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Can we neglect relativistic temperature corrections in the Planck thermal SZ analysis?

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ABSTRACT

Measurements of the thermal Sunyaev–Zel’dovich (tSZ) effect have long been recognized as a powerful cosmological probe. Here we assess the importance of relativistic temperature corrections to the tSZ signal on the power spectrum analysis of the Planck Compton-y map, developing a novel formalism to account for the associated effects. The amplitude of the tSZ power spectrum is found to be sensitive to the effective electron temperature, $\bar{T}_e$, of the cluster sample. Omitting the corresponding modifications leads to an underestimation of the $yy$ power spectrum amplitude. Relativistic corrections thus add to the error budget of tSZ power spectrum observables such as $\sigma_8$. This could help alleviate the tension between various cosmological probes, with the correction scaling as $\Delta \sigma_8/\sigma_8 \simeq 0.019 [k\bar{T}_e/5 \text{keV}]$ for Planck. At the current level of precision, this implies a systematic shift by $\pm 1\sigma$, which can also be interpreted as an overestimation of the hydrostatic mass bias by $\Delta b \simeq 0.046 (1 - b) [k\bar{T}_e/5 \text{keV}]$, bringing it into better agreement with hydrodynamical simulations. It is thus time to consider relativistic temperature corrections in the processing of current and future tSZ data.

Key words: cosmic background radiation – cosmology: observations – cosmology: theory.

1 INTRODUCTION

The thermal Sunyaev–Zel’dovich (tSZ) effect is now routinely used to detect clusters of galaxies (Sehgal et al. 2011; Planck Collaboration XX 2014). More than $10^3$ clusters have been seen through this effect and the number of Sunyaev–Zel’dovich (SZ) clusters is expected to increase by more than one order of magnitude with future experiments (e.g. Melin et al. 2018; The Simons Observatory Collaboration 2018). The tSZ effect is caused by the upscattering of cosmic microwave background (CMB) photons by thermal electrons residing in the potential wells of clusters, yielding a Compton-$\gamma$ distortion, which in the non-relativistic limit has the frequency dependence (in intensity) $Y_\nu(\nu) = (2\hbar c^2/kT_{\text{CMB}}/h)^3 \nu^2 \nu'/(\nu^2 - \nu'^2) [\text{coth}(x/2) - 4]$ (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1980). Here, $c$ denotes the speed of light and $x \equiv h\nu/kT_{\text{CMB}}$ with $h$ being the Planck constant, $k$ the Boltzmann constant, and $T_{\text{CMB}}$ the CMB blackbody temperature.

The importance of SZ clusters as a cosmological probe has long been recognized (e.g. Sunyaev & Zeldovich 1980; Rephaeli 1995a; Birkinshaw 1999; Carlstrom, Holder & Reese 2002). As the largest gravitationally bound systems, clusters are a unique tracer of the large-scale structure in the Universe. Multi-frequency observations with the Planck satellite allow us to extract valuable information about the distribution of matter on the largest scales. One example is the large-scale lensing potential, which was mapped for the first time with Planck (Planck Collaboration XV 2016). Similarly, Planck revealed the first Compton-$\gamma$ map, which through the tSZ effect informs us about the integrated electron pressure along different lines of sight (Planck Collaboration XXII 2016).

The clusters observed with Planck are massive and contain a hot electron plasma that is also seen in X-rays (Vikhlinin et al. 2006; Leccardi & Molendi 2008; Arnaud et al. 2010). The thermal velocities of electrons inside massive clusters can be appreciable, reaching a fair fraction of the speed of light ($v_b \simeq 0.1–0.2c$). In this situation, the non-relativistic approximation for the SZ signal (Zeldovich & Sunyaev 1969), commonly used in CMB analysis, no longer suffices, and relativistic temperature corrections become important (Wright 1979; Fabbri 1981; Rephaeli 1995b; Challinor & Lasenby 1998; Itoh, Kohyama & Nozawa 1998; Sazonov & Sunyaev 1998). These corrections are currently hard to detect and have been searched for in individual clusters (e.g. Hansen, Pastor & Semikoz 2002; Prukhorov & Colafrancesco 2012; Zemcov et al. 2012; Chluba et al. 2013) and through stacking analyses (e.g. Hurier 2016; Erler et al. 2018; Hincks et al. 2018). Here we consider the effect of relativistic corrections on the Planck tSZ power spectrum analysis, demonstrating that they already add to the current error budget, leading to a bias in the inferred matter power spectrum amplitude, i.e. $\sigma_8$. 

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The power spectrum of the Compton-\(y\) parameter, \(C_{\ell y}\), connects the extracted information to the underlying cosmology (e.g. Refregier et al. 2000; Komatsu & Seljak 2002). Its amplitude depends steeply on that of the matter power spectrum, parametrized by \(\sigma_8\) (Komatsu & Kitayama 1999). Using the halo model, one finds \(C_{\ell y} \propto \sigma_8^{2.1}\) for the contributions of SZ clusters (Planck Collaboration XXII 2016; Bolliet et al. 2018). Similarly, the skewness of the one-point probability distribution function (PDF) of the \(y\)-parameter was shown to scale as \(\langle y^3 \rangle \propto \sigma_8^{1.2}\) (Rubinó-Martín & Sunyaev 2003; Bhattacharya et al. 2012; Wilson et al. 2012). Therefore, tSZ measurements can be used to derive constraints on \(\sigma_8\) (Komatsu & Seljak 2002; Planck Collaboration XXII 2016; Bolliet et al. 2018), albeit with obstacles from cluster astrophysics (Battaglia et al. 2010, 2012; Shaw et al. 2010), foregrounds (Planck Collaboration XXII 2016), and systematics (e.g. Planck Collaboration XII 2014; Planck Collaboration XXII 2016).

Evidently, we do not directly measure \(C_{\ell y}\). We use multifrequency observations to obtain maps of the \(y\)-parameter, which then allow us to estimate \(C_{\ell y}\). In the intermediate steps, one of the crucial approximations is that the spectral shape of the tSZ signal, \(Y(v)\), is the same for all clusters. Thus, the tSZ power spectrum at one frequency is given by \(C_{\ell y}^{\mathrm{obs}}(v) \propto \langle |Y(v)|^2 \rangle \propto Y^2(v) C_{\ell}^{y},\) with \(C_{\ell}^{y} = \langle |y|^{2}\rangle\). An additional important simplification is that \(Y(v)\) is approximated using the non-relativistic limit, \(Y(v) \approx Y_0(v)\).

Although the first assumption is expected to have a smaller effect, both simplifications need to be revisited. When setting \(Y(v) \simeq Y_0(v)\) one implicitly assumes that the temperature of the medium responsible for the \(y\)-parameter (→ integrated pressure) fluctuations is non-relativistic \((v_{\mathrm{th}}/c \ll 10^{-4})\). However, the Planck power spectrum analysis is mostly sensitive to clusters with large masses \(M \gtrsim 3 \times 10^{14} h^{-1} M_{\odot}\) (see Fig. 1), dominating at \(\ell \approx 10^2-10^{3},\) and hence to electrons with typical temperature\(^2\) \(kT_e \gtrsim 5\) keV. A similar conclusion is reached by looking at fig. 11 of Refregier et al. (2000) and fig. 6 of Komatsu & Seljak (2002). This statement is further supported when considering SZ clusters detected by Planck at high significance. In this case, one obtains a sample-averaged cluster temperature of \(kT_e^N \simeq (6.91 \pm 0.08)\) keV (Erler et al. 2018) using measured X-ray mass–temperature scaling relations (Reichert et al. 2011), and \(kT_e^{\mathrm{SZ}} \simeq 6.3^{+3.8}_{-2.9}\) keV by stacking clusters (Erler et al. 2018). At temperatures \(kT_e \gtrsim 3-5\) keV, relativistic corrections to the tSZ signal become relevant, and hence \(Y(v) \neq Y_0(v)\). Consequently, this affects the Planck tSZ analysis, as we show here.

Relativistic temperature corrections to the SZ signal can be accurately included using SZpack (Chluba et al. 2012). Fig. 2 illustrates the variations of the tSZ signal with the electron temperature. Relativistic corrections lead to a broadening of the tSZ intensity with systematic shift towards higher frequencies, reducing its overall amplitude at fixed Compton-\(y\) parameter. This inevitably leads to an underestimation of \(y\), if \(Y_0(v)\) is used in the analysis. A similar conclusion was recently reached in Erler et al. (2018), where the effect on the considered cluster sample was \(\Delta y/\bar{y} \approx 7-14\) per cent. Hence, the amplitude of \(C_{\ell y}^{y}\) is underestimated, an effect that propagates to the tSZ observables such as \(\sigma_8\). This is further supported by the analysis of Hurier & Tchernin (2017). Similarly, relativistic tSZ should affect cluster number count statistics (Planck Collaboration XX 2014) and SZ analyses targeting neutrino masses and primordial non-Gaussianity (e.g. Hill & Pajer 2013).

2 FORMULATION OF THE PROBLEM AND RESULTS

Using a tSZ temperature moment expansion (Chluba et al. 2013) about pivot electron temperature, \(T_e\), we can express the tSZ signal, \(S(v) = y Y(v, T_e)\), across the sky using the frequency-dependent

\[^{2}\text{We used hydrostatic equilibrium expressions to estimate the cluster temperature (e.g. see Arnaud, Pointecouteau & Pratt 2005; Nagai, Kravtsov & Vikhlinin 2007b; Erler et al. 2018): } kT_e \simeq 5\text{ keV } \left[ E(z) M_{500}/3 \times 10^{14} h^{-1} M_{\odot} \right]^{2/3} \text{ with normalized Hubble factor } E(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_{\Lambda}}.\]
spherical harmonic coefficients,
\[ S_{lm}(v) \simeq Y(v, \bar{T}_e) Y_{lm}^\ast + Y^{(1)}(v, \bar{T}_e) y_{lm}^{(1)} + \frac{1}{2} Y^{(2)}(v, \bar{T}_e) y_{lm}^{(2)}, \]  
keeping terms up to second order in \( \Delta T_e = T_e - \bar{T}_e \). For convenience, we introduced the derivatives \( Y^{(k)}(v, T) \). We also defined the spherical harmonic coefficients, \( y_{lm}^{(1)} = [(T_e - \bar{T}_e) y_{lm}\rangle, \) which generally each have different spatial morphology (Chluba et al. 2013). Assuming isothermal clusters, we furthermore have \( y_{lm}^{(0,1)} = (T_e - \bar{T}_e)^0 Y_{lm}\rangle, \) an approximation that we will use below.

We still have to determine the pivot temperature \( \bar{T}_e \) introduced above. One natural choice would be the average \( y \)-weighted SZ temperature, only obtained by requiring \( \langle y_{lm}\rangle = \langle (T_e - \bar{T}_e) y_{lm}\rangle = 0 \), which yields \( \bar{T}_e^y \simeq \langle k T_{e,y}\rangle / \langle y\rangle \rangle = 0 \). Within \( \Lambda CD M \) model this has been estimated as \( k T_{e,y}^\Lambda \simeq 1.3 \text{ keV} \) with all-sky \( y \)-parameter, \( \langle y\rangle \simeq 2 \times 10^{-6} \) (Hill et al. 2015; Abitbol et al. 2017). This value for the average electron temperature is dominated by the contributions from low-mass haloes (\( M \lesssim 10^5 h^{-1} M_\odot \)). However, for the tSZ power spectrum, a different weighting is relevant, which depends on details of the cluster mass function and temperature–mass relation. This increases the effective cluster sample temperature and hence the importance of relativistic corrections relevant to the tSZ power spectrum analysis, as we illustrate next.

To obtain the tSZ power spectrum, we have to compute the ensemble average \( \langle S_{lm}^e S_{kn}^e \rangle \). Because of isotropy and homogeneity, for a spherical cluster profile this yields \( \langle y_{lm}\rangle \rightarrow \langle |Y_\ell|^2 \rangle \) (e.g. see appendix of Hill & Pajer 2013, for an explicit derivation), where \( |Y_\ell|^2 \) is the 2D Fourier transform of the projected Compton y-parameter (e.g. Komatsu & Seljak 2002; Hill & Pajer 2013). Again keeping only terms up to second order in \( \Delta T_e \), with similar arguments we find the expansion of the theoretical tSZ power spectrum:

\[
C_{\ell}^{yy}(v) \simeq Y^2(v, \bar{T}_e) \langle |y_\ell|^2 \rangle + 2Y(v, \bar{T}_e) Y^{(1)}(v, \bar{T}_e) \langle |y_\ell^{(1)}|^2 \rangle + \left[ Y^{(1)}(v, \bar{T}_e) \right]^2 \langle |y_\ell^{(1)}|^2 \rangle + Y(v, \bar{T}_e) Y^{(2)}(v, \bar{T}_e) \langle |y_\ell^{(2)}|^2 \rangle \]  

(2)

This expression shows that through relativistic corrections the tSZ power spectrum receives contributions from higher order statistics of the \( y \)-parameter and electron temperature fields. These new terms are absent if \( y(v, \bar{T}_e) \simeq y_0(v) \) and lead to additional non-trivial frequency dependence. Similar effects were previously discussed for individual clusters (Chluba et al. 2013), but here we highlight the effects for ensembles of clusters.

In equation (2), we can now chose the pivot temperature, \( \bar{T}_e \), to minimize contributions from higher order terms in \( \Delta T_e \). In fact, this makes \( \bar{T}_e \) a scale-dependent quantity, \( \bar{T}_e^{\Lambda CD M} \), which can be defined by demanding \( \langle y_\ell^{(1)}|^2 \rangle = 0 \) at each multipole \( \ell \), cancelling the leading order correction term in equation (2). It is beyond the scope of this paper to include the spatial variations of the electron temperature within each cluster (see Chluba et al. 2013, for some related discussion). However, assuming an isothermal temperature profile for each cluster (i.e. \( y_\ell^{(1)} = [T_e(M, z) - \bar{T}_e] Y_\ell \rangle \), we find

\[
kT_{e,\ell}^{yy} = \left( kT_e(M, z) \langle y_\ell \rangle \right)^2 / \langle |y_\ell|^2 \rangle = C_{\ell}^{T,yy} / C_{\ell}^{yy} \]  

(3)

to ensure \( \langle y_\ell^{(1)}|^2 \rangle = 0 \), such that in equation (2) only second-order terms in \( \Delta T_e \) remain. In the standard Planck analysis, \( kT_{e,\ell}^{yy} \) is arbitrarily set to zero. This choice biases the derived parameters since in this case higher order terms are not minimized. We remind the reader that the average \( \langle \cdots \rangle \) includes integrals over the cluster mass function and redshift. Following the formalism of Komatsu & Seljak (2002), the evaluation of \( C_{\ell}^{yy} \) boils down to replacing \( |y_\ell|^2 \) by \( kT_{e,\ell}^{yy} \) for the warm diffuse component \( |y_\ell|^2 \) in equation (1) of their work.

One can think of equation (3) as a \( C_{\ell}^{T,yy} \)-weighted temperature. In Fig. 3, we illustrate its scaling with multipole \( \ell \) as obtained by modifying \( C_{\ell}^{yy} \). This highlights that at high-\( \ell \)-low-mass, colder systems dominate, yielding \( kT_{e,\ell}^{yy} \simeq 2-3 \text{ keV} \). Around \( \ell \approx 10^2-10^3 \), which is most relevant to the Planck tSZ analysis, we find an average temperature of \( kT_{e,\ell}^{yy} \simeq 5-9 \text{ keV} \) for the \( \Lambda CD M \) cosmology. This estimate depends on the details assumed for the gas physics (e.g. the temperature–mass relation, feedback efficiencies, and redshift scalings) that will have to be computed more carefully.

These uncertainties are indicated by the green (\( \pm 20 \text{ per cent} \)) band in Fig. 3. However, our halo-model calculations further justify our statements above, and suggest that \( kT_{e,\ell} \approx 5 \text{ keV} \) provides a conservative reference value. We also note that in the computations with \( C_{\ell}^{yy} \) we only included contributions from the one-halo term, as the two-halo term is subdominant (e.g. Hill & Pajer 2013).

At large angular scales, the effective cluster temperature is expected to drop, approaching \( kT_{e,\ell}^{yy} \approx 1.3 \text{ keV} \) found for the monopole \( \ell = 0 \) (Hill et al. 2015; Abitbol et al. 2017). This is due to the presence of diffuse, warm gas (e.g. Hansen et al. 2005), which should not contribute much to \( C_{\ell}^{T,yy} \) but can increase \( C_{\ell}^{yy} \) noticeably. Using Hansen et al. (2005), we estimate this effect by adding \( 10^{22} (\ell + 1) C_{\ell}^{yy,warm} / 2\pi \lesssim 0.01 \) for the warm diffuse component to \( C_{\ell}^{yy} \). At large angular scales (\( \ell \lesssim 10^3 \)), this contribution dominates and, in spite of large uncertainties, causes \( kT_{e,\ell}^{yy} \) to decline (see blue band in Fig. 3).

A detailed study of all the associated effects on the tSZ power spectrum encoded by equation (2) will be carried out in a follow-up paper. At leading order, the impact of relativistic SZ on the tSZ

\[ kT_{e,\ell}^{yy} \]

Figure 3. Effective \( C_{\ell}^{T,yy} \)-weighted electron temperature for different multipoles computed using class-sz (Blas et al. 2011; Bollig et al. 2018) with main settings like in Fig. 1. At high-\( \ell \), low-temperature systems dominate, giving effective temperature \( kT_{e,\ell}^{yy} \approx 2-3 \text{ keV} \). Around \( \ell \approx 10^2 \), this value is most relevant to the Planck tSZ analysis, and we obtain \( kT_{e,\ell}^{yy} \simeq 6-10 \text{ keV} \). However, uncertainties in the assumed mass–temperature relation and its redshift dependence and the total amount of diffuse gas lead to large ambiguities (green and blue bands) that will have to be quantified more carefully.

3Note that \( kT_{e,\ell}^{yy} \) is generally not expected to reach \( kT_{e} \approx 1.3 \text{ keV} \), which was computed using \( kT_{e} = (kT_e(M, z) y_0)/(y_0) \) (\( y \)-weighted temperature) as opposed to \( kT_{e,\ell=0} = (kT_e(M, z) y_0)/(y_0)^2 \), which is relevant here.
power spectrum can be captured by \( C_{\ell}^{\text{SZ}}(v) \propto C_{\ell}^{\gamma} / f(T_e) \), where \( f(T_e) \) generally is scale- and frequency dependent. However, after component separation, which targets \( C_{\ell}^{\gamma} \) not \( C_{\ell}^{\text{SZ}}(v) \), we can assume one effective temperature at the current level of precision. For \( C_{\ell}^{\gamma} \propto \sigma^8 \), this implies that the Planck tSZ power spectrum analysis actually constrains \( \sigma^8 \propto \sigma / f(T_e)^{1/8.1} \) when omitting relativistic corrections. Thus, the value for \( \sigma^8 \) obtained in the analysis is lowered by

\[
\Delta \sigma^8 / \sigma^8 \simeq f(T_e)_{1/8.1} - 1, \tag{4}
\]

with \( \Delta \sigma^8 \equiv \sigma - \sigma^8 \). We show that at the current level of precision this yields a systematic shift of \( \gtrsim 1 \sigma \) towards larger \( \sigma^8 \) once relativistic corrections are included in the Planck tSZ power spectrum analysis. Since we do not know the exact value for \( \nu \), \( k_T \), it was found that after foreground marginalization the \( \nu \) constraint is driven by the 15 keV channel, indicating that more careful simulations are needed to distinguish foregrounds from the signal, and that more careful simulations are needed to quantify the effect.

In the Planck 2015 data analysis (Planck Collaboration XXII 2016), the Compton-\( \gamma \) map was estimated through a weighted linear combination of the frequency maps, with minimum variance to mitigate foreground contaminations. The weights assigned to each frequency map were determined to achieve unit response to the tSZ energy spectrum, \( Y_0(v) \), and thus ignoring relativistic corrections. In other words, it was implicitly assumed that the temperature of all clusters is \( k T_e \ll 1 \) keV (cf. Fig. 2), while here we argued that the average temperature of clusters relevant to the tSZ power spectrum analysis is \( k T_e \gtrsim 5 \) keV.

We thus revised the estimation of the Planck tSZ-\( \gamma \)-map by modifying the NILC component separation algorithm (Remazeilles, Delabrouille & Cardoso 2011; Remazeilles, Aghanim & Douspis 2013) that was adopted in Planck Collaboration XXII (2016). We used the relativistic tSZ energy spectrum, \( Y(v, T_e) \), for different temperatures \( T_e > 0 \) instead of the non-relativistic spectrum to construct the NILC filters. Bandpass averaging had no large impact on the results, although at higher sensitivity this may not be the case.

We applied our revised NILC filters to the Planck 2015 data, assuming \( k T_e = 5 \) and 10 keV, to reconstruct the tSZ-\( \gamma \)-map. We then estimated \( C_{\ell}^{\gamma} \) and the one-point PDF after foreground marginalization from the obtained \( y \)-map, as presented in Fig. 4. The amplitude of the tSZ power spectrum increases noticeably with \( T_e \), as anticipated. Similarly, the width and skewness of the PDF are modified. By comparing our results to those obtained using the non-relativistic tSZ energy spectrum we find

\[
f(T_e) \simeq C_{\ell}^{\gamma}(T_e) / C_{\ell}^{\gamma}(T_e = 0) \simeq 1 + 0.15 \left[ \frac{k T_e}{5 \text{ keV}} \right], \tag{5a}
\]

\[
g(T_e) \simeq S(T_e) / S(T_e = 0) \simeq 1 + 0.28 \left[ \frac{k T_e}{5 \text{ keV}} \right] \tag{5b}
\]

to represent the changes of the power spectrum amplitude and skewness of the one-point PDF, \( S \propto \langle y^{3} \rangle \). The result for \( f(T_e) \) can also be estimated by comparing the amplitude of \( Y_0(v) \) and \( Y(v, k T_e = 5 \text{ keV}) \) in the \( v = 353 \) GHz channel of Planck, yielding \( f(5 \text{ keV}) \simeq Y_0^{\gamma} / Y^{\gamma}(v, 5 \text{ keV}) \simeq 1.19 \). In Erler et al. (2018), it was found that after foreground marginalization the \( v = 353 \) and 143 GHz channels were indeed driving the constraints on relativistic tSZ. This is related to the ability of Planck to distinguish foregrounds from the signal, indicating that more careful simulations are needed to quantify the effect.

Various estimates of \( \sigma^8 \) exist in the literature (see Planck Collaboration XXII 2016; Bolliet et al. 2018, for references). Typical central values are \( \sigma^8 \simeq 0.8 \) with \( 1 \sigma \) error \( \simeq 0.02 \). With equations (4) and (5a), we can directly write

\[
\Delta \sigma^8 / \sigma^8 \simeq 0.019 \left[ \frac{k T_e}{5 \text{ keV}} \right] \tag{6}
\]

for the systematic shift expected in the tSZ power spectrum analysis due to relativistic corrections. Assuming a fiducial value \( \sigma^8 \simeq 0.8 \) yields \( \Delta \sigma^8 \simeq 0.015 \left[ k T_e / 5 \text{ keV} \right] \), which is comparable to the current 1\( \sigma \) uncertainty on \( \sigma^8 \). From the skewness we find \( \Delta \sigma^8 / \sigma^8 \simeq 0.025 \left[ k T_e / 5 \text{ keV} \right] \), implying \( \Delta \sigma^8 \simeq 0.02 \left[ k T_e / 5 \text{ keV} \right] \), in good agreement with equation (6).

In the Planck 2015 analysis of the Compton-\( \gamma \) map, the collaboration reported constraints on \( \sigma^8 \) that are in mild tension with the CMB anisotropy constraints, with the tSZ analysis yielding systematically lower values (Planck Collaboration XXII 2016). A detailed review of the various results and their differences is beyond the scope of this paper, but from equation (6) it follows that all the tSZ power-spectrum-derived constraints on \( \sigma^8 \) are currently biased low by about \( 1 \sigma \). This means that including relativistic temperature corrections could alleviate the tension with the CMB anisotropy data. To reduce the tension to below 1\( \sigma \), \( \Delta \sigma^8 \simeq 0.03-0.05 \) is required, implying \( k T_e \simeq 10-15 \) keV. This seems quite high, since only the most massive clusters seen in our Universe reach comparable temperatures (Menanteau et al. 2012; Chluba et al. 2013). However,
relativistic corrections play a part in the story, already adding to the total error budget at the current level of precision.

As outlined by a number of recent works (e.g. Hurier & Lacasa 2017; Bolliet 2018; Makiya, Ando &Komatsu 2018; Salvati, Douspis & Aghanim 2018), the tension between tSZ probes and CMB temperature anisotropies can be rephrased in terms of the mass bias, \( B = 1/(1 - b) \), rather than \( \sigma_8 \). Hydrodynamical simulations suggest \( b \approx 0.2 \) or \( B \approx 1.25 \) (e.g. Shi et al. 2016). This can arise due to departure from hydrostatic equilibrium (e.g. non-thermal pressure); however, other effects such as systematics in the X-ray mass calibration also contribute (e.g. see Nagai, Vikhlinin & Kravtsov 2007a; Lau, Kravtsov & Nagai 2009; Shaw et al. 2010; Shi & Komatsu 2014; Henson et al. 2017, for discussions).

A more practical approach needs to take the uncertainty in the mass bias into account. Given the current tSZ constraint on \( F = \sigma_8 (\Omega_m / B)^{0.40} h^{-0.25} \) (Bolliet et al. 2018), one finds \( B = 1.58 \pm 0.13 \) (68 per cent CL) with CMB TT-lensing, i.e. \( b = 0.37 \pm 0.05 \) (68 per cent CL). Accounting for relativistic tSZ, the mass bias is driven towards lower values, more consistent with hydrodynamical simulations. Indeed the constraint on \( F \) should be revised to \( F^* = F (T_\text{e} / 10^8 \text{keV})^{1/8} \), implying \( \Delta b \approx 0.046 (1 - b) [kT_\text{e}/5 \text{keV}] \). We also highlight that \( kT_{yy}^\| \) defined by equation (3) is relatively insensitive to mass-bias parameter, \( b \), but depends on \( \sigma_8 \) as \( kT_{yy}^\| \approx 7 \text{keV} (\sigma_8 / 0.8)^2 \) (at \( \ell \approx 10^2 - 10^3 \)), thus in principle offering a new way to break parameter degeneracies. We will explore this idea in the future.

3 CONCLUSIONS

To summarize, we took an important first step towards including the effects of relativistic temperature corrections on tSZ power spectrum analyses, providing a new formalism for capturing the associated effects, i.e. equation (2). Applying the method to Planck, we showed that this can help reduce part of the tension between different cosmological probes of \( \sigma_8 \). However, it will be important to directly estimate the average electron temperature, \( kT_\text{e} \), which has large uncertainties that need to be marginalized over. For example, cluster gas physics and feedback processes affect the temperature–mass relation and its redshift evolution. It is also clear that existing temperature estimates (e.g. X-ray/spectroscopic versus mass and \( y \)-weighted temperatures) differ significantly (e.g. Kay et al. 2008), demanding further quantification. In addition, at large angular scales, contributions from the diffuse, relatively cold gas cannot be ignored (e.g. Zhang, Pen & Trac 2004; Hansen et al. 2005; Hill et al. 2015). Degeneracies with CMB foregrounds at different scales will also have to be studied more carefully.

We highlighted that the shape of tSZ power spectrum depends on higher order statistics of the \( T_\text{e} \) and \( y \) fields (see equation 2) in a frequency-dependent manner. This is caused by weighted averages of spatially varying spectral energy distributions. Similar ideas have recently been discussed in connection with CMB foreground analyses (Chluba, Hill & Abitbol 2017). This opens a new window for exploring the statistical and physical properties of clusters in our Universe. Extracting these signals will require high sensitivity and broad spectral coverage, as discussed for space mission concepts like Cosmic Origins Explorer (CORE; Delabrouille et al. 2018; Remazeilles et al. 2018), LiteBIRD (Suzuki et al. 2018), Probe of Inflation and Cosmic Origins (PICO), and CMB-Bharat.

Finally, in this paper, as an example we highlighted the effects on the tSZ power spectrum and connections to \( \sigma_8 \). Relativistic corrections will also be relevant to tSZ constraints on the sum of neutrino masses and potentially primordial non-Gaussianity. They are furthermore expected to affect cluster number counts in a similar manner, increasing the number of clusters at a given signal-to-noise ratio threshold (see Fan & Wu 2003, for some related discussion). The refinements discussed here will also become important for the next-generation CMB experiments such as Simons Observatory, CMB-S4 (Abazajian et al. 2016) and CCAT-prime (Parshley et al. 2018), providing new science targets related to cluster astrophysics and their impact on cosmological observables.

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