Torque Vectoring and Rear-Wheel-Steering Control for Vehicle’s Uncertain Slips on Soft and Slope Terrain Using Sliding Mode Algorithm

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Abstract—Off-road vehicles always experience serious uncertain longitudinal and lateral slips when running on soft and slope terrains, and some parameters of the vehicles, such as the cornering stiffness and the slip of wheels, are always not constant. In this paper, control strategies for the torque of each wheel and the rear-wheel-steering angle are proposed to maintain a stable velocity and approach an ideal reference model for the off-road vehicle by using second-order sliding mode (SOSM) techniques. An observer is constructed to estimate the actual sideslip angle of the vehicle with the consideration of uncertainties and disturbances of the system. Then, with conditions of bounded uncertainties and disturbances, composite super-twisting (ST) controllers combined with a velocity controller are designed to generate the total torque, the differential torque, and the rear-wheel-steering angle. On this basis, the proposed controllers have been verified to lead good robustness for maintaining the stable velocity and approaching the ideal reference model by using an optimal torque allocation controller at a lower layer. In comparison with conventional yaw moment controllers without the rear-wheel-steering control, the proposed controllers are shown to be more effective.

Index Terms—Torque vectoring, rear-wheel-steering control, direct yaw moment control, sliding mode, uncertain slips, soft terrain.

I. INTRODUCTION

OFF-ROAD vehicles are considered to drive on off-paved surfaces, such as sandy beach, desert, forest and even planetary surface. Meanwhile, when running under off-road environments where deformable and rough terrains exist, dynamical characteristics of vehicles are complex [1]-[2]. As a result, wheels show easily slips and traction losses on the loose soil [3]. Moreover, some parameters of the vehicle on a slope terrain, including the cornering stiffness and the slip of wheels, are always not constants due to the variation of the normal force of each wheel. For these reasons, drivers will face more difficulties when driving the vehicle under the off-road environments than on smooth and hard-paved roads.

Some researchers have focused on precisely modeling the interaction of wheel with soft soil, which is a key part for evaluating the performance of vehicles [4], [5]. Semi-empirical models basing on Bekker’s normal stress [6] and Janosi’s shear stress equations [7] are mostly used for studying soil properties during interaction with wheels. Then, the relationship among the slip, longitudinal traction, lateral force, vertical load and torque of a single wheel can be derived [8]. On this basis, the wheel-soil model and the multibody model can be combined and utilized to describe motion dynamics of vehicles. As long as longitudinal and lateral slips of the wheel are obtained, control algorithms are able to effectively compensate the vehicle slips in order to follow a given path or approach an ideal model [9]. Furthermore, the real-time estimation for terrain parameters can be applied to improve the model accuracy and control the vehicle traversing on deformable terrains [10]. However, as positions of the vehicle on a slope are always varying, the establishment of the wheel-soil model is difficult to determine the terrain force, and there are also some compromise problems between the model accuracy and the calculation efficiency.

From another perspective, the longitudinal and lateral slips of wheels and the variations of parameters can be treated as some unknown disturbances and uncertainties in the vehicle system. Several methods have already been used for the stability control of land (ground) vehicles, and the proportional-integral-differential (PID) is one of the classic control approaches [11]-[12]. However, the PID control is hard to achieve high control performances for complicated systems under unknown conditions. Therefore, many approaches have been developed to fix the problems of complex and nonlinear systems under uncertainties and disturbances, such as the sliding mode control (SMC) [13], the model predictive control (MPC) [14], etc. For MPC methods, future states of vehicle dynamics can be predicted over a finite time horizon, and the cost functions are able to be minimized for prediction horizons...
under a set of operation constraints [15]. Therefore, the MPC approaches have been widely studied, and applied in the electronic stability control (ESC) [16], the adaptive cruise control (ACC) [17], the path following [18], and the energy management [19] for vehicle systems. However, the MPC-based approaches may cause some computational burdens in finding optimal solutions, and induce accuracy losses when using simplified models in some critical situations [20]. As an effective tool to suppress disturbances for complex systems, the SMC algorithm is proposed to be used in this study due to its satisfactory control performances for nonlinear systems and unknown disturbances.

It is well known that SMC is a strong robust technique for a variety of nonlinear applications with uncertainties and disturbances. The superior feature of the SMC methods over other robust control ones is its complete compensation of matched disturbances when the system is on the sliding surface [21]. Due to discontinuous sign functions used for the controllers, the conventional first-order SMC methods may exhibit large chattering in actuators. In order to attenuate chattering and enhance control performances, many approaches have been extensively developed, such as disturbance-observer-based control (DOBC) methods for reducing unknown parts of systems [22], and higher-order SMC (HOSMC) methods for conducting continuous controllers which further include second-order sliding-mode (SOSM) control methods [23] and arbitrary order SMC methods [24]. DOBC algorithms could reduce problems of the heavy chattering phenomenon, but need to estimate disturbances in the system with more assumptions with respect to unknown items [25]. Arbitrary order SMC approaches have advantages to achieve chattering free for the controlled system, but would bring more complexities for the controllers [23]. In comparison, the SOSM approaches are able to effectively reduce chattering with high control performances in practices, and easy to implement [26]-[27]

Thus, SOSM methods have been widely used in real applications, such as the direct yaw moment control and the four-wheel-steering (4WS) control for improving stability performances of vehicles. To four-wheel independently driven (4WID) electric vehicles (EVs), the SOSC controllers were designed to generate the yaw moment to track ideal reference models [13], [28]. To 4WS vehicles, the SMC approaches were applied to provide high control performances in practices, and easy to implement [26]-[27].

Hence, the SOSM methods have been widely used in real applications, such as the direct yaw moment control and the four-wheel-steering (4WS) control for improving stability performances of vehicles. To two-wheel independently driven (2WID) electric vehicles (EVs), the SMC controllers were designed to generate the yaw moment to track ideal reference models [13], [28]. To 2WS vehicles, the SMC approaches were applied to provide high control performances in practices, and easy to implement [26]-[27].

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Fig. 1 Dynamic model for off-road vehicle

2) To cruise at a stable velocity for the off-road vehicle, a velocity controller is designed by using ST algorithms. On this basis, the velocity controller is proposed to generate total torques for four wheels and impose longitudinal tractions on the vehicle body.

3) Composite controllers, including a differential torque controller and a rear-wheel-steering controller, are designed to track an ideal reference model. Then, the composite controllers combined with the velocity controller at a upper layer and a optimal torque allocation controller at a lower layer are proposed to exert the torque on each wheel and generate the steering angle for rear wheels.

The remainder of this paper is organized as the following. The vehicle dynamic model is established in Section II. The sidestrip angle observer construction and the controllers design are presented in Section III. In Section IV, simulations are conducted to verify the proposed controllers. Finally, the conclusions of this paper are drawn in Section V.
where  \( F_{thx} = F_{x1} - F_{x3} \),  \( F_{thv} = F_{41} - F_{33} \), and  \( F_{thw} = F_{2y} - F_{1y} \) and  \( F_{thw} = F_{4y} - F_{3y} \).

Additionally, the grade and bank angles have the following relationships with the yaw angle of the vehicle and the slope angle of the terrain

\[
\begin{align*}
\sin \alpha_g &= \sin \alpha \sin \psi \\
\sin \alpha_b &= \sin \alpha \cos \psi
\end{align*}
\]  

(4)

B. Problem Formulation

1) Uncertain longitudinal slip

According to Wong-Reece pressure distribution model when the vehicle moving on soft terrains, the longitudinal traction force and the driving torque of a single wheel have the following relationships [31]-[32]

\[
\begin{align*}
F_{x_i} &= hR\int_{\beta_i}^{\beta_{i+1}} \left( \sigma_i \cos \theta + \tau_i \sin \theta \right) d\theta \\
F_{\theta_i} &= hR\int_{\psi_i}^{\psi_{i+1}} \left( \tau_i \cos \theta - \sigma_i \sin \theta \right) d\theta \\
T_{\theta_i} &= hR\int_{\delta_i}^{\delta_{i+1}} \tau_i d\theta
\end{align*}
\]  

(5)

where  \( b, R, \sigma, \tau, \beta, \) and  \( \psi, \delta, \) are the width, radius, normal pressure, shear stress, entering angle and exit angle of wheel  \( i \), respectively;  \( T_{\theta_i} \) is the net torque imposed on wheel  \( i \).

However, the wheel model of (5) is so complex to implement on vehicle models. As a result, this model can be simplified using a fitting function basing on experimental data as follows [33]

\[
F_{x_i} = T_{\theta_i} / R - F_{x_i}(a, s_i, b_i),
\]  

(6)

where  \( s_i \) is the slip ratio of wheel  \( i \);  \( a, b_i \) are fitting coefficients.

By introducing the driving efficiency of motors  \( \eta_i \) (6) can be rewritten as

\[
F_{x_i} = (T_i - \Delta T_i) / R, \quad \Delta T_i = (1 - \eta_i)T_i + RF_i(a, s_i, b_i), \quad \eta_i \quad \text{efficiency from driving motor i to wheel i}, T_i, F_{x_i}, \eta_i
\]  

and  \( s_i \) are assumed to be bounded, and thus there exists a positive constant  \( T_5 \) such that  \( |\Delta T_i| \leq T_5 \). Furthermore, it is assumed that the tractive force  \( F_{x_i} \) and the torque  \( T_i \) will stay away from the capacity boundaries of tires and motors.

2) Uncertain lateral sideslip

The cornering stiffness of wheels is mostly treated as a constant parameter to facilitate the controller design [34], and the lateral forces of front and rear wheels can be written as

\[
\begin{align*}
F_{x_{l}} &= c_i \beta_i \\
F_{x_{r}} &= c_i \beta_i
\end{align*}
\]  

(8)

where  \( \beta_l \) and  \( \beta_r \) are the equivalent slip angles of front and rear wheels, respectively;  \( c_l \) and  \( c_r \) are the equivalent constants of front-wheel and rear-wheel cornering stiffness, respectively.

With kinematics of the vehicle, the sideslip angles of four wheels can be derived as

\[
\beta_{l,2} \approx \beta_l \approx \delta_l - \beta - \delta + \frac{1}{v} \frac{l_c}{v} \gamma, \quad \beta_{r,4} \approx \beta_r \approx \delta_r - \beta - \delta + \frac{1}{v} \frac{l_c}{v} \gamma.
\]  

(9)

However, for off-road vehicles on soft and slope terrains, the lateral force of each wheel calculated by a linear model of cornering stiffness may not accurate, and values of the cornering stiffness are dependent on terrain conditions and normal forces on wheels. With the consideration of uncertain lateral sideslips and model nonlinearities, the actual cornering stiffness of wheel  \( i \) is denoted as  \( c_i \), and the lateral force of wheel  \( i \) can be expressed as

\[
F_{x_i} = c_i \beta_i.
\]  

(10)

For the vehicle running on soft and slope terrains,  \( c_i \) is not a constant. As a result,  \( c_l \) and  \( c_r \) are, respectively, the nominal constant parts of  \( c_l \) and  \( c_r \). Then, the cornering stiffness of each wheel can be expressed as

\[
\begin{align*}
c_l &= c_{i,1} / 2 + \Delta c_{i,1}, \quad c_r = c_{i,2} / 2 + \Delta c_{i,2}, \\
c_l &= c_{i,3} / 2 + \Delta c_{i,3}, \quad c_r = c_{i,4} / 2 + \Delta c_{i,4},
\end{align*}
\]  

(11)

where  \( \Delta c_i \) is the uncertain and nonlinear part of the cornering stiffness of wheel  \( i \), and satisfies  \( |\Delta c_i| \leq c_{i,2} / 2 \).

With considerations of the uncertain cornering stiffness, the actual lateral force variables in (1)-(3) can be obtained as

\[
\begin{align*}
F_{\eta_{1}} &= (c_1 + c_2) \beta_i = (c_1 + \Delta c_1 + \Delta c_2) (\delta_1 - \beta - \delta + \frac{1}{v} \frac{l_c}{v} \gamma) \\
F_{\eta_{2}} &= (c_3 + c_4) \beta_i = (c_3 + \Delta c_3 + \Delta c_4) (\delta_2 - \beta - \delta + \frac{1}{v} \frac{l_c}{v} \gamma) \\
F_{\eta_{3}} &= (c_4 - c_3) \beta_i = (\Delta c_3 - \Delta c_4) (\delta_2 - \beta - \delta + \frac{1}{v} \frac{l_c}{v} \gamma)
\end{align*}
\]  

(12)

3) Vehicle system

As  \( \delta_l \) and  \( \delta_r \) are small, the following simplifications are used:  \( \sin \delta_l \approx \delta_l \sin \delta_r \approx \delta_r \cos \delta_l \approx 1, \cos \delta_l \approx 1 \). Let the total torque  \( T_2 = T_1 + T_2 + T_3 + T_4 \), and the differential torque  \( T_6 = T_2 - T_1 + T_3 - T_4 \). By substituting (7) and (12) into (1)-(3), the vehicle system can be obtained when  \( v > 0 \)

\[
\begin{align*}
\dot{v} &= F_{x_i}(\beta, \gamma, \nu, \delta) + F_{z_i}(\beta, \gamma, \nu, \delta) + T_i / (Rm) \\
-g \sin \alpha (\beta \cos \psi + \nu) + \epsilon_v \\
\beta &= A_1 \beta + A_2 \gamma + B_1 \delta \quad \text{for } i = l, \quad \text{and } \quad T_i = E_2 (\beta, \gamma, \nu, \psi) + \epsilon_v
\end{align*}
\]  

(13)

\[
\begin{align*}
\dot{\gamma} &= A_1 \beta + A_2 \gamma + B_2 \delta \quad \text{for } i = r, \quad \text{and } \quad T_i = E_2 (\beta, \gamma, \psi, u) + \epsilon_v
\end{align*}
\]  

\( \text{TABLE I} \)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>Vehicle mass</td>
</tr>
<tr>
<td>( l_i )</td>
<td>Distance from center of gravity (CG) to front axle</td>
</tr>
<tr>
<td>( l_i )</td>
<td>Distance from CG to rear axle</td>
</tr>
<tr>
<td>( d )</td>
<td>Distance from CG to left/right wheel</td>
</tr>
<tr>
<td>( i )</td>
<td>Wheel ID, and 1, 2, 3, 4</td>
</tr>
<tr>
<td>( F_{thi} )</td>
<td>Longitudinal/lateral force of wheel i</td>
</tr>
<tr>
<td>( I_i )</td>
<td>Vertical load of wheel i</td>
</tr>
<tr>
<td>( \beta_i )</td>
<td>Yaw moment of inertia</td>
</tr>
<tr>
<td>( h_{ih} )</td>
<td>Steering angle of front/rear wheels</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravity acceleration</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Posture angle of vehicle on slope</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Yaw rate of vehicle</td>
</tr>
<tr>
<td>( \beta_i )</td>
<td>Sideslip angle of vehicle</td>
</tr>
<tr>
<td>( \beta_i )</td>
<td>Sideslip angle of wheel i</td>
</tr>
<tr>
<td>( v )</td>
<td>Total velocity at CG</td>
</tr>
<tr>
<td>( v_i )</td>
<td>Longitudinal/lateral velocity at CG</td>
</tr>
<tr>
<td>( \gamma_i )</td>
<td>Velocity of wheel</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Lateral acceleration of vehicle</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Slope angle of terrain</td>
</tr>
<tr>
<td>( \alpha_{ia} )</td>
<td>Grade angle of vehicle on slope terrain</td>
</tr>
<tr>
<td>( \alpha_{ia} )</td>
<td>Bank angle of vehicle on slope terrain</td>
</tr>
</tbody>
</table>

\( \text{Transactions on Vehicular Technology} \)
where
\[ A = \begin{bmatrix} \frac{c_1 + c_2}{mv} & -1 + \frac{c_1 l_1 - c_2 l_1}{mv^2} \\ \frac{l_1 c_1 - l_1 c_2}{I_z} & \frac{l_1 c_2 + l_1 c_1}{I_z} \end{bmatrix}, B = \begin{bmatrix} \frac{c_1}{mv} & \frac{c_2}{mv} & 0 \end{bmatrix}, \]

\[ F_{s_1} = c_1 \left( \beta - \delta \right) \left( \delta - \frac{\gamma}{v} \right) + \frac{c_1}{m} \left( \beta \frac{\gamma}{v} \right), \]

\[ F_{s_2} = \frac{c_1 \delta^2}{m} \left( 2 \beta - \delta - \frac{\gamma}{v} \right), \]

\[ E_1 = H_1 (\gamma, \psi, v) + G_1 (\beta, \gamma, \psi, v) + \left( F_{s_1} \delta + F_{s_2} \delta \right) / (mv), \]

\[ E_2 = H_2 (\gamma, \psi, \delta, v) + G_2 (\beta, \gamma, \psi, v, \delta) + \frac{\gamma}{2} / (I_x). \]

\[ H_1 = \frac{-g \sin \alpha \cos \psi}{v} + \left( -c_1 l_1 + c_1 l_1 \right) \frac{d \gamma}{mv}, \]

\[ H_2 = \frac{-1}{v^3} \left( l_1 c_1 + l_1 c_2 d \right) \frac{d \gamma}{mv}, \]

\[ G_1 = \frac{m g \sin \alpha \sin \psi}{v} \left( c_1 + c_1 \right) \frac{d \gamma}{mv}, \]

\[ G_2 = \frac{d \beta}{v} \left( l_1 c_1 - l_1 c_2 \right) \frac{d \gamma}{mv}. \]

Additionally, due to the uncertain longitudinal and lateral slips, \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) are the unknown parts of the vehicle system as follows
\[ \varepsilon_x = \Delta F_{x_1} + \Delta F_{x_2} + \Delta T_{\psi_1} / (Rm), \]
\[ \varepsilon_\beta = \Delta A_\beta + \Delta A_\beta \gamma + \Delta B_1 \left[ \delta_i, \delta_i, \psi_1 \right] + \Delta H_1 + \Delta G_1, \]
\[ \varepsilon_\gamma = \Delta A_\gamma + \Delta A_\gamma \gamma + \Delta B_2 \left[ \delta_i, \delta_i, \psi_1 \right] + \frac{d \Delta T_{\psi_1}}{(RI_x)}, \]

with
\[ \Delta C = \frac{d}{I_x} \left( \left( \Delta c_1 - c_1 \right) \delta_i^2 + \left( \Delta c_1 - c_1 \right) \delta_i \beta + \left( \Delta c_1 - c_1 \right) \delta_i^2 \right), \]
\[ \Delta C = \frac{d}{I_x} \left( \left( \Delta c_1 - c_1 \right) \delta_i^2 + \left( \Delta c_1 - c_1 \right) \delta_i \beta + \frac{1}{v} \left( \Delta c_1 - c_1 \right) l_i \delta_i \gamma + \frac{1}{v} \left( \Delta c_1 - c_1 \right) l_i \delta_i \gamma \right). \]

4) Ideal reference model

The controlled off-road vehicle is expected to maintain consistent dynamic properties for drivers, and an ideal reference model of the vehicle running on a horizontal terrain \((a = 0)\) is defined as follows
\[ \left[ \dot{\beta}_i, \gamma \right] = A \left[ \beta_i, \gamma \right] + B \left[ \delta_i \right], \]

where
\[ B = \left[ c_1 / (mv), c_1 / I_z \right]. \]

The model (14) is a classic 2-DOF linear bicycle model [34]. Thus, the sideslip angle and the yaw rate of the off-road vehicle running on soft slopes will be controlled to track this ideal reference model.

III. CONTROL SYSTEM DESIGN

A. Observer for sideslip angle

To implement SOSM controllers, the sideslip angle needs to be known. However, the sideslip angle of the vehicle is one of the most difficult parameters to obtain, and is very expensive to use some special sensors to directly measure [35]. Therefore, many topics for the estimation of the sideslip angle have been widely discussed. Extended Kalman filter (EKF) algorithms through a single-track vehicle model could obtain the sideslip angle with low computational burdens [36]-[37]. Additionally, Lyanunov-based observers have been proposed for the estimation of the sideslip angle [38], and the gain-scheduling observer can be determined with off-line and on-line computations basing on a linear-parameter-varying (LPV) model of the vehicle [39]. To address model nonlinearities and uncertain slips, a Lyanunov-based SM observer will be designed for estimating the sideslip angle of the vehicle.

An assumption for the vehicle system is given as the following.

Assumption 1: The states and the inputs of system (13) are bounded (BIBS), and there exist some positive constants \( \bar{\beta}, \bar{\gamma}, \bar{\delta}, \) and \( \bar{F}_s \), such that
\[ (1) \ |\beta| \leq \bar{\beta}, \ |\gamma| \leq \bar{\gamma}, \ |\delta| \leq \bar{\delta}, \text{ and } |F_s| \leq \bar{F}_s; \]
\[ (2) \ |F_{s_1}|, |F_{s_2}|, |F_{s_3}|, \text{ and } |F_{s_4}| \text{ are all bounded by } \bar{F}_s / 2. \]

Remark 1: Owing to physical limits of steering mechanisms and driving motors, the steering angles and the tractions are all bounded. Actually, the steering angles are always much smaller than their physical limits when the vehicle runs at a high speed.

Remark 2: To avoid frequently switching between the driver commands of the brake or accelerator pedal and the inputs of the velocity controller at a low speed, the speed is assumed to have a lower bound when the velocity controller is effective. Due to safety concerns, the vehicle has a maximum velocity. Thus, the velocity bounded by \( \bar{v} \) and \( \bar{\bar{v}} \) is considered in this study, which is 30 km/h ≤ \( v \) ≤ 80 km/h.

Remark 3: When small front-wheel-steering angles are inputted to the vehicle system (13) by drivers, the sideslip angle is always bounded. In this study, the proposed controllers are designed to follow the ideal model (14) which is under the condition of horizontal terrains \((a = 0)\), and to provide consistent steering properties for drivers. Moreover, in some worst-cases scenarios, other controllers [40]-[41] could be considered to construct under the layer of the proposed controllers to stabilize lateral dynamics of the vehicle.

The following variables are introduced: \( x = [\beta, \gamma, \psi, \delta, u, \dot{u}, a, \dot{a}]^T, v = [\delta_i, \delta_i, \psi_1]^T. \) Additionally, \( F_{s_1} \) and \( F_{s_2} \) are treated as unknown variables. Then, the sideslip angle and the yaw rate in the vehicle model (13) can be recast as
\[ \left[ \dot{x}_1 = A_1 x_1 + A_2 x_2 + B_1 u + H_1 + G_1 + e_1, \right. \]
\[ \left. \dot{x}_2 = A_2 x_1 + A_2 x_2 + B_2 u + H_2 + G_2 + e_2 \right], \]

where
\[ e_1 = e_\beta + \delta \frac{F_{s_1} \delta_i + F_{s_2} \delta_i}{mv}, \]
\[ e_2 = \frac{\gamma}{2} / (I_x). \]

Remark 4: \( H_1, G_1, H_2, \) and \( G_2 \) are known functions of states and inputs, and \( e_1 \) and \( e_2 \) are unknown variables in the plant (15). The functions of \( H_1 \) and \( H_2 \) only include measurable variables \( x_2, \dot{u}, \psi, \) and \( v. \) Meanwhile, the functions of \( G_1 \) and \( G_2 \) include the unknown variable \( x_1 \) to be estimated.
Under the conditions of Assumption 1, $\Delta H_1, \Delta H_2, \Delta G_1,$ and $\Delta G_2$ are all bounded by some positive constants, which can be calculated as follows

$$\Delta H_1 \leq \frac{c_1 d (l_1 + l_2)^2}{m^2 v^2 (\nu - d \bar{\gamma})}, \quad \Delta G_1 \leq \frac{2c_1 d \bar{\gamma}^2}{m^2 (\nu - d \bar{\gamma})},$$

$$\Delta H_2 \leq \frac{c_2 d I_1 + I_2 + 2d \delta_s}{I_2 (\nu - d \bar{\gamma})}, \quad \Delta G_2 \leq \frac{c_2 d \bar{\gamma}^2}{I_2 (\nu - d \bar{\gamma})}.$$ $\Delta$, $\Delta$, $\Delta$, $\Delta$

Therefore, two positive constants $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ can be found, such that $\epsilon_1 \leq \bar{\epsilon}_1$ and $\epsilon_2 \leq \bar{\epsilon}_2$.

To eliminate some unmatched uncertainties for the observer, the lateral acceleration, which can be easily measured by some sensors, is expressed as follows

$$a_y = v (\dot{x}_2 + x_1) = v A_1 x_1 + v (A_2 + 1) x_2 + v B_2 u + v H_1 + v G_1 + v \epsilon_0$$

(16)

To construct the observer for the actual sideslip angle, an assumption for the parameters and variables of the vehicle is given as the following.

Assumption 2: The parameters in matrixes $A$ and $B$ are all known. Meanwhile, the slope angle of the terrain, the speed, yaw angle, yaw rate and lateral acceleration of the vehicle can be measured by some sensors, and the vehicle is understeered. Then, the bank and grade angles of the vehicle on the soft and slope terrain can be obtained. Furthermore, the parameters and variables in functions of $H_1, H_2, G_1,$ and $G_2$ are all known except the state variable of the sideslip angle.

The observer for the sideslip angle of the system (15) is constructed as

$$\dot{\hat{x}}_1 = A_1 \hat{x}_1 + A_2 \hat{x}_2 + B_2 u + H_1 (x_1, \nu, v) + (a_y - \hat{a}_y) / v + k_1 (x_2 - \hat{x}_2)$$

$$\dot{\hat{x}}_2 = A_2 \hat{x}_1 + A_2 \hat{x}_2 + B_2 u + H_2 (x_2, \nu, v) + \nu$$

$$\hat{a}_y = v A_1 \hat{x}_1 + v (A_2 + 1) x_2 + v B_2 u + v H_1 (x_1, \nu, v)$$

where $\hat{x}_1$ and $\hat{x}_2$ are the estimates of $\beta$ and $\gamma$, respectively; $\hat{a}_y$ is the estimate of $a_y$; $U$ and $k_1$ are the control variables to be designed.

Then, the following conclusion for estimating the sideslip angle of the vehicle is ready to be presented.

**Proposition 1:** If the observer is designed as (17), the control variable is given as

$$k_1 = -1 + (c_1 l_1 - c_1 l_2) (1 / (m^2 v^2) + 1 / l_2),$$

(18)

and the control law is given as

$$\nu = k_1 \text{sign} (x_2 - \hat{x}_2),$$

(19)

with

$$k_1 > \bar{\epsilon}_2 + \bar{\epsilon}_2,$$

(20)

then the estimated values $\hat{x}_1$ and $\hat{x}_2$ will track their actual values $x_1$ and $x_2$ asymptotically.

**Proof:** Let $\tilde{x}_1 = x_1 - \hat{x}_1$ and $\tilde{x}_2 = x_2 - \hat{x}_2$. Then, the time derivative of $\tilde{x}_1$ and $\tilde{x}_2$ can be obtained as

$$\dot{\tilde{x}}_1 = (A_2 - k_1) \tilde{x}_1$$

$$\dot{\tilde{x}}_2 = A_2 \tilde{x}_1 + G_2 (\hat{x}_2, \nu, v) + \epsilon_2 - \nu.$$ (21)

Select a Lyapunov function as $V (\hat{x}_1, \hat{x}_2) = \tilde{x}_1^2 / 2 + \tilde{x}_2^2 / 2.$ Taking the derivative of $V (\hat{x}_1, \hat{x}_2)$ along (21) yields

$$\dot{V} = (A_2 - k_1) \hat{x}_1 \dot{\hat{x}}_1 + \hat{x}_1 \dot{x}_2 + (A_2 \hat{x}_1 + G_2 (\hat{x}_2, \nu, v) + \epsilon_2 - \nu)$$

(22)

Under Assumption 1, there exist a positive constant $\bar{G}_1$, such that $|G_1| \leq \bar{G}_1$. Then, applying (18) and (19) into (22) yields

$$\dot{V} \leq \hat{x}_1 \left( G_2 (\hat{x}_2, \nu, v) + \epsilon_2 - k_1 \text{sign} (\hat{x}_2) \right)$$

$$\leq \hat{x}_1 \left( |G_2| \hat{x}_2 + \epsilon_2 - k_1 \hat{x}_2 \right)$$

(23)

It can be inferred from (23) that $\dot{V} < 0$ when $\hat{x}_2 \neq 0$.

According to the Lyapunov and Lasalle invariant set theorems, it is clear that the system (21) can be asymptotically stabilized by the controller, and estimated errors asymptotically converge to zero. Thus, the observer (17) for the sideslip angle of the vehicle will track its real value asymptotically. Thereby, the proof is completed.

**B. Velocity controller**

To cruise at a constant speed on soft and slope terrains using a velocity controller, the driver will only deploy steering commands to the front wheels without any brake or accelerator pedal input. Meanwhile, stability controllers will be designed to maintain consistent steering properties with the ideal vehicle travelling on a horizontal terrain ($\alpha = 0$). Therefore, to 4WS and 4WID vehicles, the inputs of the proposed controllers for the vehicle system include the total torque, the differential torque, and the rear-wheel-steering angle. Additionally, a torque allocation controller is proposed to exert the torque on each wheel and simultaneously satisfy the results of the total torque and the differential torque.

To the velocity of the vehicle, the first equation of (13) can be rewritten as

$$\ddot{v} = F_{\nu} - g \sin \alpha (\beta \cos \psi \nu + \sin \nu) + \frac{1}{mR} T_v + \epsilon_3,$$ (24)

where $\epsilon_3 = F_{\nu} + \epsilon_0$.

Under Assumption 1, the upper bound of $\epsilon_3$ can be found as

$$|\epsilon_3| \leq \frac{c_3 l}{m} \left( 2 \delta_s + 2 \bar{\gamma} + \frac{l + l}{v} \right)$$

$$+ \frac{c_3 l ^2}{m} \left( \frac{2 \bar{\gamma} + l}{v} \delta_s + \frac{4 \bar{\gamma} l}{mR} \right).$$ (25)

Unlike the virtual actuator in the observer, the chattering phenomenon is harmful to control systems. In fact, SOSM techniques can be used to reduce the chattering problem. To cruise at a stable velocity $v_0$, a SOSM-based ST controller is chosen as follows.

**Proposition 2:** If the SOSM controller is designed as...
$$T_b = \zeta_1 + \zeta_2,$$  \hspace{1cm} (26)

where

$$\zeta_1 = mgR \sin(\beta \cos \psi + \sin \psi) - R \left[ c_1 \delta_l \left( 2\hat{\beta} - \frac{l}{v} \gamma - \delta_l \right) - (c_1 + c_3) \hat{\beta}^2 - \hat{\gamma} \frac{c_1 l - c_3 l}{l} \right].$$  \hspace{1cm} (27)

$$\hat{\gamma} = -\lambda \left( v - v_d \right) \text{sign}(v - v_d) + \zeta_3,$$  \hspace{1cm} (28)

$$\dot{\zeta}_1 = -\lambda_1 \left( v - v_d \right) \text{sign}(v - v_d) + \zeta_3 \geq \zeta_{\text{md}},$$  \hspace{1cm} (29)

where

$$\alpha > \overline{\alpha} m R, \quad 0 < q < 1, \quad \hat{x}_m \geq \overline{\overline{x}_m} m R / q,$$

$$\lambda > \left( \alpha + m R \overline{x}_m \right) \left( 1 - q \right) \frac{2mR}{\sqrt{\alpha - m R \overline{x}_m}}$$  \hspace{1cm} (30)

then the vehicle will approach the stable velocity $v_d$ in a finite time when $|\hat{x}_m| \leq \overline{x}_m$ and $|\dot{x}_m| \leq \overline{\overline{x}_m}$.

**Proof:** The variables $\delta_l, \gamma,$ and $\psi$ can be obtained by some sensors under the conditions of Assumption 2, and $\hat{\beta}$ can be estimated by the observer constructed in Proposition 1. Taking time derivative of $\dot{\zeta}_1$ yields

$$\dot{\zeta}_1 = \frac{c_1}{m} \dot{\delta}_1 \left( 2\hat{\beta} - \frac{l}{v} \gamma - \delta_l \right) + \frac{c_3}{m} \dot{\delta}_1 \left( 2\hat{\beta} - \frac{l}{v} \gamma + \frac{l}{v^2} \gamma \delta_l - \delta_l \right) + \epsilon_2.$$  \hspace{1cm} (31)

Under the conditions of Assumption 1, terms given in (30) are bounded. Therefore, we can find a positive constant $\overline{\overline{x}_m}$ such that $|\dot{x}_m| \leq \overline{\overline{x}_m}$.

Choose $\epsilon_0 = v - v_d$ as the SM surface, and $\dot{v}_d = 0$. Applying (27) into system (24) leads to

$$\dot{\epsilon}_2 = \epsilon_1 + \frac{1}{m R} \epsilon_2.$$  \hspace{1cm} (30)

Then, according to the theorem of ST algorithms [26], $\epsilon_2$ and $\hat{\gamma}$ can be obtained as shown in (28) and (29). Therefore, it can be concluded that the sliding variable $\epsilon_1$ will be stabilized to the origin in a finite time under the controller (26). \hfill \blacksquare

**C. Conventional Yaw moment controller**

To the conventional yaw moment control without features of rear-wheel steering, the controller is proposed to include inputs of differential tractions. Thus, a SM surface $\sigma_\mu$ is chosen as

$$\sigma_\mu = \gamma - \gamma_d + \mu_\beta (\beta - \beta_d),$$

where $\mu_\beta$ is a weight coefficient for reflecting the proportion of the sideslip angle.

Then, the SOSM controller with the SM surface of (32) is able to be presented as follows.

**Proposition 3:** If the SOSM controller for the differential torque is designed as

$$T_b = \xi_1 + \xi_2,$$  \hspace{1cm} (33)

where

$$\xi_1 = -\frac{1}{B_{\beta 2}^2} \left[ A_1 (\hat{\beta} - \beta_d) + A_2 (\gamma - \gamma_d) + F_1 \right],$$

$$\xi_2 = -\lambda_2 \text{sign}(\hat{\beta} - \beta_d) + \zeta_3,$$

$$\dot{\xi}_1 = \left\{ \begin{array}{ll} -\xi_2 & \text{if } |\xi_1| > \zeta_{\text{md}} \\ -\sigma_\mu \text{sign}(\hat{\beta} - \beta_d) & \text{if } |\xi_1| \leq \zeta_{\text{md}} \end{array} \right\},$$

then the sliding variable $\sigma_\mu$ can be stabilized to the origin in a finite time by choosing proper $\alpha_\mu, \beta_\mu, q_\mu,$ and $\lambda_\mu$.

**Proof:** The proof is omitted for brevity since it is actually similar to the proof of Proposition 2. \hfill \blacksquare

**D. Composite Controllers**

With the consideration of longitudinal slips and gravity component effects, the velocity controller in Proposition 2 can maintain the stable velocity $v_d$. To the lateral sideslip of the vehicle, composite controllers are proposed to track the ideal reference model (14). Under the controller in Proposition 2, the model for the sideslip angle and the yaw rate in (13) can be rewritten as

$$\dot{\hat{\beta}} = A_1 \hat{\beta} + A_2 \gamma + B_1 \dot{\delta}_1 + F_1 \left( \hat{\beta}, \gamma, v_d \right) + B_{\text{md}} \sigma_\mu + \epsilon_1,$$

$$\dot{\gamma} = A_3 \hat{\beta} + A_4 \gamma + B_3 \dot{\delta}_3 + F_3 \left( \hat{\beta}, \gamma, v_d \right) + B_{\text{md}} \sigma_\mu + \epsilon_1,$$

where

$$F_1 = H_1 (\gamma, \psi, v_d), \quad G_1 (\beta, \gamma, v_d), \quad B_{\text{md}} = d^2 c_1 (I_1 - v_d \hat{v}) / I_2 (v_d^2 - d^2 \hat{v}^2),$$

$$F_2 = -d \left( I_1 c_1 v_d + I_2 c_2 d \right) / I_2 (v_d^2 - d^2 \hat{v}^2) + d \frac{c_1 l \delta_1}{I_2 (v_d^2 - d^2 \hat{v}^2)} \gamma^2,$$

$$d \frac{\gamma \hat{\beta} (I_1 c_1 v_d - l c_2 d \gamma - c_1 \delta_1 \delta_3)}{I_2 (v_d^2 - d^2 \hat{v}^2)} + \frac{d \gamma \hat{\beta} (I_1 c_1 v_d - l c_2 d \gamma - c_1 \delta_1 \delta_3)}{I_2 (v_d^2 - d^2 \hat{v}^2)}.$$  \hspace{1cm} (37)

Additionally, choose a composite disturbance variable as $e_d = e_2 - (B_{\text{md}} + B_4) \hat{\epsilon}_1 / B_{\text{md}}$. \hfill (38)

Then, the composite controllers are ready to be presented. **Proposition 4:**

(1) If the SOSM controller for the rear-wheel-steering angle is designed as

$$\dot{\delta}_1 = \zeta_1 + \zeta_2,$$  \hspace{1cm} (39)

where

$$\zeta_1 = -\frac{1}{B_{\beta 2}^2} \left[ A_1 (\hat{\beta} - \beta_d) + A_2 (\gamma - \gamma_d) + F_1 \right],$$

$$\zeta_2 = -\lambda_2 \text{sign}(\hat{\beta} - \beta_d) + \zeta_3,$$

$$\dot{\xi}_1 = \left\{ \begin{array}{ll} -\xi_2 & \text{if } |\xi_1| > \zeta_{\text{md}} \\ -\sigma_\mu \text{sign}(\hat{\beta} - \beta_d) & \text{if } |\xi_1| \leq \zeta_{\text{md}} \end{array} \right\},$$

with
\[ \alpha_0 > \frac{e_{\text{ref}}}{B_3}, \quad 0 < q_0 < 1, \quad \phi_M > \frac{e_{\text{ref}}}{q_0 B_3}, \]

\[ \lambda_0 > \sqrt{2 \left( \frac{B_3 \sigma_0 + e_{\text{ref}}}{B_3 (1-q_0)} \right)(1+q_0)}, \]

then the sideslip angle of the vehicle will approach \( \theta \) in a finite time when \( |e_| \leq \bar{\tau} \) and \( |\dot{e}_s| \leq \bar{\tau} \).

(2) If the SOSM controller for the differential torque is designed as

\[ T_b = \zeta_1 + \zeta_2, \quad (43) \]

where

\[ \zeta_1 = \frac{1}{B_3} \left[ -\left( A_{12} - \frac{B_2 + B_6}{B_3} \right) \left( \gamma - \gamma_0 \right) - F_5 + \frac{B_2 + B_6}{B_3} F_5 \right], \]

\[ \zeta_2 = -\lambda_2 \left[ e \left( \gamma - \gamma_0 \right) \right] \left( \gamma - \gamma_0 \right) + \zeta_1, \]

\[ \dot{\zeta}_1 = -\alpha_1 \left( \gamma - \gamma_0 \right) \left| \frac{\zeta_1}{\zeta_2} > \frac{\zeta_{\text{ref}}}{\zeta_{\text{ref}}} \right| \zeta_1, \]

\[ \dot{\zeta}_2 = -\alpha_2 \left( \gamma - \gamma_0 \right) \left| \frac{\zeta_2}{\zeta_{\text{ref}}} > \frac{\zeta_{\text{ref}}}{\zeta_{\text{ref}}} \right| \zeta_2, \]

with

\[ \alpha_0 > \frac{e_{\text{ref}}}{B_3}, \quad 0 < q_0 < 1, \quad \phi_M > \frac{e_{\text{ref}}}{q_0 B_3}, \]

\[ \lambda_0 > \sqrt{2 \left( \frac{B_3 \sigma_0 + e_{\text{ref}}}{B_3 (1-q_0)} \right)(1+q_0)}, \]

then the yaw rate of the vehicle will approach \( \dot{\gamma} \) in a finite time when \( |e_| \leq \bar{\tau} \) and \( |\dot{e}_s| \leq \bar{\tau} \).

By using the composite controllers in (39) and (43), the sideslip angle and the yaw rate will track the ideal reference model (14) in a finite time.

Proof: Let \( e_1 = \dot{\beta} - \dot{\beta}_s \), \( e_2 = \gamma - \gamma_0 \), and the time derivative of \( e_1 \) and \( e_2 \) can be obtained by using the ideal reference system (14) and the plant (37) as

\[ \dot{e}_1 = A_1 e_1 + A_2 e_2 + F_1 + B_1 \delta_1 + e_1 \]

\[ \dot{e}_2 = A_1 e_1 + A_2 e_2 + F_1 + B_1 \delta_1 + B_3 T_b + e_2 \].

Step 1: Choose \( e_1 \) as the first SM surface, and the controller for the rear-wheel-steering angle is constructed as (39). Applying (40) into the first equation of (47) gives

\[ \dot{e}_1 = e_1 + B_{12} \zeta_2 \].

A positive constant \( \zeta_{\text{ref}} \) can always be found such that \( |e_1| \leq \zeta_{\text{ref}} \) when \( V \leq V_s \leq V \). Moreover, based on the condition given in Assumption 1, \( e_1 \) is bounded by \( \zeta_{\text{ref}} \). Then, according to the theorem of ST algorithms, the SOSM controller for \( \zeta_2 \) and \( \zeta_3 \) can be designed as (41) and (42). Therefore, it can be concluded that the sliding variable \( e_1 \) will be stabilized to the origin in a finite time with the controller (39).

Step 2: Under the controller (39), \( e_1 \) is limited to and moving on the SM surface

\[ e_1 = 0, \quad \dot{e}_1 = 0 \].

Then, on the SM surface of (49), it implies

\[ 0 = A_1 e_1 + F_1 + B_1 \delta_1 + e_1 \]

\[ \dot{e}_2 = A_1 e_1 + F_2 + (B_2 + B_1) \delta_1 + B_3 T_b + e_2 \].

According to the first equation of (50), the equivalent control \( \delta_1 \) for the SM surface \( e_1 \) can be established as

\[ \delta_{\text{req}} = \left( A_1 e_1 + F_1 + e_1 \right) / B_3. \]

Substituting the equivalent control \( \delta_{\text{req}} \) into the second equation of (50) leads to

\[ \dot{e}_2 = [A_{12} - A_2 (B_2 + B_1)] e_2 + B_3 T_b + e_2. \]

Applying (38) and (44) into (52) leads to

\[ \dot{e}_2 = \dot{e}_2 - [B_2 + B_1] e_2 / B_3. \]

Under Assumption 1, there exists a positive constant \( \zeta_{\text{ref}} \) such that \( |e_1| \leq \zeta_{\text{ref}} \). Taking derivative of \( e_2 \) yields

\[ \dot{e}_2 = \dot{e}_2 - [B_2 + B_1] e_2 / B_3. \]

The items given in (54) are bounded, and there exists a positive constant \( \zeta_{\text{ref}} \) such that \( |e_2| \leq \zeta_{\text{ref}} \). Then, according to the theorem of ST algorithms, the SOSM controller for \( \zeta_2 \) and \( \zeta_3 \) can be designed as (45) and (46). Therefore, it can be concluded that the SM variable \( e_2 \) will be stabilized to the origin in a finite time by using the controller (43). Finally, by combining controllers (39) and (43), the plant (37) will approach the ideal reference model (14) in a finite time.

E. Torque Allocation

As a matter of fact, the 4WID vehicle is an over-actuated system with four independent driving motors, and only two controlled variables with respect to the total torque and the differential torque have been used. Thus, the controllers for vehicle motions are proposed to have the following two layers: the upper layer, which is used to calculate the total torque \( T_b \) and the differential torque \( T_b \), and the lower layer, which is used to optimally allocate the torques to the four motors and simultaneously satisfies the results from the upper layer. By using the velocity controller and the composite controllers, the control inputs of \( T_b \) and \( T_b \) have been obtained, and the following functions with respect to the torque of each wheel can be given

\[ T_b = \begin{bmatrix} \frac{T_b}{B_1} \\ \frac{T_b}{B_2} \end{bmatrix}, \quad \text{and} \quad u_3 = \begin{bmatrix} \frac{T_b}{B_1} \\ \frac{T_b}{B_2} \end{bmatrix}. \]

It implies an issue of multiple solutions in (55). In the lower layer, the optimal allocation controller is proposed to minimize the control efforts for the four motors. Then, the cost function \( J \) is defined as follows

\[ J = \frac{1}{4} \sum T_i^2 = u_i^T u_i. \]

Therefore, the proposed allocation controller is designed to minimize \( J \) subject to (55). A Lagrange function is constructed as follows

\[ \begin{bmatrix} \frac{T_b}{B_1} \\ \frac{T_b}{B_2} \end{bmatrix} = \begin{bmatrix} \frac{T_b}{B_1} \\ \frac{T_b}{B_2} \end{bmatrix}. \]
where $\lambda = [\lambda_1, \lambda_2]^T$ is a Lagrange multiplier. According to least-square solution methods, the optimization problem of the torque allocation for four wheels has been reformulated to solve the saddle point of the Lagrange function (57), and the optimality conditions can be obtained as

$$
\begin{align*}
2u'_i + B_iu^* - u_4 &= 0, \\
B_iu'_i - u_4 &= 0,
\end{align*}
$$

where $u'_i$ and $u^*$ are the saddle points of $L(u_i, \lambda)$. Since the matrix $B_iB_i^T$ is invertible, the optimal torque allocation for the motor of each wheel can be obtained using

$$
\begin{align*}
u'_i &= B_i^T(B_iB_i^T)^{-1}u_4.
\end{align*}
$$

Therefore, the allocation controller for four motors will exert the torque on each wheel in terms of the torque values given in (59).

IV. Simulation Results

The values of parameters for the controlled off-road vehicle are shown in Table II.

According to disturbances and uncertainties defined in (13) and (15), the upper bounds of $\varepsilon_1$ and $\varepsilon_2$ are, respectively, set as 0.258 rad/s and 1.780 rad/s$^2$. Then, for the composite controllers in Proposition 4, feasible regions of $\alpha_\beta$, $\alpha_\gamma$, $\alpha_\mu$, and $\alpha_{\mu_1}$ can be calculated that $\alpha_\beta > 0.1855$ and $\alpha_{\mu_1} > 230$. Under conditions of $0 < q_\beta < 1$ and $0 < q_\gamma < 1$ in Proposition 4, feasible regions of $\xi_M$, $\lambda_\beta$, $\xi_\beta$, and $\lambda_\gamma$, can be calculated as shown in Fig. 2.

The feasible regions for controllers in Propositions 2 and 3 can be similarly calculated. Then, the parameters for the velocity controller (26), the controller (33) only for the differential traction (DT), and the composite controllers (39) and (43) for the DT and the rear-wheel steering (RS) have been selected as shown in Table III.

For the composite controllers, it can be obtained that $\xi_M > 3.711$, $\lambda_\beta > 2.2193$, $\xi_\beta > 4660$, and $\lambda_\gamma > 4916$ when $q_\beta = 0.05$, $\alpha_\beta = 2.18$, $q_\gamma = 0.05$, and $\alpha_{\mu_1} = 4330$ are deployed. Therefore, it can be verified that $\xi_M = 3.8$, $\lambda_\beta = 2.3$, $\xi_\beta = 4670$ and $\lambda_\gamma = 4930$ are all in their feasible regions for the composite controllers in Proposition 4.

Moreover, the uncertainties $\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$ of the system (13) are, respectively, set as $\xi_1\sin(\pi/5)$, $\xi_2\sin(\pi/5)$, $\xi_3\sin(\pi/5)$. In order to analyze the performance of controllers, two maneuvers are considered to implement: 1) straight-line travelling without driver inputs; and 2) steering travelling with driver inputs.

A. Straight-line Running

When the off-road vehicle moves in a straight line on the soft and slope terrain, the initial position angle of $\psi$ is set as $\pi/4$ ($\psi(0) = \pi/4$), and the driver input for the front-wheel-steering angle is 0 ($\varepsilon_3 = 0$). The simulation results are shown in Fig. 3 when the velocity controller, the controller only for DT and the composite controllers for DT and RS are used.

Simulation results in Fig. 3 (a) show that the vehicle is able to maintain running at a constant speed by using the velocity controller (26). Meanwhile, the speed of the uncontrolled vehicle decreases rapidly due to the initial value of $\psi$, which implies the off-road vehicle climbs up the soft and slope terrain. Additionally, it can be seen from Figs. 3 (b), (c), and (d) that the composite controllers (39) and (43) are effective in tracking the ideal reference model (14) and following the desired straight-line path. When no steering input from the driver is used, the actual ideal sideslip angle and yaw rate of the vehicle should always be zero, and the trajectory would be a straight line. However, the controller (33) is not able to follow the ideal reference model and maintain the vehicle travelling in a straight-line path.

Simulation results in Fig. 3 (e) show that motor torque for four wheels from (39) of the composite controllers are larger than the ones from controller (33). As a matter of fact, the composite controllers need generate more torques for the wheels to compensate the additional reversed yaw moment produced by the steering angle of the rear wheels from (43) as
shown in Fig. 3 (f). Moreover, due to the continuous property of SOSM control methods, all controllers of (26), (39), (43) and (33) have almost no chattering. Figs. 3 (g), (h) and (i) take further analysis of simulation results for the velocity controller (26) and the composite controllers of (39) and (43), and show errors of the velocity, the sideslip angle and the yaw rate for keeping the constant velocity and tracking the ideal reference model. It can be seen that all the errors can be kept at a low level.

B. Steering Running

In simulations of steering running, the driver gives the steering input to front wheels as $0.04 \sin (\pi t / 5)$, and the initial position angle of $\psi$ is set as $0$ ($\psi(0) = 0$). In comparisons with simulation results by using the velocity controller, the controller of only DT, and the composite controllers of DT and RS are shown in Fig. 4.

It is clearly seen from Fig. 4 (a) that the vehicle by using the velocity controller (26) can precisely keep the constant velocity when there are some steering inputs from the driver. Meanwhile, the responses of the yaw rate, the sideslip angle, and the trajectory of the vehicle under the composite controllers (39) and (43) are able to accurately approach the ideal reference model (14), as shown in Figs. 4 (b), (c), and (d). Moreover, simulation results in Figs. 4 (g), (h) and (i) show the high performance of tracking errors. Note that, the yaw rate of the vehicle with the controller (33) can be kept in a stable region, whereas the sideslip angle is not able to follow the one of the ideal reference model.

Simulation results in Figs. 4 (e) and (f) show the torques for four wheels using (39) of the composite controllers are larger than the ones using controller (33), due to the additional yaw moment produced by (43). Furthermore, SOSM control methods for controllers (26), (39)-(43), and (33) show the
approximately no chattering of actuators.

V. CONCLUSION

In this paper, SOSM-based ST algorithms were implemented to stabilize an off-road vehicle running on a soft and slope terrain. Under conditions of uncertain longitudinal and lateral slips, the velocity controller was designed to maintain a stable velocity for the vehicle, and the composite controllers were designed to approach an ideal reference model. On this basis, the steering angle of rear wheels, the total torque, and the differential torque were obtained by using the composite controllers, and the torque allocation controller was designed to exert torque on each wheel. Compared with the conventional yaw moment control without rear-wheel steering, the composite controllers were more effective to follow the yaw rate, the sideslip angle and the trajectory of the ideal reference model, whereas more driving torque had to be implemented on each wheel to compensate the additional yaw moment produced by the rear-wheel steering.

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