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Further assessment of deceleration-time histories for occupant injury and the damage of protected object in a crash stop

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ABSTRACT

Crash signal is generally given by a deceleration-time (or a force-time) history measured in a crash stop. Occupant injury and damage of protected object are highly related to the parameters of the crash signal. This paper assesses deceleration-time histories in a crash stop from two aspects, i.e., (i) minimisation of head injury index; (ii) physical modelling using single-degree-of-freedom (SDOF) model. A mathematical description for the determination of the safest deceleration-time history for head injury is presented, and seven well-recognised equivalent crash signals are analysed with constraints of identical velocity change and stop distance. Iso-damage/injury boundaries based on both methods are constructed using non-dimensional quantities. Results show that the preferred deceleration-time history depends on the selection of the injury (or damage) criterion and the ranges of parameters in the crash stop problem. These findings may support the design of cushioning or energy absorbing systems for the crashworthiness of vehicles, air and space crafts, ships and packages.

Keywords: Deceleration-time history; Crash stop; Head injury criteria (HIC); Single-degree-of-freedom (SDOF); Impact energy absorption (IEA)

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1. Introduction

The safety of occupants and any other protected objects (e.g. goods, devices, equipment, etc.) is a critical issue in transportation, which promotes the continuous improvement of the crashworthiness safety of land, water (sea), air and space vehicles. The deceleration-time history in a crash incident has been commonly used as a crash signal for the assessment of the structural crashworthiness, occupant injury and the damage of protected objects. Deceleration-time histories may have different pulse shapes, time durations and peak values, as shown in [1] in rear impact tests. Numerical method has been adopted to evaluate the effect of crash pulse shape on occupant [2-4]. In Ref.[4], various shapes of crash signals (e.g. triangular, half-sine and square shapes) with identical velocity change (ΔV) were taken as inputs to a crash dummy model to determine how occupant reacts to different deceleration pulses where air bag and the knee bolster were applied as restraint systems in the simulation model. It showed that the time duration (t_d), the peak deceleration ($A_{\max}$), and the shape of the crash signal may influence the occupant response. When t_d is given, the square signal was found to yield smaller peak acceleration of driver chest. Zhou et al. [5] analytically modelled the deceleration pulse of FMVSS201 head impact using an asymmetric haversine wave with the consideration of rebounding and found that the HIC is sensitive to impact velocity and stop distance (ΔS).

Strong correlation has been observed between deceleration and occupant injury by analysing the real-life crashes and dynamic tests [6]. In rear impacts, it was noted that the AIS-1 neck injury in rear impacts is relevant to both the mean deceleration (A_m) and ΔV [7]. The safest crash signal shape has attracted attentions in the study of vehicle crash and restraint system [6,8-10]. Therefore, it is interesting to know the most preferred crash signal for the safety of occupants in order to develop more efficient crashworthy structure, impact energy absorber (IEA) [11-12], restraint system and shock absorber [13]. Effects of typical crash signals with identical ΔV and ΔS on head injury criteria (HIC) have been studied by Chou and Nyquist [14], Lim et al. [15,16], Huang [6], Wu and Beaudet [17] and Hutchinson et al. [18]. Wu and Beaudet [17] further proposed an ideal crash signal in the form of negative power function.

Crash stop is a common phenomenon in many natural and engineering activities, e.g., crash stop of transportation vehicles, accidental and deliberated crash-landing of containers, re-entry capsules and personal electronics, animal behaviours such as bird pecking, etc. During a crash stop, deceleration-time history ($A(t)$) of the concerned occupant, determined by the energy absorbing mechanism of the surrounding supporting structure and IEA, is usually used to assess the occupant injury or damage. The design of an IEA is normally required to bring a safe stop of the occupant for the given maximum survivable impact speed (i.e. the velocity change) and the

available stop distance. In many application fields, the stop distance is referred to the cushioning space (ΔS), which is usually limited by the product or system space available for cushioning purpose. For example, the collapse zone of rail vehicles used to absorb impact energy in crash accidents is mounted on the front-end of the underframe. This zone is restricted by other vehicle components, e.g., survival space, support beams, couplers, etc. Therefore, it is a generic problem to stop a moving object under the constraints of velocity change and stop distance.

The objective of the present work is to further assess the deceleration-time history ($A(t)$) for given stop distance (ΔS) and velocity change (ΔV) in a crash stop incident. Two different well-known methods, i.e. empirical HIC and equivalent single degree of freedom (SDOF) model, are employed. Without causing confusion and for the simplification, the term of ‘occupant’ is used for human body/part and the term of ‘protected object’, although it may include occupant, is mainly used for any other goods, products and equipment to be protected; the term of ‘injury’ is related to an occupant while the term of ‘damage’ is associated with a protected object; the term of ‘deceleration’ may include the acceleration and the term of ‘crash signal’ represents the deceleration-time (or force-time) history, if no particular notation is given.

2. Determination of deceleration-time history based on HIC

The injury or damage of a protected object is a major concern in a crash stop. A crashworthy system can be generally divided into two parts, i.e., the cushioning component (e.g. damping system, energy-absorbing structures, shock absorbers, etc.) and the protected object (e.g. human, electronic components and devices, nuclear waste containers, goods, equipment). Fig.1(a) illustrates a generic crash model, which is generally simplified into the cushioning zone consisting of cushioning components (IEA and restrains) and the concerned envelope of the protected object. Without losing generality, the protected object could be understood as either the envelope of the rigid survival space of a vehicle or the envelope of a human body (or part) depending on the statement of the problem. For a packaging problem, the cushioning in Fig.1(a) represents the package and the protected object is the product inside the package. The deceleration-time history ($A(t)$) applied to the protected object is determined by the design of cushioning zone while the safety of the protected object (or occupant) can be assessed by the associated damage (or injury) criteria in terms of $A(t)$ shown in Fig.1(b) as a typical crash signal acting on the protected object. If the damage/injury index can be expressed as a function of $A(t)$, i.e., $F(A(t))$ where $A(t) = 0$ when $t \leq 0$ and $t \geq t_d$ with t_d as the duration of the crash time, then the safest deceleration-time history $A_s(t)$ for a given stop distance (ΔS) and initial impact speed (V_0) is determined by

$$F(A_s(t)) = \min F(A(t)) \left\| \left(V_0 - \int_0^{t_d} A(t) dt = 0, \int_0^{t_d} \left[V_0 - \int_0^t A(t) dt \right] dt = S \right) \right. \quad (1)$$

Here, the residual velocity is assumed to be zero without rebounding, which means $V_0 = \Delta V$. Mathematically, the safest deceleration-time history $A_s(t)$ is determined by the minimum of $F(A(t))$ in Eq.(1).

Fig.1. (a) A generic crash senario; (b) A typical crash pulse acting on the protected object.

In a frontal vehicle crash, the deceleration $A(t)$ at the centroid of an occupant's head is a function of time (t). HIC was firstly proposed by the National Highway Traffic Safety Administration (NHTSA) in 1972. This criterion is based on the average value of the deceleration over the most critical part of $A(t)$, which is calculated using the formula below

$$HIC = \max_{t_1, t_2} \left[(t_2 - t_1) \cdot \left(\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} A(t) dt \right)^{2.5} \right] = (t_2^* - t_1^*) \cdot \left(\frac{1}{t_2^* - t_1^*} \int_{t_1^*}^{t_2^*} A(t) dt \right)^{2.5} \quad (2)$$

where t_1 and t_2 satisfy $0 \leq t_1 < t_2 \leq t_d$ and the maximum value of the HIC function in Eq.(2) is achieved when $t_1 = t_1^*$ and $t_2 = t_2^*$; deceleration $A(t)$ and time t are measured in standard gravity acceleration (g 's) and second, respectively. Normally, the interval $\Delta t = t_2^* - t_1^*$ is 36 ms [19] or 15 ms [20].

2.1 Determination of $A(t)$ by HIC formula

To elucidate the preferred $A(t)$ in a crash stop, effects of seven typical and well-recognised equivalent crash signals (ECSs) on HIC are analysed, which are equivalent square signal (ESS), equivalent positive triangular signal (EPTS), equivalent negative triangular signal (ENTS), equivalent isosceles triangular signal (EITS), equivalent half sine signal (EHSS), equivalent haversine signal (EHVS) and the equivalent negative power signal (ENPS) proposed by Wu and Beaudet [17]. It is noted that the term of 'equivalent crash wave' used in some publications

(e.g. [6]) has the same meaning of ECS. To compare their injury indices in a crash stop incident, all seven considered ECSs shown in Fig.2 will have identical ΔV and ΔS .

Fig.2. Typical shapes of ECSs: (a) ESS; (b) EPTS; (c) ENTS; (d) EITS; (e) EHSS; (f) EHVS; (g) ENPS.

The time duration t_d and maximum deceleration A_{max} may vary among the seven signals in Fig.2 for given ΔV and ΔS . The results are tabulated in Table 1, which include t_1^* and t_2^* in HIC calculations, expressions of crash signals and expressions of HIC. The normalised HIC value against each ECS is also shown in Fig.3. Details of the calculation processes are referred to Chou and Nyquist [14], Lim et al. [15], Huang [6] and Wu and Beaudet [17].

Table 1. HIC derived from ECSs with identical ΔV and ΔS . (Units of A_{max} , t_d , ΔV and ΔS are 9.8m/s^2 , sec, 9.8 m/s and 9.8m , respectively)

Crash signals	ESS	EPTS	ENTS	EITS	EHSS	EHVS	ENPS
Expression of signal	A_0	jt	$A_0 - jt$	jt ($0 \leq t \leq t_0$) $A_0 - jt$ ($t_0 \leq t \leq t_d$)	$A_0 \sin \omega t$	$\frac{A_{max}}{2} [1 - \cos \omega t]$	$ct^{-0.4}$

Times for the maximum value of HIC function	$\begin{cases} t_1^* = 0 \\ t_2^* = t_d \end{cases}$	$\begin{cases} t_1^* = \frac{3}{7}t_d \\ t_2^* = t_d \end{cases}$	$\begin{cases} t_1^* = 0 \\ t_2^* = \frac{4}{7}t_d \end{cases}$	$\begin{cases} t_1^* = \frac{3}{14}t_d \\ t_2^* = \frac{11}{14}t_d \end{cases}$	$\begin{cases} t_1^* = \frac{0.518}{\omega} \\ t_2^* = \frac{2.624}{\omega} \end{cases}$	$\begin{cases} t_1^* = 0.25t_d \\ t_2^* = 0.75t_d \end{cases}$	$\begin{cases} t_1^* = 0 \\ t_2^* = t_d \end{cases}$
$\Delta t^* = t_2^* - t_1^*$	$\frac{2\Delta S}{\Delta V}$	$\frac{6\Delta S}{7\Delta V}$	$\frac{12\Delta S}{7\Delta V}$	$\frac{8\Delta S}{7\Delta V}$	$\frac{4.212\Delta S}{\pi\Delta V}$	$\frac{\Delta S}{\Delta V}$	$\frac{8\Delta S}{3\Delta V}$
Expression of HIC	$A_0^{2.5}t_d$	$\frac{4}{7}\left(\frac{5}{7}j\right)^{2.5}t_d^{3.5}$	$\frac{4}{7j}\left(\frac{5}{7}\right)^{2.5}(A_0)^{3.5}$	$\left(\frac{7}{8}\right)^{1.5}\left(\frac{40}{49}j\right)^{2.5}\left(\frac{t_d}{2}\right)^{3.5}$	$1.303\frac{(A_0)^{2.5}}{\omega}$	$0.303(A_{max})^{2.5}t_d$	$3.586c^{2.5}$
Configurations	$\begin{cases} A_0 = \frac{(\Delta V)^2}{2\Delta S} \\ t_d = \frac{2\Delta S}{\Delta V} \end{cases}$	$\begin{cases} j = \frac{8(\Delta V)^3}{9(\Delta S)^2} \\ t_d = \frac{3\Delta S}{2\Delta V} \end{cases}$	$\begin{cases} A_0 = \frac{2(\Delta V)^2}{3\Delta S} \\ j = \frac{2(\Delta V)^3}{9(\Delta S)^2} \\ t_d = \frac{3\Delta S}{\Delta V} \end{cases}$	$\begin{cases} A_0 = \frac{2(\Delta V)^2}{\Delta S} \\ j = \frac{(\Delta V)^3}{(\Delta S)^2} \\ t_d = \frac{2\Delta S}{\Delta V} \\ t_0 = \frac{\Delta S}{\Delta V} \end{cases}$	$\begin{cases} A_0 = \frac{\pi(\Delta V)^2}{4\Delta S} \\ \omega = \frac{\pi\Delta V}{2\Delta S} \\ t_d = \frac{\Delta S}{\Delta V} \end{cases}$	$\begin{cases} A_{max} = 0.9966\frac{(\Delta V)^2}{\Delta S} \\ \omega = \frac{\pi\Delta V}{\Delta S} \\ t_d = \frac{\Delta S}{\Delta V} \end{cases}$	$\begin{cases} c = \frac{3}{5}\left(\frac{8}{3}\Delta S\right)^{-0.6}\Delta V^{1.6} \\ t_d = \frac{8\Delta S}{3\Delta V} \end{cases}$
A_{max}	$\frac{(\Delta V)^2}{2\Delta S}$	$\frac{4}{3}\frac{(\Delta V)^2}{\Delta S}$	$\frac{2}{3}\frac{(\Delta V)^2}{\Delta S}$	$\frac{(\Delta V)^2}{\Delta S}$	$\frac{\pi(\Delta V)^2}{4\Delta S}$	$0.9966\frac{(\Delta V)^2}{\Delta S}$	$+\infty$
A_{avg}	$\frac{1}{2}\frac{(\Delta V)^2}{\Delta S}$	$\frac{2}{3}\frac{(\Delta V)^2}{\Delta S}$	$\frac{1}{3}\frac{(\Delta V)^2}{\Delta S}$	$\frac{1}{2}\frac{(\Delta V)^2}{\Delta S}$	$\frac{1}{2}\frac{(\Delta V)^2}{\Delta S}$	$\frac{1}{2}\frac{(\Delta V)^2}{\Delta S}$	$\frac{3}{8}\frac{(\Delta V)^2}{\Delta S}$
Expression of HIC in terms of ΔV and ΔS	$(\Delta V)^4 \cdot (2\Delta S)^{-1.5}$	$2.15 \cdot (\Delta V)^4 \cdot (2\Delta S)^{-1.5}$	$0.76 \cdot (\Delta V)^4 \cdot (2\Delta S)^{-1.5}$	$1.39 \cdot (\Delta V)^4 \cdot (2\Delta S)^{-1.5}$	$1.28 \cdot (\Delta V)^4 \cdot (2\Delta S)^{-1.5}$	$1.7082 \cdot (\Delta V)^4 \cdot (2\Delta S)^{-1.5}$	$0.649502 \cdot (\Delta V)^4 \cdot (2\Delta S)^{-1.5}$
Expression of HIC in terms of ΔV and A_{max}	$(\Delta V) \cdot (A_{max})^{1.5}$	$0.491(\Delta V) \cdot (A_{max})^{1.5}$	$0.491(\Delta V) \cdot (A_{max})^{1.5}$	$0.491(\Delta V) \cdot (A_{max})^{1.5}$	$0.649(\Delta V) \cdot (A_{max})^{1.5}$	$0.604(\Delta V) \cdot (A_{max})^{1.5}$	-
Expression of HIC in terms of ΔV and A_{avg}	$(\Delta V) \cdot (A_{avg})^{1.5}$	$1.39647(\Delta V) \cdot (A_{avg})^{1.5}$	$1.39621(\Delta V) \cdot (A_{avg})^{1.5}$	$1.39(\Delta V) \cdot (A_{avg})^{1.5}$	$1.28(\Delta V) \cdot (A_{avg})^{1.5}$	$1.7082(\Delta V) \cdot (A_{avg})^{1.5}$	$(\Delta V) \cdot (A_{avg})^{1.5}$

127 It is shown that t_1^* and t_2^* for the calculation of HIC are dependent of the specified ECS.

128 The interval $[t_1^*, t_2^*]$ of ENPS is the largest one, which equals the whole time duration $(\frac{8S}{3\Delta V})$.

129 Interestingly, the expressions of $t_2^*-t_1^*$ derived from triangular signals are identical to $\frac{4}{7}t_d$, no
130 matter the signal is EPTS, ENTS or EITS. In other word, the HIC derived from all triangular
131 signals will be the same if t_d (rather than the stop distance ΔS) and ΔV are fixed. However, when
132 ΔV and ΔS are fixed, ENTS has the longest time duration of t_d which is 1.5 times that of ESS,
133 while ENPS has the highest peak deceleration which approaches infinite.

134 It is found that the order of HIC values is $ENPS < ENTS < ESS < EHSS < EITS < EHVS < EPTS$ for
135 given ΔV and ΔS , as shown in Fig.3. Taking the ESS as a standard signal and its $HIC = (\Delta V)^4 \cdot$
136 $(2S)^{-1.5}$ as a reference value, the normalised HIC values for ENPS, ENTS, ESS, EHSS, EITS,
137 EHVS and EPTS are 0.65, 0.76, 1, 1.28, 1.39, 1.84 and 2.15, respectively. It reveals that the ENPS
138 with high initial deceleration and longer time duration is preferable in reducing head injury, while
139 EPTS with gradually increasing deceleration and short time duration is the worst choice. These
140 observations are consistent with previous findings [14-17]. Furthermore, the orders of normalised

141 t_d (divided by $\frac{\Delta S}{\Delta V}$) and A_{avg} (divided by $\frac{(\Delta V)^2}{\Delta S}$) are also shown in Fig.3.

Fig.3. The normalised HIC , t_d and A_{avg} versus ECSs with identical ΔV and ΔS .

2.2 Determination of $A(t)$ by HIC-based injury boundary

As discussed previously, ΔV and ΔS are the two critical parameters of the crash signal to affect HIC value. Therefore, it is convenient to use an iso-injury diagram on ΔS - ΔV plane for a given critical value of HIC_{cr} (e.g. $HIC_{36}=HIC_{cr}=1000$ [21]). However, it should be noted that different values of HIC_{cr} could be given for different levels of Abbreviated Injury Scale (AIS) [7], which is applicable to following discussion on iso-injury (iso-damage) diagrams. The HIC-based ΔS - ΔV iso-injury diagram can be expressed as

$$\Delta S = \alpha \frac{(\Delta V)^{\frac{8}{3}}}{(HIC_{cr})^{\frac{2}{3}}} \quad (3)$$

according to Table 1 where α is a constant depending on the shape of ECS (Table 2).

Similar to pressure-impulse (P-I) iso-damage diagram [22], the iso-injury diagram of $A(t)$ can also be expressed on A_{max} - ΔV plane. Because the peak acceleration of ENPS is infinite, A_{avg} - ΔV iso-injury diagram is adopted here, which, however, does not prevent the use of A_{max} - ΔV iso-injury for ECSs with finite peak values. The HIC-based A_{avg} - ΔV iso-injury diagram can be expressed as

$$A_{avg} = \frac{\Delta V}{t_d} = \beta \left(\frac{HIC_{cr}}{\Delta V} \right)^{\frac{2}{3}} \quad (4)$$

according to Table 1 where β is a constant depending on the shape of ECS (Table 2).

Table 2. Coefficients (α and β) for different ECSs.

Coefficients	ESS	EPTS	ENTS	EITS	EHSS	EHVS	ENPS
α	0.5000	0.8329	0.4164	0.6228	0.5895	0.7145	0.3750
β	1.0000	0.8004	0.8006	0.8029	0.8483	0.6998	1.000

The iso-injury diagrams derived for various ECSs are presented on ΔS - ΔV plane (Fig.4(a)) and A_{avg} - ΔV plane (Fig.4(b)). In Fig.4(a), the injury area is below the iso-injury diagram while it is above the iso-injury diagram in Fig.4(b). Both iso-damage diagrams in Fig.4 show that ENPS results in the safest iso-injury diagram among those seven ECSs, which is consistent with the results presented in Wu and Beaudet [17] although the sequences of iso-injury diagrams in Fig.4(a) and Fig.4(b) for different ECSs are different. It should be noted that the iso-injury diagrams in Fig.4(b) derived from ENPS is the same as that of ESS. Iso-injury diagrams calculated from all triangular ECSs are also identical no matter the ECS is EPTS, ENTS or EITS, which can be seen clearly from Table 1. This is due to the fact that the A_{avg} - ΔV iso-injury diagram is determined without the control of same stop distance ΔS .

Fig.4. The iso-injury diagrams ($HIC_{cr}=1000$) expressed on (a) ΔS - ΔV plane; (b) A_{avg} - ΔV plane.

3. Determination of $A(t)$ based on SDOF analysis

SDOF model has been successfully used to obtain iso-damage diagrams for general structures subjected to blast loading [22-25], as well as impact loading such as package design [26-29]. Therefore, it is a useful tool for the damage assessment of a protected object subjected to deceleration during a crash stop. The SDOF is described by equivalent lump mass (m_b), elastic stiffness (k) and damping coefficient (c) and is subjected to acceleration $\ddot{x}_o(t) = -A(t)$ at its base where $A(t)$ is the deceleration-time history, as shown in Fig.5. Therefore, a SDOF model can represent a structural response inside the protected object when $A(t)$ determined by the IEA and cushion design is applied to the protected object. The dynamic response of SDOF can be described by

$$m_b \ddot{x}_b + c(\dot{x}_b - \dot{x}_o) + kx_{bo} = 0 \quad (5)$$

185 where ($z_{bo} = x_b - x_o$) is the relative displacement and $\ddot{x}_b$ represents the absolute output
 186 acceleration.

187

Fig.5. An SDOF model subjected to deceleration in a crash stop event.

189 Without losing the generality, damping effect is neglected, and the solutions of $\ddot{x}_b$ and z_{bo}
 190 can be obtained by Duhamel integral, i.e.

$$191 \quad \ddot{x}_b(t) = \omega_n \int_0^t \ddot{x}_o(t') \sin \omega_n(t - t') dt' \quad (6)$$

$$192 \quad z_{bo}(t) = -\frac{1}{\omega_n} \int_0^t \ddot{x}_o(t') \sin \omega_n(t - t') dt' \quad (7)$$

193 where t' is middle variable and $\omega_n = 2\pi f_n$ and $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$.

194 **3.1 Determination of $A(t)$ by z_{cr} -based $a-v$ diagram**

195 In addition to the HIC, the output peak deceleration ($|\ddot{x}_b|_{max}$) (e.g. of brain tissue) and the
 196 maximum relative displacement ($|z_{bo}|_{max}$) (e.g. between brain tissue and head skull) can also be
 197 used to assess the occupant injury (e.g. head). In order to evaluate the influence of crash signal $A(t)$
 198 on $|\ddot{x}_b|_{max}$ and $|z_{bo}|_{max}$, excitations (ESS, EPTS, ENTS, EITS, EHSS, EHVS and ENPS)
 199 shown in Fig.2 are also used for the present discussion. Head injury is used here to illustrate the
 200 method, which may be applied to the damage/injury of other objects (occupants). Similar to the
 201 HIC-based calculation in Section 2, all crash pulses are controlled with identical ΔV and ΔS and
 202 marked with numbers in Fig.6. The deceleration is normalised by $\frac{(\Delta V)^2}{\Delta S}$ and time is normalised by
 203 $\frac{\Delta S}{\Delta V}$.

Fig.6. Normalised crash signals with identical ΔV and ΔS .

SDOF model can be employed to construct the iso-damage diagram based on a critical relative displacement (z_{cr}). Following non-dimensional quantities are introduced to generalise the results, i.e.

$$\begin{cases} a = \frac{mA_{max}}{z_{cr}k} \\ a_{avg} = \frac{mA_{avg}}{z_{cr}k} \\ v = \frac{\Delta V}{z_{cr}\sqrt{k/m}} \end{cases} \quad (8)$$

The procedure to obtain z_{cr} -based a - v diagram is the same as that to derive P-I diagram for blast loads [23] when the pressure-time history is replaced by $A(t)$. The same way can be used to construct a_{avg} - v diagrams for all seven ECSs including ENPS with infinite maximum accelerations.

Fig.7 shows the normalised a - v diagrams derived from six ECSs in Fig.2 where the iso-injury diagram is determined by $\frac{z_{bo}}{z_{cr}} = 1$. Fig.8 shows the a_{avg} - v iso-injury diagram for seven ECSs. The a - v diagram in Fig.7 and the a_{avg} - v diagram in Fig.8 vary considerably with crash signal shapes in the impulsive regime. Both diagrams show that the safest boundary in the dynamic regime is determined by ESS and EPTS. In addition, ENPS has the lowest boundary for a_{avg} - v diagram in Regimes III, which means that occupants are most vulnerable in terms of injury defined by critical relative displacement when subjected to ENPS with the same velocity change and average acceleration. This conclusion is different from the results obtained in Sections 2. Therefore, it is important to clarify the conditions when considering the preferred deceleration-time history. In addition, the boundaries for ESS, ENTS and ENPS are stable in the quasi-static regime while those of other ECSs fluctuate initially, but eventually converge to a critical deceleration with

further increase of ΔV . The fluctuation is caused by the ascending components in EPTS, EITS, EHSS and EHVS, which has been reported in the derivation of P-I diagram for explosive loading [25].

Fig.7. The normalised a - v diagrams of six ECSs (ENPS is excluded).

Fig.8. The normalised a_{avg} - v diagrams of seven ECSs (ENPS is included).

3.2 Determination of $A(t)$ based on SRS

Shock response spectrum (SRS) is an efficient approach to assess the severity of shock environment. Various SRS definitions have been summarised in Li and Li [30], among which, the absolute acceleration SRS (AASRS) is widely used for shock test specifications [31,32]. The AASRS is a spectrum of the maximum absolute acceleration of an array of SDOF models subjected to a base excitation. The AASRS curves derived from the seven ECSs in Fig.6 are presented to evaluate their shock severity. Here, the AASRS and structural frequency (f_n) are normalised by $\frac{(\Delta V)^2}{\Delta S}$ and $\frac{\Delta V}{\Delta S}$, respectively.

As shown in Fig.9, the lowest AASRS curve is composed of the following three parts,

- 241 (i) EPTS, $0 \leq f_n/\left(\frac{\Delta V}{\Delta S}\right) \leq 0.25$;
- 242 (ii) ENPS, $0.25 \leq f_n/\left(\frac{\Delta V}{\Delta S}\right) \leq 0.35$;
- 243 (iii) ESS, $0.35 \leq f_n/\left(\frac{\Delta V}{\Delta S}\right) \leq 1.00$.

Fig.9. Normalised AASRS for various ECSs.

In addition, the relative displacement SRS (RDSRS) is also adopted for shock severity analysis, which is related to AASRS when damping is neglected [30], i.e.

$$RDSRS = \frac{AASRS}{\omega_n^2} \quad (9)$$

As shown in Fig.10, the RDSRS normalised by ΔS exhibits the same feature in AASRS. It should be noted that SRS assessment is not related to any specified occupant or protected object.

Fig.10. Normalised RDSRS plots for various ECSs.

4. Final Remarks

This study made further assessment for deceleration-time histories in a crash stop from two aspects, i.e., HIC-based calculation and SDOF-based analysis. Seven well-recognised crash signals with identical velocity change (ΔV) and stop distance (ΔS) are discussed. Strong relationship between crash signal shape and occupant injury is demonstrated. Iso-injury/damage diagrams based on both HIC calculation and SDOF analysis have been built using normalised quantities. It has been found that the ENPS is the safest deceleration-time history among the seven well-recognised crash signals in terms of HIC-based calculation, which results in the safest injury boundary in ΔS - ΔV diagram. However, EPTS, ESS and EHSS jointly construct the highest injury boundary in SDOF based a_{avg} - v diagram and SRS plots, rather than from a single crash signal. In other words, the preferred crash signal is related to the ratio between structural natural frequency and the crash signal frequency and shape. HIC is an empirical formula to assess the head injury in frontal vehicle crash. However, SDOF has broad representation of structural components, which has been widely used in structural design, such as in blast resistance design and package design. Therefore, the selection of preferred deceleration-time history in a crash stop incident depends on the priority of injury/damage criterion.

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