Decision-Dependent Uncertainty in Adaptive Real-Options Water Resource Planning

DOI:

[10.1016/j.advwatres.2019.103490](https://doi.org/10.1016/j.advwatres.2019.103490)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Erfani, T., Pachos, K., & Harou, J. J. (2020). Decision-Dependent Uncertainty in Adaptive Real-Options Water Resource Planning. *Advances in Water Resources*, 103490. <https://doi.org/10.1016/j.advwatres.2019.103490>

Published in:

Advances in Water Resources

Citing this paper

Please note that where the full-text provided on Manchester Research Explorer is the Author Accepted Manuscript or Proof version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version.

General rights

Copyright and moral rights for the publications made accessible in the Research Explorer are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Takedown policy

If you believe that this document breaches copyright please refer to the University of Manchester's Takedown Procedures [<http://man.ac.uk/04Y6Bo>] or contact uml.scholarlycommunications@manchester.ac.uk providing relevant details, so we can investigate your claim.

Decision-Dependent Uncertainty in Adaptive Real-Options Water Resource Planning

Tohid Erfania^a, Kevis Pachos^a, Julien J. Harou^{b,a}

^a*Department of Civil, Environmental and Geomatic Engineering, University College London, London, UK*

^b*School of Mechanical, Aerospace and Civil Engineering, University of Manchester, Manchester, UK*

Abstract

Staged water infrastructure capacity expansion optimization models help create flexible plans under uncertainty. In these models exogenous uncertainty can be incorporated into the optimization using an a priori hydrological and demand scenario ensemble. However some water supply intervention uncertainties cannot be considered in this way, such as demand management or technological options. In these cases the uncertainty is endogenous or ‘decision-dependent’, i.e., the optimized timing and selection of interventions determines when and which uncertainties must be considered. We formulate a multistage real-options water supply capacity expansion optimization model incorporating such uncertainty and describe its effect on cost and option selection.

Keywords: Endogenous uncertainty, Adaptive water resources planning

1. Introduction

Water security can be threatened when demand increases and climate change reduces supplies. In this case interventions (new infrastructure and/or policies) must be made to meet future demands despite the timing and extent of supply-demand changes not being known in advance. Furthermore, water infrastructures often have long lead-times, such as a decade or more. Traditionally water utilities plan system expansion on a cyclical basis (e.g. every 5 years) aiming to guarantee the supply-demand balance throughout their operating area over a long-term planning period (e.g. 25 years). Generally, given the potential large economic costs of water infrastructure, and the

uncertainties in both future supplies and demands, formal planning under uncertainty techniques aiming for robustness and/or adaptability are warranted.

Capacity expansion studies are at the heart of water resources engineering (Hsu et al., 2008; Watkins Jr and McKinney, 1998; Guo et al., 2010). In the past a typical water utility expansion plan was a cost-effective schedule of supply- and demand-side capacity expansion actions over the planning horizon (e.g. Padula et al., 2013). The decision-making under uncertainty literature has shifted the goal of water supply planning towards identifying plans that either perform well under a wide range of plausible future conditions (via robust decision making (Lempert, 2003; Lempert et al., 2006; Matrosov et al., 2013b,a)) or are adaptive (i.e., adjusted progressively as new information becomes available (Dupačová, 1995; Ray et al., 2011; Erfani et al., 2018; Hui et al., 2018)). While in the first approach the investment decisions are insensitive to the source of uncertainty, in the latter case, they are optimally activated, delayed and/or replaced so as to meet the supply and demand gap. Approaches that are both robust and adaptive can also be found in the literature (Lempert and Groves, 2010; Haasnoot et al., 2013; Kwakkel et al., 2015).

Most of the optimized water planning under uncertainty literature deals with problems where optimization decisions are independent of the uncertain parameter. That is, the uncertainty is *exogenous*; e.g. climate change impact that is independent of decisions and is not affected by them., Exogenous uncertainties are usually incorporated as a priori into the multistage optimization problem via an ensemble of scenarios. The earlier work of the authors in Erfani et al. (2018); Pachos et al. (2019) as well as Hall et al. (2012); Mortazavi-Naeini et al. (2014); Borgomeo et al. (2016); Padula et al. (2013); Matrosov et al. (2013b, 2015) are examples of exogenous uncertainty implementation.

Starting from the seminal work of Pflug (1990) and extended later on by the work of Jonsbråten et al. (1998), uncertainty can also be *endogenous*, meaning that decisions and uncertain parameters are interlinked, or otherwise said, that some uncertainties are *decision-dependent*, propagating as decisions are made. Based on the work of Pflug (1990); Goel and Grossmann (2006), endogenous uncertainty is of two types; these are described below.

1.1. *Decision-Dependent uncertainty types*

In dynamic optimization problems where decisions are optimized over time, such as the classical capacity expansion problem, there are two types of decision-dependent uncertainty (also known as ‘endogenous uncertainty’).

In the first type, intervention options’ activation decision variables and the statistical distribution from which the uncertain parameters are derived are dependent. That is, the value of the decision variables cause the alteration of the statistical distribution. This is relevant in water resource management for example for addressing reservoir effects, i.e., when increasing water supply leads to higher water demands which eventually reduce the reservoir’s initial water supply improvement (Di Baldassarre et al., 2018). Another example is the application of socio-hydrological models exploring the interplay between the impact of human interventions on drought and flood events and human responses to hydrological extremes (Di Baldassarre et al., 2015, 2017).

In the second type, intervention option activation decisions expressed as binary variables determine when the uncertainty has to be considered (i.e., the binary variables equal one at activation at which point the uncertainty is considered via pre-sampled scenarios). Notable work in this area includes Goel and Grossmann (2004) on oil field development, Viswanath et al. (2004) on network traversal problem, process planning application by Lappas and Gounaris (2016), disaster management by Poss (2014), Nohadani and Sharma (2018), and Peeta et al. (2010), and finally clinical trials modeling by Colvin and Maravelias (2008).

In this paper we modify the adaptive ‘real options’ water infrastructure planning formulation described by Erfani et al. (2018) to include endogenous uncertainty of the second type where intervention options’ activation time determine when their uncertainty must be considered. From now on in this paper, all mentions of ‘endogenous uncertainty’ refer to this endogenous uncertainty of the second type.

2. Problem description and formulation

Figure 1 shows examples of scenario tree structures for a single problem with two options O_1 and O_2 . As can be seen, uncertainty implied by the conditions of O_1 and O_2 propagates as and when the activation decisions are made resulting in different scenario tree structures.

To model this problem, we proceed as follows. Let the planning time horizon be a set of discrete time period t . Set I covers the sources of endoge-

Figure 1: Uncertainty realization for two water development options as endogenous uncertain parameters. In (a) O_2 is activated in t_1 with uncertainty over two possible realizations while O_1 is never activated accounting for two scenarios. In (b) O_1 is activated in t_1 and O_2 is activated in t_2 both with two possible realizations showing three scenarios. In (c) both options are activated in t_1 and hence produces four scenarios. These activations are during the course of optimization and are not known a priori.

nous uncertainty and θ_i represents the uncertain parameter associated with source $i \in I$. A discrete set of realizations of θ_i is represented by Θ_i . The resolution of uncertainty in uncertain parameter θ_i depends on the decision variable dS_{it} . That is, the uncertainty in θ_i is resolved in time period t if and only if $dS_{it} = 1$ and $dS_{i\tau} = 0$ for $\tau < t$. Individual scenario are indexed by $w \in \Omega$ where Ω is the set of all scenarios, and θ_i^w is the realization of θ_i in scenario w . The multi-stage stochastic programming model with endogenous uncertainty can be formulated as below.

$$\min e = \sum_{w \in \Omega, t \in T, i \in I} \frac{p_w}{(1+r)^t} [cC_i \times (dS_{t,i}^w - dS_{t-1,i}^w) + fC_i \times dS_{t,i}^w + vC_i \times S_{t,i}^w], \quad (1)$$

s.t.

$$\sum_{i \in I} S_{t,i}^w + eS_t^w \geq D_t, \quad \forall w \in \Omega, t \in T, \quad (2)$$

$$S_{t,i}^w \leq dS_{t,i}^w \times cS_i^w, \quad \forall w \in \Omega, t \in T, i \in I, \quad (3)$$

$$dS_{t+1,i}^w \leq dS_{t,i}^w, \quad \forall w \in \Omega, t \in T, i \in I, \quad (4)$$

$$dS_{1,i}^w = dS_{1,i}^v, \quad \forall w, v \in \Omega, i \in I, v \neq w \quad (5)$$

$$dS_{t+1,i}^w = dS_{t+1,i}^v \Leftrightarrow \bigwedge_{i \in D(w,v)} \bigwedge_{l < t} (1 - dS_{l,i}^w), \quad \forall w, v \in \Omega, i \in I, v \neq w \quad (6)$$

where w is a scenario with probability of occurrence of p_w , t denotes time (stages), i is a water resources development decision, r is the discount rate, cC_i , fC_i and vC_i are respectively the undiscounted capital, fixed, and variable operational costs of investment i . The optimization model minimizes the expected cost of intervention options discounted back to the present. Constraint 2 is a mass balance constraint to make sure the sum of existing supplies at time t , eS_t^w and the supply from water resource option i meets the water demand in time t , D_t . Constraint 3 allows intervention option i to be used up to its maximum capacity (cS_i^w). Constraint 4 forces an irreversible action once activated to remain active until the end of the planning horizon.

Constraint 5 and 6 introduce the endogenous uncertainty. They represent the non-anticipativity constraints (NAC) enforcing that the decisions at time t only utilize any information that is available up to that stage. They do so by linking distinguishable and indistinguishable scenarios. Two scenarios are *indistinguishable* if they are identical for all uncertain parameters' value that have been manifested up until time t . A NAC requires that for those scenarios that are indistinguishable at time t , their decisions are the same. Constraint 5 ensures that at the beginning of the first time period t_1 when no realization of uncertainty has occurred, all scenarios are indistinguishable. Constraint 6 is related specifically to endogenous uncertainty modeling and its implication is explained next.

2.1. Conditional non-anticipativity constraints

Constraint 6 is called the Conditional Non-anticipativity Constraint (c-NAC). This set of constraints formulates the relationship between the indistinguishable scenarios and the intervention options' decisions. c-NAC ensure that if scenarios are indistinguishable, then NAC is enforced and if not, they are ignored. To do so we define the set D in constraint 6 for scenario v and w in Ω as:

$$D(w, v) = \{ i | i \in I, \theta_i^w \neq \theta_i^v \}. \quad (7)$$

D represents a set of decisions in which scenario w and v differ in their possible realization. Under constraint 6, if there is no activation decision in those i that distinguish scenario w and v by time t , w and v are marked indistinguishable using $dS_{i,t}^*$.

Due to constraint 6, the proposed formulation is a logical disjunctive programming model. The logical constraint is due to the conditionality of

the NAC, and the disjunctive constraint is because of the distinguishability of scenarios. Such a model can be reformulated to mixed integer programming using the convex hull reformulation described in Williams (2013).

3. Application to a water resource planning problem

To illustrate an application of the above formulation we consider a water company with three investment decisions to implement with a five time-step planning horizon. We consider the case in which the demand growth and existing supply projection are known (Table 1). However, the intervention options include both demand management and supply expansion options (Table 2). The extra capacity added to the system is achieved via demand management (decreasing the water demand) and supply expansion options. The formulation is a least-cost aggregate supply-demand, as per Erfani et al. (2018). The uncertainties implied by the water supply-demand intervention options follow a triangular distribution. We use three realizations and mark each level as low, medium and high shown in Table 2. The distribution reflects all the possible scenarios of future realization of water availability at the time an intervention is selected. In practice such distributions of how much water a source can supply are estimated via joint hydrological and water resource systems modeling (Padula et al., 2013).

Table 1: Existing water availability and demand growth projection

	t_1	t_2	t_3	t_4	t_5
Demand (Ml/d)	2010	2024	2042	2050	2060
Water availability (Ml/d)	2000	2000	2000	2000	2000

Table 2: Decision dependent uncertainty implied by the investment options

Water availability by expanding capacity (Ml/d)					
Intervention	high	medium	low		Mean
o_1	60	42	40		47
o_2	25	20	5		17
o_3	20	18	15		18

Figure 2: (a) Solution structure for capacity expansion by considering endogenous uncertainty, (b) Utilization of options by considering endogenous uncertainty, (c) Capacity expansion deterministic solution, and (d) Utilization of options activated in deterministic solution

143 4. Results and discussion

144 The optimal activation of options and their utilization are shown in Fig-
145 ure 2. We compare the solutions of the proposed model with those of the
146 deterministic one where the mean value of the uncertain parameter is used
147 for all development options. The deterministic solution (shown in Figure 2.c)
148 suggests investing in option 1 at the beginning of the planning period and
149 to supplement the portfolio with o_3 from time period 3 onwards. In contrast
150 to the deterministic solution in which o_3 is always activated in time period 4
151 and 5, commitment to o_3 is only required as an optimal recourse decision in
152 the endogenous uncertainty model (Shown in Figure 2.a) if either o_1 and o_2
153 are realized at low level or o_1 is at its medium level. In addition, compared to
154 the deterministic solution in which option activation in o_2 is never suggested,
155 in the endogenous uncertainty model, investment in o_2 is either delayed to
156 the last stage, if o_1 and o_3 are realized at medium and low level, respectively,
157 or, o_1 is at its low level. This flexibility in options' activation is valuable
158 because by not selecting an investment option now and deferring it to the
159 next planning period, asset managers avoid its cost until more information is
160 available. Indeed, the expected cost of the proposed formulation is 10% lower
161 than the deterministic one suggesting the economic value of flexibility in our
162 case study. This highlights the value of including endogenous uncertainty,
163 and how much it is worth to postpone a decision until more information is
164 available. By not committing to o_3 in time period 3, planners can postpone
165 investment until later, when and if it is required. For the application of the
166 proposed method to a real case study it could be useful to explore the sen-
167 sitivity of optimal pathways to the selection of the probability distributions,
168 which cannot be assumed to be exact.

169 5. Extended formulation

170 In order to simplify the explanation of endogenous uncertainty our syn-
171 thetic case-study assumed no exogenous uncertainties such as the one de-
172 scribed by Erfani et al. (2018). That is, in the illustrious example provided
173 here projections of existing supply capacity and demand growth are deter-
174 ministic. To make this approach applicable for a problem with both types of
175 uncertainties, for any individual realization of exogenous uncertain parameter
176 (through the scenarios), all possible realizations of endogenous parameters
177 should be included. To formalize this, assume that ξ_t is the vector of ex-
178 ogenous uncertain parameters associated with time period t . Ξ is discrete

179 set of possible realizations for vector $\xi = (\xi_1, \dots, \xi_T)$ represents the set of
 180 all exogenous uncertainty scenarios. The scenario in a problem formulation
 181 with both exogenous and endogenous uncertainty elements corresponds now
 182 to one possible realization for vector $(\xi_1, \dots, \xi_T, \theta_1, \dots, \theta_I)$. With this amend-
 183 ment, Ω is now a set of all the endogenous and exogenous scenarios given
 184 by the Cartesian product of both exogenous and endogenous scenario sets Ω
 185 $= (\times_{i \in I} \Theta_i) \times \Xi$; i.e., for any realization of the vector of exogenous parame-
 186 ters ξ , the set of scenarios includes scenarios corresponding to all possible
 187 combinations of realizations for the endogenous parameters. θ_i^w and ξ_t^w will
 188 represent the realizations of θ_i and ξ_t , respectively, in scenario w . Note that
 189 θ_i is not time (but decision) dependant while ξ_t is independent of decisions
 190 and is resolved on given time t . We add the following set of equations to
 191 problem of section 2 to include both exogenous and endogenous uncertainty:

$$dS_{t+1,i}^w = dS_{t+1,i}^v, \quad \forall w, v \in \Xi, t \in T, i \in I, v \neq w \quad (8)$$

192 where constraint 8 is the NAC for exogenous uncertainty. If we do not
 193 have endogenous uncertainty, then Θ is an empty set and the above problem
 194 reduces to exogenous model (as explained in Erfani et al. (2018)). Similarly,
 195 if there is no exogenous parameters, then we have $\Xi = \emptyset$, $\Omega = \Theta$, and model
 196 reduces to the endogenous model (as explained by the model in Section 2).
 197 Adding both uncertainties would increase the size of the problem mainly due
 198 to the fact that the non-anticipativity constraints, which account for most
 199 constraints, grow quadratically with the number of scenarios. The size of the
 200 problem could be reduced using different theoretical approaches including the
 201 property of the set D , referring to the work of Gupta and Grossmann (2011),
 202 where an asymmetric structure of matrix D proves many NACs redundant.

203 6. Conclusion

204 This paper proposed an extension to an adaptive multistage real options
 205 water infrastructure planning optimization problem formulation for when
 206 some uncertainties are endogenous. That is, problems where water resource
 207 system intervention decisions control when additional uncertainties associ-
 208 ated with new options must be introduced. The proposed formulation is
 209 demonstrated on a synthetic problem with a small number of options show-
 210 ing how endogenous uncertainty propagates when making planning decisions

over time. The results are compared with the deterministic formulation in terms of option activations and the expected present value of the cost; the formulation with endogenous uncertainty saves 10%. For simplicity in presenting the endogenous uncertainty concept, the case-study assumed no exogenous uncertainties and referred the challenge of applying the extended formulation to cases with both exogenous and endogenous uncertainties to future work. This includes dealing with a larger multistage optimization problem as well as the correlation between uncertain parameters.

References

References

- Borgomeo, E., Mortazavi-Naeini, M., Hall, J.W., O’Sullivan, M.J., Watson, T., 2016. Trading-off tolerable risk with climate change adaptation costs in water supply systems. *Water Resources Research* .
- Colvin, M., Maravelias, C.T., 2008. A stochastic programming approach for clinical trial planning in new drug development. *Computers & Chemical Engineering* 32, 2626–2642.
- Di Baldassarre, G., Martinez, F., Kalantari, Z., Viglione, A., 2017. Drought and flood in the anthropocene: feedback mechanisms in reservoir operation. *Earth System Dynamics* 8, 1–9.
- Di Baldassarre, G., Viglione, A., Carr, G., Kuil, L., Yan, K., Brandimarte, L., Blöschl, G., 2015. Debates—perspectives on socio-hydrology: Capturing feedbacks between physical and social processes. *Water Resources Research* 51, 4770–4781.
- Di Baldassarre, G., Wanders, N., AghaKouchak, A., Kuil, L., Rangelcroft, S., Veldkamp, T.I., Garcia, M., van Oel, P.R., Breinl, K., Van Loon, A.F., 2018. Water shortages worsened by reservoir effects. *Nature Sustainability* 1, 617.
- Dupačová, J., 1995. Multistage stochastic programs: The state-of-the-art and selected bibliography. *Kybernetika* 31, 151–174.

240 Erfani, T., Pachos, K., Harou, J., 2018. Real-options water supply planning:
 241 Multistage scenario trees for adaptive and flexible capacity expansion un-
 242 der probabilistic climate change uncertainty. *Water Resources Research*
 243 .

244 Goel, V., Grossmann, I.E., 2004. A stochastic programming approach to
 245 planning of offshore gas field developments under uncertainty in reserves.
 246 *Computers & chemical engineering* 28, 1409–1429.

247 Goel, V., Grossmann, I.E., 2006. A class of stochastic programs with decision
 248 dependent uncertainty. *Mathematical programming* 108, 355–394.

249 Guo, P., Huang, G., Li, Y., 2010. An inexact fuzzy-chance-constrained two-
 250 stage mixed-integer linear programming approach for flood diversion plan-
 251 ning under multiple uncertainties. *Advances in Water Resources* 33, 81–91.

252 Gupta, V., Grossmann, I.E., 2011. Solution strategies for multistage stochas-
 253 tic programming with endogenous uncertainties. *Computers & Chemical*
 254 *Engineering* 35, 2235–2247.

255 Haasnoot, M., Kwakkel, J.H., Walker, W.E., ter Maat, J., 2013. Dynamic
 256 adaptive policy pathways: A method for crafting robust decisions for a
 257 deeply uncertain world. *Global environmental change* 23, 485–498.

258 Hall, J.W., Lempert, R.J., Keller, K., Hackbarth, A., Mijere, C., McInerney,
 259 D.J., 2012. Robust climate policies under uncertainty: A comparison of
 260 robust decision making and info-gap methods. *Risk Analysis* 32, 1657–
 261 1672.

262 Hsu, N.S., Cheng, W.C., Cheng, W.M., Wei, C.C., Yeh, W.W.G., 2008. Opti-
 263 mization and capacity expansion of a water distribution system. *Advances*
 264 *in Water Resources* 31, 776–786.

265 Hui, R., Herman, J., Lund, J., Madani, K., 2018. Adaptive water infrastruc-
 266 ture planning for nonstationary hydrology. *Advances in water resources*
 267 118, 83–94.

268 Jonsbråten, T.W., Wets, R.J., Woodruff, D.L., 1998. A class of stochastic
 269 programs with decision dependent random elements. *Annals of Operations*
 270 *Research* 82, 83–106.

- 271 Kwakkel, J.H., Haasnoot, M., Walker, W.E., 2015. Developing dynamic
272 adaptive policy pathways: a computer-assisted approach for developing
273 adaptive strategies for a deeply uncertain world. *Climatic Change* 132,
274 373–386.
- 275 Lappas, N.H., Gounaris, C.E., 2016. Multi-stage adjustable robust opti-
276 mization for process scheduling under uncertainty. *AIChE Journal* 62,
277 1646–1667.
- 278 Lempert, R.J., 2003. Shaping the next one hundred years: new methods for
279 quantitative, long-term policy analysis. Rand Corporation.
- 280 Lempert, R.J., Groves, D.G., 2010. Identifying and evaluating robust adap-
281 tive policy responses to climate change for water management agencies
282 in the american west. *Technological Forecasting and Social Change* 77,
283 960–974.
- 284 Lempert, R.J., Groves, D.G., Popper, S.W., Bankes, S.C., 2006. A general,
285 analytic method for generating robust strategies and narrative scenarios.
286 *Management science* 52, 514–528.
- 287 Matrosov, E.S., Huskova, I., Kasprzyk, J.R., Harou, J.J., Lambert, C., Reed,
288 P.M., 2015. Many-objective optimization and visual analytics reveal key
289 trade-offs for london’s water supply. *Journal of Hydrology* 531, 1040–1053.
- 290 Matrosov, E.S., Padula, S., Harou, J.J., 2013a. Selecting portfo-
291 lios of water supply and demand management strategies under uncer-
292 tainty—contrasting economic optimisation and ‘robust decision mak-
293 ing’approaches. *Water resources management* 27, 1123–1148.
- 294 Matrosov, E.S., Woods, A.M., Harou, J.J., 2013b. Robust decision making
295 and info-gap decision theory for water resource system planning. *Journal*
296 *of Hydrology* 494, 43–58.
- 297 Mortazavi-Naeini, M., Kuczera, G., Cui, L., 2014. Application of multiobjec-
298 tive optimization to scheduling capacity expansion of urban water resource
299 systems. *Water Resources Research* 50, 4624–4642.
- 300 Nohadani, O., Sharma, K., 2018. Optimization under decision-dependent
301 uncertainty. *SIAM Journal on Optimization* 28, 1773–1795.

- 302 Pachos, K., Erfani, T., Huskova, I., Matrosov, E., Harou, J., 2019. Accom-
 303 modating flexibility and adaptability in multi-objective optimised water
 304 supply planning. *Water Resources Research* (under review) .
- 305 Padula, S., Harou, J.J., Papageorgiou, L.G., Ji, Y., Ahmad, M., Hepworth,
 306 N., 2013. Least economic cost regional water supply planning–optimising
 307 infrastructure investments and demand management for south east eng-
 308 land’s 17.6 million people. *Water resources management* 27, 5017–5044.
- 309 Peeta, S., Salman, F.S., Gunnec, D., Viswanath, K., 2010. Pre-disaster
 310 investment decisions for strengthening a highway network. *Computers &
 311 Operations Research* 37, 1708–1719.
- 312 Pflug, G.C., 1990. On-line optimization of simulated markovian processes.
 313 *Mathematics of Operations Research* 15, 381–395.
- 314 Poss, M., 2014. Robust combinatorial optimization with variable cost uncer-
 315 tainty. *European Journal of Operational Research* 237, 836–845.
- 316 Ray, P., Kirshen, P., Watkins Jr, D., 2011. Staged climate change adaptation
 317 planning for water supply in amman, jordan. *Journal of Water Resources
 318 Planning and Management* 138, 403–411.
- 319 Viswanath, K., Peeta, S., Salman, S.F., et al., 2004. Investing in the links of
 320 a stochastic network to minimize expected shortest path. length. Technical
 321 Report. Purdue University, Department of Economics.
- 322 Watkins Jr, D.W., McKinney, D.C., 1998. Decomposition methods for wa-
 323 ter resources optimization models with fixed costs. *Advances in Water
 324 Resources* 21, 283–295.
- 325 Williams, H.P., 2013. Model building in mathematical programming. John
 326 Wiley & Sons.