sizes and materials are considered. Dimensional information for the tubes is provided in Figs. 11(b) and 11(c). In Fig. 11, St, Cu and Al, respectively express steel, copper and aluminium, and the first, second and third numbers specify the length, inner diameter and thickness of tubes. For example, St-30-

14.3-1 is a tube made of steel with 30 mm length, 14.3 mm inner diameter and 1 mm thickness. Table 4 and Figs. 11 reveal that inexact finite similitude is to be validated for two different designs. The first of these designs is an investigation into pure dimensional scaling along with both dimensional and material scaling. In this case the full-scale tube is made of steel and trial tubes are made of steel and copper materials. The trial tubes are scaled to half of the full-scale tube dimensions (i.e. $\beta = 2$) as indicated in Table 4. The response of full-scale tube is predicted by trial tubes for three different velocities. For the second design an investigation is made into using different materials for same size samples (i.e. $\beta = 1$). In the other words, the response of a steel tube is predicted using of same size copper tube for three different velocities. The performed designs have been conducted in such a way that the presented scaling method can be examined for pure dimensional scaling (F-S Models 1, 2 and 3 which are predicted by the T-S Models 1, 3 and 5), pure material scaling (F-S Models 4, 5 and 6 which are predicted by the T-S Models 7, 8 and 9) and simultaneously dimensional/material scaling (F-S Models 1, 2 and 3 which are predicted by the T-S Models 2, 4 and 6). The velocities in Table 4 were obtained experimentally by means of repetitive tests using the aluminium samples depicted in Fig. 11(a). This was achieved with the help of the measurement equipment identified in Fig. 8. Once the striking mass leaves the gas gun, the time recorder starts counting by cutting the first wire and on cutting the second wire at the impacted end of the specimen the time recorder stops the count. By dividing the distance travelled between two wires by the recorded time, the velocity of striking mass can be calculated. Since the second wire is placed at the impacted end of the tube it is impacted slightly as the wire breaks. Note that in the experimental tests, the velocities in Table 4 (obtained from the scaling theory) were not matched exactly, as might be anticipated in an experimental study. Aluminium tubes with 30 mm and 60 mm lengths were used for the determination of the respective velocities for tubes with 30 mm and 60 mm lengths. A selection of the aluminium tubes in their post-experimental state are depicted in Fig. 12.

7.2. Preliminary numerical testing

As apparent from the discussion in Section 5, analytical relationships for the prediction of complex tube behaviour under high-rate axial impact are somewhat limited and tend to provide only order-of-magnitude approximations. Demonstrated in this section is the ability of finite-

element based numerical simulation to predict axial impact behaviour of tubes to greater accuracy. The numerical simulations conducted also provide information for determination of approximate ranges of plastic strain and strain rate required for the inexact-similitude approach to scaled experimentation. The numerical simulation of the axially impacted circular tubes is performed using the explicit version of the finite element software Abaqus, which is ideal for transient nonlinear impact processes [37]. The distal end of the elastic-plastic tube is placed on a clamped rigid wall whereas the impacted end is simulated without any restriction. The tube is discretised using deformable C3D8R elements whereas the striking mass is assumed to be non-deformable [37]. Independence of predicted results from mesh size is obtained by using 0.25 mm and 0.50 mm mesh sizes, respectively for F-S tubes (2 mm thickness) and F-S tubes (1 mm thickness). The mesh size of scaled tubes follows immediately from the dimensional scaling factor β . The Johnson-Cook constitutive equation is used to capture the yielding, strain hardening and strain rate behaviour of the test materials. The Johnson-Cook constitutive coefficients for the steel and copper materials used in the tests are presented in Appendix A. Any contact between the folds of tubes and also between the tube and striking mass is modelled using the surface-to-surface contact method based on the penalty approach. In the analysis, rebound is allowed for defined contact between the rigid impactor and the elastic-plastic tube. All simulations are performed with the friction coefficient set to zero. The simulation details of axially impacted tubes are summarized in Fig. 13.

The experimental and numerical buckling shapes of axially impacted circular tubes made of steel and copper materials with properties listed in Table 4 are depicted in Figs. 14 and 15. Depicted in Fig. 14 are the F-S Models 1, 2 and 3 (steel tubes) and also the T-S Models 1, 3 and 5 (steel tubes), which have responded to axial impact with a mild fold at the distal end, which are predicted correctly using numerical buckling shapes. Also, the experimental buckling shapes of T-S Models 2, 4 and 6 (copper tubes) show a non-complete, complete and complete fold at the distal end, respectively whereas the numerical simulation shows a mild fold for these tubes. The location of formed fold (i.e. at the distal end) is predicted correctly using numerical simulation. Both the experimental and numerical buckling shapes of F-S Models 4, 5 and 6 (steel tubes), respectively show a complete fold, a complete fold and also a non-complete and a complete fold at the distal end. Also, the T-S Models 7, 8 and 9 (copper tubes) replicate the F-S Models 4, 5 and 6 and are predicted correctly using numerical simulation. Thus, in general it can be said that

the experimental buckling shapes of axially impacted circular tubes with properties listed at Table 4 are predicted with suitable accuracy using numerical simulations according to Figs. 14 and 15.

The shortenings and peak loads are some of the most important crashworthiness parameters of the axially impacted circular tubes [17-21], which are listed for the series of experimental tests in Table 5. The experimental and numerical shortenings of tubes properties defined in Table 4 are compared in Table 5, which shows good agreement between them. In most cases, the experimental shortenings are predicted with error less than 15%. Little rebound occurred with these tests and to a good approximation all of the input kinetic energy is absorbed by the circular tubes.

The experimental and numerical axial force versus time curves of axially impacted circular tubes of Table 4 are presented in Figs. 16 and 17. The error stated in these figures is based on the area under the curves and experimental errors appear as error bars on the curves. It is revealed in these figures that overall the experimental axial force versus time curves are predicted to acceptable accuracy by the numerical ones. In most cases, the experimental peak loads are predicted to within an error less than 15%. Fluctuations of experimental axial force versus time curves associated with “ringing” of experimental apparatus, measurement equipment and test piece are not predicted since not accounted for in the numerical model.

7.3. Determination of scaling parameters

The numerical results now validated with experimental output, provide approximate ranges for plastic strain and strain rate for the axially impacted tubes in Table 4. Application of the scaling method presented in Sections 2, 3 and 4, provide the scaling parameters listed in Table 6. Note that the calculated scaling parameters in Table 6 provide the properties for trial models listed in Table 4.

It is important to appreciate that the scaling parameter β_v is determined to satisfy Eq. (6) and has the effect of ensuring that “volume” differences that occur between the dynamic-stress surfaces $\sigma_{d\ ts}^{\text{scaled}}$ and $\sigma_{d\ ts}$ sum to zero, when plotted over specified ranges of plastic strain and strain rate. Here $\sigma_{d\ ts}^{\text{scaled}}$ is the response that is obtained in the trial space on application of the scaling theory to $\sigma_{d\ ps}$ (i.e. $\sigma_{d\ ts}^{\text{scaled}} = \alpha^\rho h^2 \beta \sigma_{d\ ps}$). Satisfying Eq. (6) ensures that $\bar{\sigma}_{d\ ts}^{\text{scaled}} = \bar{\sigma}_{d\ ts}$, i.e. mean

dynamic stresses match over the stipulated range. The essence of inexact-finite similitude is revealed here, where the scaling rules project the behaviour of the physical model into the trial space. Thus a virtual model with virtual behaviour is obtained and scaling parameters are determined to minimise the differences between actual and virtual behaviour in some sense. The scaling theory also allows for the reverse process, where trial space models are projected into the physical space and it can be readily shown that Eq. (6) ensures that $\bar{\sigma}_{d\ ps}^{\text{rev}} = \bar{\sigma}_{d\ ps}$, where $\sigma_{d\ ps}^{\text{rev}}$ is a virtual response obtained on application of the scaling rules to $\sigma_{d\ ps}$ (i.e. $\sigma_{d\ ps}^{\text{rev}} = (\alpha^\rho h^2 \beta)^{-1} \sigma_{d\ ts}$). It is worth highlighting here that the maximum plastic strain and strain rate respectively are set to be equal with 1 and 10/ms in refs. [14-15] where 10/ms typically corresponds to high-rate loading impact processes. However, as is apparent from Fig. 18, the maximum plastic strains and strain rates applied in Eq. (6) are different from those in refs. [14-15]. The ranges for plastic strain and strain rate are determined here on the basis of numerical simulation. These fall inside the domain over which the Johnson-Cook constitutive equation is defined (see Appendix A). The sensitivity of the results to these ranges has been found not to be that critical to any reasonable choice. For example, the velocity scaling factor for T-S Model 2 is equal to 0.68 (i.e. $\beta_v = 0.68$) as indicated in Table 6. Setting the maximum plastic strain rate equal to 10/ms (as opposed to 0.80/ms (see Fig. 18)) provided $\beta_v = 0.65$.

7.4. Scaled experimentation results

Inexact-finite similitude is validated experimentally in this section for the tubes identified in Table 4. Recall that, two different designs are employed, one involving full-scale tubes made of steel, whose behaviour is to be predicted using small-scale models made of steel and copper for three different velocities. The small-scale tubes are scaled to half of the full-scale tubes dimensions for this case. The second design attempts to capture the response of the steel tubes using identically sized copper tubes for three different velocities.

The buckling shapes of the tubes listed in Table 4 are depicted at Figs. 19 and 20. Fig. 19 reveals that the full-scale steel tubes respond to axial impact by forming a mild fold at the distal end. The trial models made of steel, i.e. T-S Models 1, 3 and 5 show the same buckling pattern, with a mild fold at the distal end. The trial space models made of copper material, i.e. T-S Models 2, 4 and 6, responded to axial impact respectively with a non-complete, a complete and a

complete fold at distal end, which is inconsistent with the full-scale tubes responses. However, the location of main folds in full-scale tubes is predicted correctly by trial tubes made of copper. Fig. 20 reveals that the steel tube responses to axial impact are predicted properly by the same sized copper tubes. In more detail, both of F-S Model 4 and T-S Model 7 respond to axial impact by forming a complete fold at the distal end. Both of F-S Model 5 and T-S Model 8 respond to axial impact by forming a complete fold at the distal end. Also, both the F-S Model 6 and T-S Model 9 have two folds at the distal end.

The shortenings together with error percentages of F-S Models 1, 2 and 3 and also T-S Models 1, 2, 3, 4, 5 and 6 are presented in Fig. 21. Overall the full-scale tubes shortenings are predicted using trial models made of steel with errors less than 5%. The results in Table 5, show that the shortenings of F-S Models 1, 2 and 3 are predicted with better accuracy using trial models made of steel in comparison with numerical simulation. Also, the trial tubes made of copper, i.e. T-S Models 2, 4 and 6 give a good estimation of shortening for the full-scale models. The shortening results for F-S Models 4, 5 and 6 and also T-S Models 7, 8 and 9 are presented in Fig. 22. It is revealed that the intended tubes shortenings are predicted using trial models made of copper with errors less than 15%. By referring to the Table 5, it is found that in most cases, the shortenings of F-S Models 4, 5 and 6 are predicted with better accuracy using trial models made of copper in comparison with numerical simulation.

The axial force versus time curves for the tubes are presented in Figs. 23 and 24. It is revealed that in all cases the F-S Models responses are predicted with a suitable accuracy using trial models. Recall that fluctuations in the experimental axial force versus time curves are not predicted accurately with numerical simulation. By referring to the presented axial force versus time curves in Figs. 23 and 24, it is revealed that these fluctuations are captured correctly.

The peak loads together with error percentages of F-S Models 1, 2 and 3 and also T-S Models 1, 2, 3, 4, 5 and 6 are presented in Fig. 25. It is revealed that the full-scale tubes peak loads are predicted using trial models made of steel with errors less than 12% and using trial models made of copper with errors less than 10%. By referring to the Table 5, it is found that the peak loads of F-S Models 1, 2 and 3 are predicted to greater accuracy using trial models made of steel and copper in comparison with numerical simulation.

The peak loads together with error percentages of F-S Models 4, 5 and 6 and also T-S Models 7, 8 and 9 are presented in Fig. 26. It is revealed that the intended tubes peak loads are predicted

using trial models made of copper with errors less than 10%. By referring to the Table 5, it is found that the peak loads of F-S Models 4, 5 and 6 are predicted to greater accuracy using trial models made of copper in comparison with numerical simulation.

Also the peak loads of full-scale and physical models which are obtained using analytical stress wave based model (i.e. Eq. (16)) are depicted at Figs. 25 and 26. Note that the steel used has a non-linear behaviour in the plastic range whereas Eq. (16) applies to materials with linear strain hardening. However, the nonlinear curve of used steel material in plastic range is approximated with a linear line using a least square method. It is revealed that the peak loads of full-scale and physical models are predicted with huge differences using the analytical relationship.

It is expected that various sources of error were a factor in the deviation between the results of full-scale tubes and trial-space tubes. Friction was neglected in the numerical simulation although is present and a change in materials (i.e. copper and steel) undoubtedly changed the friction coefficient. Measurement errors associated with geometry, impact velocity and force are estimated to contribute to no more than a 1.01% error in shortening, 2.25% in peak load and 3.80% in temporal response of the axial force. Note that reported in Figs. 23 and 24 are the average error percentages and experimental error bars can be found attached to the experimental curves.

8. Conclusion

A comparative study was conducted between the finite similitude and dimensional analysis based techniques in which it was revealed that the inexact-finite similitude is independent of loading type. In order to provide more evidence, the scaling numerical results of plates subjected to free air explosion was presented using finite element software LS-Dyna. Also, the practicality of inexact finite similitude theory was investigated experimentally for high-rate loading impact processes. The axially impacted circular tubes being critical impact-absorption devices were selected for the study. The tubes were impacted axially by means of a gas gun apparatus and the load at the distal end of the tubes were measured using a load cell. The commercial finite element software package Abaqus was employed for numerical simulation in which Johnson-Cook constitutive equation was used for considering the yield stress, strain hardening and strain rate of materials. The coefficients in the Johnson-Cook constitutive equation were determined by conducting static tensile test and also dynamic compression test using split Hopkinson

pressure bar at different strain rates. It was found that by determining the mass and impact velocity of scaled models (and correcting the mass of explosive charge for free air explosion) using the inexact-finite similitude theory that full-scale behaviours can be replicated with good accuracy by scaled experiments. The following conclusions can be drawn from the study:

- It was revealed that by correcting the mass of explosive charge for trial models in free air explosion, the response of full-scale plate including deflection along a specified path and plastic strain distribution can be predicted to good accuracy using small-scale trial plates made of the same or different materials.
- The experimental results of investigated circular tubes including buckling shapes, shortenings, axial force versus time curves and peak loads were predicted using numerical simulation with a suitable accuracy in most cases. However in most cases, the fluctuations of experimental axial force versus time curves were not predicted properly.
- Using of small-scale tubes made of the same and different materials in comparison with full-scale tubes and also using of same size tube which is made of a different material led to a good prediction.
- In most cases, scaled experimentation provided a better estimation of the response of the tubes (including buckling shapes, shortenings, axial force versus time curves and peak loads) in comparison with numerical simulation.
- In opposite to numerical simulation, the fluctuation of axial force versus time curves of full-scale models also were predicted properly using scaled experimentation.
- The full-scale steel tubes shortenings were predicted using small-scale trial models made of steel material with errors less than 5%. Steel tubes shortenings were predicted using same size trial models made of copper material with errors less than 15%.
- The full-scale steel tubes peak loads were predicted using small-scale trial models made of steel material with errors less than 12% and using small-scale trial models made of copper material with errors less than 10%. Also, steel-tube peak loads were predicted using same sized trial models made of copper with errors less than 10%.
- Available analytical relations for mean force and peak load prediction for tubes under high-rate axial impact are highly inaccurate in comparison with numerical simulation and especially in comparison with scaled experimentation.

Appendix A. Determination of mechanical properties

In this section, the coefficients of Johnson-Cook constitutive equation are determined by conducting static tests using static tensile testing machine and also dynamic compression tests using the split Hopkinson pressure bar at different strain rates. Using Johnson-Cook constitutive equation, the current yield stress is expressed as a function of strain hardening and strain rate according to Eq. (7). In the following, at first the Johnson-Cook coefficients A , B and n are determined based on the conducted static tensile tests and then the Johnson-Cook rate coefficient (i.e. C) has been determined based on the conducted dynamic compression tests using the split Hopkinson pressure bar. Other pertinent material properties are: Poisson ratio, density and elastic modulus, which for steel material are respectively as 0.30, 7.864×10^{-6} kg/mm³ and 200 GPa. In the case of copper, Poisson ratio, density and elastic modulus are respectively 0.34, 8.99×10^{-6} kg/mm³ and 120 GPa. The chemical composition of the copper (C11000) and steel (mild steel) materials used in the experimental testing are tabulated in Tables 7 and 8, respectively.

Determination of Johnson-Cook coefficients

The coefficient A in Eq. (7) is assumed equal with initial yield stress at reference strain rate and B and n coefficients are found using least square method [39]. This is achieved by expressing the Johnson-Cook constitutive equation in the form $Y - A = B\varepsilon^n$, which is applicable to a slow-rate process. The parameter φ is defined as $\varphi = \ln(Y - A)$, which leads to $\varphi = n \ln \varepsilon + b$, where $b = \ln B$. The static tensile test curves can be approximated to good accuracy with the relationship $y = \beta \varepsilon^\alpha$, which leads to relationships $\psi = \ln(y - A)$ and $\psi = \ln(\beta \varepsilon^\alpha - A)$. For minimization of φ and ψ in comparison with each other, the sum of the squared errors can be minimized (i.e. $\min_{n,b} \sum_{i=1}^m (\varphi_i - \psi_i)^2$), where m is the number of data points. This readily achieved on setting the derivatives $\frac{\partial}{\partial b} \sum_{i=1}^m (\varphi_i - \psi_i)^2$ and $\frac{\partial}{\partial n} \sum_{i=1}^m (\varphi_i - \psi_i)^2$ to zero. This provides the following expressions for b and n [39]:

$$b = \frac{\sum \psi_i \ln \varepsilon_i \sum \ln \varepsilon_i - \sum \psi_i \sum (\ln \varepsilon_i)^2}{(\sum \ln \varepsilon_i)^2 - m \sum (\ln \varepsilon_i)^2} \quad (19)$$

$$n = \frac{\sum \psi_i - mb}{\sum \ln \varepsilon_i} \quad (20)$$

The static stress-strain curves of used steel and copper tubes are determined according to metal tensile testing standard ASTM E8, using tensile testing machine Santam which is depicted at Fig. 27. The experimental stress-strain curves of copper and steel specimens are depicted at Fig. 27. Presented in Table 9 are the coefficients A , B and n for copper and steel materials, obtained from the least squares procedure. The Johnson-Cook constitutive response along with experimental results is presented in Fig. 27, where it is revealed that they are in good agreement.

Determination of Johnson-Cook rate coefficient

The C parameter of Johnson-Cook constitutive equation which considers the strain rate effects is calculated using the conducted dynamic compression test by split Hopkinson pressure bar at different strain rates. The current yield stress can be considered as $Y_i = A + AC \ln \dot{\varepsilon}$ where $\dot{\varepsilon}$ is normalized strain rate by the reference strain rate (i.e. $\dot{\varepsilon} = \dot{\varepsilon} / \dot{\varepsilon}_0$). Under the assumption that y_i represents the yield stresses from the experimental tests, C can be obtained using the least squares method (i.e. $\frac{\partial}{\partial C} \sum (Y_i - y_i)^2 = 0$) [39] and equals

$$C = \frac{\sum y_i \ln \dot{\varepsilon}_i}{\sum A (\ln \dot{\varepsilon}_i)} \quad (21)$$

The split Hopkinson pressure bar used in the study is depicted in Fig. 28. The dynamic stress-strain curves of the steel and copper tubes are presented in Fig. 29 at different strain rates. The values for C from the least squares procedure for copper and steel materials are presented in Table 9. Shown in Fig. 29 are the yield stresses obtained from the Johnson-Cook model along with experimental results and it is revealed that they are in a good agreement.

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Table Captions

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Table 4. The geometrical properties of tubes and also the mass and velocity of projectiles

Table 5. Experimental and numerical shortenings and peak loads together with error percentages of circular tubes with properties illustrated at Table 4

Table 6. Scaling parameters

Table 7. Chemical composition of copper C11000

Table 8. Chemical composition of mild steel [38]

Table 9. Coefficients of Johnson-Cook for copper C11000 and mild steel materials

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931 **Fig. 20.** Buckling shapes

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934 **Fig. 22.** The shortenings together with error percentages of F-S Models 4, 5 and 6 and also T-S
935 Models 7, 8 and 9

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937 **Fig. 24.** Axial force versus time curves

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944 **Fig. 28.** Testing of specimens at different strain rates

945 **Fig. 29.** Stress-strain curves and some of the yield stresses obtained from Johnson-Cook model
946 versus experimental results

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948 **Table 1.**

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Transport equation	Physical field (ψ)	Flux ($\underline{J}$)	Source (b)	Similarity identities	Explanation
Volume	ρ^{-1}	$\underline{0}$	0	$1 = \alpha^1 \beta^3$	This identity confirms that β can be set to any value (since the volume scalar α^1 is arbitrary)
				$\underline{v}_{ts}^* = h \beta^{-1} \underline{v}_{ps}^*$	Is required for the synchronisation of the control volumes
Mass	1	$\underline{0}$	0	$\rho_{ts} = \alpha^\rho \beta^3 \rho_{ps}$	The mass scalar parameter α^ρ is free to be set
Momentum	$\underline{v}$	$\underline{\underline{\sigma}}$	$\underline{b}$	$\rho_{ts} \underline{v}_{ts} = \alpha^v \beta^3 \rho_{ps} \underline{v}_{ps}$	Leads to $\alpha^v = h \beta^{-1} \alpha^\rho$ (by using the relations $\underline{v}_{ts} = h \beta^{-1} \underline{v}_{ps}$ and $\rho_{ts} = \alpha^\rho \beta^3 \rho_{ps}$)
				$\underline{\underline{\sigma}}_{ts} = \alpha^v h \beta^2 \underline{\underline{\sigma}}_{ps}$	Leads to $\underline{\underline{\sigma}}_{ts} = \alpha^\rho h^2 \beta \underline{\underline{\sigma}}_{ps}$ (by using the relation $\alpha^v = h \beta^{-1} \alpha^\rho$) where $\underline{\underline{\sigma}}$ represents the Cauchy stress tensor; Also $\underline{\underline{\sigma}}_{ts} = \alpha^\rho h^2 \beta \underline{\underline{\sigma}}_{ps}$ leads to $h = \sqrt{\frac{1}{\alpha^\rho \beta} \frac{Y_{ts}}{Y_{ps}}}$ by satisfying the von-Mises plasticity model where Y represents the yield stress
				$\rho_{ts} \underline{b}_{ts} = \alpha^v h \beta^3 \rho_{ps} \underline{b}_{ps}$	-
Movement	$\underline{u}$	$\underline{0}$	$\underline{v}$	$\alpha^\rho \underline{v}_{ts} = h \alpha^u \underline{v}_{ps}$	$\alpha^u = \beta^{-1} \alpha^\rho$ (by using the relation $\underline{v}_{ts} = h \beta^{-1} \underline{v}_{ps}$)
				$\alpha^\rho \underline{u}_{ts} = \alpha^u \beta^{-1} \underline{u}_{ps}$	$\underline{u}_{ts} = \beta^{-1} \underline{u}_{ps}$ (by using the relation $\alpha^u = \beta^{-1} \alpha^\rho$)

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962 **Table 2.**

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Model	Scaling parameters			Geometrical properties of plate (<i>mm</i>)		Explosive charge	
	β	α^{ρ} $\times 10^{-3}$	h	Radius	Thickness	Mass (<i>kg</i>) $\times 10^{-2}$	Distance (<i>mm</i>)
Full-Scale (Magnesium)	1	1000	1	500	10	500	500
Trial Model 1 (Magnesium)	10	1	10.13	50	1	0.50	50
Trial Model 2 (OFHC Copper)	10	5.06	3.98	50	1	1	50
Trial Model 3 (Aluminum 7039)	10	1.57	9.21	50	1	0.80	50
Trial Model 4 (Aluminum 1100)	10	1.53	7.77	50	1	0.70	50
Trial Model 5 (Steel 1006)	10	4.46	5.59	50	1	1.50	50

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979 **Table 3.**

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Material	Density (kg/mm^3) $\times 10^{-6}$	Elastic modulus (GPa)	Poisson ratio $\times 10^{-1}$	A (GPa) $\times 10^{-3}$	B (GPa) $\times 10^{-3}$	n $\times 10^{-1}$	C $\times 10^{-1}$	$\dot{\epsilon}$ ($1/ms$) $\times 10^{-3}$
Magnesium	1.77	45	2.90	224	380	7.61	0.12	1
OFHC Copper	8.96	120	3.40	90	292	3.10	0.25	1
Aluminum 7039	2.77	70	3.30	337	343	4.10	0.10	1
Aluminum 1100	2.70	65.76	3.00	148.36	345.51	1.83	0.01	1
Steel 1006	7.89	210	3.00	350	275	3.60	0.22	1

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Table 4.

Model	Tube dimensions				Striking mass	
	Material	Thickness (<i>mm</i>)	Inner diameter (<i>mm</i>)	Length (<i>mm</i>)	Mass (<i>kg</i>)	Impact velocity (<i>mm/ms</i>)
F-S Model 1	Steel	2	28.60	60	1.000	40.00
T-S Model 1	Steel	1	14.30	30	0.125	40.91
T-S Model 2	Copper	1	14.30	30	0.143	27.33
F-S Model 2	Steel	2	28.60	60	1.000	45.00
T-S Model 3	Steel	1	14.30	30	0.125	46.01
T-S Model 4	Copper	1	14.30	30	0.143	30.85
F-S Model 3	Steel	2	28.60	60	1.000	50.00
T-S Model 5	Steel	1	14.30	30	0.125	51.11
T-S Model 6	Copper	1	14.30	30	0.143	34.35
F-S Model 4	Steel	1	26.60	60	1.000	30.00
T-S Model 7	Copper	1	26.60	60	1.143	20.66
F-S Model 5	Steel	1	26.60	60	1.000	35.00
T-S Model 8	Copper	1	26.60	60	1.143	24.08
F-S Model 6	Steel	1	26.60	60	1.000	40.00
T-S Model 9	Copper	1	26.60	60	1.143	27.48

Table 5.

Model	Shortening (<i>mm</i>)			Peak load (<i>kN</i>)		
	Experiment	Numerical	Error%	Experiment	Numerical	Error%
F-S Model 1 (Steel)	4.10	4.63	12.93	163.01	188.79	15.82
T-S Model 1 (Steel)	2.10	2.32	10.48	41.70	49.28	18.18
T-S Model 2 (Copper)	2.40	2.41	0.42	23.76	24.51	3.16
F-S Model 2 (Steel)	5.00	5.70	14.00	169.33	193.29	14.15
T-S Model 3 (Steel)	2.60	2.86	10.00	39.17	50.36	28.57
T-S Model 4 (Copper)	3.60	3.01	16.39	22.75	24.89	9.41
F-S Model 3 (Steel)	6.70	6.94	3.58	164.28	196.86	19.83
T-S Model 5 (Steel)	3.50	3.48	0.57	42.96	51.28	19.37
T-S Model 6 (Copper)	4.30	3.71	13.72	23.76	25.19	6.02
F-S Model 4 (Steel)	10.50	8.48	19.24	68.24	77.93	14.20
T-S Model 7 (Copper)	9.00	9.73	8.11	40.44	39.00	3.56
F-S Model 5 (Steel)	11.40	12.58	10.35	72.03	79.66	10.59
T-S Model 8 (Copper)	11.20	13.14	17.32	41.70	39.38	5.56
F-S Model 6 (Steel)	15.20	15.53	2.17	73.29	80.79	10.23
T-S Model 9 (Copper)	13.60	16.66	22.50	42.96	39.79	7.38

Table 6.

Model	Scaling parameters			
	β	$\alpha^p \times 10^{-3}$	h	β_v
T-S Model 1 (Steel)	2	125.0	2.05	1.02
T-S Model 2 (Copper)	2	142.9	1.37	0.68
T-S Model 3 (Steel)	2	125.0	2.05	1.02
T-S Model 4 (Copper)	2	142.9	1.37	0.69
T-S Model 5 (Steel)	2	125.0	2.04	1.02
T-S Model 6 (Copper)	2	142.9	1.37	0.69
T-S Model 7 (Copper)	1	1143.2	0.69	0.69
T-S Model 8 (Copper)	1	1143.2	0.69	0.69
T-S Model 9 (Copper)	1	1143.2	0.69	0.69

1051 **Table 7.**

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Chemical Elements	Cu	Zn	Pb	Sn	P	Mn	Fe	Ni	Si	Mg
Percentage	99.90	<0.0030	0.0230	0.0005	0.0460	<0.0005	0.0150	<0.0003	<0.0010	<0.0005
Chemical Elements	Cr	Au	As	Sb	Cd	Bi	Ag	Co	Al	S
Percentage	<0.0003	<0.0005	<0.0004	<0.0015	0.0110	0.0110	0.0033	<0.0015	0.0031	<0.0003
Chemical Elements	Be	C	Ti	B						
Percentage	<0.0001	0.0210	0.0006	<0.0005						

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1073 **Table 8.**

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Chemical Elements	C	Si	Mn	S	Cr	Ni	Al
Percentage	0.1880	0.1855	0.8375	0.0342	0.0213	0.0217	0.0762

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1101 **Table 9.**

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Material	$A \text{ (GPa)}$ $\times 10^{-3}$	$B \text{ (GPa)}$ $\times 10^{-3}$	n $\times 10^{-3}$	C $\times 10^{-3}$	$\dot{\epsilon} \text{ (1/ms)}$ $\times 10^{-5}$
Copper	221.03	396.81	319.72	32.61	8.47
Steel	291.96	358.25	304.98	135.09	8.62

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Fig. 1.

Fig. 2.

(a) Eq. (13) compared against the experimentally obtained response of F-S Model 1

(b) Projected and actual full and trial-scale constitutive equations

(c) Predicting the full-scale tube mean force using the trial model

Fig. 3.

Fig. 4.

Fig. 5.

(a) Pressure-time curves of shock waves

(b) The specified path on the plate

(c) The deflection along the specified path

Reversed Trial Model 3 (Aluminum 7039)
Reversed Trial Model 4 (Aluminum 1100)
Reversed Trial Model 5 (Steel 1006)

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Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

Fig. 11.

(a) A total view of prepared tubes

(b) A longitudinal view of prepared tubes

(c) A cross-sectional view of prepared tubes

Fig. 12.

Fig. 13.

1303 **Fig. 14.**
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F-S Model 1 (Steel)		F-S Model 2 (Steel)	
Experiment	Numerical	Experiment	Numerical

F-S Model 3 (Steel)		T-S Model 1 (Steel)	
Experiment	Numerical	Experiment	Numerical

T-S Model 3 (Steel)		T-S Model 5 (Steel)	
Experiment	Numerical	Experiment	Numerical

T-S Model 2 (Copper)		T-S Model 4 (Copper)	
Experiment	Numerical	Experiment	Numerical

T-S Model 6 (Copper)			
Experiment		Numerical

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Fig. 15.

F-S Model 4 (Steel)		F-S Model 5 (Steel)	
Experiment	Numerical	Experiment	Numerical

F-S Model 6 (Steel)		T-S Model 7 (Copper)	
Experiment	Numerical	Experiment	Numerical

T-S Model 8 (Copper)		T-S Model 9 (Copper)	
Experiment	Numerical	Experiment	Numerical

Fig. 16.

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Fig. 17.

Fig. 18.

(a) F-S Model 1 and T-S Model 1

(b) F-S Model 1 and T-S Model 2

(c) F-S Model 4 and T-S Model 7

1382 **Fig. 19.**
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(a) F-S Model 1 and T-S Models 1 and 2

(b) F-S Model 2 and T-S Models 3 and 4

(c) F-S Model 3 and T-S Models 5 and 6

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1385 **Fig. 20.**
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(a) F-S Model 4 and T-S Model 7

(b) F-S Model 5 and T-S Model 8

(c) F-S Model 6 and T-S Model 9

Fig. 21.

Fig. 22.

1425 **Fig. 23.**

1426 (a) F-S Model 1 and reversed T-S Models 1 and 2

1428 (b) F-S Model 2 and reversed T-S Models 3 and 4

1430 (c) F-S Model 3 and reversed T-S Models 5 and 6

1432 **Fig. 24.**

1433 (a) F-S Model 4 and reversed T-S Model 7

1435 (b) F-S Model 5 and reversed T-S Model 8

1437 (c) F-S Model 6 and reversed T-S Model 9

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Fig. 25.

Fig. 26.

(a) Static tensile testing machine

(b) Specimens after conducting tensile test

(c) Stress strain curves

Fig. 28.

Fig. 29.

(a) The dynamic stress-strain curves

(b) Yield stresses