MULTIPHASE SYNCHRONOUS GENERATOR-RECTIFIER SYSTEM FOR MORE-ELECTRIC TRANSPORT APPLICATIONS

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<th>Description</th>
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<tbody>
<tr>
<td>2D</td>
<td>Two dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three dimensional</td>
</tr>
<tr>
<td>AC</td>
<td>Alternating current</td>
</tr>
<tr>
<td>AEA</td>
<td>All-electric aircraft</td>
</tr>
<tr>
<td>APU</td>
<td>Auxiliary power unit</td>
</tr>
<tr>
<td>ATRU</td>
<td>Auto-transformer rectifier unit</td>
</tr>
<tr>
<td>BEV</td>
<td>Battery-electric vehicles</td>
</tr>
<tr>
<td>BLDC</td>
<td>Brushless direct current</td>
</tr>
<tr>
<td>BTB</td>
<td>Bus-tie breaker</td>
</tr>
<tr>
<td>CF</td>
<td>Constant frequency</td>
</tr>
<tr>
<td>CHA</td>
<td>Complex harmonic analysis</td>
</tr>
<tr>
<td>CO₂</td>
<td>Carbon dioxide</td>
</tr>
<tr>
<td>CSD</td>
<td>Constant speed drive</td>
</tr>
<tr>
<td>CSM/G</td>
<td>Constant speed motor/generator</td>
</tr>
<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>DFIG</td>
<td>Doubly fed induction generators</td>
</tr>
<tr>
<td>DFIM</td>
<td>Doubly fed induction machine</td>
</tr>
<tr>
<td>DQ</td>
<td>Direct and quadrature</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive force</td>
</tr>
<tr>
<td>EPA</td>
<td>Environmental Protection Agency</td>
</tr>
<tr>
<td>EV</td>
<td>Electric vehicle</td>
</tr>
<tr>
<td>FCEV</td>
<td>Fuel cell electric vehicles</td>
</tr>
<tr>
<td>FE</td>
<td>Finite element</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite element analysis</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>FEMM</td>
<td>Finite Element Method Magnetics</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>GRS</td>
<td>Generator-rectifier system</td>
</tr>
<tr>
<td>HP</td>
<td>High pressure</td>
</tr>
<tr>
<td>HVDC</td>
<td>High-voltage direct current</td>
</tr>
<tr>
<td>IDG</td>
<td>Integrated drive generator</td>
</tr>
<tr>
<td>IM</td>
<td>Induction machine</td>
</tr>
<tr>
<td>IPM</td>
<td>Interior permanent magnet</td>
</tr>
<tr>
<td>IPMSM</td>
<td>Interior permanent magnet synchronous machine</td>
</tr>
<tr>
<td>LP</td>
<td>Low pressure</td>
</tr>
<tr>
<td>LUT</td>
<td>Look-up-table</td>
</tr>
<tr>
<td>MCT</td>
<td>Marine current turbine</td>
</tr>
<tr>
<td>MEA</td>
<td>More-electric aircraft</td>
</tr>
<tr>
<td>MMF</td>
<td>Magnetic motive force</td>
</tr>
<tr>
<td>Nm³</td>
<td>Standard cubic feet per minute</td>
</tr>
<tr>
<td>PHEV</td>
<td>Plug-in-hybrid electric vehicles</td>
</tr>
<tr>
<td>PM</td>
<td>Permanent magnet</td>
</tr>
<tr>
<td>PMG</td>
<td>Permanent magnet generator</td>
</tr>
<tr>
<td>PMSG</td>
<td>Permanent magnet synchronous generator</td>
</tr>
<tr>
<td>PMSM</td>
<td>Permanent magnet synchronous machine</td>
</tr>
<tr>
<td>PSU</td>
<td>Power supply unit</td>
</tr>
<tr>
<td>RAT</td>
<td>Ram air turbine</td>
</tr>
<tr>
<td>SPMSM</td>
<td>Surface-mounted permanent magnet synchronous machine</td>
</tr>
<tr>
<td>SR</td>
<td>Switch reluctance</td>
</tr>
<tr>
<td>THDv</td>
<td>Total harmonic distortion (phase voltage)</td>
</tr>
<tr>
<td>TR</td>
<td>Transformer rectifier</td>
</tr>
<tr>
<td>TRU</td>
<td>Transformer rectifier unit</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>VF</td>
<td>Variable frequency</td>
</tr>
<tr>
<td>VSCF</td>
<td>Variable speed constant frequency</td>
</tr>
<tr>
<td>VSI</td>
<td>Voltage source inverter</td>
</tr>
<tr>
<td>WFIG</td>
<td>Wound field induction generator</td>
</tr>
<tr>
<td>WFSG/M</td>
<td>Wound field synchronous generators/machine</td>
</tr>
<tr>
<td>WFT</td>
<td>Winding function theory</td>
</tr>
<tr>
<td>WRIG</td>
<td>Wound rotor induction generator</td>
</tr>
<tr>
<td>WRIM</td>
<td>Wound rotor induction machine</td>
</tr>
</tbody>
</table>
# NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>S.I. Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Winding slot pitch angle</td>
<td>( \text{rads} )</td>
</tr>
<tr>
<td>( \alpha_1, \alpha_2 \ldots )</td>
<td>Angular position of damper circuit 1, 2, \ldots respectively</td>
<td>( \text{rads} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Winding slot opening angle</td>
<td>( \text{rads} )</td>
</tr>
<tr>
<td>( B_g )</td>
<td>Magnetic flux density in airgap path</td>
<td>( T )</td>
</tr>
<tr>
<td>( B_i )</td>
<td>Magnetic flux density in iron path</td>
<td>( T )</td>
</tr>
<tr>
<td>( B_r )</td>
<td>Magnetic flux density produced from rotor field winding</td>
<td>( T )</td>
</tr>
<tr>
<td>( B_s )</td>
<td>Magnetic flux density produced from stator phase winding</td>
<td>( T )</td>
</tr>
<tr>
<td>( c )</td>
<td>Winding density distribution</td>
<td>( \text{rad}^{-1} )</td>
</tr>
<tr>
<td>( C^\nu, C^\mu )</td>
<td>Fourier coefficient of winding density distribution harmonic</td>
<td>( - )</td>
</tr>
<tr>
<td>( d )</td>
<td>Distance between two adjacent rotor field windings</td>
<td>( \text{rads} )</td>
</tr>
<tr>
<td>( E )</td>
<td>Back emf</td>
<td>( V )</td>
</tr>
<tr>
<td>( g )</td>
<td>Airgap distance</td>
<td>( m )</td>
</tr>
<tr>
<td>( \tilde{G}^l )</td>
<td>Fourier coefficient of airgap harmonic</td>
<td>( - )</td>
</tr>
<tr>
<td>( H )</td>
<td>Magnetic Field Strength</td>
<td>( A/m )</td>
</tr>
<tr>
<td>( i )</td>
<td>Instantaneous current</td>
<td>( A )</td>
</tr>
<tr>
<td>([\vec{i}])</td>
<td>Vector of several instantaneous current</td>
<td>( - )</td>
</tr>
<tr>
<td>( j )</td>
<td>Current density</td>
<td>( A/m^2 )</td>
</tr>
<tr>
<td>( K_b )</td>
<td>Winding slot mouth factor, subscript ( s ) denotes stator windings, ( r ) for rotor windings and ( d ) for damper windings</td>
<td>( - )</td>
</tr>
</tbody>
</table>
\( K_d \) Winding distribution factor, subscript \( s \) denotes stator windings, \( r \) for rotor windings and \( d \) for damper windings.

\( K_e \) Winding layer distribution factor, subscript \( s \) denotes stator windings, \( r \) for rotor windings and \( d \) for damper windings.

\( K_p \) Winding pitch factor, subscript \( s \) denotes stator windings, \( r \) for rotor windings and \( d \) for damper windings.

\( K_w \) Winding coefficient factor.

\( k_{sat} \) Saturation factor.

\( L \) Inductance.

\([L]\) Matrix of inductance.

\( m \) Number of phase.

\( \mu \) Harmonic order of winding density distribution.

\( \mu_0 \) Permeability of free space.

\( \mu_g \) Permeability of airgap path.

\( \mu_i \) Permeability of iron path.

\( N_c \) Number of turns in a single slot opening.

\( N_d \) Number of turns in a single damper slot opening.

\( N_g \) Number of slots in a single phase group.

\( N_{ph} \) Number of turns in a single phase group.

\( N_r \) Number of turns in a single rotor field slot opening.

\( N_s \) Number of slots in the stator.

\( \nu \) Harmonic order of winding density distribution.
\( \theta_0 \)  
Angular position of the centre of the single coil pair in stator winding  
\textit{rads}

\( \theta_m \)  
Angular position of the rotor in terms of the stator reference frame  
\textit{rads}

\( \theta_r \)  
Angular position measured from the rotor reference frame  
\textit{rads}

\( \theta_s \)  
Angular position measured from the stator reference frame  
\textit{rads}

\( \rho \)  
Number of pole pairs  
–

\( \phi \)  
Flux linkage from the energised winding  
\( \text{Wb} \)

\( \Psi \)  
Flux linkage to another winding  
\( \text{Wb} \)

\( R \)  
Stator inner radius  
\( m \)

\( R_d \)  
Damper circuit resistance  
\( \Omega \)

\( R_f \)  
Field winding resistance  
\( \Omega \)

\( R_s \)  
Stator winding resistance  
\( \Omega \)

\( [R] \)  
Matrix of resistance  
–

\( v \)  
Instantaneous voltage  
\( V \)

\( [\tilde{v}] \)  
Vector of several instantaneous voltage  
–

\( \tau \)  
Rotor field slot opening  
\textit{rads}

\( w \)  
Damper circuit slot opening  
\textit{rads}

\( W \)  
Machine stack length  
\( m \)

\( \omega_m \)  
Rotor mechanical speed  
\textit{rads}^{-1}
ABSTRACT

Growing aircraft transportation has brought the convenience for travelling around, as well as increasing benefits to transport related industries and commercial companies. However, this also has brought more greenhouse gas emission than before, leaving an urgent issue to balance between benefits and pollutions in the aircraft transport sector. More electric aircraft systems, as a solution for replacing conventional aircraft systems, help to reduce the greenhouse gas emissions, improve energy efficiency and reduce maintenance costs. Multiphase machines in aircraft systems reduce the power per electronic device, reducing its size and cost but with improved system reliability on fault condition and power density.

This thesis evaluates the performance of multiphase synchronous machines for a DC power network, where particularly harmonic components and winding configurations for the machine under healthy and fault conditions are studied.

The analytical method involved in this thesis is the complex harmonic analysis, which is a frequency domain analysis tool for machine design, considers how the harmonic components change with different machine geometry and winding configurations in order to improve the machine performance. The experimental and finite element validation for the complex harmonic analysis are conducted in this thesis to illustrate the limit of the complex harmonic analysis together with some improvements for this method to account a better fit for the machine modelling. The complex harmonic analysis has been extended with novel work addressing saliency modelling, representation of saturation and evaluation of DQ modelling. There is also new work in the end winding leakage calculations.

This thesis also presents an example of the complex harmonic analysis applied to a reconfigurable fifteen phase generator-rectifier system with different layouts to maximise the performance and fault tolerance. Specifically, the 15-phase, three sets of 5-phase and five sets of 3-phase along with series or parallel stacked diode rectifier sets are considered.

Although this thesis is focused on a specific machine, the conclusions drawn from steady state healthy and fault performance are generalised for other winding configurations, for example, the presence of zero-sequence current and lowered winding factor in short-pitched windings leads to lower peak power capability and the parallel path in parallel stacked diode rectifier sets leads to severe interference between healthy and faulted subsets.
DECLARATION

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Finally, I would like to dedicate this thesis to my grandma, for her love, unconditional encouragement.
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1.1. Background

The transport sector contributes a significant amount of CO2 emissions, leading to the problem of greenhouse gas increase worldwide. As the U.S. Environmental Protection Agency (EPA) study shows [1], the greenhouse gas emissions from the transport sector have contributed to 29% of total U.S. greenhouse gas emissions, ranking the largest in 2017 while the electricity sector ranked as second. Half of the emissions comes from passenger vehicles and light-duty trucks and 74% of the emissions comes from road vehicles [2]. When it comes to aircraft systems, CO2 emissions from aviation make up 12% of transport sector emissions in 2017. In addition, aircraft transport only takes 0.5% by amount but 35% by value in terms of world trade goods shipments, meaning the aircraft transport is appropriate for goods that are time-sensitive and for high value commodities [2]. All these facts show that it is urgent for transport systems to reduce carbon emissions and increase the efficiency of transport in terms of reduced energy usage.

The concept of more electric systems has been proposed for changing traditional transport systems into environmentally friendly systems. Due to the growing development of power electronics, more-electric systems are becoming more reliable and easier to implement in various transport systems [3]. More-electric systems aim to use electric components to replace mechanical actuators in mechanical systems, reduces equipment weight and increases systems reliability. More-electric aircraft (MEA), a concept that has grown more popular in recent years, helps to address existing problems like greenhouse gas emissions, system reliability and high maintenance costs in the
aircraft transport sector. Further information on more-electric aircraft will be discussed in the literature review section.

Road vehicles still rely heavily on oil as the main energy source, accounting for up to 94% of total energy in 2010 [4]. Replacing traditional fossil fuel-based vehicles with hybrid or all-electric vehicles helps to reduce greenhouse gas emissions. The electrical vehicles (EVs), including battery-electric vehicles (BEVs), plug-in-hybrid electric vehicles (PHEVs) and fuel cell electric vehicles (FCEVs), have larger market shares these days. There are already 1.6% of market share for EVs in UK in 2016, compared with a market share of 0.01% in 2010 [5]. Meanwhile, the number of electric buses is growing rapidly, and has more than doubled in Europe in 2016 compared with that in 2015. In Paris the aim is to replace 80% of existing fossil fuel-based buses with electric buses by 2025.

The electric transport needs generator sources, and the common ways of generating electric power is through the electrical generators. Different types of electric generation for an aircraft power system will be covered in Chapter 2. The use of multiphase synchronous generation for electric systems has been developing over recent years, especially on electric vehicles, including marine and also on wind power conversion systems [6-12]. The review on the multiphase generation technologies will also be covered in Chapter 2. MEA requires high power density and good fault tolerance capability and also DC power link to distribute and store electric power. Implementing multiphase machine removes the required transformer rectifier unit (TRU) and also reduces the size of the DC link smoothing capacitor, therefore addresses the both requirements above.

Although the multiphase machine is a favourable choice for MEA, a quick and efficient way of examining the machine geometry and winding layout design is needed to optimise its performance. The machine design at the early stage is normally done via finite element analysis (FEA) from different software packages that provide the precise solution for the machine behaviour for different applications. However, both building the FEA model and running the solver for a dynamic solution is time consuming [13]; meanwhile, the FEA solution does not provide a direct link between machine geometry and time domain harmonics to easily optimise geometry. The use of complex harmonic analysis (CHA), as the analytical computation method for machine analysis, is suitable
for determining machine geometry at the early design stage. Details of the advantages and the applications of CHA will be discussed in Chapter 2.

This thesis addresses generation systems for more electric transport applications, specifically looking at a multiphase synchronous generator-rectifier system (GRS) for DC power networks with the CHA method linked to MATLAB/Simulink and PLECS software packages.

1.2. Aims and Objectives

The aim of this research project is to improve the fault tolerance and efficiency of GRS particularly for a multiphase winding machine. The methodology is by analysing the multiphase winding configurations and rectifier connection topologies through an appropriate simulation model with experimental validation. The objectives of this research are therefore listed as

- Development of CHA method for the salient pole synchronous generator and validation of CHA results against the FEA results and improve the model accuracy.
- MATLAB/Simulink modelling for multiphase and multiple split phase windings with series and parallel-stacked rectifiers based on the CHA method.
- Upgrade of an experimental test rig in order to conduct standard generator parameter test to support the CHA model.
- Experimental testing of the prototype generator with a resistive load as well as GRS under light and heavy load conditions. Compare the experimental measurement against simulated results to identify the limit of the CHA model.
- Application of the model to evaluate the behaviour for different winding configurations in steady state and open-circuit fault conditions.

1.3. Significance of the Work

This project contributes to studies on wound field multiphase synchronous generators (WFSG) in an area where the review from recent work showed a research gap. This project extends the CHA method to the salient pole, wound field, synchronous generator, where previously the CHA method mainly focused on either induction machines or doubly fed induction generators (DFIGs) with uniform distributed airgaps.
The new CHA model of the generator can then be combined with a circuit model of the rectifier, for better understanding of how the generator winding design and rectifier circuit topology can be optimised for a GRS.

This work is important for machine designers for DC power networks for electric vehicles. Whilst the 3-phase GRS system is well-known, modernising the GRS with a multi-phase generator presents many more options in terms of winding and rectifier topologies. This thesis provides an effective way of evaluating these options in both healthy and faulted modes. The approach is necessary, and significantly different from work on actively controlled multiphase drives, because the diode rectifier introduces distorted currents. Hence this thesis provides analytic tools for evaluating the harmonic interactions within the GRS.

The CHA method in this project is validated against a particular 15-phase wound field synchronous generator but the CHA method of generator modelling is generally applicable, and is suitable for other types of machines, e.g. interior permanent magnet (IPM) machines.

1.4. Thesis Structure

This thesis contains eight chapters. The first chapter briefly introduces the MEA system in the transport applications, the aims and objectives and contribution of this project.

Chapter 2 presents a literature review of more-electric aircraft, multiphase machines and machine modelling techniques. The development of more-electric aircraft and the on-board electric generation systems are discussed. The different types of multiphase machines are reviewed and the machine modelling techniques, including CHA and winding function theory (WFT), together with saturation characteristics and diode rectifier system modelling are also reported.

Chapter 3 focuses on the implementation of CHA method for the salient pole wound field synchronous generator deriving analytical expressions for the magnetic flux density distribution, flux linkage and back-emf waveform.

Chapter 4 presents improvements to the CHA method described in chapter 3 by including the flux path in the iron and an appropriate saturation factor. The end winding
inductor terms for this particular generator are also examined through the image method.

Chapter 5 presents the MATLAB/Simulink model based on the CHA method developed in chapter 3 and chapter 4. The stationary reference frame transform to the synchronous reference frame for both multiphase stator windings and rotor damper windings are presented. The 2D FEA model in FEMM used to study saturation effects and 3D FEA model in COMSOL Multiphysics used to calculate end winding inductance are also presented in this chapter.

Chapter 6 covers the experimental test rig setup as well as the experimental validation against the CHA simulation. The validation includes open circuit back-emf, 3-phase and 5-phase balanced resistive loading for the generator alone and 15-phase configurations including the diode rectifier with a resistive load connected to the DC bus. Different excitation and load levels are selected for comparison.

Chapter 7 examines different winding configurations for alternative layout for 15-phase generators, i.e. the split-phase winding layouts. The examination is based on the CHA method and the conclusions drawn are general for the split-phase winding layouts. The steady state healthy condition and different open-circuit fault conditions are compared. Additionally, to isolate the fault that affecting other split-phase windings, a single diode rectifier set containing the fault is bypassed for the analysis of temporal operation.

Chapter 8 provides the overall summary of this project, including key contributions and potential further work in this area.
Chapter 2. Literature Review

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2.1. Introduction

The transport section produces large amounts of greenhouse gas as stated in Chapter 1 and the aircraft emissions contributes 12% in transport section. As the requirement of reducing carbon emissions, the concept of more electric systems has been proposed for changing traditional transport systems into environmentally friendly systems. Due to the growing development of power electronics, machine design topologies, batteries etc., more-electric systems are becoming more reliable and easier to implement in various transport systems.

The electric system on the aircraft has provided multiple functionalities such as navigation and flight control, and been extended for entertainment, catering and air conditioning on recent aeroplanes [14]. The generation system is also changing between different aircraft, as well as the power system architecture. Electric machines are essential for the starter/generator systems on the aircraft and the types of electric machines are different for different targeted aircraft.

This chapter reviews the advantages of more electric aircraft as well as different current aircraft electrical generation systems. Multiphase machines are also reviewed with their different types and applications. Finally, the commonly used machine modelling techniques, complex harmonic analysis and winding function theory, as well as diode rectifier models for generation systems are also covered in this chapter.
2.2. Aircraft Electrical System

With the growing development of aviation transport, aircraft have made significant progress in recent years, with more and more weight of cargo being shipped and higher speeds for long distance, non-stop journeys [3, 15]. This shows that the aircraft transport sector needs to reduce carbon emissions and increase transport efficiency either in terms of reduced energy usage or increased goods and passenger shipment.

2.2.1. More Electric Aircraft

The traditional aircraft used gas engines to produce propulsive and non-propulsive power, where the non-propulsive power was split into hydraulic, pneumatic, mechanical and electrical power [3, 17], as shown in Figure 2.1, where three hydraulic circuits (‘pump’ in Figure 2.1) are implemented to supply power to actuators [16]. The integrated drive generator (IDG) is attached to each engine to provide constant frequency (400Hz) of 115VAC to the main AC bus, where both the auxiliary power unit (APU) and constant speed motor/generator (CSM/G) also provide generation on AC power. An additional accumulator battery (Accu) is installed along with each hydraulic

![Figure 2.1 Conventional aircraft 3H (hydraulic) architecture. (Source [16])](image-url)
circuit to provide stable operation of actuators. As the backup plan, the ram air turbine (RAT) is attached to one of the hydraulic circuits when necessary. Several transformer rectifiers (TRs) are used to convert AC power to DC.

As an example of conventional aircraft power distribution system, the B767 architecture is shown in Figure 2.2 [18], where two generators provide constant frequency AC power to the main AC buses that are connected through bus-tie breaker (BTB), which is isolated when one of the generator fails. The TRUs are used to provide power for DC loads from constant frequency AC buses.

![Figure 2.2 B767 aircraft power distribution system. (Source [18])](image)

The complex interactions among different power systems increase the difficulty in maintenance and reduce the overall efficiency. Therefore, the concept of moving to a more-electric system has been introduced since World War II. The idea of the MEA was to replace redundant systems, e.g. hydraulic and mechanical systems, by an electrical system and reduce system complexity and improve reliability [3]. In the 1980s, the airplane with the concept of “Fly-By-Wire” was introduced by Airbus in the A320 series, followed by Boeing in the B777, which replaced the mechanical and hydraulic linkages by electrical control linkages [16]. With the development of power electronics for energy conversion, the concept “Power-By-Wire” or MEA became possible. The objective of introducing MEA is to replace the non-electrical power with an electrical form, in order to reduce the weight, lower the maintenance cost, improve reliability [18] and to eliminate integrated drive generator (IDG), which improves dispatch reliability [19].

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In principle, the MEA comprises generators on either high pressure (HP) or low pressure (LP) spools, which are connected via common aircraft power bus (which may be AC or DC) [20] as shown in Figure 2.3. To move from more-electric to all-electric aircraft (AEA), the biggest challenges are in the power density and energy storage, as existing all-electric aircrafts are small ones (less than 20 passengers), either private, for commercial use or to demonstrate proof-of-concept [21]. [19] pointed out that the MEA is moving to a distributed power distribution system rather than conventional centralized power distribution system. The distributed power distribution helps on weight saving, reduces maintenance cost, improves fuel efficiency and lowers total cost.

![Diagrammatic aircraft power system](source: [20])

**2.2.2. Aircraft Generation System**

The power distribution system of the more-electric aircraft system is composed of AC and/or DC systems.

Before the 1960s, DC networks were as widely used as AC networks in aircraft systems, while the electrical power was mainly focused on the instrumentation, communications, anti-icing and later for entertainment, catering. Electrical systems were rarely implemented for the environmental conditioning system, flight surface actuation or propulsion [21, 22]. The electrical system in aircraft has been changing from twin 28VDC in early 1950s to 115VAC with constant 400Hz, started from 1960s, to 115VAC variable frequency, 230VAC variable frequency and 270/350/540VDC
(proposed since 2000). Recent aircraft have used a variable frequency (300-800Hz) and 115VAC fixed voltage power system as the standard, with a conventional means for propulsion (i.e. gas turbines) and flight surface control (hydraulic and electro-hydraulic actuators) [21]. [23] proposed a conceptual architecture of universally-electric system for transport aircraft, with fully electric propulsion system connected to AC networks, a 540VDC network for subsystems such as flight control and a 28VDC network for avionics.

The generated power is normally in the form of three-phase 115VAC, either constant frequency on 400Hz or varying from 300-800Hz [24], and the generation scheme could be categorised as constant frequency (CF), variable speed constant frequency (VSCF), variable frequency (VF) and DC link.

CF (normally considered as three-phase 115VAC 400Hz) is the most widely adopted format in civil aircraft systems, like, Boeing B777, Boeing B767-400 and Airbus A340. For Boeing 767, it used the conventional CF generation system, with two generation channels have their own integrated drive generator (IDG) driven from main engine; each generator is rated at 115VAC, 400Hz, 90kVA [25]. The generator is connected to the variable speed engine shaft via a constant speed drive (CSD). The CSD usually contains a three-stage regulated synchronous generator with constant output frequency by means of a hydro-mechanical CSD [18]. The combination of the CSD and the generator into a single unity is also termed as IDG. The drive system contains sets of gearboxes, which are vital to guarantee a constant speed on generator side, but are costly, inefficient due to the mechanical coupling and need heavy maintenance [3, 26-28]. The AC/DC converters are used to convert AC power to DC form to the DC bus, where DC loads are supplied. Wound field synchronous generators are mainly used in this conventional aircraft, which are controlled based on the feedback from AC and DC voltages, to provide control signals such as firing angle for rectifiers and excitation field for the generator [28]. The electric power in this architecture is used for compressors and fans to circulate and cool air in the aircraft, in addition to avionics equipments, entertainments, lighting and catering [19].
For a VSCF generation scheme, the generators are connected to the engine directly, the output power is converted to three-phase 115VAC 400Hz form through either combined AC/DC converter and DC/AC inverters (VSCF-DC link) or cycloconverter (VSCF-Cyclo) [3, 26, 29]. The cycloconverter was introduced in 1970s to convert the generator output to constant voltage under 400Hz, as the main propulsion engine drives the generator to produce 1.2kHz to 2.4kHz frequencies [29]. The cycloconverter converts variable frequency AV power directly to fixed frequency AC, this technology is mainly used in military planes, e.g. F-18E/F fighter, as it is more efficient than VSCF-DC link and CF but requires intricate control [3, 15]. [28] stated a typical example of VSCF generation system, where the generated variable frequency AC from the generator is fed through a bi-directional power converter, the power is then converted into constant frequency AC form; while in motoring mode, the main AC system provides electric power via a bi-directional power converter to start the aircraft engine, as shown in Figure 2.4. This VSCF-DC link generation scheme used the DC bus as an intermediate connection between generator and AC bus, it is now the preferred option in some commercial and military aircraft, e.g. MD-90, C145 and B737s [30], due to is simplicity and reliability [3, 15]. Moreover, with the development of power electronics, the electric equipment in VSCF system could be distributed throughout the aircraft, rather than located close to the engine as the CSD/IDG in CF generation system [18].

The VF generation scheme has the advantages of low cost and high reliability, this commonly provides varied frequency from 360-800Hz of three-phase 115VAC or 230VAC power, depending on loads requirements, additional motor controllers maybe needed to allow motor/pump speed control [26]. VF generation scheme has been adopted in recent aircraft as Airbus A380 and Boeing B787. Take A380 as an example,

![Figure 2.4 Typical VSCF starter/generator system for aircraft. (Source [28])](image-url)
it has four main generators rated at 150kVA, generating power under frequency range
370Hz to 770Hz, while its equipment is capable of handling 360Hz-800Hz, all the
electrical sources are operated under 115VAC [31]. [31] also considered other possible
changes to the electrical system, such as moving from 115VAC to 230VAC and
applying power centrally. The doubled voltage helps to reduce the wire weight
significantly, estimated up to 32% of total; based on the 6000Nm³ (standard cubic feet
per minute) standard mission, the fuel could be reduced by approximately 800kg,
together with reduced CO2 of 2500kg. The central power control improved aircraft
safety and reliability, as in case of an event, the shut-down procedure is much easier for
this “power centre”, although the installation procedure of separate power channels
might even complex than before.

The 270VDC option is mainly used in military aircraft system, since the higher voltage
and lower current bring down cable weight and power dissipation but require higher
quality insulation. An example architecture of advanced aircraft power system with
270VDC bus is shown in Figure 2.5, where the 270VDC is generated from bi-
directional power converter with VSCF generation scheme, the DC/DC converter and
DC/AC inverter are further used to convert DC power to supply DC and AC loads. The
advantages of using 270VDC distribution system in aircraft are also stated in [25], as
easy to provide uninterruptible power when there is a battery as the back-up. Adapting
HVDC system in aircraft, as ±270VDC or 540VDC, saved approximately 364kg in
weight at the power conversion level compared with a conventional 3-phase 115VAC,
400Hz system, compressing around 60% weight saving in cabin and cargo feeder
routing weights and 50% saved weight in galley feeders [32]. Weight saving helps to
reduce fuel consumption and improve its performance. Since the electrical system in a
conventional, non-MEA aircraft consumes only 0.2% of the engine power during cruise,
there is little benefit in cutting down the electricity consumption; however, reducing the
weight of corresponding electrical equipment has a positive effect [33]. As a
compromise, a hybrid system containing 115VAC and 270VDC power generation
scheme is used in Lockheed F-22 Raptor, while 28VDC is mainly for lighting services,
other subsystem controllers and avionics systems [26].
This thesis concentrates on evaluating DC distribution performance from a variable speed prime mover and designing of a generator-rectifier system. The DC distribution network performance is examined with different winding configurations on the generator-rectifier side, since the rectifier system is passive uncontrolled devices.

2.2.3. Electrical Machines for Starter/Generator System

The wound field synchronous generators/machine (WFSG/M) is the common choice for the conventional aircraft, mechanically connected via a variable ratio transmission drive to the main engine, providing 400Hz constant frequency [34]. The WFSG/M is generally used for integrated starter/generator in aircraft because of the advantage of low cost in maintenance and ability to remove the field current under short-circuit fault [35]. [36] stated the wound field induction generator (WFIG) has been already used in A380 for its VF generation system, and in B787 for both VF generation system and starter/generator system. The B787 uses electrical environmental conditioning and thus removes the need for a bleed air system.

Conventionally, the WFSG generation includes three stages, as a permanent magnet generator (PMG), main exciter and main generator. The standard three-stage synchronous generator diagram is show in Figure 2.6. Depending the generation system, the prime mover connected to the PMG can be either constant speed from the CSD or variable speed converted from the engine. The generated electric power is then
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transferred to the main exciter used to provide stable excitation source for the main synchronous generator.

![Diagram](a)

![Diagram](b)

Figure 2.6 Three stage generation system for (a) CF and (b) VF. (Source [16])

The research in [20] studied the feasibility of a permanent magnet (PM) generator for MEA as a low-speed generator on the LP spool and suggested that full fault tolerance could be achieved with a direct drive generator. [20] also showed that the PM generator coupled with PWM voltage source converter helps increase the synchronous reactance, which meets the future requirements for MEA. [37] studied electrical machines and distribution systems for more-electric aircraft, evaluating machine types such as PM and switch reluctance (SR). The author stated that PM machines could achieve most of the criteria for a MEA generator, such as high-power density and high starting torque at zero speed but could reach high temperatures at high speeds and field excitation could not be removed during fault operation. Although SR machines could handle high temperature operation, the windage losses caused by rotor saliency becomes significant at high speed, which could be solved by filling gaps with non-magnetically permeable material, but with the cost of increased mass.

In [38], it is proposed using a high-speed open-end winding squirrel cage induction star/generator attached with an inverter/rectifier unit on the high-pressure spool and a low-speed conventional wye-connected squirrel cage induction generator with an active
rectifier unit on the low-pressure spool of the engine, both providing constant voltage DC power under variable frequency. The proposed hybrid ac/dc induction generation system removes the external exciter and reduces the complexity of hardware, compared with the 3-stage synchronous generation system.

Conventional aircraft used a 6-pulse diode rectifier system to rectify the 115V 3-phase supply, however, the behaviour of the non-linear diode rectifier results in a high level of current harmonic distortion that can cause additional heating and interference problems in the aircraft electric distribution system [39, 40]. The common solution is to use a transformer rectifier unit (TRU) or auto-transformer rectifier unit (ATRU) to regulate the DC output [39]. The commonly implemented TRU is multi-pulse, i.e. 12-pulse [41, 42], 18-pulse [43] and 24-pulse [41, 44]. Increasing the pulse number of the TRU helps to improve the DC electric system power quality and reduce harmonic distortion on the AC side, e.g. from 12-pulse to 24-pulse in [40, 41, 44] and to 36-pulse in [43], but needs increasingly complicated phase shift transformers. The implementation of multiphase generators would help get rid of heavy, bulky, expensive transformers and also simplify the rectifier connections.

2.3. Multiphase Machines

The multiphase machine has not been a new concept, it has been studied over decades and been utilised in different applications such as ship propulsion and marine current turbines [8, 45], wind energy conversion systems [46, 47] and electrical vehicles [48-52]. Five- and six-phase machines, or dual three-phase machines, are used in high power applications [53]; and the research interest has considerably increased during the past decades [54].

Multiphase machines have advantages compared with three-phase machines such as higher reliability in terms of fault tolerance [55, 56]; reduced torque pulsations [55, 57]; reduced stator losses due to suppression of 5th and 7th current harmonics [6]; less susceptible to time harmonics excitation and reduced power rating requirements for power electronic devices [56, 58] and higher torque density; since the multiphase machine has higher torque per ampere within same volume machine [53, 55, 59]. A multiphase machine also provides additional degrees of freedom, which helps to maintain the fundamental component of the air-gap flux by injecting current harmonics
via these additional degrees of freedom, this method is utilised not only in fault tolerant control, but also for balancing power flow on different winding sets and reducing DC-link voltage ripple [45, 46, 52, 53, 55-57, 59-62]. In [62], the non-traditional uses of additional degrees of freedom in multiphase machine have been reviewed, such as eliminating separate battery charging unit in electrical vehicles while charging from grid side.

2.3.1. Symmetrical and Asymmetrical Multiphase Machines

Multiphase machines can be divided into symmetrical winding distribution and asymmetrical winding distribution. The asymmetrical winding can be divided into single star point and multiple phase sets (one star point per subset).

![Symmetrical and Asymmetrical Multiphase Machines](image)

Figure 2.7 (a) symmetrical winding distribution for an m-phase machine. (b) asymmetrical (split-phase) winding distribution for a nine-phase machine.

Symmetrical multiphase machine refers to phase windings that are evenly distributed around the stator circumference, so, for an m-phase symmetrical multiphase machine, each phase winding is separated by $\frac{2\pi}{m}$ in electrical radians, shown in Figure 2.7.(a).

Most of the symmetrical winding multiphase machines presented are five-phase, seven-phase and nine-phase e.g. an example of a five-phase induction machine (IM) for drive applications has been reviewed in [52]; [60] studied fault tolerant control for five-phase IM drives; the comparison between three and five-phase PM machines was also made in [12, 63] in terms of steady-state healthy condition and diode rectifier open circuit or device failures; a vectorial approach-based control for seven-phase PM machine in [64] and space vector PWM for seven-phase drive is given in [65]; [61] analysed five and
nine-phase induction machine in terms of airgap and yoke flux density. As the equivalent to the symmetrical nine-phase PM machine, the three 3-phase submachines with separation of 40° is discussed in [66], where the PM machine is controlled by triple 3-phase voltage source inverters (VSIs) to simulate a high speed elevator. The same nine-phase structure is also proposed in [67] for design of nine-phase PMSM, where other options such as three asymmetrical sets of 3-phase with 20° separation and three in phase 3-phase submachines are also covered in this paper. A 15-phase IM was modelled in [68], but only modelled in the DQ plane.

However, the improved performance when increasing the number of phases for multiphase is limited. In terms of reduction in torque ripple for an m-phase machine with ideal sinusoidal windings, the lowest frequency of torque ripple present is 2m. As the number of phases is increased, the frequency of the torque ripple goes up, however, the general problems from torque ripple lie in lower order harmonics, meaning the benefits from increased phase number reduce. [69] stated that when the number of phases goes higher, the improvement in stator copper losses has less impact, a little improvement is made from 12-phase, 15-phase to 21-phase. The similar conclusion is found in [70] on the improvement for stator copper losses and derating factor when moving towards higher phase numbers. Meanwhile, a high number of phases increases the cost and complexity of the system, as well as power electronics devices if the machine is either supplied from voltage source inverters or coupled to converters, so the increased components count becomes dominant rather than the benefit from the desired fault tolerance feature [71].

The asymmetrical winding is commonly found in the split phase machines, usually composed of multiple sets of three-phase windings, each set usually displaced by \( \frac{\pi}{m} \), shown in Figure 2.7.(b). In the literature, the six-phase, split-phase machine, also known as a dual three-phase machine, is becoming more common in drive systems and wind energy conversion systems, as the power rating is shared in two winding sets, which brings down the power rating requirement per channel on the converter side [46, 47, 55]; meanwhile, in the aircraft applications, the split phase machine provides multiple channels for the electric power and fault compensation [71]. Comparing with symmetrical multiphase machines, asymmetrical multiphase machines provide smoother MMF than the former when the total number of phases is even [53, 55]. In [54], the
author stated that the dual three-phase machine is more sensitive to supply asymmetries, meaning differences in supply voltages for two sub-machines, than the machine asymmetries, which is the difference on the stator winding parameters due to manufacturing variation, i.e. stator resistance and leakage inductances [55].

2.3.2. Review of Multiphase Machines Technologies

Most of the published research is on PM or IM based multiphase for both generating and motoring systems. For multiphase PM machines, saliency is commonly neglected. In [47], a six-phase (dual three-phase) permanent magnet synchronous machine (PMSM), which was connected to two series connected voltage source converters, was evaluated for a wind energy conversion system. The generator was considered as a non-salient machine, similar to what has been published in [63]. [72] studied the six-phase fractional-slot concentrated winding PM brushless machine open-circuit fault tolerance performance targeted for electrical vehicles, where the machine is essentially operated as dual-three phase machines, as the second sets of windings are separated by 180° mechanical or 720° in electrical, the saliency feature was still not considered.

Multiphase induction machines are commonly applied in different areas. [60, 73] studied the five-phase induction machine for fault tolerance behaviour in drive applications. [46, 54] provided the that the dual three-phase induction machines were also been adopted in wind energy conversion systems. Multiphase machines have also been attractive in marine system. [68, 74] studied the feasibility of implementing multiphase rather than three-phase for the marine propulsion in terms of fault tolerance and torque stability. [75] discussed a six-phase induction machine design at marine electric propulsion.

Similar to the multiphase IMs, the wound field synchronous machine (WFSM) normally uses distributed stator windings. Meanwhile, the rotor saliencies and damper winding in the WFSM makes it a little different from studies on multiphase IM and PM. Not much work has been done on the multiphase WFSMs. A dual 3-phase wound rotor synchronous alternator for medium voltage DC power system was covered in [76], with a comparison in healthy operation and for open-circuit diode faults. [77] studied the modelling method for an asymmetrical nine-phase WFSG for wind power generations, where the saliency was included as maximum airgap and minimum airgap. [78] studied
a split 12-phase (4 sets of 3-phase windings) synchronous generator for DC power generator with passive rectifiers under unsaturated load operation. [7, 79, 80] considered the 3,5 and 15-phase WFSG with diode rectifier system for aircraft power systems, with healthy and open-circuit fault operations. The saliency and winding harmonics were included by externally adding terminal voltage harmonics, based on fast Fourier transform (FFT) results of back-emf from FEA analysis, however, the saturation effects were not considered.

In the areas of fault tolerance for AC/DC generation or motoring applications, the multiphase machines are usually coupled with active controlled rectifiers to minimise torque ripple and losses under faulty condition. A five-phase IM under healthy and single inverter connection open-circuit fault condition operation was reviewed in [81], where the optimal current control for voltage source inverter (VSI) was implemented to minimise the stator copper losses. The review gave a comparison of star and pentagon stator winding connection, with the conclusion that, generally, the pentagon connection has superior behaviour over the star connection under the faulted condition. [73] presented the control strategies for a five-phase IM with field-oriented control under faults with the loss of one, two or even three phases of motor windings or semiconductor failures for continuous disturbance-free operation. [49, 50] presented a new adaptive model to identify open circuit faults on inverter switches or phase windings for a five-phase PM machine (brushless DC (BLDC) motor) mainly for drive applications. The control strategy was also included for the inverter side. The control methods for two independent inverters under open-circuit and short-circuit fault scenarios of a 6-phase PM blushless machine for drive application was assessed in [72]. In [57], the author modelled a five-phase PMSM as five sets of inductive windings with self and mutual inductances, connected between two isolated inverters. This paper also proposed a control strategy for single inverter short circuit fault. In [49, 50], fault tolerant control strategies were discussed when PM machines were operated as motor drives. The fault tolerance between three-phase and five-phase permanent magnet synchronous generator (PMSG) for marine current turbine application was conducted in [45] via an marine current turbine (MCT) simulator environment, together with proposed optimal control strategy.
However, for multiphase generators with passive rectifiers the main focus is on winding/machine layout for the potential fault behaviour, but, there is not much research on this area. [70] gave a brief comparison on derating factor and stator copper losses for a multiphase motor under different winding topologies for healthy and faulty conditions, however, this was under motoring mode powered by converter. [8] analysed the short-circuit fault on single phase to ground, three phases to ground and line to line faults for a WFSG with asymmetrical 12-phase (4 sets of three-phase, rectifiers in parallel connection) layout, the modelling of the generator and the diode rectifier system was based on PSCAD/EMTDC library. The same generator winding topology was covered in [82], which implemented an analytical method for both the generator and diode rectifier system to analyse the internal phase to phase short-circuit fault.

Although the GRS system performance for split-phase machine with rectifier has already been evaluated in [78, 82] for 12-phase machine with parallel stacked diode rectifiers, this research extends the validations to 15-phase with options as five sets of 3-phase and three sets of 5-phase with both parallel and series stacked diode rectifiers.

For variable speed applications, PM for DC generation has the intrinsic issues that it needs an additional DC/DC converter to regulate the voltage (probably on aircraft DC generation system as example), while this could be solved by adjusting WFSG excitation. Moreover, the excitation can be removed to limit the fault current during AC/DC generation.

For the multiphase generator with diode rectifier system, symmetrical windings as 15-phase could be split into multiple sets of low phase number windings, i.e. 3-phase or 5-phase, their operating capability and fault tolerance have not fully covered in the literature.

The concentrated winding distribution is not considered in this thesis as it is commonly used in PM machine, whereas for the WFSG, the concentrated winding could induce additional harmonics into rotor windings which increases the harmonic loss.

This thesis identifies the split-phase WFSG with a diode rectifier as a good potential option for an aircraft DC power source, and will investigate how the number and configurations of phase windings can be used to improve the operating capability and fault tolerance of the resulting system.
2.4. Machine Modelling

The fundamental per-phase circuit model is the simplest for machine modelling, which provides enough analysis of the fundamental machine behaviour for control, especially for the non-salient rotor structure. This fundamental model is widely used in the cage-rotor induction machine and WFSG analysis for power capability analysis and power factor [83, 84].

However, when it comes to requiring more details of machine performance, like torque ripple, total voltage distortion factor, non-linear loads on the machine, the fundamental circuit model begins to show its limitations. [80] compares the open-circuit fault among 3/5/15-phase synchronous wound field machines, targeted for aircraft applications. The machine is modelled as the combination of sinusoidal fundamental voltage with harmonics obtained from a corresponding FEA model, and the saturation effects were ignored in the machine model. The model shows a relatively good prediction on short-pitched winding distribution at light loads, but less good match on fully-pitched windings.

The detailed machine modelling techniques could be mainly divided into two sections, by using computer based FEA, or by using an analytical method.

FEA was introduced in 1960s and has been widely applied to solve engineering problems [85], the FEA could successfully predict how the model reacts to real-world physical effects [86]. For machine modelling, FEA takes the non-linear effects, like magnetic or material saturation [87], into account, providing more accurate results, especially for those machines with complex or novelty geometry [85]. In exchange for accuracy, FEA consumes much longer time and computation resources. Besides, FEA only solves for a specific design pattern; a simple change in the machine geometry would result in tedious work to rebuild the model. Apart from those constraints in the fixed machine parameters, most of FEA produces static results. Obtaining a time-varying solution for dynamic behaviour from FEA would take much longer time. Meanwhile, the accuracy of FEA relies on choosing a proper element type and size and defining appropriate boundaries as well as boundary conditions [13]. The common analysis for machines at the design stage is by using 2D FEA compared with 3D FEA. Although the latter gives better accuracy by accounting for axial geometry than 2D FEA,
the 3D FEA generally needs plenty of time to build the accurate model and the 3D meshing process is longer and more difficult than 2D meshing, which leads to longer process time.

Mathematically, more detailed analysis methods that are commonly applied are WFT and CHA, where both techniques considers the space harmonics with the similar assumption of flux travelling in radial direction in the machine, neglecting stator small slotting and saturation.

2.4.1. Complex Harmonic Analysis

CHA of machine geometry is a method of improving the machine design and examining harmonic content with different machine winding layouts. CHA, based on the Fourier expansion of winding distribution, is a frequency domain analysis tool, treating the winding distribution as a periodic function in spatial domain and representing the winding distribution as a sum of Fourier series [90].

The earlier approach for CHA was to analyse fields in an induction machine to model and analysis its behaviour. The field analysis was used to compute the induction

![Diagram showing the process of CHA to compute inductance coupling for [88, 89].]
machine coupling impedance, slot leakage and effects of rotor skew, and the CHA method to represent stator/rotor winding coils, as well as the coupling emf in [88]. However, the author used the fundamental component as the magnetising impedances, and all the other harmonics were considered as leakage terms. As an improvement of [88], the CHA method in [89] used the process shown in Figure 2.8 to compute coupling inductance between stator/rotor windings for uniform air-gap induction machine. The author used a coupled impedance method to work out the voltage balance equations, to find the power factor, and gave some suggestions on induction machine design in terms of improving power factor and efficiency. The author suggested that the power factor could be improved by “means of forwards-rotating subharmonic fields”. The method proposed in [89] is also validated in [91] with a 10-pole, 26kW, 415V cage rotor induction machine, under unsaturated and a variety of unbalanced conditions, showing that a good agreement had been achieved between calculation and measurement.

As well as for modelling and analysing the IM performance, the CHA method has also been implemented as a way for condition monitoring on IM and DFIG for range of faults. For example, the method developed in [89] is applied in [92] to predict the stator winding short-circuit fault for a cage induction motor and validated on a rated 415V 10-poles induction motor (validated under 250V line rather than 415V line voltage). The validation showed a quite good agreement on stator interphase short-circuited fault, while the agreement on short-circuited current became less impressive on stator single phase to neutral short-circuited fault.

The CHA and coupled circuit approach were also implemented in [93] to analysis the dynamic performance of the brushless doubly-fed machine under healthy and fault condition in simulation, for a phase-to-neutral short-circuit fault and phase-to-phase short-circuit fault. CHA also served as a tool to predict stator/rotor unbalance and stator/rotor supply unbalance for DFIG [94], the experimental validation in the work showed the harmonic components match well, however, the harmonic components raised from saturation effects were not accounted in the analytical model. Condition monitoring on the stator current spectrum, torque spectrum and mechanical vibration spectrum for both wound rotor induction generator (WRIG) and DFIG under different additional supply harmonics (2\textsuperscript{nd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th} and 13\textsuperscript{th}) were also reviewed in [95],
where the modelled results were validated against a 4-pole 30kW wound rotor induction machine (WRIM), which could be configured as a WRIG or a type III DFIG. CHA was also explored to monitor bearing outer race fault for a WRIM targeted for wind turbine applications, together with experimental validation on a 30kW WRIM with the rotor winding short-circuited and driven by a DC machine (WRIM was operating at 1600rpm as generator), where the bearing outer race fault was modelled as periodic air-gap variation based on the bearing ball passing frequency [96]. The CHA method from [89, 91] was implemented in DFIG (3-phase, four-pole, 30kW) in [97], to monitor harmonic difference between healthy and fault conditions, however, the CHA method was used to predict potential frequency components rather than the magnitude of that component. The same application for CHA is done in [98], where the frequency components rather than magnitude is examined for CHA to predict the fault behaviour of WRIM. Both [97] and [98] did not considered the saturation effect when implementing the CHA. [99] used the CHA method to determine the winding space harmonics that producing torque ripple for brushless doubly fed induction machine (DFIM), the analytical result was then validated against 2D finite element (FE) calculation from Comsol, showing that the analytical and FE calculation had the good agreement. Both CHA and FE model were assumed under uniform air-gap and infinite permeability of iron, therefore, the saturation effect was neglected for both cases. [93-99] demonstrate that the CHA method could be implemented as a tool to predict the potential frequency components for either DFIG or WRIM, therefore, as a means of condition monitoring method for induction machine. While for the purpose of machine design, the accuracy should be improved for CHA to match not only the frequency components but also the magnitudes with corresponding experimental validation.

The CHA method is also extended to be implemented on multiphase induction machines [100] and PM machines [101]. In [100], the winding factor, stator/rotor joule losses, iron losses and pulsating torque influenced by fundamental and 17th harmonic excitation component for multi-phase induction machines (3,6,9 and 12-phase) was studied. The proposed motors were non-skewed with uniform airgap. The theoretical result showed that increasing the number of phases has benefit on reduction in stator copper losses and torque pulsation due to the attenuated phase belt harmonics, with slight decrease on rotor joule losses and no significant effect on iron losses. The method developed in [89, 91] and instantaneous Park vector module was combined in [101] to monitor PMSM
inter-turn winding fault. The author presented the simulation results only, with inter-turn coil faults limited up to 10% of phase coils.

2.4.2. Winding Function Theory

The WFT defines a function with respect to angular position that could represent the spatial total magnetic motive force (MMF) from a set of coils [90].

The WFT sums multiple coil winding functions that contribute to one circuit [102]. The WFT starts from the same winding distribution as CHA but sums over the winding distribution functions within the same circuit in the spatial domain. The current distribution is used to obtain the MMF variation across the airgap. The flux is obtained by multiplying the permeance of the airgap with the MMF. The flux linkage is found by integration in a second winding function. The accuracy of winding function theory depends on how well the initial winding function is presented. The most common and the simplest method is to consider the winding function as only the fundamental sinusoidal distributed function, as [83, 103] to compute MMF and machine winding inductance. The WFT was used to model a saturated 3-phase induction machine in [104], where the winding functions are based on the fundamentals but with added third harmonic components accounting for third harmonic MMF from saturation. The air-gap model is originally uniform, but can be changed to DC with 2nd order harmonics for the teeth saturation in rotor and stator, which introduces third harmonics in the flux linkage. In the research [105], the sum of odd harmonics for the winding function distribution were considered.

To improve the accuracy of WFT, the linear-rised MMF assumption for winding function distribution has been proposed. In [106], a linear rise MMF method with more precise air-gap permeance for a salient pole synchronous generator is presented, which helps improve accuracy as the model considers all space and time harmonics. A piece-wise winding function distribution, together with stator slotting and skewing effect have also been implemented to further increase the accuracy of WFT, but with increased complexity. As in [102], the induction machine winding inductance with the effect of rotor skew is considered and compares well with the inductance computed from [105]. In [102, 107], the piecewise function expression was implemented for each coil, which contains all space harmonics and avoid the constraint on axes of symmetry. The rotor
saliency and stator slotting effects were also considered in [108] when applying the winding function to a synchronous reluctance machine with comparison against FEA results, but [108] ignored saturation.

Meanwhile, instead of a uniform airgap [102, 105, 107], a non-uniform air-gap or with eccentricity have also been considered in WFT analysis [106, 108, 109].

As with the CHA method, the saturation effect is barely considered as the reviewed work stated above.

Besides, the WFT has also been extended to other types of machines. In research [110], the WFT was used to model a synchronous reluctance machine and test different combinations of methods to reduce torque ripple, but still did not consider saturation, as the air-gap permeance and winding functions were based on [108, 109]. For PMSM machine, WFT was used to deduce the impact of stator skew, applying the fundamental skew factor to 2D FEA computed inductance, which was to compare against 3D FEA and experimental results, showing a good match on the results [111]. The author also pointed out that for surface-mounted permanent magnet synchronous machine (SPMSM), introducing skew will decrease magnetising inductance, while for interior permanent magnet synchronous machine (IPMSM), skew results in increasing the d-axis magnetising inductance and decreasing q-axis magnetising inductance. Short-circuit inter-turn faults for 2-pole 3-phase PMSM was modelled with WFT in [112], together with experimented validation. A 5-phase PM brushless machine based on per-phase matrix equations in MATLAB/Simulink was also modelled in [113], while the loading of the simulation was based on the armature voltage reaction look-up table from experimental test, which was conducted under resistive load only.

In [114], WFT was introduced to model the synchronous generator (35MVA hydro-generator), and the results were shown from both analytical and FEA. The magnetic flux density from the rotor side is modelled by using WFT, the airgap permeance function (pole gap region) between two adjacent rotor poles has been assumed as a linear variation; the stator-slot effect on the airgap permeance function is obtained from FEA result (CEDRAT Flux 2D package). This paper also modelled damper winding bars as a network with resistance, self/mutual and stray inductances, and the external voltages are introduced from each magnetic flux density harmonic component. However,
[114] only considered the harmonic effects from stator slotting. A 12-phase WFSM with 4 sets of three-phase rectifier connected in parallel in PSCAD/EMTDC software was modelled in [8], where the rotor has two poles with saliency and the modelled is performed in DQ reference frame.

Both CHA and WFT methods are not well covered for the WFSG, although [115] covered the modelling details for WFSM with WFT, included the stator slotting effects and airgap saliency feature, both are extracted from FEA results and combined with the stator winding function. The saturation effect is still not considered. In [116-118], the WFT was used to analyse the damper winding behaviour of the salient WFSG, however, the salient feature was still obtained from FEA and implemented in WFT. Papers covering a wound rotor with CHA methods mentioned above were generally based on DFIGs with a uniformly distributed airgap. The saliency feature is not fully documented by CHA, while for WFT, it is mostly represented by up to second order harmonic or extracted from FEA results.

2.4.3. Saturation Modelling

Saturation effects in machine modelling have always been a challenge, especially with rotor saliency.

For machine models based on the fundamental circuits, because the saturation usually causes the rise of the third harmonic of MMF, the saturation is usually implemented as additional third harmonic of MMF. In [104], the saturation for an induction machine was included by superposition of 3rd harmonics to the fundamental in a synchronised phase relationship with airgap field flux in order to model saturation in the teeth path; the 3rd harmonic is also used in a phase displacement with fundamental airgap flux to model the saturation in stator and rotor core. The experimental validation (230V 4-pole induction machine) showed that the proposed method matched the experimental results pretty well under no-load condition, but less well under rated load condition.

For other more detailed analytical methods, in both CHA and WFT, a saturation factor is usually introduced to account for the saturation effect. Most of the saturation models were implemented on the DQ axis, either as a function of total flux level (a single saturation factor) [119] or considering separate saturation effects on the D and Q axes. [120] modelled synchronous reactance and iron losses dependent on the D-axis only,
but neglected the DQ axis cross coupling and Q-axis saturation on a synchronous reluctance motor. This model was used to establish the optimal rotor angle for vector control of the synchronous reluctance motor. Although this method could be used as a means of implementing the optimal angle control strategy, the validation from measured quantities showed quite significant differences from analytical predictions. As an improvement, the DQ two-axis model for synchronous machine was used in [119] and saturation factors for both the D-axis and Q-axis were introduced, although the leakage terms were assumed as constant. The author showed that using separate saturation factors for D and Q axes had almost the same result as the single saturation factor, which depends on the total air-gap flux linkage level $\lambda_t$, as shown in (2-1)

$$\lambda_t = \sqrt{\lambda_{md}^2 + \lambda_{mq}^2} \quad (2-1)$$

The single saturation factor consumes less computation time and is much simpler. The author also showed that the error was much larger when implementing a saturation factor only based on the D-axis saturation level and ignoring Q-axis saturation. Therefore, to correctly model the saturation level, it is advisable either to provide reliable both D-axis and Q-axis saturation curves separately or to evaluate total airgap flux linkage.

The state space method was implemented in [121-123] to include the saturation model, with the result showing that the saturation accuracy may also depend on the model itself. In [123], the mixed current-flux state-space model was used to model the saturation on a self-excited induction machine, where the main flux saturation was discussed in the work and the leakage inductances were assumed to be constant. The author showed that three methods of state-space models, i.e. based on stator and rotor currents, stator current combined with rotor flux linkage and rotor current with stator flux linkage had the identical result. The models generally had a good match against experiments, with around 20% errors and even higher in greater saturation region. Different state space models for the same synchronous generator were compared in [121], where these models were using different combinations of stator/field flux/current as state space variables. The author implemented a single saturation factor method to model the saturation effect in both transient and steady state. The simulation showed the same results among different models. Based on the work in [121], [122] omitted dynamic
cross-saturation model from those different models in [121]. The simulation showed that some models had worse result against the fully saturated model, and some \((i_s, i_f, \psi_m)\) indicated correct behaviour against fully saturated model, and these models become significantly simpler than the corresponding fully saturated model.

In the presence of saliency, the cross-coupling between the D and Q-axis may have a non-negligible effect on saturation. The cross-coupling saturation effect in the synchronous machine was experimentally validated in [124], the author pointed out the effect of cross-magnetization tends to reduce the magnetizing flux level for both D-axis and Q-axis, so using a single saturation factor based on total magnetizing flux saturation gave a better match against experiment than using separate D-axis and Q-axis saturation factors, as the cross-coupling effect was considered.

To include the cross-coupling in analytical methods, it is common to extract these properties from the predefined FE model, then use a set of look-up tables accounting for saturation and cross-coupling are included in the analytical calculation. As in [125], the cross-coupling properties for a PM synchronous machine were extracted from FE and made as look-up tables for the idealized model of a PM synchronous machine in the Simulink environment. Chapter 6 will show the saturation models used in this thesis.

2.4.4. Diode Rectifier Modelling

The diode rectifier or controlled thyristor combined with a generator are commonly presented in the literature, but the modelling method for diode rectifier systems are often not documented in detail [12, 46].

There are several methods of including a diode rectifier model in the machine modelling system; using a connection matrix model, an average value model or a circuit component modelling package from a simulation toolbox.

A common analytical method covered in literature when modelling a diode rectifier is called the average value method. This method was used in [126] to derive the equivalent DC side series resistance and average DC voltage, the fundamental AC currents returned from the load was also represented. This method represents the diode rectifier low frequency behaviour well, while the potential higher order harmonics, such as time harmonics due to commutation are ignored.

When it came to model the
thyristors, the average value method was commonly implemented. The average value method is favourable to model active switches, for the purpose of implementing control strategies [59]. The research [127] used an average value modelling of a diode rectifier system with a dual 3-phase synchronous machine, based on a single diode switching interval. The average value modelling was also covered in [128], in which the description of diode rectifier is limited to a period of $\pi/3$. The diode commutation has been considered. Time stepping with finite element analysis for solving the diode rectifier performance was introduced in many papers. A hybrid method combining a circuit model and finite element analysis was introduced in [129]. However, the saturation effect of the model depended on single winding excitation, which was inadequate. The diode rectifier model was still using an average value method.

Average value modelling is essentially based on calculating the lumped parameters of the equivalent circuit. Most of the methods applied assume ideal voltage waveforms and require fixed network impedances, which do not respond to load changes. For changing loads, the lumped parameters need to be recalculated. For voltage supply harmonics or when there is large leakage current ripple, the average value model is no longer suitable to predict system behaviour [130]. Meanwhile, this method averages the system behaviour over a period, which preserves slow dynamics but not the transient characteristics.

The connection matrix method represents the relationship between circuit loops through diode rectifier components, with 1 for the elements when the circuit loop relates to diode with same direction, -1 for reversed direction and 0 for the elements when there is no relationship [47]. The diodes are modelled as a small resistive load with voltage drop. The connection matrix method was implemented in [131] by combining AC and DC sides for the analysis of a DC load short-circuit fault. The diode voltage-drop and on resistance were not included, whereas the voltage drop was considered in [132] when implementing the connection matrix.

Although the connection matrix method is mathematically correct and provides more customised and flexible change for the circuit topology, this method needs careful inspection on the association matrix during coding, moreover, whenever faults occur or the winding loop changes, the matrix needs to be updated and reformed.
There is also other analytical method documented in [133, 134], where four operation modes of diode rectifiers were analysed providing a quick evaluation for the 3-phase PMSG connected to a constant DC voltage source. However, this method only considered the fundamental sinusoidal distributed back-emf, for the GRS related in this project, the back-emf is highly distorted, especially in star connected windings. Therefore, the software package analysis is implemented rather than the mathematical methods stated above.

There are other methods of modelling diode rectifier systems recorded in literature. These methods are with the help of software packages. [8, 135] modelled a multiphase synchronous generator based on PSCAD/EMTDC. In [8], the diode rectifier systems were simplified as inductances with series resistance, the LCR loads were modelled as a voltage source in series with resistors. The user defined model was validated against with standard synchronous generator model in PSCAD/EMTDC, showing a good match for a 3-phase generator system. Other software packages such as SimPowerSystem [136] and PLECS [130] for modelling diode rectifier system are also covered in the literature.

The intrinsic harmonics from the distributed stator windings and salient rotor structure make the average value method not suitable to predict system behaviour. Although the connection matrix considers harmonic effects and gives accurate results, the different fault behaviours of the system need to be examined. Updating the connection matrix for each case would be tedious work. The PLECS circuit package is selected in this project for the modelling technique for the diode rectifier, as the package is easy to handle and the PLECS circuit gives opportunity to freely add/remove faulty conditions on both generator winding side and diode rectifier side. Meanwhile, PLECS provides a feature to vectorise most of the components, with one symbol interpreted as a set of components, which could describe big circuit system compactly with a small schematic [137]. This feature makes the multiphase diode rectifier system easy to configure and manage possible changes as faults. Diode modelling is covered in Chapter 5.

2.5. Summary

Moving towards MEA or AEA has been the trend for the development from conventional aircraft. The DC electrical network for aircraft has been growing in interest compared with the conventional electrical network of 115VAC system, as the
higher level voltage DC helps to reduce the cable size and further reduce the total weight and fuel consumption. The WFSG technology is favoured in the aircraft systems because of the ability to remove field excitation during faults unlike the PMG.

The multiphase machines have been reviewed for replacing conventional three-phase machines in different areas as they have better fault tolerance, performance stability, and reduced DC ripple if connected through rectifiers. Although multipulse TRUs provides the same feature of reduced DC ripple as multiphase machines, the size and weight and complex connections from multiple transformers becomes less attractive.

The CHA method provides a way of modelling multiphase machines similar to the WFT. The ability of accounting for space harmonics means the CHA can be used to focus on the effects of geometry harmonics at the design stage. Although FEA would give better accuracy than analytical methods, the time dynamic solution is difficult for FEA. As well as the CHA, WFT and FEA modelling techniques for the machine, the saturation and diode rectifier modelling techniques are also covered in this chapter.

The rest of the thesis addresses the effects on the generator behaviour from winding design and diode rectifier topologies by using the CHA method, since the generator has no active controlled device, therefore, the generator utilisation and fault tolerance are entirely determined by how the winding is designed and the diode rectifier circuit layout.

The thesis first develops the CHA method for the WFSG and provides some improvements to the CHA. Then, the simulated generator behaviour from the CHA is compared again the experimental measurement to identify the limit of the CHA. Finally, the GRS is further modelled with different winding designs and rectifier topologies with the CHA to investigate the GRS performance and fault tolerance.
Chapter 3. Analytical Method

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3.1. Introduction

In order to analyse the geometry harmonic effects on WFSG as well as the generator performance under different winding layout and fault conditions, the WFSG should be appropriately presented in the MATLAB/Simulink environment. The analytic expressions for coupling inductances allow us to identify significant harmonics, and relate them to the machine geometry. This can be adjusted at the design stage to cancel or attenuate specific terms. The same expressions can be used to create a look-up-table (LUT) of inductances as a function of rotor angle in the circuit model of the generator rectifier system. The number of harmonics used and the angular resolution of the LUT are determined from CHA.

In this section, the CHA models for the machine winding distribution, airgap distance distribution, magnetic flux density distribution, and coupling inductance will be covered. The general idea of the CHA model presented in this section is based on the method developed [138-142] previously but the full expression for the airgap geometry is newly introduced and this part has been published in [143]. More details of the CHA models stated above can be found in A.1.Analytical Derivations.
Figure 3.1 shows the actual machine to be modelled with the CHA method and its full machine geometry data is presented in A.2. Generator Geometry. The machine is built with 30 stator slots and each slot has 3.8mm opening width, the stator slots are designed to be suitable for double layer windings with 7 turns per layer; the rotor has two salient poles with two sets of field windings, the detailed rotor geometry is also presented in Figure 3.5 in section 3.2.2. when modelling rotor field windings; four damper bars are embedded on each side of rotor arc.

![Machine geometry diagram](image)

Figure 3.1 Machine geometry diagram (cross and dot filling for field windings represent different current directions).

### 3.2. Conductor Density Distribution

This section works out the stator, rotor and damper winding conductor density distribution function as the Fourier expansion based on the CHA method. Instead of using a trigonometrical form to simply express the conductor density distribution as the fundamental sinusoidal waveform, which is commonly considered for WFT, the Fourier expansion for conductor density function considers all possible space harmonics, where choosing appropriate harmonic orders to represent the conductor density function allows for compromise between computational efficient and accuracy.
The conductor density distribution modelled stator, rotor and damper windings as magnetically coupled circuits through the magnetic flux linkage in the airgap. The coupling impedance, i.e. the mutual inductance between different windings and self-inductance for the winding itself are the essential parts to successfully implement the generator model into the MATLAB/Simulink environment.

### 3.2.1. Stator Windings

#### a. Single coil conductor density distribution

To analyse the stator phase winding conductor density distribution by using CHA method, the single coil geometry is set as the starting point. For an arbitrary coil of $N_c$ turns, viewed from the stator side, the coil distribution is shown in Figure 3.2 and equation (3-1), where the stator angle $\theta_s$, coil centre $\theta_0$, span $\alpha$ and slot opening $\beta$ are defined as shown in the Figure 3.2. Notice that CHA uses the same spatial distribution as WFT, so would be expected to give the same results.

![Figure 3.2 Single coil density distribution function](image)

The coil function in (3-1) can be expressed as the Fourier series expansion [142]

$$c(\theta_s) = \sum_{v=-\infty}^{\infty} \mathcal{C}_v e^{-jv\theta_s}$$  \hspace{1cm} (3-2)

where $\mathcal{C}_v = \frac{1}{2\pi} \int_{0}^{2\pi} c(\theta_s) e^{jv\theta_s} d\theta_s$

Equation (3-3) can be obtained by combining (3-1) and (3-2)
\[ C^\nu = -j \frac{N_c}{\pi} K_p(\nu) K_b(\nu) e^{j\nu \theta_0} \]  

(3-3)

where pitch factor and slot mouth (or same as slot opening) factor are defined as in (3-4)

\[ K_p(\nu) = \sin\left(\frac{\nu \alpha}{2}\right), \quad K_b(\nu) = \frac{\sin\left(\frac{\nu \beta}{2}\right)}{\nu \beta} \]  

(3-4)

Therefore, for a single coil, the conductor density distribution will be shown in (3-5), containing winding geometry information, e.g. winding pitch factor and winding slot mouth factor.

\[ c(\theta_s) = -j \frac{N_c}{\pi} \sum_{\nu=-\infty}^{\infty} K_p(\nu) K_b(\nu) e^{-j\nu(\theta_s - \theta_0)} \]  

(3-5)

b. Single phase group conductor density distribution

If there are \( N_s \) stator slots in total for the machine with \( m \) phases and \( \rho \) pole pairs, then the number of slots in a single phase group, where assuming the coils for a phase group are in adjacent slots, \( N_g \) is given by

\[ N_g = \frac{N_s}{2m\rho} \]  

(3-6)

The conductor density coefficient for a single \( i^{th} \) coil in this phase group can be written as

\[ \tilde{C}^{\nu,i} = -j \frac{N_c}{\pi} K_p(\nu) K_b(\nu) e^{j\nu(\theta_{o^+} + \frac{(i-1)2\pi}{N_s})}, i = 1, 2, ..., N_g \]  

(3-7)

By summing \( N_g \) slots in a phase group, the conductor density distribution coefficient of a single phase group is shown as

\[ \tilde{C}^{\nu} = \sum_{i=1}^{N_g} \tilde{C}^{\nu,i} = -j \frac{N_{p^h}}{\pi \rho} K_p(\nu) K_b(\nu) K_d(\nu) e^{j\nu \left[ \theta_{o^+} + \frac{(N_g-1)\pi}{N_s} \right]} \]  

(3-8)

where the phase winding distribution factor is defined as
\( N_{ph} = N_c N_g \rho \) and \( K_d(\nu) = \frac{\sin(\frac{\nu \theta_m}{N_s})}{\frac{\nu \theta_m}{N_s} - \sin(\frac{\nu \theta_m}{N_v})} \)

distribution factor

\[ (3-9) \]

c. Complete winding density distribution

The machine analysed in this project contains two layers of stator windings and two salient poles. A coefficient for the double layer, \( 1 - e^{j\frac{\nu \pi}{\rho}} \) can be added to (3-8) to expanding stator phase winding from single layer to double layers, where the details are presented in A.1. Analytical Derivations. Generally, the machines may have more than one pole pairs to form a single phase winding. Therefore, the factor \( \sum_{i=0}^{\rho-1} \left[ e^{j\frac{2\pi i}{\rho}} \right] \) is needed to take the multiple pole pair into account. Accordingly, the conductor density distribution for a single phase group is presented as in (3-10).

\[
\tilde{C}^\nu = -\frac{j N_{ph}}{\pi \rho} K_p(\nu) K_b(\nu) K_d(\nu) \left( 1 - e^{j\frac{\pi}{\rho}} \right) e^{j\frac{\nu}{\rho} \left[ \theta_o + \frac{(N_g - 1)\pi}{N_s} \right]} \sum_{i=0}^{\rho-1} \left[ e^{j\frac{2\pi i}{\rho}} \right] \]

For this specific single pole pair machine, the coefficient \( \sum_{i=0}^{\rho-1} \left[ e^{j\frac{2\pi i}{\rho}} \right] \) essentially equals 1. Notice that there is a condition for (3-10) to be non-zero, where \( \frac{\nu}{\rho} \) must be an odd integer, and this assumption makes (3-10) only suitable for integer rather than fractional pitch windings. In this case, (3-10) simplifies to:

\[
\tilde{C}^\nu = -\frac{j N_{ph}}{\pi} K_p(\nu) K_b(\nu) K_d(\nu) K_e(\nu) e^{j\frac{\nu}{\rho} \left[ \theta_o + \frac{(N_g - 1)\pi}{N_s} \right]} \]

where the coefficient \( K_e(\nu) = 1 - e^{j\frac{\nu \pi}{\rho}} \) is for a double layer winding distribution and \( K_e(\nu) = 1 \) when it is single layer layout.

For machines with more than one pole pair, it is usual to refer to electrical angle, \( \theta_e = \rho \theta_m \), and refer to the electrical harmonics \( \nu' = \frac{\nu}{\rho} \). However, (3-10) assumes symmetry in the current density distributions between poles within one phase, which is not necessarily valid for internal winding faults, so although (3-11) is used to check...
calculations, the code in Chapter 5 sums contributions using the per coil expression in (3-5).

Notice that the coefficient displayed in (3-11) has the following properties:

\( \tilde{C}^{-\nu} = \tilde{C}^{\nu^*}, \quad \tilde{C}^0 = 0 \)

Therefore, the single phase conductor density distribution can be expressed as

\[
c(\theta_s) = \sum_{\nu=-\infty}^{\infty} \tilde{C}^{\nu} e^{-j\nu\theta_s} = 2 \sum_{\nu=1}^{\infty} \Re\{\tilde{C}^{\nu} e^{-j\nu\theta_s}\}
\]  

(3-12)

Generally, for any arbitrary series-connected stator phase winding centred at \( \theta_n \), the winding density distribution can be expressed as

\[
c_{sn}(\theta_s) = \sum_{\nu=-\infty}^{\infty} \tilde{C}_{sn}^{\nu} e^{-j\nu\theta_s}
\]

where \( \tilde{C}_{sn}^{\nu} = -\frac{jN}{\pi} K_w(\nu) e^{j\nu\theta_n} \)  

(3-13)

and \( K_w(\nu) = K_p(\nu) K_b(\nu) K_d(\nu) K_e(\nu) e^{j\nu\left[\frac{(N_g-1)\pi}{N_s}\right]} \)

By assigning the corresponding phase number, slot number, slot pitch and pole pairs, the stator winding conductor density (turns per radian) along with its harmonic distribution of different machine topologies can be obtained. Figure 3.3 shows the stator winding distribution of 3-phase, 5-phase and 15-phase winding with fully-pitched design as examples, while its corresponding conductor density distribution is presented in Figure 3.4.

Figure 3.3 Stator winding distribution for 3-phase, 5-phase and 15-phase in fully-pitched layout (different phases been coloured differently)
The stator conductor harmonics chosen to include in Figure 3.4 are set as up to 10000, which provide precise distribution for the conductor density. In the real practice, the conductor density distribution harmonic is set to 100 due to the presence of stator skew factor that removes slotting harmonics, which will be covered in Chapter 4. Choosing 100 order harmonic as the upper limit of conductor density is only 96.3% less when choosing that as 10000. Although increasing the number of harmonics will give better representation of conductor density function, the computation time is also significantly increased.

\[ d. \text{ Harmonic cancellation within coils} \]

For the 3-phase winding, the triplen harmonics can be cancelled when windings are designed as short-pitched; a similar case could be found in 5-phase and 15-phase windings, where multiples of 5\(^{th}\) orders are eliminated in the chosen short-pitched design. This phenomenon of harmonic cancellation can be explained by applying the CHA method. In equation (3-13), \( \alpha \) in \( K_p(\nu) \) is related to winding pitch span in radians, as

\[ \alpha = \frac{2\pi}{N_s} * \frac{n_s}{2\rho} \] (where \( n_s \) is the number of slots for a single phase) \[142, 144]. Therefore, the condition for a particular harmonic to be cancelled in a short-pitched design is

\[ K_p(\nu) = 0, \text{ hence } \frac{\nu \alpha}{2} = \pi n, \] in this case

\[ \nu = \frac{n N_s \rho}{n_s}, \text{ where } n = 0, \pm 1, \pm 2 \ldots \] (3-14)

It can be concluded from equation (3-14) that particular orders of harmonics can be eliminated by proper choice of pitch factor. For example, for a 5-phase machine in a 30 stator slots, where a short-pitched design of short pitching by \( \frac{\pi}{5} \), i.e. using a span of 12 slots for a single phase, for proper choices of integer values of \( n, \nu \) is a multiple of 5.
from (3-14), therefore, multiples of 5\(^{th}\) harmonics are eliminated for this \(\frac{\pi}{5}\) short-pitched winding design.

3.2.2. Rotor Windings

The rotor field windings can be considered as two sets of concentrated windings on the rotor side, whose conductor density distribution function is shown in Figure 3.5. The rotor contains two series connected field windings with \(N_r\) turns for each part; the field coil has an effective width of \(\tau\) in radians and distributed symmetrically; two adjacent field coils are spaced \(d\) in radians. The field coil distribution in Figure 3.5.(b) is with respect the rotor angle \(\theta_r\).

Equation (3-15) shows the rotor field winding distribution in piecewise function. To conduct the Fourier transform in an easier way, the rotor coils are divided into two segments, \(c_1\) and \(c_2\).

![Rotor coils distribution and field winding distribution function](image)

Figure 3.5 Rotor coils distribution (a) and field winding distribution function (b) (Cross and dots fill in (a) represents different current directions).

\[
c_1(\theta_r) = \begin{cases} \frac{N_r}{\tau} & (\theta_1 - \frac{\tau}{2} < \theta_r < \theta_1 + \frac{\tau}{2}) \\ -\frac{N_r}{\tau} & (2\pi - \theta_1 - \frac{\tau}{2} < \theta_r < 2\pi - \theta_1 + \frac{\tau}{2}) \\ 0 & \text{elsewhere} \end{cases}
\]

\[
c_2(\theta_r) = \begin{cases} \frac{N_r}{\tau} & (\theta_2 - \frac{\tau}{2} < \theta_r < \theta_2 + \frac{\tau}{2}) \\ -\frac{N_r}{\tau} & (2\pi - \theta_2 - \frac{\tau}{2} < \theta_r < 2\pi - \theta_2 + \frac{\tau}{2}) \\ 0 & \text{elsewhere} \end{cases}
\]

(3-15)

Notice that \(\theta_2 = \theta_1 + d + \tau\).
By applying Fourier transforms on \( c_1(\theta) \) and \( c_2(\theta) \) respectively, the combined rotor field winding conductor density distribution can be expressed as

\[
c_r(\theta_r) = \sum_{\nu=-\infty}^{\infty} \tilde{C}_r^\nu e^{-j\nu\theta_r} = 2 \sum_{\nu=1}^{\infty} \Re\{\tilde{C}_r^\nu e^{-j\nu\theta_r}\}
\]

where \( \tilde{C}_r^\nu = -\frac{N_r}{j2\pi} \sin(\frac{\nu\theta_1}{2}) - \frac{N_r}{j2\pi} \sin(\frac{\nu\theta_2}{2}) \)

\[
= -\frac{2N_r}{j2\pi} \sin(\frac{\nu\theta_2}{2}) \sin(\frac{\nu(\theta_1 + \theta_2)}{2}) \cos\left(\frac{\nu(\theta_1 - \theta_2)}{2}\right)
\]  

(3-16)

For the rotor winding, a similar conductor density distribution to the stator one can be obtained, by defining \( K_{pr}(\nu) = 2 \sin\left(\frac{\nu(\theta_1 + \theta_2)}{2}\right) \cos\left(\frac{\nu(\theta_1 - \theta_2)}{2}\right) \) and \( K_{br}(\nu) = \frac{\sin(\frac{\nu\theta_2}{2})}{\frac{\nu\theta_2}{2}} \).

where \( \tilde{C}_r^\nu \) in (3-16) becomes \( \tilde{C}_r^\nu = -\frac{N_r}{j2\pi} K_{pr}(\nu) K_{br}(\nu) \).

In terms of stator reference frame, where \( \theta_r = \theta_s - \rho \theta_m \) shown in Figure 3.6, the rotor field winding conductor distribution can also be written as

\[
c_r(\theta_s) = \sum_{\nu=-\infty}^{\infty} \tilde{C}_r^\nu e^{-j\nu(\theta_s - \rho\theta_m)} = 2 \sum_{\nu=1}^{\infty} \Re\{\tilde{C}_r^\nu e^{-j\nu(\theta_s - \rho\theta_m)}\}
\]  

(3-17)

where \( \tilde{C}_r^\nu \) is the same as in equation (3-16)

![Figure 3.6 Diagram of stator reference frame (\( \theta_s \)) and rotor reference frame (\( \theta_r \)).](image)

3.2.3. Damper Windings

The damper winding modelling for salient WFSG has already covered in [116, 117] but with WFT method and FEA. The damper modelling in section is developed
independently against [116-118] with different methodology. In addition, this research is targeted on multiphase machine and [116-118] are focused on 3-phase machine, where the damper windings have different influence on 3-phase and multiphase machines and has been validated in [145].

In this section, the damper winding is rebuilt with CHA method. As shown in Figure 3.1, the four damper bars are located on both rotor pole pieces. The damper windings are modelled as single turn windings across the rotor pole faces as there are copper shorting plates connecting all the damper bars are at the two ends of the rotor [114]. Therefore, the damper circuit is modelled as eight flux loops with circulating currents, shown in Figure 3.7 [118, 146]. Each of these loops contains two bar conductors and two end connections. By applying Kirchhoff’s Voltage Law to these damper loops, as shown in (3-18), where subscripts \(n, n + 1\), and \(n - 1\) denote for three adjacent damper loops.

\[
0 = -R_di_{n-1} + 2(R_b + R_e(n))i_n - R_di_{n+1} - L_b \frac{di_{n-1}}{dt} + 2(L_b + L_e(n)) \frac{di_n}{dt} - L_b \frac{di_{n+1}}{dt}
\]

\[
+ \frac{d\Psi_d}{dt} + \frac{d\Psi}{dt}
\]

The “part I” refers the voltage drop due to damper resistances; the “part II” corresponds to the voltage drop on damper bar and end connection inductances (referred to as leakage inductances); the “part III” is the induced voltage from other damper loops \(\frac{d\Psi_d}{dt}\) and stator/rotor windings \(\frac{d\Psi}{dt}\). The damper end connections are simplified as a combination of effective resistances and stray inductances.

![Figure 3.7 Damper circuit model](image_url)

The damper resistance matrix \(R_d\), is given by
The damper leakage inductance matrix \( L_{lkd} \), is given by

\[
\begin{bmatrix}
2(L_b + L_e(1)) & -L_b & 0 & \cdots & -L_b \\
-L_b & 2(L_b + L_e(2)) & -L_b & \cdots & 0 \\
0 & -L_b & 2(L_b + L_e(3)) & -L_b & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-L_b & 0 & 0 & -L_b & 2(L_b + L_e(8)) \\
\end{bmatrix}_{8 \times 8}
\]

(3-20)

The subscripts \( b \) and \( e \) denote damper bar and end connections respectively, as shown in Figure 3.7.

To study the induced voltage term from other damper loops \( \frac{d\Psi_d}{dt} \), the damper loops are considered as sets of two conductor windings, shown in Figure 3.8, where two arbitrary damper bars form into a single damper loop effectively with positive and negative winding distributions.

Similar to the stator and rotor winding conductor density distribution, the first damper winding conductor density distribution is expressed as (3-21), shown in Figure 3.8.

\[
c_1(\theta_r) = \begin{cases} 
\frac{N_d}{w} & \left( \theta_1 - \frac{w}{2} < \theta_r < \theta_1 + \frac{w}{2} \right) \\
-\frac{N_d}{w} & \left( \theta_2 - \frac{w}{2} < \theta_r < \theta_2 + \frac{w}{2} \right) \\
0 & \text{elsewhere} 
\end{cases}
\]

(3-21)

Figure 3.8 Conductor density distribution for a single damper loop from compound damper windings.
Hence, the conductor density distribution coefficient is

\[ \tilde{C}_1 \nu = \frac{N_d}{2\pi} \sin \left( \frac{\nu W}{2} \right) \left( e^{j\nu \theta_1} - e^{j\nu \theta_2} \right) = j \frac{N_d}{\pi} \sin \left( \frac{\nu W}{2} \right) \sin \left( \frac{\nu W}{2} \right) e^{j\nu \frac{\theta_1 + \theta_2}{2}} \]  (3-22)

Similar to the rotor conductor density distribution, we can define damper pitch factor

\[ K_{pd} (\nu) = \sin \left( \nu \frac{\theta_1 - \theta_2}{2} \right) \]

and damper distribution factor \( K_{dd} (\nu) = \frac{\sin \left( \frac{\nu W}{2} \right)}{\frac{\nu W}{2}} \), so the damper coefficient becomes \( \tilde{C}_1 \nu = j \frac{N_d}{\pi} K_{pd} (\nu) K_{dd} (\nu) e^{j\nu \frac{\theta_1 + \theta_2}{2}} \), which is in the same form as stator and rotor conductor density distribution coefficients.

Therefore, for any two arbitrary damper bars with damper endings that form into a single damper loop, where the bars are located at \( \theta_1 \) and \( \theta_2 \) with opposite current polarities respectively, the corresponding damper winding density distribution is expressed as (3-23) in stator reference frame.

\[ c_d(\theta_s) = \sum_{\nu=0}^{\infty} \tilde{C}_d \nu e^{-j\nu(\theta_s - \rho \theta_m)} \]  (3-23)

where \( \tilde{C}_d \nu = j \frac{N_d}{\pi} K_{pd} (\nu) K_{dd} (\nu) e^{j\nu \frac{\theta_1 + \theta_2}{2}} \).

### 3.3. Airgap Model

The conductor density distribution from the previous section provides the way to compute the current density along specific windings. To calculate the magnetic flux density from the winding current density, the airgap permeance is needed. The airgap distance is expressed in a Fourier expansion by using the CHA method, similar to the expression for the conductor density distribution. To simplify the analytical method, the rotor spreader has been simplified, the rotor is assumed to be of circular section and stator slots are ignored. Additionally, all flux lines are assumed to be travel in radial direction through the rotor core. The simplified model is shown in Figure 3.9.
The reciprocal of airgap distance can be expressed as equation (3-24), based on the geometry shown in Figure 3.9, where angles of \( \theta_1 \) and \( \theta_2 \) are based on rotor equivalent measurements.

\[
\frac{1}{g(\theta_r)} = \begin{cases} 
\frac{1}{g_1} & (0 < \theta_r < \theta_1) \cup (\pi - \theta_1 < \theta_r < \pi) \\
\frac{1}{g_2} & (\theta_1 < \theta_r < \theta_2) \cup (\pi - \theta_2 < \theta_r < \pi - \theta_1) \\
\frac{1}{g_3} & (\theta_2 < \theta_r < \pi - \theta_2)
\end{cases}
\]  

(3-24)

Transfer equation (3-24) into Fourier series form:

\[
\frac{1}{g(\theta_r)} = \sum_{l=-\infty}^{\infty} \bar{G}^l e^{-j l \theta_r}
\]

where \( \bar{G}^l = \frac{2}{\pi l} \left( \frac{1}{g_1} - \frac{1}{g_2} \right) \sin (l \theta_1) + \left( \frac{1}{g_2} - \frac{1}{g_3} \right) \sin (l \theta_2) \), \( l = \pm 2, \pm 4, \pm 6 \ldots \)  

(3-25)

\( \bar{G}^l = 0 \) for \( l = \pm 1, \pm 3, \pm 5 \ldots \) and \( \bar{G}^0 = \frac{2}{\pi} \left( \frac{\theta_1}{g_1} + \frac{\theta_2 - \theta_1}{g_2} + \frac{\pi - \theta_2}{g_3} \right) \)

Since \( \bar{G}^{-l} = \bar{G}^l \), equation (3-25) could also be expressed as

\[
\frac{1}{g(\theta_r)} = \bar{G}^0 + 2 \Re \left( \sum_{l=1}^{\infty} \bar{G}^l e^{-j l \theta_r} \right)
\]  

(3-26)

or in the stator reference frame
\[
\frac{1}{g(\theta_s)} = G_0 + 2R \left\{ \sum_{l=1}^{\infty} G_l e^{-jl(\theta_s-\rho \theta_m)} \right\} \quad \text{(3-27)}
\]

### 3.4. Magnetic Flux Density

In this section, the magnetic flux density along the airgap contour from both stator windings and rotor windings are computed. The magnetic flux density is the intermediate quantity to compute the winding coupled impedance from winding conductor density distribution and airgap distribution. By applying Amperes’ Law, the magnetic flux density combines the winding conductor density distribution and the airgap distribution.

#### 3.4.1. Stator Winding Energised

Considering put an arbitrary time-varying \( q^{\text{th}} \) harmonic current \( i_{s_n}^q(t) \) into \( n^{\text{th}} \) stator winding, the current density distribution of this \( n^{\text{th}} \) stator winding is

\[
\mathbf{j}_{s_n}^q(t) = i_{s_n}^q(t) \mathbf{c}_{s_n}(\theta_s) = i_{s_n}^q(t) \sum_{\nu=-\infty}^{\infty} \tilde{c}_{s_n}^\nu e^{-j\nu \theta_s} \quad \text{(3-28)}
\]

There are several assumptions need to be made before using the current density distribution to compute the magnetic flux density from that winding [144]:

1. The windings are infinitely thin so that the current density does not vary axially.
2. The flux always travels the airgap in radial direction (therefore \( \frac{\partial H}{\partial \theta} = \frac{dH}{d\theta} \)).
3. The permeability of iron is infinity, so that the iron path is dominated by the airgap path.
4. The stator slot is ignored when the flux travels in the airgap.

By applying airgap harmonic expression (3-25), Amperes Law (3-29) and a Taylor expansion for the magnetic field strength \( H \), approximated for small angle \( \Delta \theta \) (3-30), and neglecting the iron path and second order effects, the magnetic flux density \( B_{gs}(\theta_s) \) produced from stator current can be expressed as (3-31) [144], where subscript \( g \) means the magnetic flux density in the airgap and \( s \) denotes for magnetic flux density from stator windings.
\[ \oint H \, dl = \sum l \quad (3-29) \]

\[ H(\theta + \Delta \theta) = H(\theta) + \frac{\partial H}{\partial \theta} \Delta \theta \quad (3-30) \]

\[ B_{gs}(\theta_s) = -\mu_0 i_{sn}^q(t) \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \tilde{c}_{sn}^\nu G_l e^{-j(\nu+1)\theta_s} \frac{j\nu}{j\nu} e^{j\nu \rho m} \quad (3-31) \]

where \( \mu_0 \) is the permeability of air.

A detailed solution for (3-29) - (3-31) can be found in A.1. Analytical Derivations.

From equation (3-31), it is clear that the harmonics of magnetic flux density depends not only on the stator winding harmonics not also the airgap harmonics.

Equation (3-31) can be expanded as follows by combining (3-12) and (3-27)

\[ B_{gs}(\theta_s) = -\mu_0 i_{sn}^q(t) 4\Re \left\{ \sum_{l=1}^{\infty} \sum_{\nu=1}^{\infty} \tilde{c}_{sn}^\nu G_l e^{-j(\nu+1)\theta_s} \frac{j\nu}{j\nu} e^{j\nu \rho m} \right\} \quad (3-32) \]

\[ -\mu_0 i_{sn}^q(t) 2\Re \left\{ \sum_{\nu=1}^{\infty} \tilde{c}_{sn}^\nu e^{-j\nu \theta_s} \frac{j\nu}{j\nu} \right\} \quad \text{part II} \]

Therefore, it can be concluded that the magnetic flux density produced by energising the stator windings consists of two components: the magnetic flux density from the uniformly distributed airgap ("part II" in (3-32)) and the magnetic flux density from airgap harmonics due to the rotor saliency ("part I" in (3-32)).

### 3.4.2. Rotor Winding Energised

In the same way, the magnetic flux density from the rotor excitation can be derived based on equation (3-17), (3-30) and (3-31), where subscript \( r \) in (3-33) denotes magnetic flux density from rotor windings.
\[
B_{gr}(\theta_r) = -\mu_0 i_r^q(t) \sum_{l=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \frac{\bar{C}^v \bar{G}^l e^{-j(v+l)\theta_r}}{jv}
\]

or in terms of stator reference frame as

\[
B_{gr}(\theta_s) = -\mu_0 i_r^q(t) \sum_{l=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \frac{\bar{C}^v \bar{G}^l e^{-j(v+l)(\theta_s-\rho \theta_m)}}{jv}
\]

Similarly to the magnetic flux density with the stator winding energised, (3-34) can be split into components associated with the mean airgap (“part II” in (3-35)) and with harmonic due to airgap saliency (“part I” in (3-35)) parts, as

\[
B_{gr}(\theta_s) = -\mu_0 i_r^q(t) 4\Re \left\{ \sum_{l=1}^{\infty} \sum_{v=1}^{\infty} \frac{\bar{C}^v_{sn} \bar{G}^l e^{-j(v+l)(\theta_s-\rho \theta_m)}}{jv} \right\} \\
-\mu_0 i_r^q(t) 2\Re \left\{ \sum_{v=1}^{\infty} \frac{\bar{C}^v_{sn} \bar{G}^0 e^{-jv(\theta_s-\rho \theta_m)}}{jv} \right\}
\]

### 3.5. Flux Linkage and Inductance

The flux linkage between arbitrary two windings is computed in this section. The flux linkage investigates the mutual coupling between two arbitrary windings, as well as the magnetic reluctance between two magnetic circuits. The flux linkage is computed by integrating the magnetic flux density from the excitation winding along the area of the induced winding.

#### 3.5.1. Flux Linkage and Inductance in Stator Windings

The flux linkage coupling from the magnetic flux density produced from a single-phase stator winding, \(B_{gs}(\theta_s)\) in (3-31), into another stator phase winding can be deduced by integrating the magnetic flux density along the area of the second phase winding, shown as (3-36)-(3-38), where \(\phi\) is the flux linked from one phase winding to another, \(A\) is the area of the second phase winding and effectively \(dA = RWd\theta\), where \(R\) is the stator inner radius and \(W\) is the machine stack length.
In general:

\[ \phi = \int B_{gs} dA = \int BRW d\theta = RW \int Bd\theta \quad (3-36) \]

\[ \Psi = N\phi \quad (3-37) \]

The flux linkage in the stator winding \( s_2 \) induced from stator winding \( s_1 \) can be expressed as in (3-38) from (3-36) to (3-37), where the detailed calculation process is attached in the A.1. Analytical Derivations.

\[ \psi_{s_2s_1} = 2\pi \mu_0 i_{sn}^q (t) RW \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^{v} C_{sn2}^{l} G_{l}}{v(v + l)} e^{jl\rho_m} \quad (3-38) \]

and the coupled inductance could be computed as (3-38) divided by the excitation current \( i_{sn}^q (t) \).

\[ L_{s_2s_1} = \frac{\psi_{s_2s_1}}{i_{sn}^q (t)} = 2\pi \mu_0 RW \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^{v} C_{sn2}^{l} G_{l}}{v(v + l)} e^{jl\rho_m} \quad (3-39) \]

with the condition of \( v + \mu + l = 0 \) for (3-38) and (3-39).

Taking the self-inductance as the example and similar as the magnetic density flux in (3-32) and (3-35), the DC component and the harmonic order of self-inductance is shown in (3-40)

\[ L_{s_2s_1} = 2\pi \mu_0 RW \left\{ 2\Re \left\{ \sum_{l=1}^{\infty} \sum_{\nu=1}^{\infty} \frac{C_{sn1}^{v} C_{sn2}^{-(\nu + l)} G_{l}}{v(v + l)} e^{jl\rho_m} \right\} \right\} + 2\Re \left\{ \sum_{l=1}^{\infty} \sum_{\nu=1}^{\infty} \frac{C_{sn1}^{v} C_{sn2}^{-(\nu - l)} G_{l}}{v(v - l)} e^{-jl\rho_m} \right\} \quad (3-40) \]

where the “part I” is the component due to winding and airgap harmonics and “part II” is due to winding harmonics only.

By only considering fundamental stator winding component and up to the second order harmonic in airgap [7], the self-inductance can be expressed from (3-40) as
\[ L_{s_2s_1} = L_2 \cos(2p\theta_m - 2\theta_{s_1} - \gamma) + L_0 \cos(\gamma) \]  

(3-41)

where \( \theta_{s_1} \) is the angular position of the first winding \( s_1 \) that produces flux, \( \gamma \) is the phase difference between two windings \( s_1 \) and \( s_2 \). Both \( L_0 \) and \( L_2 \) represent the simplified coefficients in equation (3-40). More specifically, \( L_0 \) term is related to the fundamental harmonic in the reciprocal of airgap distance and \( L_2 \) term is related to the second harmonic in airgap distance. In this case, the winding mutual inductance is in the same form as stated in the [14, 147], which is commonly referred as the mutual stator inductance for the WFSGs.

### 3.5.2. Flux Linkage and Inductance between Stator and Rotor Windings

Similarly to the analysis process for calculating flux and inductance in stator windings, the flux linkage from rotor windings to stator windings and corresponding mutual inductance between stator and rotor windings are expressed in (3-42) and (3-43), based on the stator conductor density distribution (3-12), rotor field winding distribution (3-17) and rotor magnetic flux density distribution (3-34).

\[
\Psi_{sr} = 2\pi \mu_0 i_r^q(t)RW \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \frac{\tilde{C}_r^\nu \tilde{C}_sn^\mu \tilde{G}_l^i}{v(v + l)} e^{j(v+l)\rho \theta_m} 
\]  

(3-42)

\[
L_{sr} = 2\pi \mu_0 RW \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \frac{\tilde{C}_r^\nu \tilde{C}_sn^\mu \tilde{G}_l^i}{v(v + l)} e^{j(v+l)\rho \theta_m} 
\]  

(3-43)

where the conditions for (3-42) and (3-43) are \( \nu + \mu + l = 0 \).

In general, the same analysis can be applied to compute the coupled inductance between two arbitrary windings \( w_1 \) and \( w_2 \).

\[
L_{w_2w_1} = 2\pi \mu_0 RW \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \frac{\tilde{C}_{w1}^\nu \tilde{C}_{w2}^\mu \tilde{G}_l^i}{v(l + v)} e^{j\kappa \rho \theta_m} 
\]  

(3-44)

where coefficient \( \kappa \) is a linear combination of indices \( \nu, \mu \) and \( l \), its dependency on rotor position is shown in Table 3.1.
3.6. Back EMF

In this section, the back-emf induced to stator windings from rotor winding energised is deduced. The back-emf is easily accessible from experiment measurement to compare against the analytical expression for validation purpose. Although the coupling inductance from the previous section is essential for the generator modelling with loading operation, the validation on no load condition makes it useful for initial validation and adding proper improvements to the CHA.

The back emf in stator windings is the time derivative of flux linkage from rotor winding energised, the induced back emf is shown in (3-45)

$$E_{sn} = \frac{d\Psi_{sr}}{dt} = \frac{d\Psi_{sr}}{d\theta} \frac{d\theta}{dt}$$

$$= 2\pi \mu_0 i_r^d(t) R W p \omega_m \sum_{l=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} (v + l) \frac{\bar{C}_r \bar{C}_{sn} \bar{G}_l e^{j(v+l)p\theta_m}}{v(v+l)}$$

(3-45)

where the condition $v + \mu + l = 0$ remains the same as in (3-44).

Notice that (3-45) shows that back-emf harmonics in the stator can come from the $v^{th}$ rotor space harmonics with average airgap (when $l = 0$), but also can get coupling at the same frequency from combinations of other rotor harmonics ($v$) with the $l^{th}$ airgap harmonic. This can be a problem where the additional harmonics, which are designed to be cancelled with specific machine geometry, can be re-introduced in the machine.

3.7. Validation Against 2D FEA

For the initial validation of the CHA method, the back-emf waveform from 2D FEA software package FEMM is simulated, where the details of the 2D FEA model implementation are covered in Chapter 5.
An example of the 3-phase back-emf comparison between CHA and FEA under unsaturated condition (2A field current with 1800rpm) is presented in Figure 3.10, where FEA1 refers to the extracted back-emf from 2D FEA and CHA1 refers to equation (3-45). The full comparison for 5-phase and 15-phase as well as saturated condition are documented in Chapter 6.

![Figure 3.10 Back-emf comparison for 3-phase between FEA and CHA method under fully-pitched (left) and short-pitched (right) winding configuration.](image)

The initial comparison shows the CHA method overestimates the back-emf than the FEA, this might come from the neglected flux iron path since it is assumed as infinity in section 3.4. This is considered as an improvement method for the CHA that presented in Chapter 4. Meanwhile, both CHA and FEA showed quite noticeable slotting effect harmonics from the back-emf waveform, which is normally not presented in the real machine geometry due to the stator skew technology. The implementation of stator skew is also documented in Chapter 4.

### 3.8. Summary

The winding mutual coupling inductance as well as the stator back-emf is derived by using CHA method in this chapter. The CHA method finds the conductor density distribution harmonics \( \bar{C} \) for each coil from (3-5) and sums over coils in a series-connected winding to compute the stator winding \( \bar{C}_s \) and rotor winding \( \bar{C}_r \) density distribution harmonics. The airgap harmonics \( \bar{G} \) is found in (3-25).

For the validation purpose, (3-44) is essential for the generator modelling, which can be calculated off-line into a look-up table from (3-1)-(3-43) with proper justification on the number of harmonics to be included for stator winding \( \bar{C}_s \), rotor winding \( \bar{C}_r \), damper
winding $\tilde{C}_d$ and airgap harmonics $\tilde{G}$. For the initial validation, stator back-emf comparison from (3-45) against FEA showed the CHA is overestimating the result. The possible solutions as including flux iron path and skew factor are implemented and covered in Chapter 4.

The problem stated in the back-emf section, where additional time harmonics are re-introduced from the combination of winding and airgap harmonics, can be solved by either short-pitching the winding or remove circulating current path. Short-pitching cancels the specific harmonics within the winding but can cause the reduction in winding factor, giving reduced terminal voltages. Furthermore, short-pitching only works at the coil level, which does not work if the supply injects harmonics. Removing the circulating current path can be achieved by using a star connection with an isolated neutral point and avoiding parallel paths. The features of short-pitching and removing circulating current path are further investigated in Chapter 7.
4.1. Introduction

The idealized CHA method for the salient pole WFSG has been presented in last chapter. In practice, the generator is manufactured with stator skew to eliminate stator slotting effect, which is not presented in idealized CHA model. The research from [148] used the asymmetric design of damper cages that minimised the slotting effect for the WFSG, which is equivalent to stator skew method. However, alternating damper cage has been validated as having less effect on the multiphase machines in [145], and the validation plots for 15-phase WFSG with the CHA method developed in this thesis are also presented in A.3. Damper Winding Influence to Multiphase Model. Therefore the normal stator skew method is considered in this thesis. Meanwhile, the simplified airgap model considers the flux path in the airgap permeance and neglects the flux path in the rotor and stator iron due to large relative permeability of iron. Furthermore, to solve the generator model based on the inductance coupling from (3-39), the leakage inductance, especially for the stator leakage inductance, is essential to avoid the singularity problem. In this chapter, the skew factor for salient pole WFSG is covered. The improved airgap including flux iron path is presented as well as the calculation of stator end winding leakage inductance and this work has been published in [143].
4.2. Skew Factor

The generator is designed with stator skew to remove the slotting effects from stator side. As shown in Figure 4.1, the winding harmonics repeat every 30 harmonics, which causes pulsating torque and may damage to the generator or shorten its lifetime.

Figure 4.1 shows the slotting effect on the 3-phase and 5-phase conductor density harmonics and back emf waveform, as this is common in literature [103, 149] for non-ideal windings. However, due to the salient feature of the rotor geometry, the stator inductance coupling is also affected. Therefore, instead of simply scaling the inductance by the length of the generator, it is necessary to integrate from one end to the other to compute the effect of skew on salient airgap geometry.

Figure 4.1 Example of slotting effect on stator conductor density harmonics for (a) 3-phase and (b) 5-phase, and back emf waveform for (c) 3-phase and (d) 5-phase.
For an arbitrary position of \( w' \) along the generator axis, the revised stator conductor density distribution function is

\[
c_{sn}(\theta, w') = \sum_{\nu=-\infty}^{\infty} \tilde{c}_{sn}^{v} e^{-j\nu\theta} e^{-j\frac{\alpha}{W}w'}
\] (4-1)

where \( \alpha \) is the skew angle in electrical.

The coupled flux linkage between any two stator windings \( sn_1 \) and \( sn_2 \) at arbitrary axial position of \( w' \) will be

\[
\Delta \phi_{sn_2 sn_1}(w') = 2\pi \mu_0 l_{sn}^q(t) R \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{\tilde{c}_{sn_1}^{\nu} \tilde{c}_{sn_2}^{\mu} \tilde{g}^{l}}{v(v + l)} e^{j\rho \theta_m} e^{-j(\nu + \mu + l)\frac{\alpha}{W}w'}
\] (4-2)

Integrating from one end to the other end of the generator, the total flux linkage will be

\[
\Psi_{sn_2 sn_1} = \int_{-\frac{W}{2}}^{\frac{W}{2}} \Delta \phi_{sn_2 sn_1}(w')
\]

\[
= 2\pi \mu_0 l_{sn}^q(t) R \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{\tilde{c}_{sn_1}^{\nu} \tilde{c}_{sn_2}^{\mu} \tilde{g}^{l}}{v(v + l)} K_{sk}(\mu + \nu) e^{j\rho \theta_m}
\]

where \( K_{sk}(\mu + \nu) = \frac{\sin \left( \frac{(\mu + \nu)\alpha}{2} \right)}{\frac{(\mu + \nu)\alpha}{2}} \).

For the back-emf from equation (3-45), the modified back-emf with stator skew implemented will be

\[
E_{sn} = 2\pi \mu_0 l_{r}^q(t) R W \rho_0 m \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} (v + l) \frac{\tilde{c}_{w_1}^{\nu} \tilde{c}_{wn_2}^{\mu} \tilde{g}^{l}}{v(v + l)} K_{sk}(\mu) e^{j(v + l)\rho \theta_m}
\] (4-4)

In general, for any two arbitrary windings, the coupled inductance will be in form of

\[
L_{wn_2 w_1} = 2\pi \mu_0 R W \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \frac{\tilde{c}_{w_1}^{\nu} \tilde{c}_{w_2}^{\mu} \tilde{g}^{l}}{v(l + v)} K_{sk} e^{j\kappa \rho \theta_m}
\]

where coefficient \( \kappa \) and skew factor \( K_{sk} \) are combinations of indices \( \nu, \mu \) and \( l \), shown in Table 3.1 and repeated here to also show \( K_{sk} \).
CHAPTER 4 IMPROVEMENTS FOR COMPLEX HARMONIC ANALYSIS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>Stator Stator Stator Rotor Rotor Rotor Damper</td>
</tr>
<tr>
<td>$w_2$</td>
<td>Stator Rotor Damper Stator Rotor Damper Damper</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$l$ $\mu + l$ $\mu + l$ $v + l$ 0 0 0</td>
</tr>
<tr>
<td>$K_{sk}$</td>
<td>$K_{sk}(\mu + v)$ $K_{sk}(v)$ $K_{sk}(v)$ $K_{sk}(\mu)$ 1 1 1</td>
</tr>
</tbody>
</table>

Table 4.1 Mutual inductance dependency on rotor position and skew factor.

After applying the skew factor, the slotting effect in stator conductor density distribution, shown in Figure 4.2, has been removed.

![Graphs showing harmonic analysis](image)

Figure 4.2 Example of applying skew to remove slotting effect on stator conductor density harmonics for (a) 3-phase and (b) 5-phase, and back emf waveform for (c) 3-phase and (d) 5-phase.

The full comparison plots of stator skew implementation are detailed in Chapter 6.

4.3. Airgap Modification

In Chapter 3, the iron path is assumed to have infinitely permeability, therefore, as the equation (4-6) shows, the flux in iron path is ignored. Equation (4-6) is the extension of
(3-29) from Chapter 3, recommended by [149]. When considering the flux path travelling in iron with permeability of $\mu_i$ relative to air permeability $\mu_0$ and assuming the magnetic flux density $B$ is continuous across the interface, then $B_i \approx B_g$. Consequently, the resultant magnetic flux density from iron path and airgap path is expressed as (4-7)

$$\oint (H_i dl_i + H_g dl_g) = \oint \left( \frac{B_i}{\mu_i \mu_0} dl_i + \frac{B_g}{\mu_g \mu_0} dl_g \right) = \sum_k I_k \quad (4-6)$$

which can be rearranged to

$$B_g = \frac{\mu_0}{g_e(\theta)} \sum_k I_k \quad (4-7)$$

where $g_e(\theta)$ is the equivalent airgap with the combination of iron path $\frac{l_i}{\mu_i}$ and airgap path $l_g$ and

$$\frac{g_e(\theta)}{\mu_0} = \frac{g_i(\theta)}{\mu_i \mu_0} + \frac{g_g(\theta)}{\mu_g \mu_0} \quad (4-8)$$

where $g_g$ is the physical airgap distance and $g_i$ is the distance the flux travels in iron part, $\mu_g$ is the relative permeability of air so $\mu_g = 1$.

As (4-7) shows, ignoring the iron path decreases the effective airgap length, causing $B_g$
to be overestimated. Figure 4.3 shows the simplified iron path for the magnetic flux in the rotor and stator iron, where \( l_b \) refers the distance the magnetic flux travels in the stator back iron.

Similar to the airgap distribution function, the iron path distribution function can be expressed as

\[
\frac{1}{g_i(\theta_r)} = \begin{cases} 
\frac{1}{R - g_1 + l_b} & (0 < \theta_r < \theta_1) \cup (\pi - \theta_1 < \theta_r < \pi) \\
\frac{1}{R - g_2 + l_b} & (\theta_1 < \theta_r < \theta_2) \cup (\pi - \theta_2 < \theta_r < \pi - \theta_1) \\
\frac{1}{R - g_3 + l_b} & (\theta_2 < \theta_r < \frac{\pi}{2}) \cup (\pi - \frac{\pi}{2} < \theta_r < \pi)
\end{cases}
\]  \tag{4-9}

where the stator back iron path length \( l_b = \pi(R + \Delta l_b) + 2\Delta l_b \), illustrated in Figure 4.3.(a).

Therefore, the effective airgap is

\[
\begin{align*}
g'_{1} &= \frac{g_1}{\mu_g} + \frac{R - g_1 + l_b}{\mu_i} \\
g'_{2} &= \frac{g_2}{\mu_g} + \frac{R - g_2 + l_b}{\mu_i} \\
g'_{3} &= \frac{g_3}{\mu_g} + \frac{R - g_3 + l_b}{\mu_i}
\end{align*}
\]  \tag{4-10}

<table>
<thead>
<tr>
<th>( \frac{g_i'}{g_i} )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{g_1'}{g_1} )</td>
<td>111.9%</td>
</tr>
<tr>
<td>( \frac{g_2'}{g_2} )</td>
<td>100.4%</td>
</tr>
<tr>
<td>( \frac{g_3'}{g_3} )</td>
<td>102.9%</td>
</tr>
<tr>
<td>( \frac{G_0'}{G_0} )</td>
<td>89.2%</td>
</tr>
<tr>
<td>( \frac{G_2'}{G_2} )</td>
<td>88.5%</td>
</tr>
</tbody>
</table>

Table 4.2 Comparison of effective airgap against original airgap \( \frac{g_i'}{g_i}, i = 1, 2, 3 \) along with their harmonics \( \frac{G_i'}{G_i}, i = 0, 2 \)
Because of the relatively high value of $\mu_i$, the iron path has a much smaller effect. This can be seen by comparing the ratio of $g_i' / g_i$ ($i = 1, 2, 3$), where $g_i'$ refers the modified airgap distance and $g_i$ to the original airgap distance, shown in Table 4.2. Accounting for the iron path has increased the effective airgap distance, particularly $g_1$ which increases by 11.9%. This results in a reduction in fundamental component of airgap harmonic $G_0$ of 10.8% and second order harmonic $G_2$ of 11.5%, which effectively reduces airgap flux.

![Figure 4.4 Reciprocal of airgap distance comparison between original (blue trace) and modified (red trace) along with their harmonic contents.](image)

The harmonic analysis shows that including the iron path decreases the DC component and second order harmonic of airgap harmonics more significantly than other harmonics, shown in Figure 4.4.

It should be mentioned that the iron path does not have much effect as this is scaled by $\mu_i$; also, the iron path is simplified and assumed as the same path for all magnetic flux and the fringing effect is neglected. [150] has discussed the fringing effect, which been applied to induction machines modelling fringing effect between stator and rotor slots. However, the separation between $\theta_1$ and $\theta_2$ is more significant than the rotor slots considered in [150], so simply applying Carter’s coefficient could result in inaccurate results.

### 4.4. Leakage Inductance

End winding inductance is an important part of the winding leakage inductance. The stator end winding is only considered in this section, as for the rotor windings, the
excitation is DC, where leakage inductance does not have an effect when the machine is the steady state operation. The accuracy of the leakage term from damper circuits is less significant, as the damper windings have little effect on the overall machine behaviour when it comes to a higher number of phases than three, this phenomenon has also been validated in [145]. An example of damper winding and its leakage inductance influence on the steady state performance on the GRS is presented in A.3.Damper Winding Influence to Multiphase Model.

There are many methods of computing end winding inductance. Mathematically, the Neuman integral method and image method are applied for calculating end winding inductance [151]. FE analysis of a 3D model of the end winding provides better accuracy for specific machine winding geometry; however, the model setup and processing is time consuming.

The IEC standard [152] removes the rotor structure completely for stator leakage measurement, where the measured stator leakage inductance includes slotting leakage inductance, end winding inductance and inductance due to the flux passing through the stator bore [151, 152]. The latter inductance is usually obtained through the FE method with a linear circuit model, as the test of stator leakage inductance is usually under the unsaturated region [152].

4.4.1. Analytical Calculation of End Winding Inductance

The research from [153] provides an image method that could be applied to end winding inductance calculations, where the image method is used to simplify the magnetic flux distribution, as it is then distributed in a single media with the same permeability, making it easier to compute the flux linkage between two end windings.

![Figure 4.5 Two simplified end windings.](image-url)
To simplify the model, the end winding is assumed to be rectangular shaped, as shown in Figure 4.5. The two adjacent end windings conductors (with radius of \( r \)) are separated by a horizontal distance \( d \) and a vertical distance \( y \). After applying the image method, as shown in Figure 4.6.(a), the effect of the stator iron is removed, where the end winding is effectively distributed in air, with \( \mu_0 \) as permeability. The image method proposes a mirrored current \( I_2 \) according to (4-11) [153]

\[
I_2 = \frac{\mu_r - \mu_0}{\mu_r + \mu_0} I_1
\]

(4-11)

For a material with infinitely permeability, the mirrored current \( I_2 \) is same as the original excitation current \( I_1 \); for zero permeability material, meaning no magnetic flux in area 2, the mirrored current \( I_2 \) has the same magnitude with \( I_1 \) but with reversed direction. It must be emphasized that the image method showing in Figure 4.6.(a) can only be used to solve the flux distribution in area 1. If the analysis on the flux distribution in area 2 is needed, the new image method should be applied as shown in Figure 4.6.(b), where \( I_2 \) now becomes (4-12) [153]

\[
I_2 = \frac{2\mu_0}{\mu_r + \mu_0} I_1
\]

(4-12)

Figure 4.6 Method of images applied to remove the effect of the stator iron when analysing the magnetic flux distribution of (a) area 1 and (b) area 2.

When calculating mutual inductance, the conductors are assumed to be infinitely thin, ignoring their radius. The end winding mutual magnetic flux linkage calculation is based on computing the integral magnetic potential along the second conductor, while
the magnetic potential is given by solving $\vec{A}$ in (4-13) [153]. The mathematical model with image method is shown in Figure 4.7.

$$\vec{A} = \frac{\mu_0 I_1}{4\pi} \int_{l_2} \frac{1}{r_1} d\vec{l}_1 + \frac{\mu_0 I'_1}{4\pi} \int_{l_2'} \frac{1}{r'_1} d\vec{l}_1$$

$$\phi = \int_{l_2} \vec{A} d\vec{l}_2 = \frac{\mu_0 I_1}{4\pi} \int_{l_2} \int_{l_1} \frac{1}{r_1} d\vec{l}_1 d\vec{l}_2 + \frac{\mu_0 I'_1}{4\pi} \int_{l_2'} \int_{l_1'} \frac{1}{r'_1} d\vec{l}_1' d\vec{l}_2$$

(4-13)

The inductance is then calculated by summing flux linkage from different segments and dividing by the excitation current, the number of turns could be applied if considering multi-turn conductor bundle.

$$L = \frac{N^2 c \sum \phi}{I}$$

(4-14)

For self-inductance calculations, the conductors are assumed to have an effective radius $r$ when calculating self-inductance of these end windings. The same image method is applied as before, where the flux linkage is calculated by integrating the magnetic flux potential along the conductor itself.

$$\phi = \frac{\mu_0 I_1}{4\pi} \int_{l_1} \int_{l_1'} \frac{1}{r'} d\vec{l}_1 d\vec{l}_1'$$

(4-15)

The detailed mutual inductance and self-inductance calculations are attached in the A.4. Image Method.
The total analysed end winding inductance for 3, 5 and 15-phase windings are shown in Table 4.3, where the computed leakage inductance are normally less than 5% of the magnetising inductance $L_{sd}$ computed from the CHA.

<table>
<thead>
<tr>
<th>Winding layout</th>
<th>Computed end winding inductance (mH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fully-pitched</td>
</tr>
<tr>
<td>3-phase</td>
<td>2.38</td>
</tr>
<tr>
<td>5-phase</td>
<td>0.92</td>
</tr>
<tr>
<td>15-phase</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 4.3 Calculated end winding inductance from image method for different phase windings.

4.4.2. Validation against Experimental Measurement and 3D FEA

The validation for end winding inductance is done in both FEA modelling and experimental measurements, where the 3D FEA model setup and experiment test rig setup are presented in Chapter 5 and Chapter 6 respectively.

The experimental measurement is conducted when the entire rotor is removed, as suggested in [152], which eliminates the magnetising path so the measured stator inductance will only be the combination of slot inductance and end winding inductance. The inductance term is measured by connecting winding terminals to LCR meter (Agilent 4284A) for each phase; the measurement is done under 3, 5 and 15-phase winding layouts separately. Figure 4.8 shows a simplified diagram for the end winding measurement. The result is averaged among all measured phases and results are presented in Table 4.4.

Figure 4.8 Simplified diagram of leakage inductance measurement.
The measurement shows the total leakage inductance for stator windings, including both ending winding inductance and stator slot leakage inductance, which was obtained from 2D FEA and also presented in Table 4.4. The comparison showed that the main difference from analytical image method is due to the lack of slot leakage inductance. The combination of FEA slot leakage and end winding inductance from image method is generally overestimating the overall leakage than that from experimental measurement, especially for the short-pitched case.

<table>
<thead>
<tr>
<th>Winding layout</th>
<th>Measured leakage inductance (mH)</th>
<th>3D FEA computed leakage inductance (mH)</th>
<th>Computed end winding inductance (mH)</th>
<th>FEA computed slot leakage inductance (mH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fully-pitched</td>
<td>Short-pitched</td>
<td>Fully-pitched</td>
<td>Short-pitched</td>
</tr>
<tr>
<td>3-phase</td>
<td>3.48</td>
<td>2.35</td>
<td>3.33</td>
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</tr>
<tr>
<td>5-phase</td>
<td>1.62</td>
<td>1.20</td>
<td>1.64</td>
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</tr>
<tr>
<td>15-phase</td>
<td>0.31</td>
<td>0.25</td>
<td>0.27</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.4 Leakage inductance measurement for different phase windings.

During the measurement, it was found that the phase A winding has general large leakage term than other phase windings for short-pitched layout, as listed in Table 4.5. This is caused by the winding layout for phase A, where the spatial span for phase A is same for both fully-pitched and short-pitched, which is $\pi$ in spatial, however, phase B and C in short-pitched layout have smaller spatial span ($\frac{2\pi}{3}$ for 3-phase winding and $\frac{4\pi}{5}$ for 5-phase and 15-phase winding). An example of 3-phase phase A and B in short-pitched layout is presented in Figure 4.9.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Measured impedance (in the form of $R(\Omega)+j\omega L(mH)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-phase</td>
</tr>
<tr>
<td>A</td>
<td>0.74+j$\omega$3.45</td>
</tr>
<tr>
<td>B</td>
<td>0.74+j$\omega$3.49</td>
</tr>
<tr>
<td>C</td>
<td>0.75+j$\omega$3.50</td>
</tr>
<tr>
<td>D</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>-</td>
</tr>
<tr>
<td>F</td>
<td>-</td>
</tr>
<tr>
<td>G</td>
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<tr>
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</tbody>
</table>

Table 4.5 Measured leakage term for different phase windings.
Although the image method calculation overestimates the end winding inductance, it proves that it could be used as a way of predicting the end winding inductance as in a similar order of magnitude. To guarantee the accuracy of the simulation result with CHA method, the measured leakage inductance is implemented in the Simulink model. Meanwhile, the simulation finds it is less sensitivity to the stator leakage terms and this part of work has been published in [143].

The FEA validation is based on the 3D model that set up in the COMSOL® Multiphysics. Due to the time consuming model setup, only the fully-pitched winding of 3, 5 and 15-phase of synchronous machine model is set up, as shown in Figure 4.10, where the red traces refer to the stator windings. The extracted leakage inductance from models in Figure 4.10 is presented in Table 4.4. The short-pitched winding of 3, 5 and 15-phase of 3D FEA models are not presented as the model setup process is time consuming and this thesis only shows the rough comparison between experiment, image method and 3D FEA, where the fully-pitched winding is enough to demonstrate the principle.

The 3D FEA provides better results than analytical methods, however, because the end winding inductance is only a small part of the synchronous inductance, the accuracy of that becomes less important. In conclusion, the image method offers a way to compute...
end winding inductance but lacks accuracy; 3D FEA gives a better match but requires knowledge of the complete machine winding geometry and takes more setup time.

![3D model in COMSOL® Multiphysics with a single phase fully-pitched winding for (a) 3-phase, (b) 5-phase and (c) 15-phase.](image)

**Figure 4.10** 3D model in COMSOL® Multiphysics with a single phase fully-pitched winding for (a) 3-phase, (b) 5-phase and (c) 15-phase.

### 4.5. Summary

The skew factor considering rotor saliency for WFSG, airgap with iron path and stator end winding inductance has been covered in this chapter.

The skew factor and airgap modifications help to improve the accuracy of the CHA model, where skew factor removes the stator slotting effect and including airgap flux iron path effectively increases the airgap distance. Compared with an IM, the combination of harmonic indices is different regarding different winding flux linkage due to the rotor saliency.

The end winding inductance from image method provides an order of magnitude accuracy, while the 3D FEA gives a closer result against the experimental measurement but the model setup is time consuming. The stator leakage inductance composes less than 5% of the magnetising inductance but is essential to avoid the singularity problem (the singularity problem is common for the machine modelling, where the standard books [83, 84] usually adds the leakage terms. The detailed demonstration is presented in A.5.Singularity Problem of Machine Stator Winding Inductance Matrix) while solving the generator model. A rough assumption of leakage inductance could be achieved when using image method at the designing stage, while more accurate result would be recommended to get from either detailed 3D FEA or directly experimental measurement.
Chapter 5. Simulation Models

5.1. Introduction

In this chapter, the transformation from stationary reference frame to synchronous reference frame is covered, including conventional 3-phase and multiphase windings. As well as the symmetrical winding transformation, the asymmetrical winding transformation, i.e. for damper windings is also presented. The complete Simulink model, combined with PLECS circuit model is also presented, as well as the reason for not using DQ reference frame model. Finally, the 2D FEA and 3D FEA model setup process is covered in the end.

5.2. Stationary Reference Frame to Synchronous Reference Frame

Machines are usually modelled in the synchronous reference frame, i.e. DQ reference frame. DQ modelling is a common technique especially for machine control, in steady-state it converts the time varying AC quantities to DC quantities via Clarke and Park transform matrices [154-156]. The DQ transformation changes three or multi-phase connections to a two-axis connection and weakens or cancels the dependency on rotor position of those time-varying quantities. This approach transfers multiphase time-
varying, fixed spatially distributed AC variables into time-varying rotating DC variables. The DQ transformation helps to simplify control that relates to the machine systems, as the DC quantities are much easier to implement in control than time varying AC quantities, especially when the control system is based on observer design, because the DQ transformation reduces the number of input references and variables to be controlled. An m\textsuperscript{th} order multiple input multiple output (MIMO) system can be changed into DQ sets and higher order sets, where generally the higher order sets have zero references in terms of control. The fundamental DQ sets can be split into two single input single output (SISO) system using an appropriate decoupling method. In addition, it is much easier to set reference and control DC variables rather than directly control AC variables.

The classical DQ modelling depends on the assumptions of sinusoidal distributed windings or sinusoidal distributed flux, as well as linear magnetic circuits [154], therefore, the saturation and cross-coupling effects are ignored. For concentrated windings, especially in PMs, the DQ modelling is also commonly used, with an assumption of sinusoidal distributed PM flux [157]. For fractional slot concentrated windings, the DQ modelling technique is also implemented with additional modification [158].

5.3. DQ Transformation for a Symmetric Winding Layout

5.3.1. 3-phase Winding Transform

The Clarke and Park transform theory are the basics of transforming to a synchronous reference frame. The aim of the Clarke and Park transform was originally to transform 3-phase time varying AC quantities to two-axis DC quantities and remove underlying couplings. The traditional three-phase transform matrix for Clarke (C) and Park (P) is shown in (5-1) and (5-2), with the convention of power invariance applied.

\[
C = \begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{2}{\sqrt{3}} & 1 & \cos \left( \frac{2\pi}{3} \right) \cos \left( \frac{4\pi}{3} \right) \\
0 & \sin \left( \frac{2\pi}{3} \right) & \sin \left( \frac{4\pi}{3} \right)
\end{bmatrix}
\]  (5-1)
\[
P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_r & -\sin \theta_r \\ 0 & \sin \theta_r & \cos \theta_r \end{bmatrix}
\]  
(5-2)

where the electrical angular position in \( P \) is presented in rotor reference frame, and \( \theta_r = \theta_s - \rho \theta_m \).

Because of the orthogonal property of matrices \( C \) and \( P \), the inverse transform matrix of \( (5-1) \) and \( (5-2) \) are their transpose matrix respectively.

The transformation of stator self/mutual inductances to DQ inductances is based on the transformation of flux. The process is shown as following

\[
\Psi_{abc} = L_{ss}i_{abc}
\]  
(5-3)

\[
\Psi_{\alpha\beta\gamma} = CL_{ss}i_{abc} = CL_{ss}C^T i_{\alpha\beta\gamma} = L_{\alpha\beta\gamma}i_{\alpha\beta\gamma}
\]  
(5-4)

\[
\Psi_{dqy} = PL_{\alpha\beta\gamma}i_{\alpha\beta\gamma} = PL_{\alpha\beta\gamma}P^T i_{dqy} = L_{dqy}i_{dqy}
\]  
(5-5)

where the \( \alpha\beta \) refer to stationary two axis reference frame and \( \gamma \) to zero sequence axis.

Combining equation (5-3) - (5-5), the DQ inductance term can be obtained as

\[
L_{dqy} = PCL_{ss}C^T P^T
\]  
(5-6)

The 3-phase stator inductance matrix, which is based on equation (3-41), rewritten as

\[
L_{s2s1} = L_2 \cos(2p\theta_m - 2\theta_{s1} - \gamma) + L_0 \cos(\gamma),
\]

is transformed as

\[
L_{dq} = \frac{3}{2} L_0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{3}{2} L_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & L_d & 0 \\ 0 & 0 & L_q \end{bmatrix}
\]  
(5-7)

\[
L_d = \frac{3}{2} (L_0 + L_2)
\]

\[
L_q = \frac{3}{2} (L_0 - L_2)
\]

The \( L_0 + L_2 \) and \( L_0 - L_2 \) terms are the maximum and minimum values of the stator magnetising inductance in (3-41). Therefore, the DQ stator inductance terms are scaled with a factor of 1.5 with respect to the maximum/minimum of the stator per phase stator magnetising inductance for 3-phase windings. This is the usual dynamic model for WFSG in [84].
5.3.2. **Multiphase Winding Transform**

When extending to multiphase windings, [159] described the generalised transform method into the synchronous reference frame for induction machines from an m-phase stator winding and n-phase rotor winding to convert to two-phase stationary stator and rotor components. This decoupling method can also be found in [56, 160] for stator windings with more than 3-phases. The proposed matrix is in (5-8) which shows the transform for a symmetric winding with even number of phases, and the second row is deleted for an odd number of phases.

\[
\mathbf{T} = \sqrt{\frac{2}{m}} \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\
1 & \cos \delta & \cos 2\delta & \cos 3\delta & \cos 4\delta & \ldots \\
0 & \sin \delta & \sin 2\delta & \sin 3\delta & \sin 4\delta & \ldots \\
1 & \cos 2\delta & \cos 4\delta & \cos 6\delta & \cos 8\delta & \ldots \\
0 & \sin 2\delta & \sin 4\delta & \sin 6\delta & \sin 8\delta & \ldots \\
1 & \cos 3\delta & \cos 6\delta & \cos 9\delta & \cos 12\delta & \ldots \\
0 & \sin 3\delta & \sin 6\delta & \sin 9\delta & \sin 12\delta & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\]

(5-8)

where \( \delta = \frac{2\pi}{m} \) and \( m \) is the number of phase windings.

Assume the \( m \)-phase quantity \( x \), the transformed stationary reference frame component is \( x_5 \), where

\[
x_5 = \begin{bmatrix}
x_{0+} \\
x_{0-} \\
x_\alpha \\
x_\beta \\
x_{1+} \\
x_{1-} \\
x_{2+} \\
x_{2-} \\
\vdots
\end{bmatrix} = \mathbf{T}x
\]

(5-9)

The \( x_{0+} \) and \( x_{0-} \) term refer to positive and negative zero sequence components and the negative zero sequence component only exists when \( m \) is even, the transformed \( \alpha \beta \)
term are \(x_\alpha\) and \(x_\beta\) respectively, \(x_{xi}\) and \(x_{yi}\) terms are the higher order quantities that composed the matrix \(\mathcal{T}\) into a square matrix, similar to the form for 3-phase winding. Notice that the sequence of equation (5-8) and (5-9) from [159] has been rearranged so that the transformed stationary quantities are in the sequence of zero-sequence plane, \(\alpha\beta\) plane and higher order plane.

Specifically, for the 5-phase and 15-phase in this projection that considered, the Clarke matrix can be \(C_5\) and \(C_{15}\) respectively.

\[
C_5 = \sqrt{\frac{2}{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 1 & \cos \delta & \cos 2\delta & \cos 3\delta & \cos 4\delta \\ 0 & \sin \delta & \sin 2\delta & \sin 3\delta & \sin 4\delta \\ 1 & \cos 2\delta & \cos 4\delta & \cos 6\delta & \cos 8\delta \\ 0 & \sin 2\delta & \sin 4\delta & \sin 6\delta & \sin 8\delta \end{bmatrix}
\]

\[
C_{15} = \sqrt{\frac{2}{15}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} & \cdots & \sqrt{2} \\ 1 & \cos \delta & \cos 2\delta & \cdots & \cos 14\delta \\ 0 & \sin \delta & \sin 2\delta & \cdots & \sin 14\delta \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cos 7\delta & \cos 7\times 2\delta & \cdots & \cos 7\times 14\delta \\ 0 & \sin 7\delta & \sin 7\times 2\delta & \cdots & \sin 7\times 14\delta \end{bmatrix}
\]

where \(\delta = \frac{2\pi}{5}\).

For an \(m\)-phase winding, it can be shown that the \(nm^{th}\) time harmonics map to the zero-sequence plane, \(nm \pm 1^{th}\) harmonics map to the \(\alpha\beta\) plane, \(nm \pm 2^{th}\) harmonics map to \(x_{1y_1}\) plane, \(nm \pm 3^{th}\) harmonics maps to \(x_2y_2\) plane etc., where \(n = 0,1,2 \ldots\) and \(m\) is the phase number as stated above. The rotating direction for each adjacent plane is reversed. The relation of the different harmonic order that been mapped into \(\alpha\beta\) or \(x_{1y_1}\) plane for 3-phase, 5-phase and 15-phase is listed in the Table 5.1 as an example, assuming quantities in the \(\alpha\beta\) plane rotate in the forward direction.
### Table 5.1 Harmonics mapped to different axis after the stationary reference frame transformation $\mathcal{T}$ for 3-phase, 5-phase and 15-phase windings.

<table>
<thead>
<tr>
<th>Zero-sequence</th>
<th>3-phase</th>
<th>5-phase</th>
<th>15-phase</th>
<th>Rotating direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha \beta$</td>
<td>DC, 3rd, 6th …</td>
<td>DC, 5th, 15th …</td>
<td>DC, 15th …</td>
<td>Stationary</td>
</tr>
<tr>
<td>$x_1y_1$</td>
<td>-</td>
<td>3rd, 7th, 13th, 17th …</td>
<td>13th, 17th …</td>
<td>Backward</td>
</tr>
<tr>
<td>$x_2y_2$</td>
<td>-</td>
<td>-</td>
<td>3rd, 27th, 33rd …</td>
<td>Forward</td>
</tr>
<tr>
<td>$x_3y_3$</td>
<td>-</td>
<td>-</td>
<td>11th, 19th …</td>
<td>Backward</td>
</tr>
<tr>
<td>$x_4y_4$</td>
<td>-</td>
<td>-</td>
<td>5th, 25th, 35th …</td>
<td>Forward</td>
</tr>
<tr>
<td>$x_5y_5$</td>
<td>-</td>
<td>-</td>
<td>9th, 21st …</td>
<td>Backward</td>
</tr>
<tr>
<td>$x_6y_6$</td>
<td>-</td>
<td>-</td>
<td>7th, 23rd …</td>
<td>Forward</td>
</tr>
</tbody>
</table>

Based on the harmonic content and rotating direction listed above, the extended Park transform for 3-phase, 5-phase and 15-phase are the $P_3$, $P_5$ and $P_{15}$ respectively.

### Equations

$$P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$  \hspace{1cm} (5-12)

$$P_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 & 0 \\ 0 & \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 0 & \cos(-3\theta) & -\sin(-3\theta) \\ 0 & 0 & 0 & \sin(-3\theta) & \cos(-3\theta) \end{bmatrix}$$  \hspace{1cm} (5-13)

$$P_{15} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \cos \theta & -\sin \theta & 0 & 0 & 0 & 0 & \cdots \\ 0 & \sin \theta & \cos \theta & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cos(-13\theta) & -\sin(-13\theta) & 0 & \cdots \\ 0 & 0 & 0 & \sin(-13\theta) & \cos(-13\theta) & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$  \hspace{1cm} (5-14)
Where $\theta$ is the instantaneous angular displacement between the $\alpha\beta$ axes and the $dq$ axes. For transforming from the stationary to synchronous reference frame, shown in Figure 5.1, $\theta = \int \omega_s \, dt$, where $\omega_s$ is the synchronous electrical angular speed.

![Diagram of transforming from $\alpha\beta$ plane to $dq$ plane.]

**5.4. DQ Transformation for an Asymmetric Winding Layout**

Instead of the symmetrical distributed windings, the damper windings are normally not symmetrically distributed along the stator natural reference frame.

Unlike the periodic distribution of the stator windings, the damper bar arrangement is symmetrical with respect to the D- and Q- axes of the rotor, but the flux distribution from each damper loop is not equally spaced along the airgap contour. The extended Clarke and Park transform matrix from (5-8) and (5-12)-(5-14) no longer applies to these damper circuits. [161, 162] suggested an approach for mapping a non-sinusoidal phase distribution to DQ axis, which is to solve the matrix based on the orthogonal relation of each sub-row vectors. Mathematically, there is a method called the QR decomposition, which decomposes a matrix into a product of an orthogonal matrix and a supplementary matrix (upper triangular matrix if the original matrix is a square matrix), and the MATLAB has in-built code for computing the QR decomposition [163]. Based on this method, the damper circuits are mapped to DQ axis easily. Equation (5-15) shows the dedicated transform matrix for damper windings, for the test machine with 8 damper loops.
\[
T = \begin{bmatrix}
T_d \\
T_q \\
x_1 \\
y_1 \\
x_2 \\
y_2 \\
x_3 \\
y_3 \\
\end{bmatrix}
\]

(5-15)

The transform matrix is generated by introducing the two vectors that transform the damper circuits into D-axis and Q-axis \((T_d\) and \(T_q\)), since the transform matrix should satisfy the condition that every pair of axes are orthogonal in space, shown in equation sets (5-16).

\[
x_i \cdot y_i = 0
\]

\[
T_d \cdot x_i = 0
\]

\[
T_d \cdot y_i = 0
\]

\[
T_q \cdot x_i = 0
\]

\[
T_q \cdot y_i = 0
\]

(5-16)

where \(x_i\) and \(y_i\) represent the higher order axes in space and \(i = 1,2,3\).

With the convention of power invariance, the matrix \(T\) should also satisfy

\[
T \cdot T^T = I
\]

(5-17)

where \(I\) represents for unit matrix.

The transform vector \(T_d\) and \(T_q\) can be solved by mapping damper circuit loops into D- and Q-axis separately, where the damper loops are illustrated in the Figure 3.7. In the test machine, the damper circuits are formed into eight loops with \(\theta = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 & \theta_8 \end{bmatrix}\), where \(\theta_i\) is the centre of loop flux produced by the circulating loop current \(I_{di}\), shown in Figure 3.7, therefore

\[
T_d = \begin{bmatrix}
\cos \theta_1 \\
\cos \theta_2 \\
\ldots \\
\cos \theta_8 \\
\end{bmatrix}
\]

(5-18)
\[ T_q = [\sin \theta_1 \sin \theta_2 \ldots \sin \theta_k] \]

To solve for matrix \( T \) in (5-15) subject to the conditions of (5-16)-(5-18), the mathematical method, QR decomposition is used; the result is shown in (5-19).

\[
QR = \begin{bmatrix} T_d^T & T_q^T \end{bmatrix} \tag{5-19}
\]

where \( Q^T \) is the dedicated orthogonal matrix (8 by 8) for the damper winding, which links to vector set \( \begin{bmatrix} T_d \\ T_q \end{bmatrix} \); matrix \( R \) is a supplementary matrix with size 8 by 2. Therefore, the transform matrix for damper winding is in the same form as (5-15).

### 5.5. Simulation Model

In order to predict the behaviour of the generator combined with a diode rectifier, a complete system model is required. The MATLAB® Simulink environment is chosen to simulate the generator model, as it is widely used to simulate drive applications. The Simulink model includes the generic generator circuit model, online and offline processing of the CHA method and a PLECS circuit model built in Simulink. A brief description of FEMM model to validate the CHA method in earlier stage is attached at the end of this section. The Simulink solver is set as ode23t with variable step and 1e-4 of error tolerance, the maximum and minimum step size are set as ‘auto’ by default. The zero crossing detection settings also use the as default setup as ‘local’ settings of zero crossing control, with a nonadaptive algorithm, \( 2.8 \times 10^{-13} \) of time tolerance and count of 1000 consecutive zero crossings.

#### 5.5.1. Generic Generator Circuit Model

The equivalent circuit model for the modelled generator is shown in (5-20), where the equation is under the generating convention where positive current and power are delivered out of the machine terminals.

\[
\bar{V} + [R] \bar{I} + \frac{d}{dt} ([L] \bar{I}) = 0 \tag{5-20}
\]

where \( \bar{V}, \bar{I} \) are vectors and \([R], [L] \) are the matrix, these all contains stator, rotor and damper quantities. A more detailed circuit equation in matrix form is listed in (5-21)
where subscript \( s \) denotes for stator side quantities, subscript \( f \) denotes for rotor field side quantities and \( d \) for damper winding side quantities. Therefore, \( \tilde{V}_s \) and \( \tilde{I}_s \) are the vectors for stator quantities; \( V_f \) and \( I_f \) are scalar quantities for rotor windings; \( \tilde{V}_d \) and \( \tilde{I}_d \) are vectors for damper circuit quantities, since damper circuits are effectively short circuited, there is no external voltage source in damper circuits, therefore \( \tilde{V}_d = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ T]^T \).

5.5.2. DQ representation and Reasons for Not Using It

When transforming the generator model from the natural reference frame into the synchronous reference frame, equation (5-21) is further split into stator side, field winding side and damper winding side equations as

\[
[\tilde{V}_s] + [R_s]I_s + \frac{d}{dt}(L_{ss}I_s + L_{sf}I_f + L_{sd}I_d) = 0
\]

\[
V_f + R_fI_f + \frac{d}{dt}(L_{fs}I_s + L_{ff}I_f + L_{fd}I_d) = 0
\]

\[
[\tilde{V}_d] + [R_d]I_d + \frac{d}{dt}(L_{ds}I_s + L_{df}I_f + L_{dd}I_d) = 0
\]

Transforming equation (5-22) from the natural reference frame into the synchronous reference frame, the Clarke (\( \mathcal{C} \)) and Park (\( \mathcal{P} \)) transform matrix are applied whenever the stator side quantities are involved, damper winding transform matrix (\( T_d \)) is applied whenever the damper windings are involved. Therefore, equation (5-22) becomes (5-23), where the subscript \( s \) denotes for variables in synchronous reference frame and the detailed derivation is attached in the A.6. Simulation Model and MATLAB Codes.

\[
[\tilde{V}_s^s] + [R_s]^s\tilde{I}_s^s + P \frac{dP^{-1}}{dt}(I_{ss}^s\tilde{I}_s^s + I_{sf}^sI_f + I_{sd}^sI_d) + \frac{d}{dt}(I_{ss}^s\tilde{I}_s^s + I_{sf}^sI_f + I_{sd}^sI_d) = 0
\]

\[
V_f + R_fI_f + \frac{d}{dt}(L_{fs}^sI_s + L_{ff}^sI_f + L_{fd}^sI_d) = 0
\]

\[
[\tilde{V}_d^s] + [R_d]^s\tilde{I}_d^s + \frac{d}{dt}(L_{ds}^sI_s + L_{df}^sI_f + L_{dd}^sI_d) = 0
\]
In terms of stator side equations, for the commonly modelled D-axis (with super script \(sd\)) and Q-axis (with super script \(sq\)) synchronous reference frame, the stator side equation from (5-23) becomes (5-24) and zero sequence quantities (with super script \(s0\)) becomes (5-25).

\[
\begin{bmatrix}
V_{sd}^s \\
V_{sq}^s
\end{bmatrix} + \begin{bmatrix}
R_s^d & 0 \\
R_s^q & L_s^q
\end{bmatrix} \begin{bmatrix}
I_{sd}^s \\
I_{sq}^s
\end{bmatrix} + \omega_s \begin{bmatrix}
0 & -L_{ss}^s \\
L_{ss}^s & 0
\end{bmatrix} \begin{bmatrix}
I_{sd}^s \\
I_{sq}^s
\end{bmatrix} + \omega_s \begin{bmatrix}
L_{sf}^s \\
L_{sf}^q
\end{bmatrix} I_f^s \\
\]

\[
+ \omega_s \begin{bmatrix}
0 & -L_{sd}^s \\
L_{sd}^s & 0
\end{bmatrix} \begin{bmatrix}
I_{sd}^s \\
I_{sq}^s
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
L_{sd}^s & 0 \\
0 & L_{sd}^q
\end{bmatrix} \begin{bmatrix}
I_{sd}^s \\
I_{sq}^s
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
L_{sf}^s & 0 \\
0 & L_{sf}^q
\end{bmatrix} I_f^s = 0
\]

(5-24)

\[
V_{s0}^s + R_s^s I_{s0}^s + \frac{d}{dt} \left( L_{ss} I_{s0}^s + L_{sf} I_{s0}^s \right) = 0
\]

(5-25)

The rotor side equation from (5-23) becomes (5-26) when only considering D, Q-axis and zero sequence for the synchronous reference frame.

\[
V_f + R_f I_f + \frac{d}{dt} \left( L_{f0} I_{f0} + L_{fs} I_{fs} + L_{sf} I_{sf} \right) = 0
\]

(5-26)

Since the generator behaviours are analysed under steady state conditions, the damper windings are ignored in order to simplify the modelling procedure on D and Q-axis synchronous reference frame. Therefore, equations from (5-24) to (5-26) become (5-27).

\[
\begin{bmatrix}
V_{sd}^s \\
V_{sq}^s
\end{bmatrix} + \begin{bmatrix}
R_s^d & 0 \\
R_s^q & L_s^q
\end{bmatrix} \begin{bmatrix}
I_{sd}^s \\
I_{sq}^s
\end{bmatrix} + \omega_s \begin{bmatrix}
0 & -L_{ss}^s \\
L_{ss}^s & 0
\end{bmatrix} \begin{bmatrix}
I_{sd}^s \\
I_{sq}^s
\end{bmatrix} + \omega_s \begin{bmatrix}
L_{sf}^s \\
L_{sf}^q
\end{bmatrix} I_f^s \\
\]

\[
+ \frac{d}{dt} \begin{bmatrix}
L_{ss}^s & 0 \\
0 & L_{ss}^q
\end{bmatrix} \begin{bmatrix}
I_{sd}^s \\
I_{sq}^s
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
L_{sf}^s & 0 \\
0 & L_{sf}^q
\end{bmatrix} I_f^s = 0
\]

(5-27)

\[
V_{s0}^s + R_s^s I_{s0}^s + \frac{d}{dt} \left( L_{ss} I_{s0}^s + L_{sf} I_{s0}^s \right) = 0
\]

\[
V_f + R_f I_f + \frac{d}{dt} \left( L_{f0} I_{f0} + L_{fs} I_{fs} + L_{sf} I_{sf} \right) = 0
\]

For the 3-phase generator model, the standard DQ model shows no difference from the result from the natural reference frame model, since both the DQ synchronous reference frame model and natural reference frame have the same order of dimensions on the solved generator currents, where the waveform plots are presented in Figure 5.2, where \(Vs\), \(Is\) (generator) and \(Is\) (diode) refer to the generator voltage, current and diode current.
quantities for the same phase, and Vdc and Idc are the resistive load voltage and current waveforms.

\[ V_s, V_{dc}, I_s \text{(generator)}, I_{dc} \text{(diode)} \]

Figure 5.2 Simulated results from DQ synchronous reference frame (left) and natural reference frame (right) for 3-phase polygon short-pitched generator-rectifier system, under 8A field excitation and 10Ω DC resistive load.

The generator waveform from these two models match well, although there are some noises on the generator voltage waveform in the DQ synchronous reference frame, which implies the alternative way of modelling generator-rectifier behaviour for 3-phase windings.

However, when it comes to 5-phase and 15-phase windings, the results from the DQ synchronous reference frame are significantly different from those in the natural reference frame, shown in Figure 5.3. The simulated generator waveforms are completely different, where nicely sinusoidal generator current and diode current waveforms arise from the DQ synchronous reference frame model.

The reason for the significant differences between DQ synchronous reference frame model and natural reference frame model is that the solver is generator current based, and the generator voltages are extracted from the PLECS model based on the solved generator current. The standard DQ-synchronous reference frame assumes that the winding inductances are nicely sinusoidal distributed, which produces sinusoidal waveform of generator currents. Besides, the standard DQ-synchronous reference frame ignores the other higher order harmonics, forcing them to zeros. However, this is not applicable for the uncontrolled rectifier systems with higher phase number, where the
higher order harmonics are a significant part of the generator voltage and current waveforms. For the controlled rectifier systems, the standard DQ-synchronous reference frame model could be applicable if the control algorithm could force the generator current to form a sinusoidal waveform. In [14], the author used this DQ model, but added additional voltage sources to represent the harmonic back-emfs. This gave much better results than Figure 5.3, but neglected harmonic stator-stator coupling.

Figure 5.3 Simulated results from DQ synchronous reference frame (left) and natural reference frame (right) for (a) 5-phase polygon short-pitched and (b) 15-phase polygon short-pitched generator-rectifier system, under 8A field excitation and 10Ω DC resistive load respectively.

Extending higher order harmonic equations in DQ synchronous reference frame does not simplify the model compared with the natural reference frame but will result a complicated model with algebraic loops in Simulink environment. For example, considering $x_1y_1$ and $x_2y_2$ planes for the 15-phase generator model, additional
equations would be as the following, where (5-28) is for \(x_1y_1\) plane and (5-29) is for \(x_2y_2\) plane. To fully represent the 15-phase winding model, 6 additional higher order harmonic planes (from \(x_1y_1\) to \(x_6y_6\)) requiring equations similar as (5-28) and (5-29) are needed, which increases the model complexity against the natural reference frame model based on (5-21).

\[
\begin{bmatrix}
V_{sx1} \\
V_{sy1}
\end{bmatrix} + \begin{bmatrix}
R_{s} & 0 \\
0 & R_{s}
\end{bmatrix}\begin{bmatrix}
I_{sx1} \\
I_{sy1}
\end{bmatrix} + \omega_s\begin{bmatrix}
0 & 13L_{ss}^{y_1} \\
-13L_{ss}^{x_1} & 0
\end{bmatrix}\begin{bmatrix}
I_{s} \\
I_{s}
\end{bmatrix} \\
+ \omega_s\begin{bmatrix}
13L_{sf}^{y_1} \\
-13L_{sf}^{x_1}
\end{bmatrix}I_f + \frac{d}{dt}\left(\begin{bmatrix}
I_{sx1} \\
I_{sy1}
\end{bmatrix}\begin{bmatrix}
I_{s} \\
I_{s}
\end{bmatrix} + \begin{bmatrix}
L_{sf}^{x_1} \\
L_{sf}^{y_1}
\end{bmatrix}I_f\right) = 0
\] (5-28)

\[
\begin{bmatrix}
V_{sx2} \\
V_{sy2}
\end{bmatrix} + \begin{bmatrix}
R_{s} & 0 \\
0 & R_{s}
\end{bmatrix}\begin{bmatrix}
I_{sx2} \\
I_{sy2}
\end{bmatrix} + \omega_s\begin{bmatrix}
0 & 3L_{ss}^{y_2} \\
-3L_{ss}^{x_2} & 0
\end{bmatrix}\begin{bmatrix}
I_{s} \\
I_{s}
\end{bmatrix} + \omega_s\begin{bmatrix}
-3L_{sf}^{y_2} \\
3L_{sf}^{x_2}
\end{bmatrix}I_f \\
+ \frac{d}{dt}\left(\begin{bmatrix}
I_{sx2} \\
I_{sy2}
\end{bmatrix}\begin{bmatrix}
I_{s} \\
I_{s}
\end{bmatrix} + \begin{bmatrix}
L_{sf}^{x_2} \\
L_{sf}^{y_2}
\end{bmatrix}I_f\right) = 0
\] (5-29)

5.5.3. Simulink Model

Transforming the machine circuit equation matrix in (5-20) into Simulink model, where the simplified block diagram is shown in Figure 5.4, the integral method on solving current vectors was used, as shown in (5-30).

![Simplified block diagram of Simulink model for generator based on equation (5-30).](image)
\[ I = [L]^{-1} \int (-\dot{\varphi} - [R]I) \, dt \] (5-30)

The Simulink model system diagram is shown in Figure 5.5. The WFSG part solves equation (5-30); inputs to the block are system voltages and the output of synchronous machine block is a vector of currents. The diode rectifier system is modelled in the PLECS® circuit model, where the full description is in section 5.5.4. The stator currents are modelled as controlled current sources in PLECS with the controlled signals coming from outputs of (5-30), the measured phase voltages from these current sources are fed back to the synchronous generator model. Snubber resistors are added to ensure the systems remains fully defined when diode is off. The stator current sources are connected to the diode rectifier with a resistive load on the DC side.
The inductance matrix \( [L] \) is calculated from the offline process, via a MATLAB® script, which is based on the inductance calculation with CHA method detailed described in (3-44) in Chapter 3 (the MATLAB® code is attached in the A.6. Simulation Model and MATLAB Codes), and is copied as the following

\[
L_{w_2w_1} = 2\pi\mu_0 R W k \sum_{l=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \frac{\tilde{C}_{w_1} \tilde{C}_{w_2} \tilde{G}_l^l}{v(l + v)} e^{jk\rho \theta_m}
\]

where \( k \) is a ratio depends on saturation condition and will be covered in Chapter 6; the rest of the symbols are defined in Chapter 3. An example figure of 3-phase, 5-phase and 15-phase stator-stator and stator-rotor winding inductance variation against rotor position is presented in Figure 5.6, more inductance plots are presented in A.7. Inductance Plots.

Figure 5.6 Inductance variation against rotor position for (a) 3-phase phase A stator winding inductance, (b) 3-phase magnetising inductance; (c) 15-phase phase A stator winding inductance and (d) 15-phase magnetising inductance.
The stator, rotor and damper harmonics are selected up to 100 order, where $\nu, \mu = \{-100, -99, ..., -1, 0, 1, ..., 99, 100\}$ and the airgap harmonics is chosen up to 50 order, where $l = \{-50, -49, ..., -1, 0, 1, ..., 49, 50\}$ to compute the inductance matrix $[L]$ so that the CHA model could cover all potential harmonics.

The inductance matrix $[L]$ is made as a look-up table in the Simulink with 512 points over a single electrical cycle, since $2^n$ samples allows efficient and accurate discrete Fourier transform for harmonic analysis. Too many points of resolution would result in high computation time when calculating the offline inductance matrix. 512 sampling points gives enough resolution ($0.7^\circ$ of increment) and an acceptable computation time (3 hours for 3-phase winding inductance, 4 hours for 5-phase and 13 hours for 15-phase).

5.5.4. PLECS Circuit Model

The diode rectifier is modelled in PLECS® circuit built in Simulink in MATLAB® instead of implementing connection matrix method [47] or average value model [126]. The average value model considers the fundamental model only where the CHA includes higher order of harmonics; the connection matrix method for fault tolerance feature analysis is harder to implement, as the connection matrix needs to be changed for all varieties of fault behaviours that need to be assessed. Using a software based model method provides an easier way to change circuit properties.

![Diode rectifier bridge system model with (a) polygon connection and (b) star connection.](image)

Figure 5.7 Diode rectifier bridge system model with (a) polygon connection and (b) star connection.
The original diode rectifier system is shown in Figure 5.7. For each controlled current source, the current signals are from equation (5-30) and the parallel connected source impedances are included to prevent over-specification of the system. Each of the impedances is sized as 1kΩ to limit current errors to less than 0.5% of rated current.

The diode parameter settings in the simulation are based on the physical diode rectifier module (VS-60MT120KPbF), with 1V forward conduction voltage drop and 1.07 V diode turn-on voltage threshold [164], but diode reverse recovery characteristics have been omitted. The experiment measured that the diode reverse recovery has reverse recovery time of 24.6us and peak current of 0.672A under 16A of field excitation, 22% of rated load at 1800rpm, where diode current is at 11.2A peak and 5.2Arms, shown in Figure 5.8 Experimentally measured diode reverse recovery effect for phase A under 16A rotor field excitation along with 2Ω DC load.

Figure 5.9 Simulated generator waveform under 80% of rated power condition without diode recovery (left) and with diode recovery (right).
Figure 5.8. In the example comparison in Figure 5.9, diode recovery is shown to have little effect.

To simplify the configuration of the diode rectifiers, a vectorization technique is implemented for the diode rectifier, which allows easy control on each diode parameters [137]. The simplified diode rectifier system for polygon and star connection from Figure 5.7 is shown in Figure 5.10, where the signal size is shown in square brackets [].

![Figure 5.10 Vectorization on diode rectifier system for (a) polygon connection and (b) star connection in PLECS circuit model.](image)

### 5.5.5. FEMM Model and 3D FEA

The 2D FEA and 3D FEA models for the WFSG are introduced in this section. Initially, the 2D FEA model was used to examine the back-emf waveform extracted from the CHA model. The 2D FEA also provides an idea of how the generator saturates at different winding excitation. The 3D FEA model was implemented to validate the analytical calculation of end winding inductance. The FEA results are useful with combination of the CHA model to evaluate the generator behaviour at the design stage.

#### a. 2D FEA

The FEA model was originally based on [14], starting from the 3-phase geometrical VariCAD drawing. FEA was used to analysis the saturation characteristic under different winding layouts. The model winding was re-designed to account for 3-phase, 5-phase and 15-phase with both fully-pitched and short-pitched layouts.

The model is in a square shaped boundary of side 400mm, where the magnetic potential $A = 0$ at the boundary. The default meshing size is used for this model, and is chosen...
automatically by the program. The default solver ‘Succ. Approx AC’ is chosen with $1 \times 10^{-8}$ of solver precision. The material for generator stator core and rotor are defined as steel. Since we have no data on the exact steel used for the generator, the choice was based on other papers about Cummins generators with a reasonable match to the back-emf [7, 14, 165]. The BH characteristic of the selected steel is shown in Figure 5.12. It can be identified that saturation begins at approximate 1.5T.

Figure 5.11 Figure of FEMM model, where the outer square is the boundary condition (Dirichlet typed) for the solver.

Figure 5.12 BH characteristic of steel used for generator stator and rotor material in FEMM.
b. 3D FEA

The 3D FEA model was setup in COMSOL® Multiphysics to validate the end winding inductance. The geometry was initially drawn in Solidworks® as a 3D model, shown in Figure 5.13, and imported to COMSOL® Multiphysics. To simplify the mesh process and avoid manual adjustment on fine element meshing, the winding in a single slot was modelled as a single coil with effective 7 turns setup in COMSOL® Multiphysics. The winding diameter was set as 5mm instead of filling the whole stator slot, since the larger winding diameter was found to have an issue that the stator slot mesh and winding mesh could intersect. The mesh size was adjusted as a normal sized tetrahedron with a maximum size of 40mm and 7.2mm minimum size.

A sphere-shaped boundary was setup in COMSOL® Multiphysics with 200mm radius, where the generator model was located in the centre. The magnetic potential \( A \) is defined as 0 at the boundary.

The Magnetic Field physics was added and Ampere’s Law was applied to the entire model. The coil feature was implemented for the single phase winding and a small lumped cylinder port (10mm in length and 5mm in diameter) was manually added to
simulate current injection into the phase windings. The Coil Geometry Analysis study was added to further compute end winding inductance through the magnetic energy, based on the equation shown below

\[ L = \frac{2W}{I^2} N^2 \]  

(5-31)

where \( W \) is the total magnetic energy from coil, \( I \) refers the coil current and \( N \) for number of turns per coil.

The results of the 3D-FEA computed leakage inductance have already presented in Chapter 4.

### 5.5.6. Fourier Analysis

The Fourier analysis implemented throughout the project is done via the discrete Fourier transform, shown as (5-32) [166]

\[ F(n) = \sum_{k=0}^{N-1} f[k] e^{-j\frac{2\pi nk}{N}}, \quad n = 0: N - 1 \]  

(5-32)

where \( N \) is the number of sample points for function \( f \), and is chosen to span an integer number of cycles.

The phase of the harmonic content is found by comparing the relative phase against the fundamental component, and it is assigned as zero whenever the harmonic component is smaller than 1% of the fundamental component.

### 5.6. Summary

The simulation models, including complete GRS model in the Simulink combined with PLECS, 2D and 3D FEA models, are presented in this chapter. Vectorization of PLECS circuit helps to simplify the parameter configurations on the diodes. The diode reverse recovery effect can be ignored in this thesis as the diode is operating under lower switching frequency.

The conventional synchronous reference frame DQ model is suitable for 3-phase system where there are no additional higher order sets. For an uncontrolled GRS, the DQ model
is no longer suitable for the systems with higher number of phases, unless the higher order sets are included in the model. Implementing higher order sets has the same order of dimension as in the stationary reference frame, but needs to consider the cross coupling between different sets and avoid algebraic loops. Therefore, it is easier and safe to present the uncontrolled GRS in stationary reference frame rather than synchronous reference frame.
Chapter 6. Experimental Validation

6.1. Introduction

This chapter describes the experimental test rig. Experimental results are compared with model predictions to check end winding inductance values. Back-emf measurements are used to find a saturation factor. The generator is first tested with a three-phase AC load to validate saturation modelling. The diode rectifier is then added to evaluate the full generator-rectifier system modelling.

6.2. Experimental Test Rig

The experimental test rig, shown in Figure 6.1, was previously set up for evaluating active converter control of the multiphase generator [167]; the wiring system has now been changed to by-pass the multiphase inverter system, and the system schematic structure is presented in Figure 6.2.

The Cummins BCI162G generator module (the multiphase generator in Figure 6.2) used in this project, was originally a 3-phase, single pole pair machine with concentric winding layout short-pitched by \( \frac{\pi}{3} \), rated at 3000rpm, 50Hz, 22kVA and 380V, with an automatic voltage regulator (AVR) SX460 mounted on the stator side to control the rotor field current, where the output of AVR is fed to the main rotor winding through a brushless exciter and 3-phase full wave bridge rectifier [168], shown in Figure 6.3, where the functionality of the excitation system is documented in the research [169, 170].
The stator windings of the original generator have been rewound to a reconfigurable distributed double layer winding, with 7 turns per coil rather than 8 turns as before [14]. The field winding excitation system was originally brushless under the control of AVR, but has been replaced with a brushed contactor allowing for direct control of field current, as shown in Figure 6.4. The generator stator winding terminals are brought out to a fixed panel for setting connections as either fully-pitched or short-pitched winding distribution for 3/5/15-phase, where the generator stator winding diagrams are attached.

![Figure 6.3 Photo of original Cummins generator BCI162G with rotor been taken out.](image)

Figure 6.4 Rewound generator stator winding (left) and rotor winding with brushed contactor (right).

The multiphase generator is driven by a DC motor whose armature winding is powered by another DC generator in a Ward Leonard arrangement shown in Figure 6.2. The DC generator is driven by the 3-phase induction motor through the grid. Two DC power supply units (PSUs) provide the field current for the DC motor and DC generator separately. An additional circuit breaker is connected between the armature winding and DC motor for overspeed protection, where the breaker is controlled by the field current for the DC motor. The detail of overspeed protection is attached in A.9. Test Rig Configurations.
The speed control for the DC motor (2) is open loop by adjusting the field current of DC generator (1) manually. There is no speed controlled feedback or optimised speed control in either the experiment or simulation model.

Five sets of 3-phase diode rectifier blocks (VS-60MT120KpbF), rated at 60A peak, are fixed to a patch panel where the multiphase generator stator windings can be reconfigured. Multiple resistor banks (PENTAGON) are used as the DC link load for the diode rectifier set and can be configured as a minimum of 2Ω (all in parallel), with 4kW and 10Ω for each resistor bank so 20kW in total.

Three differential voltage probes (TA041) are used to measure generator phase voltage and DC link voltage, where phase voltages are measured under the range of ±70V (±70V\(_{\text{peak}}\) for AC or 70V\(_{\text{rms}}\) for DC) with 0.7mV\(_{\text{rms}}\) noise level. Five current transducers (LA55-P), capable of measuring 70A peak, are fixed in the panel for generator stator current and diode rectifier current measurement. Meanwhile, a handheld current probe (LEM PR 30), maximum of 30A peak measurement, as a supplementary current measurement, is used to handle current measurements below 30A peak. All the measured voltage and current results are displayed and recorded on a Tektronix oscilloscope (TDS2024C), and further processed by MATLAB®.

The speed measurement is based on the speed sensor (RADIO-ENERGIE RE.0444 R1B 0.06EG) mounted on the DC motor, where the precision is 60V/1000rpm, where the readings are displayed on a multi-meter (FLUKE 115). To avoid mechanical system resonance, the generator is set to operate under the limit up to 1800rpm. A torque limiter (Autogard series 320) is for mechanical coupling on the shafts between DC motor and multiphase generator.

An encoder (HENGSTLER R158-O/5000AS.41RB, single pole pair) is mounted on the multiphase generator shaft, whose index output signal is used to trigger the data logging on the oscilloscope as well as identifying the D-axis position of the rotor for the Park transform in the further analysis.
6.3. Position Initialisation

The rotor D-axis position (zero position) is found by comparing the back-emf waveform between experiment and the CHA method. From the oscilloscope recording, the back-emf is triggered at $t=0$s in Figure 6.5; while from CHA method, the rotor D-axis position is in the centre of the zone where $V_{emf}=0$. Therefore, the D-axis positions on experimental measured back-emfs are marked as light blue for 3-phase, pink for 5-phase and yellow for 15-phase in Figure 6.5. The oscilloscope records 2500 data points through a full-time scale, e.g. 2500 samples in 50ms at 1800rpm operation, gives 1667 samples in a single period. The data from the oscilloscope is then over-sampled as 5000 samples within 50ms in MATLAB® with linear interpolation to increase the resolution when phase shifting by points, leaving 0.11 degree of resolution per sample.
6.4. Calculation of DQ Quantities

DQ quantities are used in the model to compute the saturation factor. Measured DQ values are used to validate the models. Because the test rig does not have 15 channels of simultaneous current and voltage measurements, DQ values are derived using two channels measuring the phase shift between two adjacent phases and all phase sequence are further manipulated by assuming the balanced generator operation.

Figure 6.6 Example of manipulated 15-phase voltage waveform and corresponding DQ quantities under 10.23kVA, 1800rpm operating condition.

The waveform is then duplicated and phase shifted to create all the phase sequence. This assumes the measurement is under balanced condition. An example figure of manipulated phase winding back-emf against rotor position waveforms is presented in Figure 6.6. The DQ quantities are then extracted from the manipulated waveforms based on the Clark and Park transforms in Chapter 5.

To validate the correctness of the DQ-axis mapping, the voltages are investigated instead of current, since the modelled current may be affected by the accuracy of

<table>
<thead>
<tr>
<th>Polygon connection</th>
<th>Vd (V)</th>
<th>Vq (V)</th>
<th>Star connection</th>
<th>Vd (V)</th>
<th>Vq (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim 15-phase FP</td>
<td>56.22</td>
<td>98.37</td>
<td>Sim 15-phase FP</td>
<td>12.33</td>
<td>161.84</td>
</tr>
<tr>
<td>EXP 15-phase FP</td>
<td>52.47</td>
<td>98.17</td>
<td>EXP 15-phase FP</td>
<td>16.35</td>
<td>167.07</td>
</tr>
<tr>
<td>Error</td>
<td>7.1%</td>
<td>0.2%</td>
<td>Error</td>
<td>24.6%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Sim 15-phase SP</td>
<td>52.25</td>
<td>99.09</td>
<td>Sim 15-phase SP</td>
<td>17.47</td>
<td>156.12</td>
</tr>
<tr>
<td>EXP 15-phase SP</td>
<td>49.04</td>
<td>96.56</td>
<td>EXP 15-phase SP</td>
<td>17.74</td>
<td>161.77</td>
</tr>
<tr>
<td>Error</td>
<td>6.5%</td>
<td>2.6%</td>
<td>Error</td>
<td>1.5%</td>
<td>5.9%</td>
</tr>
</tbody>
</table>

Table 6.1 DQ-axis voltage comparison of 15-phase generation with diode rectifier loading under 80% of rated loading at 1800rpm.
modelling changes in the magnetising inductance due to saturation.

From Table 6.1, it can be identified that the DQ-axis mapping method provides a good match against the experiment under heavy loading where saturation occurs, where the voltage difference is consistent which gives around 4 to 5V either on Vd or Vq.

6.5. Experiment Validation

The validation of the CHA method mainly involves comparison of model result with experimental data for both open-circuit and loading conditions:

1. For the open-circuit condition, the back-emf waveform is compared between experiment, CHA method and FEA results.
2. The generator is configured for 3-phase and 5-phase with a balanced load of 10Ω per phase, showing CHA is capable of predicting generator behaviour with resistive loading.
3. The generator is then operated with a passive diode rectifier with a resistive load to simulate the DC system operation in the aircraft generation system. The validation is split into light load (up to 33% of rated load at 1800rpm) and heavy load (up to 80% of rated load) condition for FP and SP, star and polygon connection.

6.5.1. Back-emf Validation

In this section, the simulated back-emf from the CHA method is compared against the experimental measured result. The experimental measurement is split into two sections, measuring stator terminal back-emf when the generator is at no load and computing the total induced stator voltage when the generator is operating under balanced resistive load. The comparison of experimental measured results includes the simulations from the CHA and FEA methods. The computation of saturation factor is firstly presented, followed by the open-stator terminal back-emf waveform comparison against the CHA and FEA; the total induced stator voltage for 3-phase and 5-phase under balanced resistive load is also presented in this section.
a. No load

The back-emf measurement is based on standard criteria [84], where the measurement is of a single phase winding under star connection for both fully-pitched and short-pitched layout. Using a Power Analyser (LEM NORMA D6000) with 1MHz bandwidth, the extracted harmonic magnitudes from the Power Analyser for different excitation field currents (up to 12A) are presented in Figure 6.7 for a 15-phase winding. The generator is operating under 1.0krpm for this measurement. The measured results are presented as rms magnitudes.

![Figure 6.7 15-phase back-emf along with its harmonic content variation against field excitation for fully-pitched (left) and short-pitched (right) winding layout under 1.0krpm.](image)

For the fully-pitched condition, the total harmonics (V_{rms}) and fundamental (V_{1rms}) follow the same trends, which begin to show saturation after 4A of field excitation. For the other harmonics, i.e. 5th harmonic (V_{5rms}) and 7th harmonic (V_{7rms}), the saturation characteristic happens earlier than the fundamental component, at around 3A of field excitation, but still follows similar trends to fundamental voltage. It is obvious that the 3rd harmonic shows a different saturation behaviour, as this is general case for core saturation where the 3rd harmonic (V_{3rms}) becomes the most significant part that influences the shape of waveform [171]. The short-pitched condition has similar behaviour to the fully-pitched condition, except the 5th harmonic (V_{5rms}) has been suppressed due to the \( \pi/5 \) short-pitched layout. Besides, the fundamental voltage (V_{1rms}) is closer to the total voltage (V_{rms}) for the short-pitched winding, as the magnitude of higher harmonics (V_{3rms}, V_{7rms}) are generally smaller for the fully-pitched windings, which are slightly suppressed by the pitch factor in Chapter 3.
\[ K_p(v) = \sin \left( \frac{v\alpha}{2} \right) \]  \hspace{1cm} (6-1)

where \( \alpha = \pi \) for fully-pitched and \( \alpha = \frac{4\pi}{5} \) for short-pitched in 15-phase, therefore, the 3rd harmonic is suppressed by 59% and 7th by 59%.

In Chapter 3, the saturation factor has not been considered, as from the CHA model, the computed back-emf is proportional to the field excitation, which does not match from the experiment measurement. The saturation factor is therefore introduced by comparing the back-emf between CHA model and experiment, shown in Figure 6.8, where the experimental measurement of rms back-emf is extended with the field excitation up to 15A.

Figure 6.8 Comparison between CHA calculated back-emf and experimental measured one.

[104] suggests that the both rotor and stator teeth saturation and the core saturation cause the rise of the third harmonic MMF, however, the teeth saturation results in a flattened air-gap flux and the core saturation results in a peak shaped air-gap flux, due to different phase relationship for the third harmonics.

[119, 124] have been suggested either using two separate saturation factors for the D- and Q-axes or a single saturation factor depending on total magnetic flux vector, \( \lambda_t = \sqrt{\lambda_d^2 + \lambda_q^2} \). The saturation factor is computed as (6-2), where the ratio \( k_{sat} \) is presented in Figure 6.9 as a function of the field excitation.

\[ k_{sat}(i_f) = \frac{E_{exp}}{E_{CHA}} \]  \hspace{1cm} (6-2)

where \( i_f \) is the field current.
Figure 6.9 Saturation ratio against field current from equation (6-2) for different phase windings (left) and modified saturation ratio for fully-pitched and short-pitched windings, with linear extrapolation (right).

Under low excitation condition, due to the hysteresis effect and the accuracy of the measurement, the computed saturation factor varies between 0.9 and 1.0, while for increasing field excitation, the saturation factor for different phases and windings layout shows similar results, indicating that a single saturation factor could account for different winding layouts either fully-pitched and short-pitched, as shown in the right side plot in Figure 6.9.

As [104, 119, 124] suggested, the generator saturation level depends on both field excitation and stator referred current, including D- and Q-axis. To compute the total saturation level, the stator to rotor referring turns ratio is needed, where the calculation is based on the magnetic flux density with stator/rotor winding energised from CHA, as in (3-31) and (3-34) in Chapter 3.

To simplify the calculation process, the fundamental stator/rotor winding harmonic and up to 2nd order of airgap harmonics are considered, where the detailed calculation process is covered in A.1 Analytical Derivations. The list of referring turns ratio is presented in Table 6.2 where \(i_{sd}\) and \(i_{sq}\) representing stator D- and Q-axis current separately. It shows consistency against results from [165], which are produced from combining winding factor and FEA.
Stator to rotor referring turns ratio \( k_{sr} \)

\[
\sqrt{i_{sd}^2 + i_{sq}^2} = k_{sr} i_r
\]

<table>
<thead>
<tr>
<th>3-phase</th>
<th>Fully-pitched</th>
<th>4.4903</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-pitched by ( \frac{\pi}{3} )</td>
<td>5.1867</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5-phase</th>
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<th>5.6306</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-pitched by ( \frac{\pi}{5} )</td>
<td>5.9172</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>15-phase</th>
<th>Fully-pitched</th>
<th>9.6061</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-pitched by ( \frac{\pi}{5} )</td>
<td>10.1112</td>
</tr>
<tr>
<td></td>
<td>Short-pitched by ( \frac{\pi}{3} )</td>
<td>11.0988</td>
</tr>
</tbody>
</table>

Table 6.2 List of stator to rotor referring turns ratio.

Therefore, the total magnetising current for the saturation factor is

\[
i_f' = i_r + \frac{1}{k_{sr}}(i_{sd} + j i_{sq})
\]

(6-3)

Depending on the Figure 6.9, the saturation factor is obtained from total magnetising current \( i_f' \).

\[
k_{sat} = k_{sat}(|i_f'|)
\]

(6-4)

Figure 6.10 Measured 15-phase back-emf waveform with different field excitation for fully-pitched (left) and short-pitched (right) under 1.8krpm.
The back-emf waveform was then recorded on an oscilloscope, as shown in Figure 6.10. The corresponding harmonic magnitudes and phases are also extracted for a single phase in Figure 6.11.

From Figure 6.11, it is apparent that the short-pitched winding generally has smaller higher harmonic components than fully-pitched. There is a slight drop in percentage magnitude with increasing field current for 5th and 7th harmonic, which indicates that the fundamental for higher field currents is increasing more than the higher order harmonics. The phases of the harmonics do not vary much with the increasing field current.

The back-emf measurement is then compared against FEA and CHA methods, where the back-emf from FEA is extracted by computing the derivative of phase winding flux linkage with respect to rotor position, as in (6-5).

\[ E = \frac{d\psi}{dt} = \frac{d\psi}{d\theta} \frac{d\theta}{dt} = \omega_s \frac{d\psi}{d\theta} \]  

(6-5)

In Figure 6.12, CHA1 refers to the CHA method from (3-45) stated in Chapter 3, which is without both stator skew and improved airgap algorithm; CHA2 refers to the CHA method with stator skew applied only, as equation (4-4); CHA3 refers to the CHA method with both stator skew and improved airgap algorithm; the FEA2 refers to the FEA results from (6-5) with combination of four skew slices (3° shift each sub-slice), while FEA1 refers to the non-skewed FEA results. The original FEA results from FEA1 shows the significant slotting effect which is been removed in experiment, CHA2 and CHA3 due to the stator skew, as well as slicing the FEA model. Under the unsaturated
condition, the improved CHA method from Chapter 4 provides sufficient accuracy against the experiment, although some details on the waveform are different, particularly, the flat top from CHA compared with the peak shape from experiment, which is probably due to the rotor arc saturation and stator slot saturation. The magnetic flux density plot for the condition in Figure 6.12 is presented in Figure 6.13.

Figure 6.12 The comparison on back-emf under unsaturated operation (2A of field current) for (a) 3-phase, (b) 5-phase and (c) 15-phase, fully-pitched (left column) and short-pitched (right column).
In Figure 6.13, the rotor arc is around the knee point for magnetic flux saturation. Since the FEA under-predicts back-emf compared with experiment, it can be concluded that the saturation level on rotor arc point would be higher in the experiment.

![Magnetic flux density plot under 2A of field current excitation.](image)

The back-emf comparison among experiment, CHA and FEA with generator operating under saturated condition (8A of field current) is also presented in Figure 6.14.

The legends in Figure 6.14 are the same as in Figure 6.12, with addition of CHA4 which refers to the CHA method with the saturation factor from Figure 6.9 applied. It is clear from Figure 6.14 that the saturation factor now dominates under this operating condition and the CHA method with saturation factor provides better accuracy than the FEA results, where the FEA gives a slightly lower magnitude than experiment under both unsaturated and saturated condition. It is believed that the under prediction from FEA might be caused from the incorrect material characteristics, since the precise model of actual generator is difficult to obtain. Besides, the stator lamination manufacturing process could also change the material BH characteristic [172].

Both Figure 6.12 and Figure 6.14 indicate CHA method is usable to predict generator behaviour for open-circuit condition with the improvement methods applied.
Figure 6.14 The comparison on back-emf under saturated operation (8A of field current) for (a) 3-phase, (b) 5-phase and (c) 15-phase with fully-pitched (left column) and short-pitched (right column).

b. 3-phase and 5-phase under balanced load

The back emf and saturation modelling is also checked for operation in a loaded condition. Due to the limited number of resistor banks, the validation is only made for 3-phase and 5-phase connections. For each winding layout, either 3 or 5-phase, fully-pitched or short-pitched, the generator stator windings are loaded with balanced 10Ω
resistive load per phase. The star connection for stator and load side is considered in this section as it is easier to access the back-emf term in star winding configuration than delta/polygon configuration due to the presence of circulating zero sequence current.

The comparison is done by computing the total induced voltage in the stator windings, which is \( \frac{d}{dt} ([L] \tilde{I}) \) in (5-20) from the CHA method. The experiment takes measurements of generator stator voltage and current which are used to compute the total induced back-emf as \( V_s + R_{load} I_s \) at the generator stator winding side, where \( R_{load} \) is the 10Ω load.

The FEMM simulation is set up using the pre-defined constant rotor field current. For the load test, stator currents are set to match experimental measurements, accounting for up to 9\(^{th}\) order time harmonics as well as the phase displacement. A zero magnetic potential boundary condition is defined outside the stator outer boundary of FEMM model. To make a reasonable comparison between the three different methods, the total induced back-emf \( \omega_s \frac{d}{d\theta} (\psi) \) is used, where \( \psi \) is the flux in a single-phase winding calculated from the FEMM circuit model and \( \omega_s \) is the synchronous speed. The numerical differentiation used a position resolution of 0.7° and a forward difference method.

The Simulink model solves equation (5-30) and feeds the generator currents into a PLECS circuit model, shown in Figure 6.15. The signals for the controlled current source are the solved generator currents and 1kΩ (0.5% of load current) parallel resistors are placed around the current source, operating as a source impedance, which is required from PLECS to run the simulation successfully.

![Figure 6.15](image-url)
An example of comparison of total induced voltage from the CHA method, experimental measurement and FEA analysis for a 5-phase star connection under fully-pitched condition with balanced load is presented in Figure 6.16.

![Total induced back-emf comparison for CHA, FEA and experiment test for 5-phase fully-pitched winding layout under unsaturated (2A field current, left) and saturated (8A field current, right) condition.](image)

Generally, the CHA can predict the induced voltage with a relatively good match. The FEA method under-predicts the voltage waveform. The CHA has a better match on the unsaturated condition while under saturated condition, the CHA also under-predicts the voltage waveform than experiment, but is slightly higher than the FEA. The waveforms between CHA and experiment are quite similar under unsaturated conditions. Due to the harmonic variations under the saturated case, the waveform from CHA shows the voltage peak appearing at a different rotor position.

Further comparisons between CHA and experiment are presented in the following, where the generator voltage and current waveform are compared in Figure 6.17 - Figure 6.18 and Table 6.3 - Table 6.4, where the ‘Vs’ and ‘Is’ refer to phase A generator phase

<table>
<thead>
<tr>
<th></th>
<th>Vs (V/fund)</th>
<th>Vs (V)</th>
<th>Is (A/fund)</th>
<th>Is (A)</th>
<th>Vd (V)</th>
<th>Vq (V)</th>
<th>Id (A)</th>
<th>Iq (A)</th>
</tr>
</thead>
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<td>14.5</td>
<td>108.1</td>
<td>217.3</td>
<td>10.9</td>
<td>21.9</td>
</tr>
<tr>
<td>EXP 3-phase FP</td>
<td>151.4</td>
<td>155.2</td>
<td>14.8</td>
<td>15.1</td>
<td>112.9</td>
<td>237.9</td>
<td>11.2</td>
<td>23.1</td>
</tr>
<tr>
<td>Error</td>
<td>7.7%</td>
<td>7.6%</td>
<td>4.7%</td>
<td>4.0%</td>
<td>4.3%</td>
<td>8.7%</td>
<td>2.7%</td>
<td>5.2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Vs (V)</th>
<th>Is (A)</th>
<th>Vd (V)</th>
<th>Vq (V)</th>
<th>Id (A)</th>
<th>Iq (A)</th>
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</thead>
<tbody>
<tr>
<td>CHA 3-phase SP</td>
<td>134.8</td>
<td>135.0</td>
<td>13.6</td>
<td>81.7</td>
<td>219.2</td>
<td>8.2</td>
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<tr>
<td>EXP 3-phase SP</td>
<td>145.0</td>
<td>145.1</td>
<td>14.2</td>
<td>85.8</td>
<td>236.0</td>
<td>8.5</td>
</tr>
<tr>
<td>Error</td>
<td>7.0%</td>
<td>7.0%</td>
<td>4.2%</td>
<td>4.8%</td>
<td>7.1%</td>
<td>3.5%</td>
</tr>
</tbody>
</table>

Table 6.3 Voltage and current rms data from Figure 6.17 in comparison between CHA method and experiment measurement.
voltage and current respectively; in Table 6.3, ‘fund’ refers to the fundamental term and ‘Vd’, ‘Vq’, ‘Id’ and ‘Iq’ refer to the DQ-axis quantities in terms of generator stator voltages and currents; all the data in Table 6.3 and Table 6.4 are presented as rms values.

Figure 6.17 Voltage and current waveform for 3-phase star connection with balanced load under saturated condition (8A of field) in fully-pitched (top) and short-pitched (bottom) layouts for measurements (left) and CHA (right).

The 3-phase under saturated field supply with balanced load, where waveform is shown in Figure 6.17, the CHA method predicts both voltage and current waveform in a good match in terms of general trends and peak magnitudes, the in-depth analysis on the voltage and current rms data in Table 6.4 showed the CHA provides reasonable prediction of the machine behaviour under balanced load with saturation occurs, where the errors all lie within 10%.

The same conclusion is found for 5-phase under saturated condition with balanced load, where waveform and corresponding rms data are shown in Figure 6.18 and Table 6.4 respectively.
Figure 6.18 Voltage and current waveform for 5-phase star connection with balanced load under saturated condition (8A of field) in fully-pitched (top) and short-pitched (bottom) layouts for measurements (left) and CHA (right).

<table>
<thead>
<tr>
<th></th>
<th>Vs (V/fund)</th>
<th>Vs (V)</th>
<th>Is (A/fund)</th>
<th>Is (A)</th>
<th>Vd (V)</th>
<th>Vq (V)</th>
<th>Id (A)</th>
<th>Iq (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHA 5-phase FP</td>
<td>100.2</td>
<td>103.7</td>
<td>10.1</td>
<td>10.4</td>
<td>65.4</td>
<td>214.5</td>
<td>6.6</td>
<td>21.7</td>
</tr>
<tr>
<td>EXP 5-phase FP</td>
<td>109.1</td>
<td>111.2</td>
<td>10.7</td>
<td>10.9</td>
<td>60.0</td>
<td>236.7</td>
<td>6.0</td>
<td>23.2</td>
</tr>
<tr>
<td>Error</td>
<td>8.2%</td>
<td>6.7%</td>
<td>5.6%</td>
<td>4.6%</td>
<td>9.0%</td>
<td>9.4%</td>
<td>10.0%</td>
<td>6.5%</td>
</tr>
<tr>
<td>CHA 5-phase SP</td>
<td>98.4</td>
<td>99.2</td>
<td>9.9</td>
<td>10.0</td>
<td>58.4</td>
<td>212.4</td>
<td>5.9</td>
<td>21.5</td>
</tr>
<tr>
<td>EXP 5-phase SP</td>
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<td>105.4</td>
<td>10.3</td>
<td>10.4</td>
<td>62.0</td>
<td>225.7</td>
<td>6.2</td>
<td>22.3</td>
</tr>
<tr>
<td>Error</td>
<td>6.1%</td>
<td>5.9%</td>
<td>3.9%</td>
<td>3.8%</td>
<td>5.8%</td>
<td>5.9%</td>
<td>4.8%</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Table 6.4 Voltage and current rms data from Figure 6.18 in comparison between CHA method and experiment measurement.

The further harmonic analysis is presented in Figure 6.19. The main difference between CHA and experiment lies in 5th harmonic content, where CHA significantly over predicts than experiment, and this can be a reason for slight difference in waveform.
The comparison of the 3-phase and 5-phase generator with balanced loading condition demonstrates that the CHA method is capable of predicting generator behaviour with the harmonics that are intrinsic in the generator itself, i.e. no other time harmonics produced from loading. The saturated condition shows the single saturation factor from equation (6-2) and Figure 6.9 is sufficient for the generator under balanced load. Notice that there is a slight difference in the shape of waveform, which results from assumptions of the CHA model itself, as flux is deemed to travel in a radial path and rotor arc saturation and stator tooth saturation have been ignored.

6.5.2. Generator Rectifier System Validation

The previous section shows the CHA method is capable of predicting generator behaviour with a balanced linear load, where no additional time harmonics been introduced. To evaluate the aircraft DC power generation, the generator is tested with a passive diode rectifier in this section. Due to lab restrictions, only the resistive loading on the DC side is considered.

In this section, the 15-phase generator CHA model is assessed. Both polygon and star winding connections are validated for different loading conditions: ‘light load’ as 20% to 33% of rated generator power and ‘heavy load’ as 80% of rated generator power, for both fully-pitched and short-pitched winding layouts. The stator output was connected to 15 parallel-connected diode rectifier sets, along with a resistive load on the DC side. For this validation, the generator performs not only under intrinsic harmonic content from the design layout and saturation effects, but also the time harmonics from diode
rectifier, because of the distortion and commutation effects on the current waveform that are caused by the diode rectifier. The further validation results for the multiple 3-phase and multiple 5-phase configurations are presented in A.10. Additional Experiment Plots.

a. Light load condition

The generator is firstly monitored under light load condition. The power rating for light condition is set at around 3kVA for star connection, which is 33% of rated power under 1800rpm, and 2kVA for polygon connection, which is the 20% of rated power. Both winding connections provide similar DC side power output. The measured powers for four validations are listed in the Table 6.5. For polygon connection, the field current was set as 8A and the DC side load was 40Ω; the same field current was used with the star connection but with a 5Ω DC load to provide similar power output. Notice that the DC load power is higher than the generator active power for polygon fully-pitched case, this is because the measurement is conducted at different time, so the small variations could affect the accuracy.

<table>
<thead>
<tr>
<th>Winding Layout</th>
<th>Generator apparent power S (kVA)</th>
<th>Generator active power (kW)</th>
<th>DC load power (kW)</th>
<th>Power factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon FP</td>
<td>1.83</td>
<td>1.55</td>
<td>1.58</td>
<td>0.85</td>
</tr>
<tr>
<td>Polygon SP</td>
<td>1.86</td>
<td>1.65</td>
<td>1.58</td>
<td>0.89</td>
</tr>
<tr>
<td>Star FP</td>
<td>3.67</td>
<td>2.06</td>
<td>2.04</td>
<td>0.56</td>
</tr>
<tr>
<td>Star SP</td>
<td>3.13</td>
<td>2.09</td>
<td>2.04</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 6.5 Power levels for different winding layout under light load condition.

The measured waveform for generator voltage, generator current, diode current on phase A, as well as DC side voltage and current within two periods of measurement under different winding layouts are presented in Figure 6.20, where Vs, Is (generator) and Is (diode) refer to the generator voltage, current and diode current quantities for the same phase, and Vdc and Idc are the resistive load voltage and current waveforms. The corresponding simulation results are attached on the right-hand side in Figure 6.20 to compare against the experiment measurements. The measured data in rms values for both experiment and simulation are presented in Table 6.6, where ‘fund’ refers to the fundamental component and Idd to the diode current.

The Table 6.6 illustrates that the CHA provides good match compared with the experimental measurement for both fundamental component and rms values. The errors
The waveform in Figure 6.20 shows the diode current and generator current match well, and so do DC side current and voltages, especially under polygon condition. The waveforms from the CHA generally follows the trends against the waveform from the experiments. The main difference lies in the peak shape of phase voltages, as the flat top in the CHA and peak top in the experiment. For the star connection, the phase relationship between stator voltage and stator current show some mismatch, as in Figure 6.20.(c), however, in terms of rms values presented in Table 6.6 shows quite good matches. Further tests show when increasing field excitation, the diode current waveform shows quite big changes in terms of waveform and phase relationship, where detailed analysis is covered in A.11.Limits of the CHA.

Table 6.6 Comparison between experimental test and simulation under light load condition, where ‘poly’ refer to polygon connection and ‘star’ to star connection.
Figure 6.20 Experimental tests (left) compare against simulation results (right) for 15-phase generator under light load condition with (a) polygon fully-pitched, (b) polygon short-pitched, (c) star fully-pitched and (d) star short-pitched.
From the harmonic analysis of the polygon connected generator phase voltage in Figure 6.21, the mainly difference in fully-pitched windings lies in 3rd, 5th and 7th harmonics. In the polygon connection layout, the core saturation causes the 3rd harmonic content is higher in experiment than the CHA prediction, while in the star connected layout, the 3rd harmonic content between CHA and experiment shows a similar level. Figure 6.21 also shows that both 5th and 7th harmonics in fully-pitched winding have higher magnitude in CHA prediction than that from experiment, which is similar to the case for the generator operating under balanced load. The reason for the slight voltage waveform difference between CHA predictions and experiments is that the phase relationships of generator voltage, especially the 3rd, 5th and 7th harmonic, are different, where the plots are presented in A.11.Limits of the CHA.

![Figure 6.21 Stator voltage harmonics comparison on polygon connected windings (left) and star connected windings (right) under light load condition.](image)

In general, the generator current waveforms between CHA and experiment are a close match under the light load condition. This shows the CHA model operates well when the machine is less saturated; however, the machine drives usually works under rated condition, where the machine is highly saturated. The validation on heavy loading condition is presented in the following section.

**b. Heavy load condition**

The validation is repeated under the heavy load condition, where generator is operating around 80% of its rating at the speed of 1800rpm, with proper adjustment on the resistor bank. The power quantities are listed in Table 6.7. To maintain similar level of power output, the generator under polygon connection was operating under 12A of field supply with 3.3Ω DC load and 16A of field supply with 2Ω DC load under star connection.
CHAPTER 6 EXPERIMENTAL VALIDATION

<table>
<thead>
<tr>
<th></th>
<th>Generator apparent power $S$ (kVA)</th>
<th>Generator active power (kW)</th>
<th>DC load power (kW)</th>
<th>Power factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon</td>
<td>FP 10.23 7.89 7.72 0.77</td>
<td>SP 10.12 8.13 7.81 0.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Star</td>
<td>FP 9.92 6.21 6.06 0.63</td>
<td>SP 8.93 6.49 6.29 0.73</td>
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<td></td>
</tr>
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</table>

Table 6.7 Power levels for different winding layout under heavy load condition.

The measured and simulated waveforms for generator phase voltages, current, diode current and DC side voltage and current are presented in Figure 6.22, as well as the corresponding data in Table 6.8, where the legends and notations remain the same as the ones presented in ‘light load’ condition section.

The Figure 6.22 and Table 6.8 show that the polygon connection of short-pitched has the best match. The generator phase current in polygon fully-pitched and generator phase voltage in star connections show quite similar waveforms. Although the DC side voltages and currents are generally lower in simulation than that in experiment for star connection, this can be considered within the error of tolerance, since the resistance of resistor bank could rise due to high temperature, which depends on the time span of the experiment.

<table>
<thead>
<tr>
<th></th>
<th>Vs $\text{(V/fund)}$</th>
<th>Vs $\text{(V)}$</th>
<th>Is, Idd $\text{(A/fund)}$</th>
<th>Is, Idd $\text{(A)}$</th>
<th>Vdc $\text{(V)}$</th>
<th>Idc $\text{(A)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly CHA FP</td>
<td>29.5 34.2 21.6 22.3 8.9 11.3 167.5 50.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poly EXP FP</td>
<td>28.8 32.8 20.2 20.8 8.2 10.6 165.3 46.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Poly CHA SP</td>
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<td></td>
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</table>

Table 6.8 Comparison between experimental test and simulation under heavy load condition.
Figure 6.22 Experimental tests (left) compare against simulation results (right) for 15-phase generator under heavy load condition with (a) polygon fully-pitched, (b) polygon short-pitched, (c) star fully-pitched and (d) star short-pitched.
Compared with the light load condition, the rms data in Table 6.8 show the CHA has better match compared with the experiments in heavy load condition for the star connections. The generator voltage and current quantities have the best fit compared with other quantities. The highly distorted diode current still gives relative higher error. The overall errors are still within 10% and showing the CHA can predict the behaviour of the GRS well.

It is also obvious that the polygon fully-pitched phase voltage and diode current waveforms are different between the CHA and experiment, where both phase voltage and diode current in experiment show double peak feature while this is not found in CHA. The diode current in star connections also show differences between experiment and CHA: the diode current peak location of star fully-pitched is phase advanced in experiment while this is not in CHA; the diode current has wider time span but lower peak for star short-pitched in experiment while the CHA shows narrower but high peak diode current. The further analysis of diode current variation is covered in A.11. Limits of the CHA.

From FEA analysis, shown in Figure 6.23, the core saturation level is lower in the fully-pitched winding layout than that in short-pitched winding layout, while both winding layouts have high saturation levels in the rotor arc tips. Since the saturation factor from equation (6-1) depends on the core saturation, mainly from the D-axis but does not consider saturation in teeth, the CHA method provides better flux density prediction for

![Figure 6.23 FEA magnetic flux density plot for generator operation polygon FP (left) and polygon SP (middle) and together with colour legend (right), where the rotor position is aligned to stator D-axis.](image)
the short-pitched polygon connection than that on fully-pitched polygon connection in Figure 6.22.

The harmonics in Figure 6.24 shows the CHA follows the same trends as from experiments, and the main difference lies in $5^{th}$ harmonic which could be the insufficient modelling of saturation. Notice that although the short-pitched winding layout is carefully designed to cancel $5^{th}$ harmonic in the stator winding and the rotor geometry is designed to remove $5^{th}$ space harmonic from rotor field, there is still a $5^{th}$ time harmonic component in generator voltage for the short-pitched winding.

![Figure 6.24 Stator voltage harmonics comparison on polygon connected windings (left) and star connected windings (right) under heavy load condition.](image)

More comparisons on different generator loading conditions are covered in the A.10.Additional Experiment Plots.

### 6.6. Summary

The lab test rig and validation results have been presented in this section. Validation of both open-circuit back-emf measurements and light and heavy load measurements show quite good match. The CHA method generally provides a close match against experiment under light loads and an acceptable match for heavy loads. The unsaturated condition gives a more accurate result for CHA while the saturation factor could not predict the machine behaviour as well with highly saturated teeth. At the detailed design stage, FEA would be expected to better accuracy than CHA, but with longer computation times than CHA. Other assumptions, including accuracy of leakage inductance values and assumption of radial flux in the airgap may also contribute to the mismatch to a lesser extent. Despite those minor mismatches, the model fit is good
enough to allow a comparison between generator connection topologies for initial designs.

The back-emf validation indicates that it is necessary to model saturation to improve the CHA. The static FEA results are acceptable to find saturation factor if the experimental values are not accessible. The single saturation factor is sufficient to model the salient generator at the power rating analysis level, but the cross coupling could be non-negligible when there are high D-axis and Q-axis currents in the generator. Further study is recommended on the saturation modelling for the wound field salient machines.

GRS models show the single saturation factor is adequate but not perfect, although the FEA provides more accuracy, the dynamic analysis from FEA is tedious work and time consuming, where the CHA method begins to show its advantage. The model identifies some harmonic mismatch, particularly 5\textsuperscript{th} harmonic. The CHA model works well for the generator with light load, where similar results have been shown in [14] with a simpler model. Under heavy load conditions, discrepancies lie in the waveform shapes and harmonics, which are sensitive to the excitation level, which might be caused from simplifying assumptions such as radial flux path, a single saturation factor, neglect of stator slotting and teeth saturation effects, but the CHA model is still capable of predicting fundamental behaviour and DC quantities since the saturation factor takes the fundamental behaviour into account. In general, the validation in this chapter indicates the CHA model is good enough to let us compare the generator behaviour under different winding layouts in the next chapter. In future work, the detailed behaviour of generators could be obtained by combining FEA results.
Chapter 7. Complex Harmonic Analysis Model Application

7.1. Introduction

In this chapter, the CHA method in combination with the Simulink models is further expanded to predict the behaviour of generator with different winding configurations. In a diode rectifier system, winding connections are the only things to modify to investigate the systems behaviours, once the generator design is fixed. Both the split-phase and the 15-phase winding configurations are considered under healthy and different open-circuit fault conditions. The generator current and diode current variations are compared for different layouts under fault conditions. In order to continue using the generator with the faulted winding excluded, the split-phase winding layouts are examined with the faulted winding set disconnected from the diode rectifier sets. Diode rectifier arrangements of series stack and parallel stack for different split-phase windings are also examined.
7.2. Rectifier Connection Options

The different winding layouts, i.e. split-phase windings with series or parallel stacked diode rectifier sets are considered. For the controllable generator converter systems, the current waveforms could be manipulated by controlling the active controlled power electronic devices; for the uncontrollable passive diode rectifiers, the only way to

Figure 7.1 Winding arrangement for three sets of 5-phase in (a). polygon connection, (b). star connection and five sets of 3-phase in (c). delta connection and (d). star connection.

Figure 7.2 Diode rectifier with series stacked for (a). three sets of 5-phase and (b). five sets of 3-phase, as well as parallel stacked for (c). three sets of 5-phase and (d). five sets of 3-phase.
change the GRS performance is how the windings are set up and this is the easier way to change the GRS performance without redesigning the generator.

The model is therefore used as a design tool to analyse machine behaviour under different winding layout. The 15-phase model is used as a baseline for comparison. The machine can be reconfigured as 5-phase machine with $\frac{2\pi}{5}$ phase shift between each adjacent phase and three sub-machines in total $(3 \times 5\varphi)$, each sub-machine is shifted by $\frac{2\pi}{15}$; another configuration is five sets of 3-phase machines $(5 \times 3\varphi)$, where each phase is separated by $\frac{2\pi}{3}$ in 3-phase machine and $\frac{2\pi}{15}$ phase shift between adjacent sub-machine.

For short-pitched winding connection, the each 5-phase sub-machine is short-pitched by $\frac{\pi}{5}$ and $\frac{\pi}{3}$ for each 3-phase sub-machine. The different winding connections are shown in Figure 7.1. notice that in the star connection for split-phase models, each phase set has individual ground point, and these ground points are not commonly connected. There is another option for 5-phase winding, pentacle connection [70, 173, 174], but is omitted for in this thesis as the model is illustrating the principle that the CHA model could roughly compare GRS performance under different winding layouts.

The diode rectifier connection is set as either in series or parallel connection among different sub-machine sets, shown in Figure 7.2.

### 7.3. Model Setup

The winding inductances are kept unchanged from the ones computed in Chapter 3 and Chapter 6 chapter while the reconnections for the windings and diode rectifier sets are made in the PLECS circuit instead.

#### 7.3.1. Saturation Ratio in Split-phase Model

Since each sub-machine phase winding consist of only two coils, as in the 15-phase machine, the stator referred ratio is chosen the same as the 15-phase one found in section 6.5.1.

For the 15-phase model,

$$I_r = |I_f + k_1I_{sd} + jk_1I_{sq}|$$

(7-1)
For three sets of 5-phase (3×5φ) model,

\[ I_r = |I_f + k_2(I_{sd1} + jI_{sq1}) + k_2(I_{sd2} + jI_{sq2})e^{j\gamma} + k_2(I_{sd3} + jI_{sq3})e^{j2\gamma}| \]  

where \( \gamma = \frac{2\pi}{15} \), \( I_{sd1,2,3} \) and \( I_{sq1,2,3} \) are defined as the DQ currents in each subset and each subset is phase shifted by \( \gamma \).

For five sets of 3-phase (5×3φ) model,

\[ I_r = |I_f + k_3(I_{sd1} + jI_{sq1}) + k_3(I_{sd2} + jI_{sq2})e^{j\gamma} + \ldots + k_3(I_{sd5} + jI_{sq5})e^{j4\gamma}| \]  

where the coefficient \( k_1 = k_2 = k_3 \) for fully-pitched winding arrangement, and \( k_1 = k_2 \neq k_3 \) for short-pitched winding layout, as the 3-phase model is short-pitched by \( \frac{\pi}{3} \) rather than \( \frac{\pi}{5} \) for 5-phase and 15-phase model. Different stator/rotor referring turns ratio \( k_i \) are listed in Table 6.2 in the Chapter 6.

### 7.3.2. Simulated DC Link Power vs Load Resistance

To compare the machine behaviour between different winding and rectifier arrangements, the targeted DC link power was set as 15kW, and the field current was set to operate at 20A, which is slightly under the maximum level (23A) due to the thermal ratings, as some of the winding/rectifier arrangements could not achieve targeted DC power level under lower field current. The load resistance for target power is selected, so that the system with different winding layouts are operating under same output DC power but with different load resistance. An example of power capability curve for different topologies under series stacked is shown in Figure 7.3. The curve is obtained by computing DC side power for various DC side resistances with liner interpolation curve fitting. From Figure 7.3, it is obvious that each topology provides two operating points for targeted DC power; only the ones with lower generator current and higher generator voltage have been selected to compare since the other operating point exceeds the stator maximum current for thermal ratings. Figure 7.3 also allows the peak power capability to be compared.
7.4. Model Comparison with 15kW DC Consumption under Healthy Operation

In this section, the generator-diode rectifier system performance with different winding configurations, i.e. 15-phase and split-phase layout, are compared under healthy condition. The comparison focused on the generator-diode rectifier waveforms, generator voltage/currents and diode current as well as DC voltage/currents. The generator power quality including stator copper loss and power factor are also compared in this section. The comparison provides the rough ideas on how the winding configurations could affect the generator diode rectifier performance. The conclusions drawn from the comparison are in general and not specifically limited to the generator in this project, therefore these conclusions could be applicable to other multiphase machines with these certain winding configurations.

7.4.1. 15-phase Model

For the 15-phase winding stator geometry, the conventional 15-phase winding layout is initially examined under 15kW operating condition.
Figure 7.4 Simulated generator waveform for 15-phase with (a) polygon connection and (b) star connection, where the left column is for fully-pitched winding layout and the right column for short-pitched layout.

The waveforms are shown in Figure 7.4, where the blue trace represents generator stator terminal phase voltage, the green trace is for generator phase current, red for phase diode current, brown for DC side voltage and cyan for DC side current. All the waveforms are presented in 150V per division of voltage scale and 20A per division of current scale. The time base for the plots is the same, as 10ms per division.

<table>
<thead>
<tr>
<th></th>
<th>Vs (V/f)</th>
<th>Vs (V)</th>
<th>Is (A/f)</th>
<th>Is (A)</th>
<th>Id (A/f)</th>
<th>Id (A)</th>
<th>Vdc (V)</th>
<th>Idc (A)</th>
<th>DC ripple</th>
<th>THDv (Vs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FP</td>
<td>15Φ</td>
<td>72.4</td>
<td>75.6</td>
<td>14.7</td>
<td>15.9</td>
<td>6.0</td>
<td>10.6</td>
<td>460.2</td>
<td>32.5</td>
<td>3.0%</td>
</tr>
<tr>
<td>SP</td>
<td>15Φ</td>
<td>71.5</td>
<td>72.3</td>
<td>14.3</td>
<td>15.7</td>
<td>6.0</td>
<td>10.9</td>
<td>473.0</td>
<td>31.7</td>
<td>3.3%</td>
</tr>
<tr>
<td>Star</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FP</td>
<td>15Φ</td>
<td>69.2</td>
<td>71.7</td>
<td>15.2</td>
<td>23.2</td>
<td>15.1</td>
<td>23.2</td>
<td>173.9</td>
<td>86.1</td>
<td>6.4%</td>
</tr>
<tr>
<td>SP</td>
<td>15Φ</td>
<td>67.9</td>
<td>68.8</td>
<td>16.1</td>
<td>24.1</td>
<td>16.0</td>
<td>24.1</td>
<td>175.4</td>
<td>85.5</td>
<td>5.0%</td>
</tr>
</tbody>
</table>

Table 7.1 Voltage and current quantities for healthy operation with 15-phase winding layout.
It shows quite similar waveforms to that presented in Chapter 6. The polygon connections give higher DC voltage but less DC current, which is reversed in star connections. The relevant voltage and current data as well as power quantities are presented in Table 7.1 and Table 7.2, where the total harmonic distortion of generator phase voltage \( THD_v \) is defined as

\[
THD_v = \sqrt{\frac{V_{2rms}^2 + V_{3rms}^2 + \cdots}{V_{1rms}}} \times 100\% \quad (7-4)
\]

The generator active power in Table 7.2 is found by averaging the instantaneous power of the generator over a cycle as

\[
P_{inst} = \sum_{i=1}^{m} v_i(t)i_i(t) \quad (7-5)
\]

where \( m \) is the total phase number.

The short-pitched winding gives significant improvement on the voltage harmonic distortion for both polygon and star connections; the polygon connections have slightly smaller DC ripple than the star connections, however, the differences are within error tolerance of the model. With the same power dissipation on the DC side, the polygon connections have less stator copper loss than the star connections, and consequently higher power factor. It can be noticed that the 15-phase star connections have much higher diode peak current as well as rms currents and this leads to further higher losses in the both diode rectifiers and generators.
The peak power capability shows the polygon connections generally have higher peak power capability than the corresponding star connections and that the short-pitched windings are better than the fully-pitched windings. The polygon connection short-pitched 15-phase has the highest peak power capability at 26.9kW while the star connection fully-pitched has the lowest peak power capability at 21.7kW.

7.4.2. Series Stacked Split-phase Model

The healthy condition of series stacked diode rectifier sets for split-phase GRS is shown in this section, with the waveforms shown in Figure 7.6, corresponding data in Table 7.3 and Table 7.4 and peak power capability plot in Figure 7.7.

Figure 7.6 shows that there is high circulating zero-sequence current in the polygon fully-pitched winding, which is not present in either short-pitched or star connections, and this leads to the higher stator losses and lower power factor, listed in Table 7.4. The polygon fully-pitched 3×5φ and 5×3φ have much higher generator rms currents than the other winding layouts but with less fundamental generator current, leaving them with high stator copper loss and low power factor and peak power capability. Therefore, these two windings are excluded from further discussion.
In general, the short-pitched winding layout reduces the THDv compared with the corresponding fully-pitched winding layout, since the higher order harmonics are generally attenuated due to the short-pitched winding factor. The generator voltage THD can be a crucial factor which may affect the operation of any power electronic devices directly connected to the generator AC voltages [175]. Specifically, the 3×5φ polygon short-pitched winding layout provides the smallest THDv while the worst THDv happens in 3×5φ star fully-pitched layout.

In terms of generator rms phase currents, polygon connected 3×5φ with fully-pitched layout have similar current ratings to the 15-phase star connections. The 5×3φ for both polygon and star connections generally require less current than that in 3×5φ. For the split-phase topologies, the diode current rating in polygon connections requires higher level than in star connections, where the 5×3φ with polygon short-pitched gives the highest and star fully-pitched gives the lowest diode current among these eight split-phase topologies.
Figure 7.6 Simulated generator waveform for $3\times 5\phi$ with (a) polygon connection and (b) star connection; $5\times 3\phi$ with (c) polygon connection and (d) star connection, where the left column is for fully-pitched winding layout and the right column for the corresponding short-pitched layout.
In terms of power quality, both polygon connected $5\times3\varphi$ in short-pitched and star connected $5\times3\varphi$ in fully-pitched have the highest power factor, and the worst solution is polygon connected $3\times5\varphi$ in fully-pitched. The star connected $5\times3\varphi$ in fully-pitched gives the lowest stator copper loss.

As seen in the 15-phase GRS, higher rms generator current with less fundamental generator current results in high apparent power but low power factor and high copper loss. This feature applies to $3\times5\varphi$ and $5\times3\varphi$ polygon fully-pitched configurations due to high circulating currents.

In terms of DC ripple requirement [176], the polygon connected $3\times5\varphi$ provides the best solution on the DC power ripple. The DC ripple for arrangements as star connected $3\times5\varphi$ in fully-pitched, $5\times3\varphi$ in short-pitched are beyond the limit allowed in [165].

The Figure 7.7 shows that the $5\times3\varphi$ star in fully-pitched winding connection provides the highest peak power capability, while the corresponding short-pitched winding connection only provides 76% of peak power capability. In general, the star connected short-pitched winding connection has lower peak power capability than corresponding fully-pitched winding connection, and the polygon connected short-pitched winding connection has higher peak power capability than corresponding fully-pitched winding connection. The difference in peak capability is much larger in $5\times3\varphi$ than that in $3\times5\varphi$. 

![Figure 7.7 DC link power capability for series stacked split-phase machine.](image-url)
Overall, the star connected $5\times3\phi$ in fully-pitched layout provides the highest peak power capability, power factor and lowest stator copper loss, with acceptable DC side ripple and phase voltage harmonic distortion. While the polygon connected $3\times5\phi$ in short-pitched gives the better solution on DC side ripple and phase voltage harmonic distortion, with relative higher power factor and low stator copper loss, however, the peak power capability is only 83% of star connected $5\times3\phi$ in fully-pitched.

7.4.3. Parallel Stacked Split-phase Model

Instead of series stacking the diode rectifiers for the split-phase sequence, the parallel stacking of diode rectifier would also be a choice [78]. The parallel stacked topologies retain the same winding configuration as the series stacked topologies, but with a difference on the diode rectifier configurations.

![Graph showing DC link power overload capability for parallel stacked under healthy condition.](image)

Figure 7.8 DC link power overload capability for parallel stacked under healthy condition.

The trends for DC power overload capability among different split-phase arrangements show the highest peak power capability happens on $3\times5\phi$ with star fully-pitched winding layout, shown in Figure 7.8, which is different from the conclusion drawn from the series stacked topologies. The polygon fully-pitched $5\times3\phi$ still provides the least peak power capability among these split-phase arrangements. Additionally, both the polygon short-pitched and star fully-pitched layout in $5\times3\phi$ under parallel stacked
Figure 7.9 Simulated generator waveform for 3×5φ with (a) polygon connection and (b) star connection; 5×3φ with (c) polygon connection and (d) star connection, where the left column is for fully-pitched and the right column for short-pitched layout under parallel stacked topologies.
layout have lower overload capability than under series stacked layout, the overload capability of other split-phase windings are barely changed with parallel stacked diode rectifiers.

<table>
<thead>
<tr>
<th>Winding Layout</th>
<th>Voltage (Vs) (V)</th>
<th>Current (I) (A)</th>
<th>DC Current (Id) (A)</th>
<th>DC Voltage (Vdc) (V)</th>
<th>DC Ripple (Idc) (A)</th>
<th>THDv (Vs)%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly 3×5φ FP</td>
<td>70.7</td>
<td>24.7</td>
<td>22.5</td>
<td>100.1</td>
<td>2.2%</td>
<td>22%</td>
</tr>
<tr>
<td>Poly 3×5φ SP</td>
<td>68.6</td>
<td>17.4</td>
<td>22.5</td>
<td>100.1</td>
<td>2.2%</td>
<td>11%</td>
</tr>
<tr>
<td>Poly 5×3φ FP</td>
<td>73.8</td>
<td>16.9</td>
<td>16.8</td>
<td>196.1</td>
<td>8.6%</td>
<td>32%</td>
</tr>
<tr>
<td>Poly 5×3φ SP</td>
<td>69.2</td>
<td>18.0</td>
<td>17.9</td>
<td>179.1</td>
<td>1.1%</td>
<td>16%</td>
</tr>
<tr>
<td>Star 3×5φ FP</td>
<td>73.8</td>
<td>16.9</td>
<td>16.8</td>
<td>196.1</td>
<td>8.6%</td>
<td>32%</td>
</tr>
<tr>
<td>Star 3×5φ SP</td>
<td>69.2</td>
<td>18.0</td>
<td>17.9</td>
<td>179.1</td>
<td>1.1%</td>
<td>16%</td>
</tr>
<tr>
<td>Star 5×3φ FP</td>
<td>74.1</td>
<td>19.7</td>
<td>27.3</td>
<td>157.8</td>
<td>7.3%</td>
<td>26%</td>
</tr>
<tr>
<td>Star 5×3φ SP</td>
<td>65.9</td>
<td>20.0</td>
<td>34.7</td>
<td>183.1</td>
<td>5.2%</td>
<td>21%</td>
</tr>
<tr>
<td>Star 5×3φ FP</td>
<td>68.8</td>
<td>17.5</td>
<td>17.5</td>
<td>166.5</td>
<td>5.7%</td>
<td>26%</td>
</tr>
<tr>
<td>Star 5×3φ SP</td>
<td>66.5</td>
<td>17.7</td>
<td>17.6</td>
<td>149.8</td>
<td>10.0%</td>
<td>26%</td>
</tr>
</tbody>
</table>

Table 7.5 Voltage and current quantities for healthy operation with different winding layout with parallel stacked diode rectifiers.

For the split-phase topologies, the parallel stacked arrangement has the similar magnitude level of generator voltage/current and diode current but slightly higher compared with the series stacked diode rectifier layout, and the slightly higher generator current on the parallel stacked topologies result in higher stator losses and lower power factor than the series stacked topologies, shown in Table 7.6. However, the parallel stacked GRS have higher DC current level and lower DC voltage than series stacked GRS, since the series stacked topologies add voltage up while the parallel stacks current up.

<table>
<thead>
<tr>
<th>Winding Layout</th>
<th>Stator Copper Loss (W)</th>
<th>Generator Active Power (kW)</th>
<th>Generator Apparent Power (kVA)</th>
<th>Power Factor</th>
<th>DC Peak Power Capability (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly 3×5φ FP</td>
<td>1274.3</td>
<td>15.32</td>
<td>26.87</td>
<td>0.57</td>
<td>21.0</td>
</tr>
<tr>
<td>Poly 3×5φ SP</td>
<td>644.6</td>
<td>15.30</td>
<td>18.06</td>
<td>0.85</td>
<td>22.7</td>
</tr>
<tr>
<td>Star 3×5φ FP</td>
<td>602.3</td>
<td>15.32</td>
<td>19.65</td>
<td>0.78</td>
<td>23.9</td>
</tr>
<tr>
<td>Star 3×5φ SP</td>
<td>675.2</td>
<td>15.30</td>
<td>18.92</td>
<td>0.81</td>
<td>23.2</td>
</tr>
<tr>
<td>Poly 5×3φ FP</td>
<td>807.5</td>
<td>15.35</td>
<td>22.63</td>
<td>0.68</td>
<td>19.1</td>
</tr>
<tr>
<td>Poly 5×3φ SP</td>
<td>827.4</td>
<td>15.51</td>
<td>18.35</td>
<td>0.85</td>
<td>21.1</td>
</tr>
<tr>
<td>Star 5×3φ FP</td>
<td>621.5</td>
<td>15.32</td>
<td>18.66</td>
<td>0.82</td>
<td>23.0</td>
</tr>
<tr>
<td>Star 5×3φ SP</td>
<td>644.0</td>
<td>15.24</td>
<td>18.23</td>
<td>0.84</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Table 7.6 Power quantities for healthy operation with different winding layout with parallel stacked topologies.

For the split-phase machines with parallel stacked diode rectifiers, the 3×5φ with short-pitched polygon connection gives the least voltage harmonic distortion and fairly small
DC ripples, with highest power factor, acceptable stator copper less and peak power capability, which is 94% of the $3\times5\phi$ with star fully-pitched connection.

7.4.4. Summary of GRS at Healthy Operation

For the same required DC load power output, the 15-phase polygon short-pitched connection seems to be a better option than other 15-phase connections due to reduced THDv, DC ripple, lower peak diode current, high power factor and lower stator copper loss, as well as high peak power capability.

When it comes to split-phase GRS, the series stacked generally performs better than parallel stacked in terms of power factor and peak power capability. The high peak diode current at healthy operation in parallel stacked split-phase GRS makes it less attractive than series stacked GRS, especially for $5\times3\phi$ polygon connections. However, the $3\times5\phi$ polygon parallel stacked short-pitched connection could be an option for the low THDv and DC ripple, high power factor and peak power capability and if the DC load requires higher level of current.

The present of zero-sequence current makes the polygon fully-pitched winding layout the worst option with high THDv and stator copper loss and low power factor.

For the selection of series stacked GRS, the trade-off between higher peak power capability and smaller DC ripple and THDv should be made when selecting either star series stacked $5\times3\phi$ in fully-pitched or polygon series stacked $3\times5\phi$ in short-pitched.

In the general case, the polygon connected winding has higher peak power capability in short-pitched than fully-pitched, due to the presence of the zero-sequence current, while the star connection has higher peak power capability in fully-pitched than short-pitched, since the short-pitched reduces the winding factor and so the terminal voltage.

7.5. Open Circuit Fault Tolerance

The healthy operation from previous section shows the 15-phase polygon short-pitched connection. The generator fault tolerance for different winding configurations should also be examined. Three open-circuit fault conditions are considered in this thesis: the
stator winding open-circuit fault represents stator winding loose connection or broken stator winding; the diode bridge connection open-circuit fault represents the loss of a wiring connection between the generator and diode rectifier system or the complete single diode leg broken; the diode open-circuit fault represents a single broken diode.

The fault condition is simulated by using pre-controlled MOSFETs in the PLECS circuits, shown in Figure 7.10, where three MOSFETs, controlled by pre-defined constants (‘1’ for switching on and ‘0’ for switching off indicating open-circuit fault), are implemented to simulated generator phase winding open-circuit (through ‘winding fault control signals’), generator to diode rectifier bridge connection open-circuit (through ‘bridge fault control signals’) and single diode open-circuit (through ‘diode fault control signals’) conditions. Therefore, only the steady state fault condition is analysed in this research. Notice that for star connections, the stator phase winding open-circuit fault condition at steady state is the same as the bridge open-circuit fault.

Figure 7.10 PLECS circuit diagram of fault location for polygon connection (top) and star connection (bottom).
condition. The DC loads for each fault condition of different winding layouts are at the same resistance as the generator under healthy operating condition at 15kW.

The different fault analyses of different winding layouts are further split into series-stacked diode rectifier sets and parallel stacked diode rectifier sets, which is the similar process as the analysis for GRS at healthy condition.

7.5.1. Single Phase Winding Open-circuit Fault

In this section, the GRS fault tolerance of different winding layouts to a single phase winding open-circuit fault is analysed. Specifically, phase A of the winding set 1 for a split-phase layout is considered as open-circuit, where this corresponds to phase A winding open-circuit fault for a 15-phase winding, where the fault location diagram is shown in Figure 7.11.

Figure 7.11 Single phase winding open-circuit fault location for polygon (left) and star (right) connections.

a. 15-phase model

The generator phase current and diode current variation for 15-phase GRS under single phase A winding open-circuit fault condition is presented in Figure 7.12 and Figure 7.13 respectively. Since the phase current is the same as the diode current for star connections, Figure 7.13 only shows the diode current variation for polygon connections. It is observed that the phase current rises gradually from the faulted winding to the adjacent phases and the maximum is reached at the opposite phase in polygon connections. However, the phase current with greatest change happens on the adjacent phases to the faulted one and the fault has less impact on the opposite phase in
the star connections. However, in terms of diode current variations, the adjacent phases are mainly affected for polygon connections.

![Diagram of phase current variation](image1.png)

**Figure 7.12** Phase current variation under single phase winding open-circuit condition for 15-phase with polygon connections (top two plots) and star connections (bottom two plots), as in fully-pitched (left column) and short-pitched (right column)

![Diagram of diode current variation](image2.png)

**Figure 7.13** Diode current variation under single phase winding open-circuit condition for 15-phase with polygon connections (top two plots), as in fully-pitched (left column) and short-pitched (right column)
Table 7.7 shows the phase current and diode current increase as well as DC side voltage, ripple and power change against the healthy operation in Table 7.1. The ‘maximum phase/diode current’ is found by comparing the highest phase/diode current shown in Figure 7.12 against the corresponding healthy operation. The percentage is found by \( \frac{I_{\text{max}}}{I_{\text{healthy}}} \times 100\% \) with the current difference presented in the brackets, with a similar expression for the DC voltage. The DC power change is found by \( \frac{P_{\text{fault}}}{P_{\text{healthy}}} \times 100\% \).

<table>
<thead>
<tr>
<th></th>
<th>Phase current change</th>
<th>Diode current change</th>
<th>DC power change</th>
<th>DC ripple (%)</th>
<th>DC average voltage change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon</td>
<td>15φ FP</td>
<td>179% (+12.6A)</td>
<td>103% (+0.4A)</td>
<td>95.4%</td>
<td>12.6%</td>
</tr>
<tr>
<td></td>
<td>15φ SP</td>
<td>182% (+13.0A)</td>
<td>107% (+0.7A)</td>
<td>94.4%</td>
<td>9.5%</td>
</tr>
<tr>
<td>Star</td>
<td>15φ FP</td>
<td>123% (+5.5A)</td>
<td>124% (+5.5A)</td>
<td>100.3%</td>
<td>28.4%</td>
</tr>
<tr>
<td></td>
<td>15φ SP</td>
<td>137% (+9.0A)</td>
<td>137% (+9.0A)</td>
<td>98.9%</td>
<td>23.8%</td>
</tr>
</tbody>
</table>

Table 7.7 15-phase single phase winding open-circuit fault comparison.

It can be identified from Table 7.7 that the short-pitched in polygon connection has little bit higher phase current and diode current rise than that in the fully-pitched polygon connection. The current rise for phase current is much smaller in star connections than the polygon connections, while this case is reversed for diode current rise. It is also found that the star connection has better fault tolerance in terms of DC power change than polygon connection but results in a much higher level of DC ripple.

\textit{b. Series stacked split-phase model}

A similar comparison process for series stacked split-phase models is conducted in this section for the 15-phase models. The phase current and diode current variations for phase A in winding set 1 open-circuit fault in series stacked split-phase model are presented in Figure 7.14 to Figure 7.17, as well as the corresponding data in Table 7.8.
Figure 7.14 Phase current variation under single phase winding open-circuit condition for 3×5φ series stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

The split-phase configurations show the same trends as the 15-phase machine on phase current rises gradually from the faulted winding to the adjacent phases and the maximum is reached at the opposite phase; it can also be identified that this feature is more significant for a higher number of phases by comparing 15φ, 3×5φ and 5×3φ polygon connections. Meanwhile, the trend that short-pitched polygon has higher phase current but lower diode current rise than fully-pitched polygon in 15φ is also retained in the series stacked split-phase configuration. This is because the generator current in the healthy fully-pitched polygon configurations is higher due to circuiting zero sequence currents, so the change in rms current is lower than short-pitched polygon configurations.

Specifically, the split-phase has the feature that the phase current only varies significantly in the split-phase set where the winding fault occurs, the other split-phase machines are operating at the healthy conditions.
Figure 7.15 Phase current variation under single phase winding open-circuit condition for 5×3φ series stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

Figure 7.16 Diode current variation under single phase winding open-circuit condition for 3×5φ series stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).
Figure 7.17 Diode current variation under single phase winding open-circuit condition for 5×3φ series stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).

Similar as the generator current, the diode current variation follows the same trend as in 15φ model, where for the polygon connection the diode current variations are smaller than that in the generator phase current. However, for the split-phase, the star connections have higher maximum current rise than the corresponding polygon connections. Meanwhile, polygon short-pitched winding layout has less current change than the polygon fully-pitched winding layout. The highest current rise happens in 3×5φ polygon fully-pitched layout while the least change happens in 5×3φ polygon short-pitched connection.

For the DC side power, same as the 15φ models, the star connections have more DC power drop than the polygon connections and the worst case happens on the short-pitched winding, star connection of 5×3φ, the DC side power drop up to 8.3% at a 26.2% ripple. Comparing against 15φ models, both 3×5φ and 5×3φ, polygon connected short-pitched winding have better performance than the 15-phase polygon connected on the

<table>
<thead>
<tr>
<th></th>
<th>Phase current change</th>
<th>Diode current change</th>
<th>DC power change</th>
<th>DC ripple (%)</th>
<th>DC average voltage change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Polygon</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3×5φ FP</td>
<td>121% (+5.0A)</td>
<td>105% (+0.9A)</td>
<td>100.2%</td>
<td>20.2%</td>
<td>100% (+0.5V)</td>
</tr>
<tr>
<td>3×5φ SP</td>
<td>128% (+13.3A)</td>
<td>101% (+0.2A)</td>
<td>98.0%</td>
<td>5.2%</td>
<td>99% (+0.46V)</td>
</tr>
<tr>
<td>5×3φ FP</td>
<td>129% (+4.6A)</td>
<td>129% (+4.6A)</td>
<td>94.4%</td>
<td>21.8%</td>
<td>97% (+16.8V)</td>
</tr>
<tr>
<td>5×3φ SP</td>
<td>123% (+4.0A)</td>
<td>124% (+4.0A)</td>
<td>95.9%</td>
<td>13.9%</td>
<td>98% (+11.3V)</td>
</tr>
<tr>
<td><strong>Star</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3×5φ FP</td>
<td>129% (+4.6A)</td>
<td>129% (+4.6A)</td>
<td>94.4%</td>
<td>21.8%</td>
<td>97% (+16.8V)</td>
</tr>
<tr>
<td>3×5φ SP</td>
<td>123% (+4.0A)</td>
<td>124% (+4.0A)</td>
<td>95.9%</td>
<td>13.9%</td>
<td>98% (+11.3V)</td>
</tr>
<tr>
<td>5×3φ FP</td>
<td>135% (+7.0A)</td>
<td>101% (+0.3A)</td>
<td>99.2%</td>
<td>14.0%</td>
<td>99% (+2.0V)</td>
</tr>
<tr>
<td>5×3φ SP</td>
<td>172% (+12.4A)</td>
<td>100% (+0.1A)</td>
<td>98.8%</td>
<td>6.7%</td>
<td>99% (+2.4V)</td>
</tr>
</tbody>
</table>

Table 7.8 Series stacked single phase winding open-circuit fault comparison.
DC ripple.

In conclusion, if protection of the machine windings has the highest priority, the 5×3φ in star connection with either fully-pitched or short-pitched winding connection would be the best choice. For protection of the diode rectifier, the polygon connected 5×3φ in short-pitched winding layout is a more reasonable selection. Depending on the power dissipation, polygon 3×5φ in short-pitched winding layout would be sensible pick if the load system requires the same power or less ripple and harmonics.

c. Parallel stacked split-phase model

Using the same analysis process as the previous sections, the phase current and diode current variations for parallel stacked split-phase models are presented in Figure 7.18 to Figure 7.21 and relevant data in Table 7.9.

Both 3×5φ and 5×3φ in polygon connection with parallel stacked provides better fault tolerance in terms of maximum winding current rise than the corresponding layouts in series stacked. However, the star connections perform worse than the corresponding series stacked layout. In terms of diode current, the parallel-stacked layout gives worse results than series-stacked ones.

<table>
<thead>
<tr>
<th></th>
<th>Phase current change</th>
<th>Diode current change</th>
<th>DC power change</th>
<th>DC ripple (%)</th>
<th>DC average voltage change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3×5φ FP</td>
<td>115% (+3.67A)</td>
<td>125% (+5.53A)</td>
<td>100.2%</td>
<td>15.5%</td>
<td>100% (+0.1V)</td>
</tr>
<tr>
<td>3×5φ SP</td>
<td>118% (+3.16A)</td>
<td>117% (+3.90A)</td>
<td>99.1%</td>
<td>7.4%</td>
<td>100% (-0.7V)</td>
</tr>
<tr>
<td>Star</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3×5φ FP</td>
<td>139% (+6.68A)</td>
<td>139% (+6.68A)</td>
<td>97.5%</td>
<td>21.5%</td>
<td>99% (-2.5V)</td>
</tr>
<tr>
<td>3×5φ SP</td>
<td>139% (+7.00A)</td>
<td>139% (+7.00A)</td>
<td>98.4%</td>
<td>12.4%</td>
<td>99% (-1.5V)</td>
</tr>
<tr>
<td>Polygon</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5×3φ FP</td>
<td>116% (+3.16A)</td>
<td>113% (+3.48A)</td>
<td>100.2%</td>
<td>19.4%</td>
<td>100% (+0.1V)</td>
</tr>
<tr>
<td>5×3φ SP</td>
<td>110% (+1.98A)</td>
<td>110% (+3.48A)</td>
<td>99.1%</td>
<td>7.6%</td>
<td>100% (-0.4V)</td>
</tr>
<tr>
<td>Star</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5×3φ FP</td>
<td>123% (+3.97A)</td>
<td>123% (+3.97A)</td>
<td>98.8%</td>
<td>10.5%</td>
<td>99% (-1.0V)</td>
</tr>
<tr>
<td>5×3φ SP</td>
<td>120% (+3.44A)</td>
<td>120% (+3.44A)</td>
<td>98.8%</td>
<td>17.9%</td>
<td>99% (-0.9V)</td>
</tr>
</tbody>
</table>

Table 7.9 Parallel stack single phase winding open-circuit fault comparison.

In contrast to the series stacked split-phase connections, the currents in other subsets have increased currents due to the faulted subset, as shown in Figure 7.18 to Figure 7.21. This is due to the parallel path between different subsets in parallel stacked rectifiers providing a path for faulted current to circulate among other subsets.
Figure 7.18 Phase current variation under single phase winding open-circuit condition for $3 \times 5\phi$ parallel stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

In terms of DC power, the parallel stacked configuration gives generally better fault tolerance performance than the series stacked configuration.

In conclusion, the polygon short-pitched of $5 \times 3\phi$ gives the best fault tolerance in terms of generator current and diode current rise and DC side power quality during a single phase winding fault.
Figure 7.19 Phase current variation under single phase winding open-circuit condition for 5×3φ parallel stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

Figure 7.20 Diode current variation under single phase winding open-circuit condition for 3×5φ parallel stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).
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Figure 7.21 Diode current variation under single phase winding open-circuit condition for 5x3φ parallel stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).

7.5.2. Bridge Connection Fault

Another open-circuit condition considered is if the wiring breaks between generator terminals and power electronic devices resulting in the loss of a diode phase bridge. Notice that, for star connected topologies, this fault is same as the single phase winding open-circuit fault in the PLECS circuit simulation. The fault is simulated at the phase A in winding set 1 for split-phase models, where the fault location is shown in Figure 7.22.

Figure 7.22 Single diode bridge open-circuit fault location for polygon (left) and star (right) connections.
The phase current variations are presented in Figure 7.24 - Figure 7.25 and diode current variations in Figure 7.26 - Figure 7.28, while the corresponding data are summarised in Table 7.10. Since the star connections share the same fault as in previous section, only the polygon phase current and diode current variations are presented in Figure 7.24 - Figure 7.28. Notice that the data are copied from single phase winding open-circuit fault for the star connections for easy comparison.

Both $3 \times 5\varphi$ and $15\varphi$ in polygon connection have better performance than the corresponding star connection, while the case is reversed for $5 \times 3\varphi$ connections. The polygon connected $15\varphi$ configuration shows the best tolerance, as the DC link output power is barely changed, the DC side ripple only rises by around 2%. In general, the $3 \times 5\varphi$ has better tolerance in terms of DC power change than the $5 \times 3\varphi$ windings, similar as the DC ripple. However, for all polygon connections, the diode current rise is much

<table>
<thead>
<tr>
<th>Phase current change</th>
<th>Diode current change</th>
<th>DC power change</th>
<th>DC ripple (%)</th>
<th>DC average voltage change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>15-phase</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polygon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$15\varphi$ FP</td>
<td>101% (+0.2A)</td>
<td>139% (+4.11A)</td>
<td>100.1%</td>
<td>5.7%</td>
</tr>
<tr>
<td>$15\varphi$ SP</td>
<td>101% (+0.1A)</td>
<td>132% (+3.46A)</td>
<td>99.6%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Star</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$15\varphi$ FP</td>
<td>123% (+5.5A)</td>
<td>124% (+5.45A)</td>
<td>100.3%</td>
<td>28.4%</td>
</tr>
<tr>
<td>$15\varphi$ SP</td>
<td>137% (+9.0A)</td>
<td>137% (+9.00A)</td>
<td>98.9%</td>
<td>23.8%</td>
</tr>
<tr>
<td><strong>Series stacked split-phase</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polygon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \times 5\varphi$ FP</td>
<td>106% (+1.3A)</td>
<td>125% (+5.18A)</td>
<td>97.2%</td>
<td>15.9%</td>
</tr>
<tr>
<td>$3 \times 5\varphi$ SP</td>
<td>111% (+1.8A)</td>
<td>101% (+5.42A)</td>
<td>97.0%</td>
<td>16.7%</td>
</tr>
<tr>
<td>Star</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \times 5\varphi$ FP</td>
<td>129% (+4.6A)</td>
<td>129% (+4.61A)</td>
<td>94.4%</td>
<td>21.8%</td>
</tr>
<tr>
<td>$3 \times 5\varphi$ SP</td>
<td>123% (+4.0A)</td>
<td>124% (+4.00A)</td>
<td>95.9%</td>
<td>13.9%</td>
</tr>
<tr>
<td><strong>Parallel stacked split-phase</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polygon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \times 5\varphi$ FP</td>
<td>113% (+3.19A)</td>
<td>130% (+6.68A)</td>
<td>98.8%</td>
<td>15.0%</td>
</tr>
<tr>
<td>$3 \times 5\varphi$ SP</td>
<td>121% (+3.77A)</td>
<td>139% (+8.71A)</td>
<td>99.0%</td>
<td>14.6%</td>
</tr>
<tr>
<td>Star</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \times 5\varphi$ FP</td>
<td>139% (+6.68A)</td>
<td>139% (+6.68A)</td>
<td>97.5%</td>
<td>21.5%</td>
</tr>
<tr>
<td>$3 \times 5\varphi$ SP</td>
<td>139% (+7.00A)</td>
<td>139% (+7.00A)</td>
<td>98.4%</td>
<td>12.4%</td>
</tr>
</tbody>
</table>
| **Table 7.10** 15-phase, series/parallel stacked split-phase single diode bridge open-circuit fault comparison.
higher under a single phase to diode open-circuit fault than that under single phase winding open-circuit fault, while this is reversed for the phase winding current increase.

Figure 7.23 Phase current variation under single bridge open-circuit condition for 15-phase with polygon connections, as in fully-pitched (left) and short-pitched (right)

Figure 7.24 Phase current variation under single bridge open-circuit condition for $3 \times 5\phi$ with polygon connection series stacked (top two) and parallel stacked (bottom two), as in fully-pitched (left column) and short-pitched (right column).
Figure 7.25 Phase current variation under single bridge open-circuit condition for 5×3φ with polygon connection series stacked (top two) and parallel stacked (bottom two), as in fully-pitched (left column) and short-pitched (right column).

Figure 7.26 Diode current variation under single bridge open-circuit condition for 15-phase with polygon connections, as in fully-pitched (left) and short-pitched (right)
For the phase current variations, the star connections give worse performance, however, for the diode currents, the conclusion drawn above is reversed. The series stacked topologies generally have less current rise than the parallel connections for both phase and diode current. Specifically, the polygon fully-pitched of $5\times3\varphi$ gives better fault tolerance on the winding current and diode current under single phase diode connection fault.

The DC power and ripple shares the same behaviour between parallel and series stacked topology, where the polygon short-pitched $5\times3\varphi$ and star fully-pitched $5\times3\varphi$ have the lower power variation than the other winding layouts.
7.5.3. Diode Open-circuit Fault

The diode open-circuit fault simulates the single diode failure condition, where the generator phase current and diode current, as well as power quality are examined. The fault is located at the top diode connected to the phase A in winding set 1, as shown in Figure 7.29.
The analysis follows the same pattern as in single phase winding open-circuit fault condition, the $15\phi$ as well as $3\times5\phi$ and $5\times5\phi$ in both series stacked and parallel stacked conditions are considered. The plot legends for phase current and diode current variation plots remain the same as in the previous fault analysis section.

**a. 15-phase model**

The generator phase current and diode current variations for 15-phase GRS under single diode open-circuit fault condition is presented in Figure 7.30 and Figure 7.31 respectively.

The influence of single diode leg open-circuit on overall phase and diode currents are much smaller than that from a single phase winding fault. The phase current in polygon connection does not vary as much as in the star connection, while for the star connection, the faulted phase causes the adjacent phase current and diode current to rise but to much smaller degree than the fault caused from single phase winding open-circuit. The similar feature can be found in diode current variation for both polygon and star connections.
Figure 7.30 Phase current variation under single diode open-circuit condition for 15-phase with polygon connections (top two plots) and star connections (bottom two plots), as in fully-pitched (left column) and short-pitched (right column).

Figure 7.31 Diode current variation under single diode open-circuit condition for 15-phase with polygon connections, as in fully-pitched (left column) and short-pitched (right column).
Table 7.11 15-phase, series/parallel stacked split-phase single diode open-circuit fault comparison.

The current variation data and DC power change for 15-phase and series/parallel stacked split-phase are summarised in Table 7.11 for easier comparison. DC side ripple in polygon connection is still better than star connection in the case for a single phase winding open-circuit fault for 15-phase, as shown in Table 7.11.

b. Series stacked split-phase model

The phase current and diode current variation plots for series stacked split-phase are shown in A.12.Model Application Plots.

Referring to Table 7.11 and plots in A.12.Model Application Plots, the polygon connection performs better than the corresponding star connection for 3×5φ, while the situation is reversed in 5×3φ topology, and the polygon fully-pitched 3×5φ has the lowest phase current rise. For the diode current variations, the polygon connection
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suffers more from the diode leg open-circuit fault than the phase winding open-circuit fault, while the star connection has better tolerance on the diode leg open-circuit fault. Similar to the fault characteristic of diode current variation from single phase winding open-circuit fault, the adjacent phase diode current rises and the other phase windings keep around the healthy condition when the single diode leg open-circuit fault occurs.

The DC power is barely changed under the diode leg open-circuit fault, however, this fault causes higher DC side ripple for the split-phase machines than that from single phase winding fault. It is observed from Table 7.11 that the polygon 15φ connections has the better performance on the DC ripple than other topologies.

In conclusion, polygon connection of 3×5φ would be better if the machine winding derating is important under this fault condition; 5×3φ in either winding layout would give a stable diode current for the power electronic devices; however, 15-phase polygon connection is the sensible choice for the stable DC power that required from load system.

c. Parallel stacked split-phase model

The phase current and diode current variation plots for series stacked split-phase are shown in A.12.Model Application Plots.

In comparison with the series stacked topologies, the diode leg open-circuit fault would result in higher current rise in both generator and diode rectifiers for 3×5φ sets, while the generator behaviour remain the same for 5×3φ sets under this fault condition. The power variation is reversed.

The polygon short-pitched 5×3φ gives the lowest percentage current rise in terms of generator and diode currents with sensible DC ripple and fairly low change on DC power.

It can be concluded that under diode rectifier connections faults, either diode leg open-circuit fault or single phase diode connection open-circuit fault, the 15-phase with polygon connections have better performance in terms of generator current variation and DC power variation. The 3×5φ polygon short-pitched connection is good for the single phase diode open-circuit fault is the power electronic device ratings are vital.
7.5.4. Summary of GRS at Faulted Operation

This section analysed the fault tolerance for different GRS system; although it is specifically focused on a reconfigurable 15-phase GRS, the general principles that suitable for other machines are:

1. The phase winding open-circuit fault causes the phase current rises gradually from the faulted location and reaches the maximum at the opposite phase, this feature becomes significant when the number of phase in a submachine set is higher. This feature can be concluded as the generalised feature for the multiphase GRS, as it is repeated on 15-phase and split-phase either series/parallel connections.

2. The winding failure causes a higher current in polygon than star, while this is reversed for the bridge connection failure in the series stacked topology. The diode current adjacent to the faulted location in the same subset for series stacked GRS shows the highest current rise.

3. The series stacked configuration generally has better fault isolation than the parallel stacked configurations due to the elimination of the parallel path.

Specifically, depending on the fault tolerance requirement on the phase winding, diode rectifier or the DC power variation, the selected fault tolerance winding configurations are summarised in the following table. The ‘stator winding and diode device protection’ are based on the least current rise under faulted condition, while the ‘DC load variation’ comes to the least DC power change.

<table>
<thead>
<tr>
<th>Single phase winding fault</th>
<th>Stator winding protection</th>
<th>Diode device protection</th>
<th>DC load variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5×3φ parallel stacked polygon SP (or)</td>
<td>3×5φ series stacked polygon SP</td>
<td>15φ polygon SP (or) 3×5φ series stacked polygon SP</td>
<td></td>
</tr>
<tr>
<td>5×3φ series stacked star SP</td>
<td>3×5φ series stacked polygon SP</td>
<td>15φ polygon SP (or) 3×5φ series stacked polygon SP</td>
<td></td>
</tr>
<tr>
<td>Single diode fault</td>
<td>15φ polygon SP (or)</td>
<td>3×5φ series stacked polygon SP</td>
<td>15φ polygon SP (or) 3×5φ series stacked polygon SP</td>
</tr>
<tr>
<td>5×3φ parallel stacked polygon SP</td>
<td>3×5φ series stacked polygon SP</td>
<td>15φ polygon SP (or) 3×5φ series stacked polygon SP</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.12 Selected possible configurations for different fault tolerance.
7.6. Single Split-phase Bypass after Fault

From the analysis and the data presented above, it is found that the other split-phases are barely affected when there is fault in one split-phase, especially in series stacked rectifiers. To isolate a fault in a single split-phase machine, the diode rectifier set from the faulted split-phase must be bypassed, since the power electronic devices are uncontrolled diode rectifiers in this research. While bypassing the faulted diode rectifier set, the DC load resistance is kept the same to check to GRS performance.

The phase arrangement is now an asymmetric split-phase layout, with two sets of 5-phase machines and four sets of 3-phase machines. Each phase set is separated by $\frac{2\pi}{15}$. There are two conditions for a diode fault with the polygon connection, where the first diode rectifier set could be bypassed either with the disconnected winding set open-circuited or not, classified as two conditions, condition 1 and condition 2, shown in Figure 7.32. These two conditions appear same in star connections.

7.6.1. Series Stacked Split-phase Models

From the Table 7.13, the peak power capability of all eight types of connections is lowered by losing one split-phase set. The most significant drop on the maximum power capability happens on the polygon short-pitched four sets of 3-phase winding and the least happens on the polygon fully-pitched four sets of 3-phase and star short-pitched four sets of 3-phase.

<table>
<thead>
<tr>
<th></th>
<th>Maximum power capability (kW)</th>
<th>Peak power drop percentage (%)</th>
<th>Maximum power capability (kW)</th>
<th>Peak power drop percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Healthy condition</td>
<td>condition 1</td>
<td>condition 2</td>
<td>condition 1</td>
</tr>
<tr>
<td>Poly</td>
<td>3×5φ FP</td>
<td>21.4</td>
<td>20.0</td>
<td>93.5%</td>
</tr>
<tr>
<td></td>
<td>3×5φ SP</td>
<td>22.3</td>
<td>21.1</td>
<td>94.6%</td>
</tr>
<tr>
<td>Star</td>
<td>3×5φ FP</td>
<td>24.7</td>
<td>22.7</td>
<td>91.9%</td>
</tr>
<tr>
<td></td>
<td>3×5φ SP</td>
<td>23.0</td>
<td>22.0</td>
<td>95.7%</td>
</tr>
<tr>
<td>Poly</td>
<td>5×3φ FP</td>
<td>19.1</td>
<td>18.7</td>
<td>97.9%</td>
</tr>
<tr>
<td></td>
<td>5×3φ SP</td>
<td>26.1</td>
<td>23.7</td>
<td>90.8%</td>
</tr>
<tr>
<td>Star</td>
<td>5×3φ FP</td>
<td>26.8</td>
<td>25.0</td>
<td>93.3%</td>
</tr>
<tr>
<td></td>
<td>5×3φ SP</td>
<td>20.0</td>
<td>19.5</td>
<td>97.5%</td>
</tr>
</tbody>
</table>

Table 7.13 Peak power capability for series stack with first diode rectifier set bypassed.
Figure 7.32 Winding arrangement for first diode rectifier bypassed under condition 1: the phase winding does not appear open-circuit fault with (a). \(2 \times 5\phi\) polygon connection and (b). \(4 \times 3\phi\) polygon connection; condition 2: the phase winding appears open-circuit fault with (c). \(2 \times 5\phi\) polygon connection and (d). \(4 \times 3\phi\) polygon connection; together with (e). \(2 \times 5\phi\) star connection and (f). \(4 \times 3\phi\) star connection.

It is also observed that there is not much difference when the first diode rectifier set bypassed under condition 1 and condition 2, and the peak power capability curve showed quite similar between these two conditions, as presented in Figure 7.33, therefore, only the steady state behaviour of condition 1 is considered for the following part, where the relevant data and peak power capability are presented in Table 7.13 and Figure 7.34.
Figure 7.33 Peak power capability when the first diode rectifier set bypassed under condition 1 and condition 2 for polygon connections.

Figure 7.34 DC link power peak capability for series stacked with first diode rectifier set bypassed.
### CHAPTER 7 COMPLEX HARMONIC ANALYSIS MODEL APPLICATION

#### Table 7.14

<table>
<thead>
<tr>
<th>Type</th>
<th>Condition</th>
<th>Vs (V)</th>
<th>Vs (V/f)</th>
<th>Is (A)</th>
<th>Is (A/f)</th>
<th>Id (A)</th>
<th>Id (A/f)</th>
<th>Vdc (V)</th>
<th>Idc (A)</th>
<th>DC ripple</th>
<th>THDv (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly</td>
<td>2×5φ FP</td>
<td>75.5</td>
<td>77.0</td>
<td>10.9</td>
<td>21.9</td>
<td>12.7</td>
<td>14.8</td>
<td>326.1</td>
<td>24.0</td>
<td>15.1%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>2×5φ SP</td>
<td>73.4</td>
<td>74.0</td>
<td>10.8</td>
<td>11.7</td>
<td>12.8</td>
<td>15.1</td>
<td>326.1</td>
<td>24.1</td>
<td>15.0%</td>
<td>12%</td>
</tr>
<tr>
<td>Star</td>
<td>2×5φ FP</td>
<td>77.3</td>
<td>80.0</td>
<td>9.0</td>
<td>10.6</td>
<td>8.9</td>
<td>10.5</td>
<td>400.0</td>
<td>17.0</td>
<td>14.3%</td>
<td>27%</td>
</tr>
<tr>
<td></td>
<td>2×5φ SP</td>
<td>73.5</td>
<td>74.1</td>
<td>10.5</td>
<td>12.3</td>
<td>10.5</td>
<td>12.3</td>
<td>385.4</td>
<td>19.8</td>
<td>9.9%</td>
<td>12%</td>
</tr>
<tr>
<td>Poly</td>
<td>4×3φ FP</td>
<td>77.0</td>
<td>79.9</td>
<td>12.6</td>
<td>16.0</td>
<td>21.6</td>
<td>22.5</td>
<td>392.7</td>
<td>27.8</td>
<td>15.1%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>4×3φ SP</td>
<td>63.3</td>
<td>64.9</td>
<td>14.2</td>
<td>14.8</td>
<td>24.6</td>
<td>25.6</td>
<td>353.1</td>
<td>31.5</td>
<td>8.8%</td>
<td>23%</td>
</tr>
<tr>
<td>Star</td>
<td>4×3φ FP</td>
<td>72.7</td>
<td>74.8</td>
<td>12.2</td>
<td>12.7</td>
<td>12.1</td>
<td>12.6</td>
<td>709.0</td>
<td>15.6</td>
<td>9.2%</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>4×3φ SP</td>
<td>67.7</td>
<td>70.2</td>
<td>13.6</td>
<td>14.2</td>
<td>13.5</td>
<td>14.1</td>
<td>609.2</td>
<td>17.4</td>
<td>10.9%</td>
<td>27%</td>
</tr>
</tbody>
</table>

#### Table 7.15

<table>
<thead>
<tr>
<th>Type</th>
<th>Condition</th>
<th>Stator copper loss (W)</th>
<th>Generator active power (kW)</th>
<th>Generator apparent power (kVA)</th>
<th>Power factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon</td>
<td>2×5φ FP</td>
<td>907.4</td>
<td>8.00</td>
<td>24.19</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>2×5φ SP</td>
<td>190.6</td>
<td>8.04</td>
<td>8.72</td>
<td>0.92</td>
</tr>
<tr>
<td>Star</td>
<td>2×5φ FP</td>
<td>157.6</td>
<td>6.95</td>
<td>8.60</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>2×5φ SP</td>
<td>211.70</td>
<td>7.79</td>
<td>9.20</td>
<td>0.85</td>
</tr>
<tr>
<td>Polygon</td>
<td>4×3φ FP</td>
<td>449.7</td>
<td>11.22</td>
<td>17.27</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>4×3φ SP</td>
<td>367.1</td>
<td>11.45</td>
<td>11.61</td>
<td>0.99</td>
</tr>
<tr>
<td>Star</td>
<td>4×3φ FP</td>
<td>269.0</td>
<td>11.24</td>
<td>11.44</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>4×3φ SP</td>
<td>333.5</td>
<td>10.79</td>
<td>11.94</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 7.14 Voltage and current quantities for condition 1 (a) parallel stacked with first diode rectifier set bypassed and (b) comparison against corresponding healthy operation in Table 7.3.

Table 7.15 Power quantities for condition 1 with series stacked with first diode rectifier set bypassed.

Table 7.14 and Table 7.15 show the performance of bypassed series stacked split-phase. The comparisons in Table 7.14.(b) are made by \( \frac{x_{\text{bypass}}}{x_{\text{healthy}}} \times 100\% \), where \( x_{\text{bypass}} \) refers to the quantities in bypassed condition and \( x_{\text{healthy}} \) to the quantities in healthy condition. ‘Pdc’ refers to DC link power.
Figure 7.35 Simulated generator waveform for $2 \times 5\varphi$ with (a) polygon connection and (b) star connection; $4 \times 3\varphi$ with (c) polygon connection and (d) star connection, where left column for fully-pitched and right column for short-pitched layout under series stacked topologies.
From Table 7.14, it can be found that the phase voltage after bypassing does not change much, so the voltage harmonic distortion (THDv) remains the same.

The existence of zero-sequence current causes a severe problem at both healthy and bypassed conditions, with increased stator copper loss, high apparent power and low power factor, as indicated in Table 7.15. The generator waveforms are presented in Figure 7.35, where it is obvious that the voltage distortion and the circulating zero sequence current are significant in polygon fully-pitched connections. Meanwhile, the peak diode current is much smaller in star connections than the polygon bypassed split-phase windings.

Compared with the original healthy state, the generator is operating at lower power level. Although one of three sets is shutdown in 3×5φ split-phase GRS system, the DC link power is almost halved compared with the corresponding healthy state rather than losing $\frac{1}{3}$ of the power, indicating it is severe for 3×5φ GRS to lose one subset. For 5×3φ GRS, losing one of the subset makes the system operates at 70% of its original power. This concludes that the higher number of sets for the split-phase machine, the less severe it is for the system to lose a subset.

When comparing the generator performance, it is again the star series stacked 4×3φ fully-pitched winding layout has the low stator copper loss, the highest peak power capability and highest power factor. It also gives the lowest DC side ripple among the new configurations. This connection also gives fairly average generator current and diode current rise.

7.6.2. Parallel Stacked Split-phase Models

Same as the bypass process from series stacked topology, the first diode rectifier set containing the open-circuit fault has been disconnected in parallel case, where the two conditions described in the series stacked case has been considered. The comparison on these two conditions show quite similar performance to the case in series stacked topology, as in Table 7.16, therefore, only the peak power capability for condition 1 is plotted in Figure 7.36 as well as the generator behaviours in Table 7.17, Table 7.18 and Figure 7.37.
Figure 7.36 DC link power peak capability for parallel stacked under one of the split-phase disconnected.

Similar to the case for series stacked condition, the maximum power capability of all disconnected split-phase is lowered, shown in Figure 7.36 and Table 7.16.

<table>
<thead>
<tr>
<th></th>
<th>Maximum power capability (kW)</th>
<th></th>
<th>Maximum power capability (kW)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Healthy condition</td>
<td>Condition 1</td>
<td>Peak power drop percentage (%)</td>
<td>Condition 2</td>
</tr>
<tr>
<td>Poly</td>
<td>3×5φ FP</td>
<td>21.1</td>
<td>94.3%</td>
<td>20.0</td>
</tr>
<tr>
<td></td>
<td>3×5φ SP</td>
<td>22.7</td>
<td>93.4%</td>
<td>21.2</td>
</tr>
<tr>
<td>Star</td>
<td>3×5φ FP</td>
<td>23.9</td>
<td>94.1%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>3×5φ SP</td>
<td>23.2</td>
<td>95.3%</td>
<td>-</td>
</tr>
<tr>
<td>Poly</td>
<td>5×3φ FP</td>
<td>19.0</td>
<td>98.9%</td>
<td>18.8</td>
</tr>
<tr>
<td></td>
<td>5×3φ SP</td>
<td>20.9</td>
<td>102.9%</td>
<td>21.5</td>
</tr>
<tr>
<td>Star</td>
<td>5×3φ FP</td>
<td>22.9</td>
<td>96.5%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>5×3φ SP</td>
<td>20.0</td>
<td>97.5%</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.16 Peak power capability for parallel stack with first diode rectifier set disconnected. (FP: fully-pitched SP: short-pitched)
Figure 7.37 Simulated generator waveform for $2 \times 5 \varphi$ with (a) polygon connection and (b) star connection; $4 \times 3 \varphi$ with (c) polygon connection and (d) star connection, where left column for fully-pitched and right column for short-pitched layout under parallel
stacked topologies.

<table>
<thead>
<tr>
<th></th>
<th>Vs (V/f)</th>
<th>Vs (V)</th>
<th>Is (A/f)</th>
<th>Is (A)</th>
<th>Id (A/f)</th>
<th>Id (A)</th>
<th>Vdc (V)</th>
<th>Idc (A)</th>
<th>DC ripple (Vs)</th>
<th>THDv (Vs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly</td>
<td>2×5φ FP</td>
<td>70.2</td>
<td>72.2</td>
<td>21.7</td>
<td>30.1</td>
<td>25.8</td>
<td>31.8</td>
<td>148.5</td>
<td>98.4</td>
<td>17.4%</td>
</tr>
<tr>
<td></td>
<td>2×5φ SP</td>
<td>67.9</td>
<td>68.5</td>
<td>22.0</td>
<td>30.2</td>
<td>27.4</td>
<td>32.1</td>
<td>148.2</td>
<td>98.8</td>
<td>14.5%</td>
</tr>
<tr>
<td>Star</td>
<td>2×5φ FP</td>
<td>73.1</td>
<td>76.9</td>
<td>19.3</td>
<td>23.8</td>
<td>19.3</td>
<td>23.7</td>
<td>191.4</td>
<td>75.1</td>
<td>22.9%</td>
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<td></td>
<td>2×5φ SP</td>
<td>68.7</td>
<td>69.6</td>
<td>21.7</td>
<td>25.1</td>
<td>21.6</td>
<td>25.0</td>
<td>175.4</td>
<td>82.4</td>
<td>10.3%</td>
</tr>
<tr>
<td>Poly</td>
<td>4×3φ FP</td>
<td>73.8</td>
<td>76.4</td>
<td>17.8</td>
<td>22.2</td>
<td>30.7</td>
<td>33.4</td>
<td>94.1</td>
<td>156.8</td>
<td>12.7%</td>
</tr>
<tr>
<td></td>
<td>4×3φ SP</td>
<td>59.5</td>
<td>60.8</td>
<td>20.4</td>
<td>23.2</td>
<td>35.3</td>
<td>40.2</td>
<td>81.6</td>
<td>181.3</td>
<td>10.5%</td>
</tr>
<tr>
<td>Star</td>
<td>4×3φ FP</td>
<td>68.5</td>
<td>70.7</td>
<td>17.4</td>
<td>20.0</td>
<td>17.3</td>
<td>19.9</td>
<td>165.2</td>
<td>89.8</td>
<td>10.6%</td>
</tr>
<tr>
<td></td>
<td>4×3φ SP</td>
<td>65.9</td>
<td>68.2</td>
<td>20.0</td>
<td>21.5</td>
<td>19.9</td>
<td>21.4</td>
<td>148.4</td>
<td>98.9</td>
<td>15.9%</td>
</tr>
</tbody>
</table>

Table 7.17 Voltage and current quantities for condition 1 (a) parallel stacked with first diode rectifier set disconnected and (b) comparison against corresponding healthy operation in Table 7.5.

<table>
<thead>
<tr>
<th></th>
<th>Vs (fund %)</th>
<th>Vs (V)</th>
<th>Is (fund %)</th>
<th>Is (A)</th>
<th>Id (fund %)</th>
<th>Id (A)</th>
<th>Vdc (V)</th>
<th>Idc (A)</th>
<th>Pdc (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly</td>
<td>2×5φ FP</td>
<td>99.3%</td>
<td>99.7%</td>
<td>144.7%</td>
<td>121.9%</td>
<td>146.6%</td>
<td>141.3%</td>
<td>98.5%</td>
<td>97.2%</td>
</tr>
<tr>
<td></td>
<td>2×5φ SP</td>
<td>99.0%</td>
<td>99.3%</td>
<td>152.7%</td>
<td>144.8%</td>
<td>153.9%</td>
<td>142.7%</td>
<td>98.7%</td>
<td>97.5%</td>
</tr>
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<td>Star</td>
<td>2×5φ FP</td>
<td>99.1%</td>
<td>99.2%</td>
<td>141.9%</td>
<td>140.8%</td>
<td>141.9%</td>
<td>141.1%</td>
<td>97.6%</td>
<td>95.3%</td>
</tr>
<tr>
<td></td>
<td>2×5φ SP</td>
<td>99.3%</td>
<td>99.1%</td>
<td>145.6%</td>
<td>139.4%</td>
<td>145.9%</td>
<td>139.7%</td>
<td>97.9%</td>
<td>96.0%</td>
</tr>
<tr>
<td>Poly</td>
<td>4×3φ FP</td>
<td>99.6%</td>
<td>99.7%</td>
<td>126.2%</td>
<td>115.7%</td>
<td>126.9%</td>
<td>122.3%</td>
<td>99.4%</td>
<td>98.7%</td>
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<td>4×3φ SP</td>
<td>99.3%</td>
<td>99.3%</td>
<td>122.9%</td>
<td>116.0%</td>
<td>122.6%</td>
<td>115.9%</td>
<td>99.0%</td>
<td>98.1%</td>
</tr>
<tr>
<td>Star</td>
<td>4×3φ FP</td>
<td>99.6%</td>
<td>99.2%</td>
<td>121.7%</td>
<td>114.3%</td>
<td>121.8%</td>
<td>113.7%</td>
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<td>98.5%</td>
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<td>4×3φ SP</td>
<td>99.1%</td>
<td>99.1%</td>
<td>129.0%</td>
<td>121.5%</td>
<td>129.2%</td>
<td>121.6%</td>
<td>99.1%</td>
<td>98.1%</td>
</tr>
</tbody>
</table>

(b)

Table 7.18 Power quantities for condition 1 with series stacked with first diode rectifier set disconnected.

Similar to the series stacked case, the disconnect condition does not change generator voltage and THDv much, illustrated in Table 7.17. However, instead of decreasing phase and diode currents, the parallel stacked split-phase GRS has both phase current and diode current rise after disconnection. The 4×3φ topologies generally have lower
current rise on the generator and diode rectifier (around 20% rise) than the 2\( \times 5\phi \) topologies (around 40% rise), whereas the lowest change happens on star fully-pitched parallel stacked 4\( \times 3\phi \) layout.

The advantage of the parallel stacked split-phase GRS is that it remains at a similar level of DC power output and power factor, but with significant rise on the DC ripple after disconnection.

The same conclusions drawn as from the series stacked case, where the star fully-pitched parallel stacked 4\( \times 3\phi \) layout performs best with lower stator copper loss, high power factor and relative low current rise among other split-phase topologies.

### 7.6.3. Summary of GRS at One Subset Bypassed Condition

To isolate the faulted location completely, the faulted subset is bypassed/disconnected and this is not possible for 15-phase GRS. In this case, the parallel stacked GRS begins to show its advantage of continuing to operate at same DC output power over the series stacked GRS. The star parallel stacked fully-pitched 4\( \times 3\phi \) configuration gives the best option with relative high power factor.

### 7.7. Summary

The multiphase GRS with different winding configurations in terms of healthy and fault tolerance operation is discussed in this chapter. The parallel and series stacked diode rectifier for split-phase winding layouts, 3\( \times 5\phi \) and 5\( \times 3\phi \), as well as 15-phase winding layouts under health, different types of open-circuit faults and one of the split-phase set bypassed/disconnected conditions are analysed above.

For the specific discussion on this reconfigurable 15-phase GRS, the series stacked diode rectifier generally has slightly lower current magnitude of the generator and diode currents than that in parallel stacked diode rectifiers under healthy condition, where lower generator current result in lower stator copper loss. Considering the lower generator voltage harmonic distortion, DC ripple and generator currents, the polygon short-pitched 3\( \times 5\phi \) in series stack, star short-pitched 3\( \times 5\phi \) in parallel stack and polygon short-pitched 15\( \phi \) share good performance, listed in Table 7.19. Besides, the peak power capability of 15\( \phi \) is relative high among the other winding layouts.
Table 7.19 Selected possible configurations.

However, both the polygon short-pitched 3×5φ in series stack and polygon short-pitched of 15φ winding layout suffer significant high phase current rise under single phase winding open-circuit fault, while the star connection short-pitched of 3×5φ in parallel stack only has half of the generator current rise under the single phase open-circuit fault of the other two selected winding layouts, but with much higher DC power distortion and DC ripple. In terms of generator winding protection, the star connection of 5×3φ could be a good option in case of generator phase winding open-circuit fault.

Under single diode leg open-circuit fault and single phase diode connection open-circuit fault, the 15φ polygon short-pitched connection provides the better fault tolerance on generator current, diode current and DC power quality than other winding layouts. However, the open phase winding fault is a severe potential threat resulting in high
generator currents and the torque vibration would also be a potential threat to the generator mechanical system unbalanced condition.

To isolated the influence from the faulted phase sequence, one of the possible method is to bypass/disconnect the faulted split-phase sequence. In terms of constant power delivery to the load side, the parallel stacked GRS provides a good solution, particularly, star 4×3φ fully-pitched in parallel stack gives the high power factor and low DC ripple becomes attractive. While the polygon 2×5φ SP in parallel stack provides less DC power and high DC ripple but with low THDv on the generator side and a similar level of power factor. This makes it another option at bypassed condition if the power electronic devices require low THDv but can stand higher peak current.

The comparison from the CHA model shows the existence of zero-sequence current in the polygon connection makes the polygon fully-pitched the worst configuration, and this confirms the analysis from the simpler analysis in [14], which uses the sinusoidal voltage model with harmonic contents to represent the WFSG and SimPowerSystem to include the diode rectifier model. The fault condition that results in phase current rise from the CHA prediction also confirms the analysis in [14]. Additionally, the feature of diode open-circuit fault induced phase current rise for the 12-phase parallel stacked GRS [78] is also validated by the CHA method, where the author uses the connection matrix for the diode rectifier model.

In addition to the validation of the previous papers, the comparison of different winding configurations using the CHA model also adds consideration of the peak power capability and the steady state performance under healthy, fault and bypassed/disconnected conditions.

Although the analysis in this chapter is focused on the specific reconfigurable GRS up to 15-phase, the features that drawn from either 15-phase or split-phase under healthy, fault and bypassed/disconnected conditions are not limited to this specific machine. For example, the peak power capability in polygon short-pitched is higher than fully-pitched for the elimination of zero-sequence current and star fully-pitched has higher peak power than short-pitched due to the increased winding factor. Meanwhile, the current variation feature is also validated on the other types of machines, i.e. single 3-phase, 5-phase in [14] and 12-phase in [78].
8.1. Introduction

This project has focused on the complex harmonic modelling of a salient pole multiphase wound field synchronous machine intended for the electrical system on an aircraft DC power network. The project uses CHA circuit modelling of the GRS as the main modelling tool. Supplementary FEA modelling, including 2D FEA from FEMM and 3D FEA from COMSOL Multiphysics, was used to refine the CHA methods. The experimental validation and the further applications of the CHA are also presented.

8.2. Review of the Research

This section briefly describes the work that has been carried out in each of the chapter from this thesis.

Chapter 2 reviewed the development of the aircraft electric power systems where the big picture targets the DC power system on the aircraft. A review of research on analysis and modelling of multiphase machines indicated that modelling of multiphase, WFSG with saliency is still a research gap and needs further contributions. This thesis identifies the CHA method as capable of representing time and space harmonics in the WFSG. Methods for saturation modelling and diode rectifier system modelling are also reviewed in this chapter.

Chapter 3 introduced the CHA methods by considering the salient pole structure of the multiphase generator. In this chapter, the magnetic flux density, flux linkage from different windings, back EMF from CHA method are derived. Analytically, geometry of
the test generator was carefully designed to cancel specific harmonic content with rotor arc length and stator short-pitching, while from the CHA analysis, the generator can still produce harmonic content at these frequencies, which comes from the combination of stator winding harmonics and airgap harmonics. The initial comparison against FEA identified the need to improve accuracy of the CHA, for instance, including flux iron path, implementing stator skew, where these improvements are covered in the following chapter.

Chapter 4 improves the CHA methods presented in chapter 3 by considering the flux path in the rotor and stator iron, as well as the stator skew factor. Adding flux path in rotor and stator iron makes the effective airgap a bit larger than the original, which reduces the DC airgap distance by 10.8% and second order harmonic by 11.5%. The end winding inductance is also evaluated by using the image method in this chapter. The analytical method overestimates leakage terms compared with the measured values. The experimental measured leakage terms are used in GRS simulation, although it is found out the accuracy of leakage terms are less impact on the accuracy of the simulation.

Chapter 5 detailed the Simulink model setup based on the CHA method described in Chapter 3. The generator is modelled by using the natural reference frame rather than the standard 2-axis DQ synchronous reference frame, as the latter was shown to represent the generator less well since the uncontrolled higher order harmonics are ignored. Implementing in the extended synchronous reference frame requires other higher order harmonic planes, which are coupled through the saliency terms, and requires the same dimensions of matrix as the model from the natural reference frame, so the extended reference frame was also shown not to simplify the model itself. The 2D FEA and 3D FEA models are also presented in this chapter. The 2D FEA model is used to validated the saturation characteristic of generator and compare the result against the CHA and experiment under a simple resistive load in the following chapter. 3D FEA model is used to validate the stator leakage inductance, including the end winding inductance and slot leakage inductance, against the combination of analytical result from image method and FEA simulation. The comparison showed the 3D FEA has better accuracy but requires a complex set up process. The image method could provide a rough estimation of the end winding inductance, which is shown to be sufficient, since
model results are only weakly dependent on leakage inductance. For better accuracy, either detailed 3D FEA or experimental measurement is recommended.

Chapter 6 describes how the experiment test rig is set up. The rewound Cummins generator BCI162G has a flexible stator terminal configuration suitable for 3-phase, 5-phase and 15-phase connection. The measured experimental results of different generator behaviors, back-emf waveform, resistive loading and diode rectifier loading, are compared against the simulated results from CHA method. In general, the CHA methods under predicts by 3% of generator voltage under light load condition and over-predicts by 4% of generator voltage under heavy load condition. The details of the waveform shapes match better at light load than at full load and the errors are within 5% on most variables and 10% on highly distorted diode currents. The saturation factor modelling and inclusion of iron path in the CHA methods significantly improves accuracy and is as good as the 2D FEA results. Generally trends show the CHA can predict the GRS behavior well at the design stage.

Chapter 7 applied the CHA model to compare different winding configurations with diode rectifier systems. The common split-phase structures for a 15-phase winding, 3×5φ and 5×3φ, are taken as an illustrative examples in this section under both healthy and faulted conditions. Both the series stacked and parallel stacked diode rectifier systems are also evaluated in this chapter. In general, the series stacked diode configuration helps to reduce the generator current magnitude and hence reduce the copper loss with respect to the parallel stacked diode rectifier systems under same winding configuration. However, the parallel stacked diode rectifier systems have the ability to maintain the similar level of DC power after disconnecting a faulted subset but with increased phase current in the remaining subsets, where the DC power in series stacked is either halved (3×5φ topologies) or 30% reduced (5×3φ topologies). The selection of winding arrangement is a compromise between the demand for either generator fault tolerance or DC power quality.

8.3. Contribution

This project has been focused on analysing and modelling a synchronous generator-rectifier system. The significance of this work is summarised in the following as an overview of the contribution of this project.
8.3.1. Modelling of Saliency

The detailed harmonic modelling of saliency including harmonics greater than 2\textsuperscript{nd} order is a new contribution. This allows identification of problem harmonics to be linked to machine geometry. For example, this is significant for the machine studied in this thesis, as the 4\textsuperscript{th} airgap saliency led to 5\textsuperscript{th} back-emf harmonic which appeared as zero sequence in the 5-phase configurations.

8.3.2. Higher Order DQ Axis Modelling of the WFSG

Full orthogonal transforms of the multiphase generator model are developed for WSFG, including for the damper circuits. Implementing a multiphase WFSG in the extended synchronous reference frame needs the same dimensions as in the per-phase natural reference frame and fails to decouple the DQ and higher-order sets, unless the higher-order sets in the synchronous reference frame are actively controlled to be zeros. Although it is shown that the DQ synchronous reference frame for multiphase WFSG does not give any computational benefits compared with per-phase natural reference frame, this finding is still valuable to future researchers.

8.3.3. Singularity Clarification

This thesis also states the underlying problem when implementing synchronous reference frame to multiphase machines, i.e. the matrix singularity problem, which is not fully stated in the previous research work \cite{83, 84}. The singularity problem is inevitable for the symmetrical distributed windings and can be simply solve by adding extra leakage terms, as the demonstration stated in the appendix A.5.

8.3.4. A Methodology for Comparing Symmetrical and Split-phase topologies under healthy and open-circuit faults

This project compared different winding configurations as alternatives to 15-phase layout, i.e., the split-phase winding layouts, and their performance in terms of generator/diode currents, voltage harmonic distortion and DC load power stability under healthy and open-circuit fault conditions are discussed. The comparison provides ideas of how the generator performance is affected by choosing different winding and diode rectifier layouts at the design stage. This part of research confirms that the 12-phase work by \cite{78} applies to a 15-phase machine and split phase combinations, and extends it
to the series stacked diode rectifier system. Modelling of the open-circuit fault conditions identifies the generator fault behaviour for different winding layouts and how to choose the best winding configurations to deal with the mostly likely faults. The use of representative field excitation and evaluation of peak power capability under both steady-state and fault conditions extends existing work. Although the comparison in this project focused on a specific generator geometry, the modelling process and the conclusions drawn are general and applicable for other machine geometries.

8.3.5. Saturation factor

Much of the previous work on multiphase GRS had been done at light loads, under unsaturated conditions [14], this project shows it is essential to model saturation in the WFSG for operation at rated field and load. The implementation of a single saturation factor accounting for both D-axis and Q-axis saturation shows sufficient accuracy for comparing topologies.

8.3.6. End winding leakage

The image method reviewed from [153] is extended in this project to evaluate the end winding leakage inductance of a distributed winding for a WFSG. Although the measured stator leakage inductance is smaller than the estimation, it still provides the general idea of how the end winding inductance could be estimated as an accurate order of magnitude. This work also shows that 3D FEA gives a much better estimation of leakage inductance despite the time-consuming set up process.

8.4. Future work

This project covered the aims and objectives listed in the introduction chapter, while the further potential area of work, which listed as the following, could be carried out as an extension for this project.

8.4.1. Saturation modelling

As presented in chapter 6, the experimental validation against the CHA method shows the single saturation factor presenting an overall saturation characteristic improves accuracy, but with residual errors, and cross saturation effects could not be neglected. Separate D-axis and Q-axis saturation factors should be included depending on their
respective characteristics. In addition, local saturation significantly changes higher harmonics, which are not accounted by a single scale factor.

8.4.2. Damper bar behaviours

The review of behaviour of damper bar for synchronous generator has been documented in [116-118, 145] for their modelling and steady state behaviour in three-phase machines. However, as [145] shows, there is still a research gap on the transient behaviour of damper windings for synchronous generators. The study on transient effects from damper bars, especially under faulted condition should be continued, although in the steady state condition the damper circuits are shown to be insignificant in this project.

8.4.3. Short-circuit fault analysis

As well as open-circuit faults, short-circuit fault analysis should also be investigated to evaluate generator fault tolerance between different winding configurations. Compared with the open-circuit faults, the short-circuit faults can be severe due to the high current that could potentially damage the insulation of winding. Meanwhile, inter-turn winding short-circuit fault could cause additional flux linkage loops which add extra dimensions for the CHA model. The fault current resulting from a short-circuited DC bus is also important for protection sizing of devices.

8.4.4. Capacitor and Constant Current Loads

Common DC loads such as capacitive and constant current loads should also be considered. The GRS may have an issue with stability when the resistive load is replaced by the constant current load. More work should be done to enable the PLECS model to study DC network stability with capacitive and constant current loads.

8.5. Publications


X. Zhang, A. Semjonovs, Y. Zbede, A. Bodrov, and J. Apsley, "Speed Sensorless Control of a Surface-mounted Permanent Magnet Drive," presented at the 10th
References


REFERENCES


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A.1. Analytical Derivations

A.1.1. CHA of Stator Winding

The mathematical expression of the stator conductor density distribution, referred to Figure 3.2, is expressed as following [89, 142]:

\[
c(\theta) = \begin{cases} 
\frac{N_c}{\beta} \left( \theta_0 - \frac{\alpha}{2} - \frac{\beta}{2} < \theta < \theta_0 - \frac{\alpha}{2} + \frac{\beta}{2} \right) \\
-\frac{N_c}{\beta} \left( \theta_0 + \frac{\alpha}{2} - \frac{\beta}{2} < \theta < \theta_0 + \frac{\alpha}{2} + \frac{\beta}{2} \right) \\
0 & \text{elsewhere}
\end{cases}
\]
\[ \sum c(e^{j\theta}) \, d\theta = \frac{N_c}{2\beta\pi} \int_{\theta_0 - \frac{\alpha}{2}}^{\theta_0 + \frac{\alpha}{2}} e^{j\theta} \, d\theta = \frac{N_c}{2\beta\pi} \int_{\theta_0 - \frac{\alpha}{2}}^{\theta_0 + \frac{\alpha}{2}} e^{j\theta} \, d\theta \]

\[ = \frac{N_c}{j2\nu\beta \pi} e^{j\nu \theta_0} \times 4 \sin \left( \frac{\nu\alpha}{2} \right) \sin \left( \frac{\nu\beta}{2} \right) = \frac{N_c}{j\pi} e^{j\nu \theta_0} \sin \left( \frac{\nu\alpha}{2} \right) \sin \left( \frac{\nu\beta}{2} \right) \]

Let \( K_p(v) = \sin \left( \frac{\nu\alpha}{2} \right) \) (pitch factor) where \( K_p(-v) = -K_p(v) \) and

\[ K_b(v) = \frac{\sin \left( \frac{\nu\beta}{2} \right)}{\nu\beta} \] (slot mouth factor) where \( K_b(-v) = K_b(v) \)

\[ \Rightarrow \bar{C}^v = \frac{jN_c}{\pi} K_p(v)K_b(v) e^{j\nu \theta_0} \quad \text{or} \quad \bar{C}^v = -\frac{iN_c}{\pi} K_p(v)K_b(v) e^{j\nu \theta_0} \]

Extending to a single layer phase group, where \( N_g \) slots in a single phase group.

\[ \bar{C}^v = -\frac{jN_c}{\pi} K_p(v)K_b(v) \sum_{i=1}^{N_g} e^{j\nu (\theta_0 + \frac{2\pi (i-1)}{N_g})} \]

\[ \Rightarrow \sum_{i=1}^{N_g} e^{j\nu (\theta_0 + \frac{2\pi (i-1)}{N_g})} = e^{j\nu \theta_0} \sum_{i=1}^{N_g} e^{j\nu \frac{2\pi (i-1)}{N_g}} = \frac{\sin \left( \frac{\nu N_g \pi \theta_0}{N_g \pi} \right)}{\sin \left( \frac{\nu \pi}{N_g} \right)} e^{j\nu \theta_0} \]

\[ \Rightarrow \bar{C}^v = -\frac{jN_c}{\pi} K_p(v)K_b(v) \frac{\sin \left( \frac{\nu N_g \pi \theta_0}{N_g \pi} \right)}{\sin \left( \frac{\nu \pi}{N_g} \right)} e^{j\nu \theta_0} \]

Let \( K_d(v) = \frac{\sin \left( \frac{\nu N_g \pi \theta_0}{N_g \pi} \right)}{\sin \left( \frac{\nu \pi}{N_g} \right)} \) (distribution factor) where \( K_d(-v) = K_d(v) \)

\[ \Rightarrow \bar{C}^v = -\frac{jN_c N_g}{\pi} K_p(v)K_b(v)K_d(v) e^{j\nu \theta_0} \]

\[ = -\frac{jN_p}{\pi_p \rho} K_p(v)K_b(v)K_d(v) e^{j\nu \theta_0}, \text{where } N_{ph} = N_c N_g \rho \]

Extending to double layer multiple pole pairs with an angular separation of \( \frac{2\pi}{\rho} \)
\[\bar{C}_v = -\frac{jN_{ph}}{\pi \rho} K_p(v)K_b(v)K_d(v) \left(1 - e^{j\frac{\pi}{\rho}}\right) e^{jv\left[\theta_0 + \frac{(N_g - 1)\pi}{N_s}\right]} \sum_{i=0}^{\rho-1} e^{jv\left(\frac{2i\pi}{\rho}\right)}\]

Conductor density (rad\(^{-1}\))

![Diagram of conductor density](image)

Figure A.1 Double layer stator conductor distribution

The layout of double layer is shown in Figure A.1.

The term \(\sum_{i=0}^{\rho-1} e^{jv\left(\frac{2i\pi}{\rho}\right)}\) = \(\frac{1-e^{j2\pi\frac{\rho}{i}}}{1-e^{j\frac{2\pi}{\rho}}} = 0\), where the condition is when \(\frac{v}{\rho} \neq integer\).

To ensure the summation is not zero, the condition must be \(\frac{v}{\rho} = integer\), therefore the summation becomes \(\sum_{i=0}^{\rho-1} e^{jv\left(\frac{2i\pi}{\rho}\right)}\) = \(\rho\)

\[\Rightarrow \bar{C}_v = -\frac{jN_{ph}}{\pi} K_p(v)K_b(v)K_d(v) e^{jv\left[\theta_0 + \frac{(N_g - 1)\pi}{N_s}\right]}\], same as shown in equation (3-11).

A.1.2. CHA of Rotor Field Winding

The rotor field winding distribution is shown as follows, where the winding distribution is shown in Figure 3.5.(b).

For the first winding group

\[c_1(\theta_r) = \begin{cases} \frac{N_r}{\tau} & \left(\theta_1 - \frac{\tau}{2} < \theta_r < \theta_1 + \frac{\tau}{2}\right) \\
\frac{N_r}{\tau} - \frac{N_r}{\tau} & \left(2\pi - \theta_1 - \frac{\tau}{2} < \theta_r < 2\pi - \theta_1 + \frac{\tau}{2}\right) \\
0 & \text{elsewhere} \end{cases}\]

\[\Rightarrow \bar{C}_1^v = \frac{N_r}{2\tau\pi} \int_{\theta_1 - \frac{\tau}{2}}^{\theta_1 + \frac{\tau}{2}} e^{jv\theta} d\theta - \frac{N_r}{2\tau\pi} \int_{2\pi - \theta_1 - \frac{\tau}{2}}^{2\pi - \theta_1 + \frac{\tau}{2}} e^{jv\theta} d\theta\]
\[-\frac{N_r}{j\nu \pi} \times 4 \sin(\nu \theta_1) \sin\left(\frac{\nu \tau}{2}\right) = -\frac{N_r}{j \pi} \sin(\nu \theta_1) \frac{\sin\left(\frac{\nu \tau}{2}\right)}{\frac{\nu \tau}{2}}\]

For the second winding group

\[
c_2(\theta_r) = \begin{cases} \frac{N_r}{\tau} & \left(\frac{\theta_2 - \tau}{2} < \theta < \theta_2 + \frac{\tau}{2}\right) \\ -\frac{N_r}{\tau} & \left(2\pi - \theta_2 - \frac{\tau}{2} < \theta < 2\pi - \theta_2 + \frac{\tau}{2}\right) \\ 0 & \text{elsewhere} \end{cases}
\]

\[\Rightarrow \bar{C}_2^\nu = -\frac{N_r}{j \pi} \sin(\nu(\theta_1 + d + \tau)) \frac{\sin\left(\frac{\nu \tau}{2}\right)}{\frac{\nu \tau}{2}}\]

Therefore

\[
c_1(\theta) = \sum_{\nu = -\infty}^{\infty} C_1^\nu e^{-j\nu \theta} = 2 \sum_{\nu = 1}^{\infty} \Re\{C_1^\nu e^{-j\nu \theta}\}
\]

\[
c_2(\theta) = \sum_{\nu = -\infty}^{\infty} \bar{C}_2^\nu e^{-j\nu \theta} = 2 \sum_{\nu = 1}^{\infty} \Re\{\bar{C}_2^\nu e^{-j\nu \theta}\}
\]

\[\Rightarrow c_r(\theta) = c_1(\theta) + c_2(\theta) = \sum_{\nu = -\infty}^{\infty} \bar{C}_r^\nu e^{-j\nu \theta} = 2 \sum_{\nu = 1}^{\infty} \Re\{\bar{C}_r^\nu e^{-j\nu \theta}\}\]

where \( \bar{C}_r^\nu = C_1^\nu + C_2^\nu = -\frac{N_r}{j \pi} \sin(\nu \theta_1) \frac{\sin\left(\frac{\nu \tau}{2}\right)}{\frac{\nu \tau}{2}} - \frac{N_r}{j \pi} \sin(\nu(\theta_1 + d + \tau)) \frac{\sin\left(\frac{\nu \tau}{2}\right)}{\frac{\nu \tau}{2}} \) same as shown in equation (3-16).

**A.1.3. CHA of Airgap Model**

The airgap distance distribution function is shown as following, where the distribution is referred to Figure 3.9.
\[
\frac{1}{g(\theta)} = \begin{cases} 
\frac{1}{g_1} & (0 < \theta_r < \theta_1) \cup (\pi - \theta_1 < \theta_r < \pi) \\
\frac{1}{g_2} & (\theta_1 < \theta_r < \theta_2) \cup (\pi - \theta_2 < \theta_r < \pi - \theta_1) \\
\frac{1}{g_3} & (\theta_2 < \theta_r < \pi - \theta_2)
\end{cases}
\]

\[
\bar{G}^l = \frac{1}{2\pi} \int_{\theta_r}^{\theta_r + 2\pi} \frac{1}{g(\theta)} e^{j\theta} d\theta
\]

\[
= \frac{1}{2\pi} \left[ \int_{\theta_r}^{\theta_r + \theta_1} \frac{1}{g_1} e^{j\theta} d\theta + \int_{\pi + \theta_r - \theta_1}^{\pi + \theta_r} \frac{1}{g_1} e^{j\theta} d\theta + \int_{\theta_r + \theta_1}^{\theta_r + \theta_2} \frac{1}{g_2} e^{j\theta} d\theta \\
+ \int_{\pi + \theta_r - \theta_2}^{\pi + \theta_r - \theta_1} \frac{1}{g_2} e^{j\theta} d\theta + \int_{\theta_r + \theta_2}^{\pi + \theta_r - \theta_2} \frac{1}{g_3} e^{j\theta} d\theta \right]
\]

\[
\Rightarrow \bar{G}^l = \frac{1}{2\pi l} e^{j\theta_r} \left( 1 + e^{j\pi l} \right) \left( \frac{1}{g_1} - \frac{1}{g_2} \right) \left( e^{j\theta_1} - e^{j\pi} e^{-j\theta_1} \right)
\]

\[+ \left( \frac{1}{g_2} - \frac{1}{g_3} \right) \left( e^{j\theta_2} - e^{j\pi} e^{-j\theta_2} \right) - \frac{1 - e^{j\pi l}}{g_1} \]

For odd harmonic numbers, $\bar{G}^l = 0$; for even harmonic numbers, $\bar{G}^l = \frac{2}{\pi l} e^{j\theta_r} \left( \frac{1}{g_1} - \frac{1}{g_2} \right) \sin (l\theta_1) + \left( \frac{1}{g_2} - \frac{1}{g_3} \right) \sin (l\theta_2)$.

$g(\theta)$ and $\bar{G}^l$ in equation above is expressed in term of $\theta_s$.

In the special case when $l = 0$ (average airgap distance)

\[
\bar{G}^0 = \frac{2}{2\pi} \left[ \int_{\theta_r}^{\theta_r + \theta_1} \frac{1}{g_1} d\theta + \int_{\pi + \theta_r - \theta_1}^{\pi + \theta_r} \frac{1}{g_1} d\theta + \int_{\theta_r + \theta_1}^{\theta_r + \theta_2} \frac{1}{g_2} d\theta + \int_{\pi + \theta_r - \theta_2}^{\pi + \theta_r - \theta_1} \frac{1}{g_2} d\theta \\
+ \int_{\theta_r + \theta_2}^{\pi + \theta_r - \theta_2} \frac{1}{g_3} d\theta \right] = \frac{1}{\pi} \left[ \frac{2\theta_1}{g_1} + \frac{2(\theta_2 - \theta_1)}{g_2} + \frac{\pi - 2\theta_2}{g_3} \right]
\]
A.1.4. CHA of Stator Energised Magnetic Flux Density

Referred to the text stated in Chapter 3, the calculation process of stator magnetic flux density is shown as following, where a cross section view is shown in Figure A.2.

\[
\oint H \, dl = \sum I
\]

\[
\Rightarrow \oint H_i \, dl_i + \oint H_g \, dl_g = \int j_{sn}^q(\theta, t) \, d\theta
\]

\[
\Rightarrow \oint \frac{B_i}{\mu_i} \, dl_i + \oint \frac{B_g}{\mu_g} \, dl_g = \int j_{sn}^q(\theta, t) \, d\theta
\]

Since \( \mu_i \gg \mu_g \), so neglect iron path. For small section of \( \Delta \theta \)

\[
H(\theta + \Delta \theta)g(\theta + \Delta \theta) - H(\theta)g(\theta) = j(\theta, t)\Delta \theta
\]

\[
\Rightarrow \left( H(\theta) + \frac{\partial H}{\partial \theta} \Delta \theta \right) \left( g(\theta) + \frac{\partial g}{\partial \theta} \Delta \theta \right) - H(\theta)g(\theta) = j(\theta, t)\Delta \theta
\]

\[
\Rightarrow H(\theta)g(\theta) + H(\theta) \frac{\partial g}{\partial \theta} \Delta \theta + g(\theta) \frac{\partial H}{\partial \theta} \Delta \theta + \frac{\partial g}{\partial \theta} \frac{\partial H}{\partial \theta} \Delta \theta^2 - H(\theta)g(\theta) = j(\theta, t)\Delta \theta
\]

Neglect \( \Delta \theta^2 \) term.

\[
\Rightarrow H(\theta) \frac{\partial g}{\partial \theta} + g(\theta) \frac{\partial H}{\partial \theta} = j(\theta, t)
\]
⇒ \frac{\partial [g(\theta)H(\theta)]}{\partial \theta} = j(\theta, t)

⇒ H(\theta) = \frac{1}{g(\theta)} \int j(\theta, t) \, d\theta

⇒ H(\theta) = \frac{1}{g(\theta)} \int i^q_{sn}(t)c_{sn}(\theta) \, d\theta

⇒ H(\theta) = \frac{1}{g(\theta)} i^q_{sn}(t) \sum_{\nu=-\infty}^{\infty} \tilde{C}_{sn}^\nu \left( \frac{e^{-jv\theta_s}}{-jv} \right)

∴ B(\theta) = -\frac{\mu_o i^q_{sn}(t)}{g(\theta)} \sum_{\nu=-\infty}^{\infty} \frac{\tilde{C}_{sn}^\nu e^{-jv\theta_s}}{jv} e^{jlp\theta_m}

A.1.5. CHA of Flux linkage and Inductance in Stator Winding

Consider the flux linkage in a single coil of phase winding \( c_{sn2}(\theta_2) \) from the energised phase winding \( c_{sn1}(\theta_1) \), where \( c_{sn2}(\theta_2) \) is located at the angular position from \( \theta_2 \) to \( \theta_2 + \alpha \).

\[ \Phi(\theta_2) = \int BdA = \int B \, d(rt\theta) \]

\[ = -\mu_o i^q_{sn1}(t)RW \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{\tilde{C}_{sn1}^\nu \tilde{G}^l e^{jlp\theta_m}}{j\nu} e^{-j(\nu+l)\theta_1} \, d\theta_1 \]

\[ = -\mu_o i^q_{sn1}(t)RW \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{\tilde{C}_{sn1}^\nu \tilde{G}^l e^{jlp\theta_m}}{\nu(\nu + l)} e^{-j(\nu+l)\theta_2} \left( e^{-j(\nu+l)\alpha} - 1 \right) \]

Integrate the flux linkage over a whole distribution of phase winding \( c_{sn2}(\theta_2) \)
\[ \Psi = n \Phi = \int c_{sn2}(\theta_2) \Phi(\theta_2) d\theta_2 = \frac{1}{2} \int_0^{2\pi} c_2(\theta_2) \Phi(\theta_2) d\theta_2 \]

\[ = \frac{1}{2} \int_0^{2\pi} c_{sn2}(\theta_2) \left( -\mu_o i_{sn1}^q(t) r l \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu \tilde{G}_l e^{j l p \theta_m}}{\nu + l} e^{-j(\nu l + 1)\theta_2} \left( e^{-(\nu + 1)\alpha} - 1 \right) \right) d\theta_2 \]

\[ = \frac{1}{2} \int_0^{2\pi} c_{sn2}(\theta_2) \left( -\mu_o i_{sn1}^q(t) r l \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu \tilde{G}_l e^{j l p \theta_m}}{\nu + l} e^{-j(\nu l + 1)\theta_2 + \alpha} \right) d\theta_2 \]

\[ - \frac{1}{2} \int_0^{2\pi} c_{sn2}(\theta_2) \left( -\mu_o i_{sn1}^q(t) r l \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu \tilde{G}_l e^{j l p \theta_m}}{\nu + l} e^{-j(\nu l + 1)\theta_2} \right) d\theta_2 \]

Since \( c_{sn2}(\theta_2) = -c_{sn2}(\theta_2 + \alpha) \) and \( d(\theta_2) = d(\theta_2 + \alpha) \), integrating \( \theta_2 \) and \( \theta_2 + \alpha \) over the \( 2\pi \) radian will result in computing same flux twice, therefore, the flux in conductor winding \( c_{sn2}(\theta_2) \) is

\[ \Rightarrow \Psi = -\frac{1}{2} \int_0^{2\pi} c_2(\theta_2 + \alpha) \left( -\mu_o i_{sn1}^q(t) r l \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu \tilde{G}_l e^{j l p \theta_m}}{\nu + l} e^{-j(\nu l + 1)(\theta_2 + \alpha)} \right) d(\theta_2 + \alpha) \]

\[ - \frac{1}{2} \int_0^{2\pi} c_2(\theta_2) \left( -\mu_o i_{sn1}^q(t) r l \sum_{l=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu \tilde{G}_l e^{j l p \theta_m}}{\nu + l} e^{-j(\nu l + 1)\theta_2} \right) d\theta_2 \]

\[ = \frac{1}{2} \mu_o i_{sn1}^q(t) R W \int_0^{2\pi} \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu C_{sn2}^\mu \tilde{G}_l e^{j l p \theta_m}}{\nu + l} e^{-j(\nu + 1 + \mu)(\theta_2 + \alpha)} d(\theta_2 + \alpha) \]

\[ + \frac{\mu_o i_{sn1}^q(t) R W}{2 g_0} \int_0^{2\pi} \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu \tilde{G}_l e^{j l p \theta_m}}{\nu + l} e^{-j(\nu + 1 + \mu)\theta_2} d\theta_2 \]

The condition for integral is non zero only when \( \nu + l + \mu = 0 \), therefore

\[ \Psi = 2\pi \mu_o i_{sn1}^q(t) R W \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu C_{sn2}^\mu \tilde{G}_l e^{j l p \theta_m}}{\nu + l + \nu} \]

The corresponding inductance is

\[ L_{s_{s1}} = \frac{\Psi}{i_{sn1}^q(t)} = 2\pi \mu_o R W \sum_{l=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} \frac{C_{sn1}^\nu C_{sn2}^\mu \tilde{G}_l e^{j l p \theta_m}}{\nu(l + \nu)} \]
A.1.6. CHA of Stator Inductance with Reduced Harmonic Components

Considering up to the second harmonics for airgap distribution, the stator inductance can be simplified as

\[ L_{ss} = 2\pi \mu_o R W \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \tilde{C}_{\nu}^{\nu} \left( \frac{\tilde{C}_{\nu}^{2-\nu} G^{2-\nu}}{\nu(-2+\nu)} e^{-2j\mu\theta_m} + \frac{\tilde{C}_{\nu}^{2-\nu} G^{2}}{\nu(2+\nu)} e^{2j\mu\theta_m} + \frac{\tilde{C}_{\nu}^{2-\nu} G^{0}}{\nu^2} \right) \]

Consider the conductor density distribution product for mutual inductance between different phases of a 3-phase machine:

\[ \tilde{C}_a^\nu \tilde{C}_b^{-l-\nu} = \left( \frac{jN_{ph}}{\pi} \right)^2 \frac{K_p(\nu)K_p(-l-\nu)K_b(\nu)K_b(-l-\nu)K_d(\nu)K_d(-l-\nu)}{e^{-jl\left(\theta + \frac{(N_g-1)\pi}{N_s}\right)} e^{-j(l+1)2\pi}} \]

Let \( f_1(\nu) = \frac{jN_{ph}}{\pi} K_p(\nu)K_b(\nu)K_d(\nu) \),

and \( f_2(-l-\nu) = \frac{jN_{ph}}{\pi} K_p(-l-\nu)K_b(-l-\nu)K_d(-l-\nu) \), notice that \( f(-\nu) = -f(\nu) \).

For the fundamental components, where \( \nu = \pm 1 \)

When \( l = -2 \)

\[ \Rightarrow L_{ss(-2)} = 2\pi \mu_o R W \left( \frac{f_1(-1)f_2(3)}{3} e^{2j\left[\theta + \frac{(N_g-1)\pi}{N_s}\right] e^{j(3)2\pi/3}} + \frac{f_1(1)f_2(1)}{-1} e^{2j\left[\theta + \frac{(N_g-1)\pi}{N_s}\right] e^{j(1)2\pi/3}} \right) G^{-2} e^{-2j\mu\theta_m} \]

When \( l = +2 \)
⇒ \( L_{ss(2)} = 2\pi \mu_0 RW \left( \frac{f_1(-1)f_2(-1)}{-1} e^{-2j\left[ \theta_0 + \frac{(N_g-1)\pi}{N_s} \right]} e^{\frac{j(-1)2\pi}{3}} \right) \)

\[ + \frac{f_1(1)f_2(-3)}{3} e^{-2j\left[ \theta_0 + \frac{(N_g-1)\pi}{N_s} \right]} e^{\frac{j(-3)2\pi}{3}} \G^2 e^{2j\theta_m} \]

When \( l = 0 \)

⇒ \( L_{ss(0)} = 2\pi \mu_0 RW \left( \frac{f_1(-1)f_2(1)}{1} e^{\frac{j(1)2\pi}{3}} + \frac{f_1(1)f_2(-1)}{1} e^{\frac{j(-1)2\pi}{3}} \right) \G^0 \)

The summation of \( L_{ss(-2)} + L_{ss(2)} + L_{ss(0)} \) is

\[ L_{ss} = 2\pi \mu_0 RW \left( \frac{f_1(-1)f_2(3)}{3} e^{\frac{2j\left[ \theta_0 + \frac{(N_g-1)\pi}{N_s} \right]}{\frac{j(3)2\pi}{3}}} \right) \]

\[ + \frac{f_1(1)f_2(1)}{-1} e^{\frac{2j\left[ \theta_0 + \frac{(N_g-1)\pi}{N_s} \right]}{\frac{j(1)2\pi}{3}}} \G^{-2} e^{-2j\theta_m} \]

\[ + \left( \frac{f_1(-1)f_2(-1)}{-1} e^{-2j\left[ \theta_0 + \frac{(N_g-1)\pi}{N_s} \right]} e^{\frac{j(-1)2\pi}{3}} \right) \G^2 e^{2j\theta_m} \]

\[ + \left( \frac{f_1(1)f_2(-3)}{3} e^{-2j\left[ \theta_0 + \frac{(N_g-1)\pi}{N_s} \right]} e^{\frac{j(-3)2\pi}{3}} \right) \G^0 \]

Assume \( K_1 = \frac{f_1(1)f_2(3)}{3}, K_2 = f_1(1)f_2(1), K_3 = \theta_0 + \frac{(N_g-1)\pi}{N_s} \) and \( K_4 = \frac{2\pi \mu_0 r l}{G^0} \)

⇒ \( L_{ss} = K_4 \left( -K_1 e^{2jK_3} e^{\frac{3j2\pi}{3}} - K_2 e^{2jK_3} e^{\frac{j2\pi}{3}} \left( \frac{G^{-2}}{G^0} \right) e^{-2j\theta_m} + \left( -K_2 e^{-2jK_3} e^{-\frac{3j2\pi}{3}} - K_1 e^{-2jK_3} e^{-\frac{j2\pi}{3}} \right) \G^2 e^{2j\theta_m} + \left( -K_2 e^{-\frac{j2\pi}{3}} - K_2 e^{-\frac{j2\pi}{3}} \right) \right) \)

Since \( \G^{-2} = \G^2 \) and \( \G^2 \) is a real number from equation (3-25), the summation becomes:
Let $C = 2K_3$

$$L_{ss} = -2K_4 \frac{\tilde{G}^2}{G_0} \left( K_1 \cos \left( 2K_3 - 2p\theta_m + 3 \cdot \frac{2\pi}{3} \right) + K_2 \cos \left( 2K_3 - 2p\theta_m + \frac{2\pi}{3} \right) \right)$$

$$+ \tilde{G}_0 \frac{K_2}{G^2} \cos \left( \frac{2\pi}{3} \right)$$

Generally, for arbitrary different phase, the stator mutual inductance can be expressed as:

$$L_{ss21} = -2K_4 A_1 \left( K_1 \cos (C - 2p\theta_m + 3\gamma) + K_2 \cos (C - 2p\theta_m + \gamma) + \frac{K_2}{A_1} \cos (\gamma) \right)$$

where $A_1 = \frac{\tilde{G}^2}{G^2}$.

For variable airgap distance, the stator mutual inductance depends on not only the phase difference ($\gamma$), but also the rotor position ($\theta_m$) and the reference phase ($C$). ($\gamma = \phi(\text{phase}_2) - \phi(\text{phase}_1)$)

Neglect 3rd harmonic components of stator winding, where $K_4 = 0$:

$$\Rightarrow L_{ss} = -2K_4 \frac{\tilde{G}^2}{G_0} K_2 \cos (C - 2p\theta_m + \gamma) - 2K_4 K_2 \cos (\gamma)$$

$$= -2K_4 \frac{\tilde{G}^2}{G_0} K_2 \cos (2p\theta_m - C - \gamma) - 2K_4 K_2 \cos (\gamma)$$

$$= -2K_4 \frac{\tilde{G}^2}{G_0} K_2 \cos \left( 2p\theta_m - 2 \left( \theta_0 + \frac{(N_g - 1)\pi}{N_s} \right) - \gamma \right) - 2K_4 K_2 \cos (\gamma)$$

Since $\theta_0$ is the centre position of the first conduct coil in the reference phase group, effectively, the $\theta_0 + \frac{(N_g - 1)\pi}{N_s}$ can be regarded as the angle position of the reference phase. Take $\theta_{s1} = \theta_0 + \frac{(N_g - 1)\pi}{N_s}$

Assume that $-2K_4 \frac{\tilde{G}^2}{G_0} K_2 = L_2$ and $-2K_4 K_2 = L_0$
\[ L_{s2s1} = L_2 \cos(2p\theta_m - 2\theta_{s1} - \gamma) + L_0 \cos(\gamma) \]

where \( \theta_{s1} \) is the phase starting angle for stator winding s1 and \( \gamma = \theta_{s2} - \theta_{s1} \) is the phase difference. The expression of \( L_{s2s1} \) now is similar as the equation for mutual inductance of three-phase WFSG that stated in [84].
## A.2. Generator Geometry

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>permeability of vacuum</td>
<td>$4\pi \times 10^{-7}$ H/m</td>
</tr>
<tr>
<td>$W$</td>
<td>machine axial length</td>
<td>$0.14 \times 0.98$ m</td>
</tr>
<tr>
<td>$r$</td>
<td>stator inner radius</td>
<td>$0.0875$ m</td>
</tr>
<tr>
<td>$N_s$</td>
<td>total number of slots in stator</td>
<td>30</td>
</tr>
<tr>
<td>$N_c$</td>
<td>No. of conductors per stator slot</td>
<td>7</td>
</tr>
<tr>
<td>$w_s$</td>
<td>stator slot opening</td>
<td>$0.0038$ m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>number of rotor pole pairs</td>
<td>1</td>
</tr>
<tr>
<td>$D_{rod}$</td>
<td>rotor outer diameter</td>
<td>$0.1726$ m</td>
</tr>
<tr>
<td>$D_{spod}$</td>
<td>spreader outer diameter</td>
<td>$0.165$ m</td>
</tr>
<tr>
<td>$D_{cod}$</td>
<td>minimum rotor coil outer diameter</td>
<td>$0.1078$ m</td>
</tr>
<tr>
<td>$w_d$</td>
<td>damper slot opening</td>
<td>$0.00292$ m</td>
</tr>
<tr>
<td>$N_d$</td>
<td>No. of conductors per damper slot</td>
<td>1</td>
</tr>
<tr>
<td>$d_r$</td>
<td>angle between adjacent rotor coils</td>
<td>$12.66^\circ$</td>
</tr>
<tr>
<td>$w_r$</td>
<td>single rotor coil width</td>
<td>$0.4072$ rads</td>
</tr>
<tr>
<td>$N_r$</td>
<td>No. of turns per rotor coil</td>
<td>195</td>
</tr>
<tr>
<td>$m$</td>
<td>number of phases</td>
<td>15</td>
</tr>
<tr>
<td>$R_s$</td>
<td>stator phase winding resistance</td>
<td>$0.1388$ $\Omega$</td>
</tr>
<tr>
<td>$R_f$</td>
<td>rotor total coil resistance</td>
<td>$1.7279$ $\Omega$</td>
</tr>
<tr>
<td>$R_b$</td>
<td>damper circuit bar resistance</td>
<td>$0.1199$ m$\Omega$</td>
</tr>
<tr>
<td>$R_e$</td>
<td>damper circuit end resistance</td>
<td>$R_e(1,3,5,7) = 0.0966$ m$\Omega$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_e(2,6) = 0.177$ m$\Omega$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_e(4,8) = 0.665$ m$\Omega$</td>
</tr>
</tbody>
</table>

Table A.1 Cummins BCI162G generator geometry data.
A.3. Damper Winding Influence to Multiphase Model

It has been pointed out in [145] that the damper windings are less significant for the multiphase winding machines. Specifically, for this 15-phase GRS, the validation is done under fully-pitched polygon connection with 12A of field excitation and 3.3Ω DC load. The following figure shows the simulated GRS waveforms for different damper winding leakage inductance, the plot legends are the same as in Chapter 6. It can be found that for 15-phase polygon GRS, the damper windings do not have influence on the steady state behaviour.

Figure A.3 Damper winding and leakage inductance influence to the 15-phase polygon GRS with damper leakage inductance $L_{lk}$ of (a) $0.5L_{lk}$ (b) $L_{lk}$ (c) $2L_{lk}$ and (d) no damper windings.
A.4. Image Method

Note: Calculations are based on [153], assuming infinite permeability of the iron circuit and the end windings are assumed to be rectangular shape.

A.4.1. Mutual Inductance

After applying image method on the end winding, shown in Figure A.4, the windings are divided into segments as \( \overline{ABCD}, \overline{AEFD} \) and \( \overline{abcd} \).

![Image Method Diagram]

Figure A.4 Image method applied when calculating end winding inductance

Starting from calculating magnetic potential at point \( P(x, y) \) energised from segment \( AB \)

\[
A_1 = \frac{\mu_0 l_1}{4\pi} \int_{A}^{B} \frac{1}{\sqrt{(x)^2 + (y)^2}} \, dx \, dy = \frac{\mu_0 l_1}{4\pi} \int_{A}^{B} \frac{1}{\sqrt{(abx - ABx)^2 + (y)^2}} \, dy
\]

\[
= -\frac{\mu_0 l_1}{4\pi} \sinh^{-1} \left( \frac{y}{abx - ABx} \right) \bigg|_{A}^{B} = -\frac{\mu_0 l_1}{4\pi} \left( \sinh^{-1} \left( \frac{y - Ay}{abx - ABx} \right) - \sinh^{-1} \left( \frac{y - Ay}{abx - ABx} \right) \right)
\]

Integrating magnetic potential \( A_1 \) along segment \( ab \) gives magnetic flux on segment \( ab \)

\[
\phi_1 = -\frac{\mu_0 l_1}{4\pi} \int_{a}^{b} \left( \sinh^{-1} \left( \frac{y - By}{abx - ABx} \right) - \sinh^{-1} \left( \frac{y - Ay}{abx - ABx} \right) \right) \, dP_x \, dP_y
\]

\[
= -\frac{\mu_0 l_1}{4\pi} \int_{a}^{b} \left( \sinh^{-1} \left( \frac{y - By}{abx - ABx} \right) - \sinh^{-1} \left( \frac{y - Ay}{abx - ABx} \right) \right) \, dP_y
\]

\[
= -\frac{\mu_0 l_1}{4\pi} \left\{ \left( x - By \right) \sinh^{-1} \left( \frac{y - By}{abx - ABx} \right) - \left( x - ABx \right) \left( \frac{y - By}{abx - ABx} \right)^2 + 1 \right\} \bigg|_{P_y = y}^{P_y = a_y}
\]

\[
-\left( x - Ay \right) \sinh^{-1} \left( \frac{y - Ay}{abx - ABx} \right) - \left( x - ABx \right) \left( \frac{y - Ay}{abx - ABx} \right)^2 + 1 \right\} \bigg|_{P_y = y}^{P_y = a_y}
\]

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Let \( f(x) = x \sinh^{-1} x - \sqrt{(x)^2 + 1} \)

Therefore

\[
\phi_1 = -\frac{\mu_0 I_1}{4\pi} (ab_x - AB_x) \left( f \left( \frac{b_y - B_y}{ab_x - AB_x} \right) - f \left( \frac{a_y - B_y}{ab_x - AB_x} \right) - f \left( \frac{b_y - A_y}{ab_x - AB_x} \right) + f \left( \frac{a_y - A_y}{ab_x - AB_x} \right) \right)
\]

Same method can be applied to calculate the magnetic flux along segments \( \overrightarrow{bc}, \overrightarrow{cd} \) from energised segments \( \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{AE}, \overrightarrow{EF} \) and \( \overrightarrow{FD} \).

The total mutual inductance between \( \overrightarrow{ABCD} \) and \( \overrightarrow{abcd} \) would be

\[
L_m = \frac{N_c^2 \Sigma \phi_i}{I}
\]

same as equation (4-14), where \( N_c \) is the number of turns, \( I \) is the energising current.

### A.4.2. Self-Inductance

Same method is used when calculating self-inductance, while the conductors are assumed with effective radius of \( r \), the calculated self-flux linkage for segment \( \overrightarrow{AB} \) is

\[
\phi'_1 = \frac{\mu_0 I_1}{4\pi} r \left( 2X \left( \frac{B_y - A_y}{r} \right) - 2X \left( \frac{0}{r} \right) \right)
\]

Similarly, the total self-inductance for the end winding \( \overrightarrow{ABCD} \) would be

\[
L_s = \frac{N_c^2 \Sigma \phi'_i}{I}
\]

For total end winding inductance would be

\[
L_{end} = 2(\Sigma L_s + \Sigma L_m)
\]

as the slot inductance is so small, it is omitted in end winding inductance calculation.
A.5. Singularity Problem of Machine Stator Winding Inductance Matrix

For the arbitrary distributed winding stator inductance matrix $L_{ss}$, with corresponding Clark and Park transform matrix $C, P$ as well as their inverse matrix $C^{-1}, P^{-1}$.

Since $C^{-1}P^{-1}PC = I$, therefore $L_{ss} = C^{-1}P^{-1}PCL_{ss}C^{-1}P^{-1}PC = C^{-1}P^{-1}L_{ss}^{dq}PC$, the determinant of $L_{ss}$ is then equivalent as $|L_{ss}| = |C^{-1}P^{-1}L_{ss}^{dq}PC| = |C^{-1}P^{-1}||L_{ss}^{dq}||PC|$. Since $|L_{ss}^{dq}| = 0$ under the condition that the mutual coupling between synchronous reference frame is zero or near zero and there is no or little zero sequence inductance, therefore $|L_{ss}| = 0$, which proves to be a singular matrix, and this is common for the machine modelling but can be addressed by adding leakage inductance.

Assume a constant leakage term $L_{slk}$ added to each part of stator self-inductance term, the stator inductance matrix becomes $L_{ss} + L_{slk}diag(I)$, where $diag(I)$ represents the diagonal unit matrix with the same dimension as $L_{ss}$. The extra leakage inductance causes the determinant of $L_{ss} + L_{slk}diag(I)$ is no longer zero, which avoids the singularity problem.
A.6. Simulation Model and MATLAB Codes

A.6.1. Natural Reference Frame to Synchronous Reference Frame

The natural reference frame for WFSG is shown as

\[
\begin{align*}
[\tilde{V}_s] + [R_s]\tilde{I}_s + \frac{d}{dt}(L_{ss}\tilde{I}_s + L_{sf}\tilde{I}_f + L_{sd}\tilde{I}_d) &= 0 \\
V_f + R_f\tilde{I}_f + \frac{d}{dt}(L_{fs}\tilde{I}_s + L_{ff}\tilde{I}_f + L_{fd}\tilde{I}_d) &= 0 \\
[\tilde{V}_d] + [R_d]\tilde{I}_d + \frac{d}{dt}(L_{ds}\tilde{I}_s + L_{df}\tilde{I}_f + L_{dd}\tilde{I}_d) &= 0
\end{align*}
\]

By applying transform matrix \(C\) and \(P\) for stator windings and \(T_d\) for damper windings,

\[
\begin{align*}
PC[\tilde{V}_s] + PC[R_s]\tilde{I}_s + PC\frac{d}{dt}(L_{ss}\tilde{I}_s + L_{sf}\tilde{I}_f + L_{sd}\tilde{I}_d) &= 0 \\
V_f + R_f\tilde{I}_f + \frac{d}{dt}(L_{fs}\tilde{I}_s + L_{ff}\tilde{I}_f + L_{fd}\tilde{I}_d) &= 0 \\
T_d[\tilde{V}_d] + T_d[R_d]\tilde{I}_d + T_d\frac{d}{dt}(L_{ds}\tilde{I}_s + L_{df}\tilde{I}_f + L_{dd}\tilde{I}_d) &= 0
\end{align*}
\]

For the stator winding side

\[
\begin{align*}
PC[\tilde{V}_s] + PC[R_s]\tilde{I}_s + PC\frac{d}{dt}(L_{ss}\tilde{I}_s + L_{sf}\tilde{I}_f + L_{sd}\tilde{I}_d) &= 0 \\
\Rightarrow PC[\tilde{V}_s] + PC[R_s]C^{-1}P^{-1}PC\tilde{I}_s + PC\frac{d}{dt}(C^{-1}P^{-1}PC_{ss}C^{-1}P^{-1}PC\tilde{I}_s) + PC\frac{d}{dt}(C^{-1}P^{-1}PC_{sf}\tilde{I}_f) + PC\frac{d}{dt}(C^{-1}P^{-1}PC_{sd}\tilde{I}_d) &= 0 \\
\Rightarrow [\tilde{V}_s^s] + [R_s^s]\tilde{I}_s + PC\frac{d}{dt}(C^{-1}P^{-1}L_{ss}^s\tilde{I}_s) + PC\frac{d}{dt}(C^{-1}P^{-1}L_{sf}^s\tilde{I}_f) + PC\frac{d}{dt}(C^{-1}P^{-1}L_{sd}^s\tilde{I}_d) &= 0 \\
+PC\frac{d}{dt}(C^{-1}P^{-1}L_{sd}^s\tilde{I}_d) &= 0 \\
\Rightarrow [\tilde{V}_s^s] + [R_s^s]\tilde{I}_s + PC\frac{d}{dt}(C^{-1}P^{-1}L_{ss}^s\tilde{I}_s) + \frac{d}{dt}(L_{sd}^s\tilde{I}_d) + PC\frac{d}{dt}(C^{-1}P^{-1}L_{sf}^s\tilde{I}_f) + PC\frac{d}{dt}(C^{-1}P^{-1}L_{sd}^s\tilde{I}_d) &= 0 \\
+PC\frac{d}{dt}(C^{-1}P^{-1}L_{sd}^s\tilde{I}_d) &= 0
\end{align*}
\]
\[ [\dddot{V}_s^s] + [R_s^s] \dddot{I}_s^s + P \frac{d^{P-1}}{dt} \left( L_{ss}^s \dddot{I}_s^s + L_{sf}^s I_f + L_{sd}^s \dddot{I}_d^s \right) + \frac{d}{dt} \left( L_{ss}^s \dddot{I}_s^s + L_{sf}^s I_f + L_{sd}^s \dddot{I}_d^s \right) = 0 \]

For the rotor winding side

\[ V_f + R_f I_f + \frac{d}{dt} \left( L_{fs} C^{-1} P^{-1} P C \dddot{I}_s + L_{ff} I_f + L_{fd} T_a^{-1} T_a \dddot{I}_d \right) = 0 \]

\[ V_f + R_f I_f + \frac{d}{dt} \left( L_{fs} \dddot{I}_s^s + L_{ff} I_f + L_{fd} \dddot{I}_d^s \right) = 0 \]

For the damper winding side

\[ [\dddot{V}_d] + [R_d] \dddot{I}_d + \frac{d}{dt} \left( L_{ds} \dddot{I}_s^s + L_{df} I_f + L_{dd} \dddot{I}_d^s \right) = 0 \]

\[ T_d [\dddot{V}_d] + T_d [R_d] T_a^{-1} T_a \dddot{I}_d + T_d \frac{d}{dt} \left( L_{ds} \dddot{I}_s^s + L_{df} I_f + L_{dd} \dddot{I}_d^s \right) = 0 \]

\[ T_d [\dddot{V}_d] + T_d [R_d] T_a^{-1} T_a \dddot{I}_d + \frac{d}{dt} \left( T_d L_{ds} C^{-1} P^{-1} P C \dddot{I}_s + T_d L_{df} I_f + T_d L_{dd} T_a^{-1} T_a \dddot{I}_d \right) = 0 \]

\[ [\dddot{V}_d] + [R_d] \dddot{I}_d + \frac{d}{dt} \left( L_{ds} \dddot{I}_s^s + L_{df} I_f + L_{dd} \dddot{I}_d^s \right) = 0 \]

A.6.2. MATLAB Codes

\textit{a. Stator winding coefficients}

\texttt{function} \ [C] = Conductor_skew(row,v) 
\texttt{Dstator=0.175; \hspace{1cm} \%Diameter of stator in meter} 
\texttt{Rstator=Dstator/2; \hspace{1cm} \%Radius of stator in meter} 
\texttt{Bwidth=0.0038; \hspace{1cm} \%Slot mouth width in meter} 
\texttt{Pp=1; \hspace{1cm} \%Pole pairs} 
\texttt{m=3; \hspace{1cm} \%Number of phases} 
\texttt{Ns=30; \hspace{1cm} \%Number of slots in a whole radians} 
\texttt{Nc=7; \hspace{1cm} \%Number of conductors in a single coil} 
\texttt{pitch=15; \hspace{1cm} \%Slot pitch} 
\texttt{alph=2*pi/Ns*pitch; \hspace{1cm} \%Pitch angle in radian}
beta=Bwidth/Rstator;  %Single slot opening mouth in radians
Ng=Ns/m/Pp/2;  %Number of slots in a single phase group
O_starting=-alph/2-(Ng-1)*pi/Ns;  %Starting angle for phase A first slot
O_0=O_starting+alph/2;  %Center of single coil
s_alpha=12/180*pi;  %Stator slot skew angle in electrical
%Phase difference when calculating the stator to stator mutual inductance
gamma=2*pi/m;
%defining the factors
Kp_v=sin(v*alph/2);
Kb_v=(sin(v*beta/2))/(v*beta/2);
Kd_v=(sin(v*Ng*pi/Ns))/(v*Ng*pi/Ns)/(v*pi/Ns)/(sin(v*pi/Ns));
Nph=Nc*Ng*Pp;
double_layer=1-exp(1i*v*pi);
Polepairs=exp(1i*v*2*pi/Pp)*ones(1,Pp-1);
skew=sin(v*s_alpha/2)/(v*s_alpha/2);
%Stator conductor density coefficient
C=
1i*Nph/pi/Pp*Kp_v*Kb_v*Kd_v*double_layer*exp(1i*v*((O_0+gamma*(row-1)+(Ng-1)*pi/Ns))*1+sum(cumprod(Polepairs)))*skew;
%O0+2*pi/m*(row-1): phase shift for other phases
end

b. Rotor winding coefficients

function [ Cr ] = Rotor(v)
distance=12.66*pi/180;  %rotor coil span
O1=72*pi/180;
tau=23.34*pi/180;
Nr=195;  %Number of turns for single field winding group
%rotor winding coefficients
Kb_nu=(sin(v*tau/2))/(v*tau/2);
C1=-Nr/(1i*pi)*sin(v*(O1))*Kb_nu;
C2=-Nr/(1i*pi)*sin(v*(O1+distance+tau))*Kb_nu;
Cr=[C1;C2];
end
c. Airgap coefficients

function [ Gl ] = Airgap_Co( l )
    Dstator=0.175;  %Diameter of stator in meter
    Rstator=Dstator/2;  %Radius of stator in meter
    nui=4416;  %Relative permeability of iron
    ODrmax=0.1726;  %Rotor max outer diameter
    IDs=0.175;  %Stator inner diameter
    ODsp=0.165;  %Spreader outer diameter
    ODcm=0.1078;  %Minimum coil outer diameter
    theta1=60.33*pi/180;  %starting of the rotor field winding
    theta2=(60.33+23.34)*pi/180;  %end of the single rotor field winding

    g1=(IDs-ODrmax)/2;  %Airgap distance near the d axis
    g2=(IDs-ODcm)/2;  %Airgap distance near the field windings
    g3=(IDs-ODsp)/2;  %Airgap distance near the spreader
    g1i=Rstator-g1;  %Iron path distance near the d axis
    g2i=Rstator-g2;  %Iron path distance near the field windings
    g3i=Rstator-g3;  %Iron path distance near the spreader

    Iron_back=0.05272;  %Stator back iron distance in FEMM (max)
    Iron_DC=pi/2*(Rstator+Iron_back)+Iron_back;

    g1=g1+(g1i+Iron_DC)/nui;
    g2=g2+(g2i+Iron_DC)/nui;
    g3=g3+(g3i+Iron_DC)/nui;

    Gl=1i/(2*pi*l)*(1+exp(1i*l*pi))*((1/g1-1/g2)*(exp(1i*l*theta1)-exp(1i*l*theta2))+(1/g2-1/g3)*(exp(1i*l*theta1)-exp(1i*l*theta2))-(1-exp(1i*l*pi))/g1);
end
d. **Damper winding coefficients**

```matlab
function [ Cd ] = Damper(alpha1, alpha2, v )
    ODrmax=0.1726; %Rotor max outer diameter
    Bdwidth=0.00292; %Damper slot mouth width in meter
    w=Bdwidth/ODrmax/2; %Damper winding coil opening in radius
    Nd=1; %Number of turns

    %Damper coefficients
    Kb=sin(v*w/2)/(v*w/2);
    Cd=2*Nd/pi*Kb*(exp(1i*v*alpha1)-exp(1i*v*alpha2));
end
```

e. **Inductance matrix calculation**

```matlab
for v=-Conductor_Har:Conductor_Har
    for l=-Air_Har:Air_Har
        Airgap_l=Airgap_Co(l);
        % Stator – Stator inductance
        for phase1=1:m
            for phase2=1:m
                if (v==0)
                    Lss(phase1,phase2)=Lss(phase1,phase2);
                elseif ((v+l)==0)
                    Lss(phase1,phase2)=Conductor_skew(phase1,v)*Conductor_skew(phase2,-v-l)/v/(v+l)*Airgap_l*exp(1i*l*theta_m)+Lss(phase1,phase2);
                elseif (l==0)
                    Lss(phase1,phase2)=Conductor_skew(phase1,v)*Conductor_skew(phase2,-v-l)/v/(v+l)*G0+Lss(phase1,phase2);
                end
            end
        end
    end
% Stator-Rotor inductance
for row=1:m
    if (v==0)
        Lsf(row)=Lsf(row);
    ```
Lfs(row) = Lfs(row);
elseif ((v+l)~=0 && l~=0)
    Lsf(row) = squeeze(sum(Rotor(v),1))*Conductor_skew(row,v-l)/v/(v+l)*Airgap_l*exp(1i*(l+v)*theta_m)+Lsf(row);
    Lfs(row) = squeeze(sum(Rotor(v),1))*Conductor_skew(row,v-l)/v/(v+l)*Airgap_l*exp(1i*(l+v)*theta_m)+Lfs(row);
elseif (l==0)
    Lsf(row) = squeeze(sum(Rotor(v),1))*Conductor_skew(row,v-l)/v/(v+l)*G0*exp(1i*v*theta_m)+Lsf(row);
    Lfs(row) = Conductor_skew(row,v)*squeeze(sum(Rotor(v),1))/v/(v+l)*G0*exp(1i*v*theta_m)+Lfs(row);
end

% Stator-Damper inductance
for phase1=1:m
    for row=1:8
        alpha1 = DamperAngle(row);
        if (row<8)
            alpha2 = DamperAngle(row+1);
        else
            alpha2 = DamperAngle(1);
        end
        if (v==0)
            Lds(row,phase1) = Lds(row,phase1);
            Lsd(phase1,row) = Lsd(phase1,row);
        elseif ((v+l)~=0 && l~=0)
            Lds(row,phase1) = Conductor_skew(phase1,v)*Damper(alpha1,alpha2,v-l)/v/(v+l)*Airgap_l*exp(1i*(l+v)*theta_m)+Lds(row,phase1);
            Lsd(phase1,row) = Damper(alpha1,alpha2,v)*Conductor_skew(phase1,v-l)/v/(v+l)*Airgap_l*exp(1i*(l+v)*theta_m)+Lsd(phase1,row);
        elseif (l==0)
            Lds(row,phase1) = Lds(row,phase1);
            Lsd(phase1,row) = Lsd(phase1,row);
        elseif (l==0)

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Lds(row,phase1)=Conductor_skew(phase1,v)*Damper(alpha1,alpha2,-v-l)/v/(v+l)*G0*exp(1i*(-v)*theta_m)+Lds(row,phase1);

Lsd(phase1,row)=Damper(alpha1,alpha2,v)*Conductor_skew(phase1,-v-l)/v/(v+l)*G0*exp(1i*(v)*theta_m)+Lsd(phase1,row);

end

end

end

% Rotor-Rotor inductance
if (v==0)
    Lff=Lff;
elseif ((v+l)~=0 & l~=0)
    Lff=squeeze(sum(Rotor(v),1))*squeeze(sum(Rotor(-v-l),1))/v/(v+l)*Airgap_l+Lff;
elseif ((v+l)==0)
    Lff=Lff;
elseif (l==0)
    Lff=squeeze(sum(Rotor(v),1))*squeeze(sum(Rotor(-v-l),1))/v/(v+l)*G0+Lff;
end

% Rotor-Damper inductance
for row=1:8
    alpha1=DamperAngle(row);
    if (row<8)
        alpha2=DamperAngle(row+1);
    else
        alpha2=DamperAngle(1);
    end

    if (v==0)
        Ldf(row)=Ldf(row);
        Lfd(row)=Lfd(row);
    elseif ((v+l)~=0 & l~=0)
        Ldf(row)=squeeze(sum(Rotor(v),1))*Damper(alpha1,alpha2,-v-l)/v/(v+l)*Airgap_l+Ldf(row);
        Lfd(row)=Damper(alpha1,alpha2,v)*squeeze(sum(Rotor(-v-l),1))/v/(v+l)*Airgap_l+Lfd(row);
    elseif ((v+l)==0)
        Ldf(row)=Ldf(row);
        Lfd(row)=Lfd(row);
elseif (l==0)
    Ldf(row)=squeeze(sum(Rotor(v),1))*Damper(alpha1,alpha2,-v-l)/v/(v+l)*G0+Ldf(row);
    Lfd(row)=Damper(alpha1,alpha2,v)*squeeze(sum(Rotor(-v-l),1))/v/(v+l)*G0+Lfd(row);
end
end

% Damper-Damper inductance
for row1=1:8
    for row=1:8
        alpha1=DamperAngle(row1);
        if (row1<8)
            alpha2=DamperAngle(row1+1);
        else
            alpha2=DamperAngle(1);
        end
        alpha3=DamperAngle(row);
        if (row<8)
            alpha4=DamperAngle(row+1);
        else
            alpha4=DamperAngle(1);
        end
        if (v==0)
            Ldd(row1,row)=Ldd(row1,row);
        elseif ((v+l)~=0 & & l~==0)
            Ldd(row1,row)=Damper(alpha1,alpha2,v)*Damper(alpha3,alpha4,-v-l)/v/(v+l)*Airgap_l+Ldd(row1,row);
        elseif ((v+l)==0)
            Ldd(row1,row)=Ldd(row1,row);
        elseif (l==0)
            Ldd(row1,row)=Damper(alpha1,alpha2,v)*Damper(alpha3,alpha4,-v-l)/v/(v+l)*G0+Ldd(row1,row);
        end
    end
end
end
end

Lss = Lss * Rstator * 2 * pi * nu0 * Width;
Lsf = Lsf * Rstator * 2 * pi * nu0 * Width;
Lfs = Lfs * Rstator * 2 * pi * nu0 * Width;
Lff = Lff * Rstator * 2 * pi * nu0 * Width;
Lfd = Lfd * Rstator * 2 * pi * nu0 * Width;
Ldf = Ldf * Rstator * 2 * pi * nu0 * Width;
Lsd = Lsd * Rstator * 2 * pi * nu0 * Width;
Lds = Lds * Rstator * 2 * pi * nu0 * Width;
Ldd = Ldd * Rstator * 2 * pi * nu0 * Width;
A.7. Inductance Plots

A.7.1. 3-phase Winding Inductance

a. Fully-pitched winding

Figure A.5 Winding inductance plot against rotor position for 3-phase fully-pitched
coupling inductance, (d). stator-damper coupling inductance.
b. Short-pitched winding

A.7.2. 5-phase Winding Inductance

a. Fully-pitched winding

Figure A.7 Winding inductance plot against rotor position for 5-phase fully-pitched:

(a). stator-stator self-inductance,
(b). stator-stator mutual inductance,
(c). stator-rotor coupling inductance,
(d). stator-damper coupling inductance.
b. Short-pitched winding

A.7.3. 15-phase Winding Inductance

a. Fully-pitched winding

Figure A.9 Winding inductance plot against rotor position for 15-phase fully-pitched (a). stator-stator self-inductance, (b). stator-rotor coupling inductance, (c) and (d). stator-stator mutual inductance, (e). stator-damper coupling inductance.
b. Short-pitched winding

Figure A.10 Winding inductance plot against rotor position for 15-phase short-pitched (a). stator-stator self-inductance, (b). stator-rotor coupling inductance, (c) and (d). stator-stator mutual inductance, (e). stator-damper coupling inductance.
A.7.4. Damper Winding Inductance

Figure A.11 Winding inductance plot against rotor position for (a).rotor-damper coupling inductance, (b).damper-damper coupling inductance.
A.8. Generator Stator Terminal Winding Diagram

In this section, the winding connection diagrams are presented for 3-phase, 5-phase and 15-phase options, and notice that solid lines in this section are for stator panel connections and dashed lines for stator inner winding connections.

A.8.1 3-phase Stator Winding Diagram

3-phase fully-pitched winding diagram:

Figure A.12 Fully-pitched 3-phase winding diagram.
3-phase short-pitched winding diagram:

Figure A.13 Short-pitched 3-phase winding diagram.
A.8.2. 5-phase Stator Winding Diagram

5-phase fully-pitched winding diagram:

Figure A.14 Fully-pitched 5-phase winding diagram.
5-phase short-pitched winding diagram:

![Diagram](image)

Figure A.15 Short-pitched 5-phase winding diagram.
A.8.3. 15-phase Stator Winding Diagram

15-phase fully-pitched winding diagram:

![Diagram of a 15-phase fully-pitched stator winding]

Figure A.16 Fully-pitched 15-phase winding diagram.
15-phase short-pitched winding diagram:

Figure A.17 Short-pitched 15-phase winding diagram.
A.9. Test Rig Configurations

A.9.1. Overspeed Protection

As there is no further protection on the shared DC link between the two DC armature windings, the armature voltages for the two DC machine follow the relation as

\[ V_1 = k_1 I_{f1} \omega_1 \]
\[ V_2 = k_2 I_{f2} \omega_2 \]

where the subscript 1 and 2 refer to the DC generator ① and DC motor ② in Figure 6.2 respectively. Neglecting voltage drops on the DC link due to armature resistance, \( V_1 \) and \( V_2 \) should be same. Therefore, the shaft speed of the DC motor would be

\[ \omega_2 = \omega_1 \frac{k_1 I_{f1}}{k_2 I_{f2}} \]

Therefore, if the field current of the DC motor is lost, the motor shaft would overspeed and could damage the motor. The circuit board uses a TRACO POWER (TEN3-2413N) DC-DC converter to supply the controllers from the singe phase AC supply. A LM311 is used as comparator to control the relay (G6D-1A-ASI) that connected to the relay (Ghisalba) on the armature winding of DC motor ② in Figure 6.2. A current sensor (LTS 6NP) measuring field current from DC motor ② provides input signal to the comparator. The comparator is adjusted to capable of switching-off whenever the field current of DC motor ② drops below 0.8A and switching-on whenever it raises above 2.1A. The armature winding circuit breaker schematic diagram is shown in Figure A.18.
Figure A.18 Schematic diagram of circuit breaker.
A.10. Additional Experiment Plots

Three sets of 5-phase and five sets of 3-phase short-pitched winding layout, under 16A rotor field excitation and 2Ω DC load condition.

![Waveform](image)

Figure A.19 3×5φ short-pitched winding layout waveform (top) and 5×3φ short-pitched winding layout waveform from experimental measurement (left plots) and CHA simulation (right plots).

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Vs (V)</th>
<th>Vs (A/fund)</th>
<th>Is (A)</th>
<th>Is (A/fund)</th>
<th>Idd (A)</th>
<th>Idd (A/fund)</th>
<th>Vdc (V)</th>
<th>Idc (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly 3×5φ CHA SP</td>
<td>41.8</td>
<td>41.9</td>
<td>7.2</td>
<td>9.5</td>
<td>7.5</td>
<td>10.1</td>
<td>92.7</td>
<td>46.1</td>
</tr>
<tr>
<td>Poly 3×5φ EXP SP</td>
<td>43.7</td>
<td>43.9</td>
<td>7.1</td>
<td>10.0</td>
<td>8.1</td>
<td>12.8</td>
<td>94.7</td>
<td>45.6</td>
</tr>
<tr>
<td><strong>Error</strong></td>
<td>4.3%</td>
<td>4.6%</td>
<td>1.4%</td>
<td>5.0%</td>
<td>7.4%</td>
<td>21.1%</td>
<td>2.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Poly 5×3φ CHA SP</td>
<td>39.7</td>
<td>40.8</td>
<td>2.7</td>
<td>3.6</td>
<td>4.5</td>
<td>6.2</td>
<td>55.7</td>
<td>27.9</td>
</tr>
<tr>
<td>Poly 5×3φ EXP SP</td>
<td>40.3</td>
<td>40.7</td>
<td>2.6</td>
<td>3.2</td>
<td>4.0</td>
<td>5.2</td>
<td>55.4</td>
<td>26.4</td>
</tr>
<tr>
<td><strong>Error</strong></td>
<td>1.5%</td>
<td>0.2%</td>
<td>3.8%</td>
<td>12.5%</td>
<td>12.5%</td>
<td>19.2%</td>
<td>0.5%</td>
<td>5.7%</td>
</tr>
</tbody>
</table>

Table A.2 Comparison between experimental test and simulation for polygon 3×5φ short-pitched and polygon 5×3φ short-pitched winding configuration.
A.11. Limits of the CHA

In this section, the limits of the CHA method are presented, including mismatch on the harmonic phase and diode current shape variation due to different excitation level.

A.11.1. GRS Voltage Harmonic Phase Relationship

Starting with the harmonic phase relationship for the generator voltage under light and heavy load condition. It can be identified from the following figure that the 3rd, 5th and 7th harmonics are showing quite significant phase mismatch between the CHA and experiments.

Figure A.20 Stator voltage harmonics phase comparison on polygon connected windings (left) and star connected windings (right) under light load condition.

Figure A.21 Stator voltage harmonics phase comparison on polygon connected windings (left) and star connected windings (right) under heavy load condition.
For the heavy load condition, in terms of diode current waveform, the phase and magnitude of 5\textsuperscript{th} and 7\textsuperscript{th} harmonics for fully-pitched polygon connection shows considerable mismatch between CHA and experiment, which could be the cause for the difference on the diode current waveform, shown in Figure A.22 and Figure A.23. While for star connection, the main different of harmonic components happens on the phase relationship rather than the magnitude, which could the reason for shifted conducting time for star fully-pitched and lower peak diode current for star short-pitched layout between CHA and experiment.

Figure A.22 Diode current harmonic magnitude (left) and phase (right) for generator operation under heavy load polygon connection.

Figure A.23 Diode current harmonic magnitude (left) and phase (right) for generator operation under heavy load star connection.
A.11.2. Evaluation of CHA Generator-Rectifier Models for Different Field Excitation Conditions

From the light load and heavy load validation, the main differences in prediction of the current waveforms are diode currents in fully-pitched windings, either for polygon (with double peak) or star connection (with shifted phase) compared with the experimental result. Further tests investigate how this feature changes as the field excitation varies.

Under the 15-phase diode rectifier loading test, where the generator is operating at 1.0krpm with fixed 3.3Ω DC load for polygon connections and 2Ω DC load for star connections, it is found that the diode current waveform changes as the generator experiences different field excitation. The diode current conducting time is reduced by nearly 50% as the field current increases, as shown in Figure A.24.(left).

Similar trends in the diode current waveform from simulated results are presented in Figure A.24 (right), where the diode conduction time is also reduced by nearly 50%. However, the double peak feature is less obvious on the simulated results. The harmonic analysis, from Figure A.25, shows the diode current waveform double peak is mainly linked to the phase differences and magnitude of 5th and 7th harmonics. The diode current waveform in the simulation shows a slight double peak characteristic at low field. As the field excitation increases, the double peak characteristic quickly disappears, which happens faster than in the experimental measurement. As the field current further increases, the double peak characteristic completely disappears from both experiment and simulation, and the simulation again matches against the experiment, as the

Figure A.24 The diode current waveform for the polygon connection with fully-pitched winding layout under different field excitation comparing between experimental measurement (left) and simulation result (right), where $I_r$ in the legends refers to field excitation.
magnitude of 5th and 7th harmonic content become consistent between experiment and simulation. So the CHA method is correctly predicting the double width conduction at low excitation, moving to shorter conduction with higher current at higher excitation, but is not accurate in waveform double peak shape.

Figure A.25 Harmonic plot for diode current in terms of magnitude (top) and (middle) as percentage respect to fundamental, as well as phase (bottom) for polygon connected with fully-pitched winding layout, where $I_r$ in the legends refers to field excitation.

From the analysis of the generator currents for the polygon connection, the d-axis and q-axis currents are increasing at a different rate, shown in Figure A.26 (left), with a similar trend for both experiment and simulation.

Figure A.26 Variation of d-axis and q-axis generator currents under polygon (left) and star (right) connection for fully-pitched; experiment (solid line) and simulated (dashed line).

Sensitivity to excitation level is also found in the star connection, where the diode current waveforms are presented in the Figure A.27 together with their harmonic analysis in Figure A.28. In the star connection, the diode conduction time from experimental measurement is phase advanced when field excitation increases. However,
this is not seen in the simulation result. The harmonic analysis of the diode current waveform shows that the both the magnitude and phase variation are different between experiment and simulation. The percentage of odd harmonics drops firstly but then increases later in the experiment, whereas it only drops in the simulation.

\[
\begin{align*}
I_r &= 2A \\
I_r &= 4A \\
I_r &= 6A \\
I_r &= 8A \\
I_r &= 10A \\
I_r &= 12A \\
I_r &= 14A
\end{align*}
\]

**Figure A.27** The diode current waveform for a star connected fully-pitched winding layout under different field excitation comparing between experimental measurement (left) and simulation result (right), where \(I_r\) in the legends refers to field excitation.

**Figure A.28** Harmonic plot for diode current in terms of magnitude (top) and (middle) as percentage respect to fundamental, as well as phase (bottom) for star connected with fully-pitched winding layout, where \(I_r\) in the legends refers to field excitation.

Additionally, the proportions of D- and Q-axis currents are different between experiment and simulation, shown in Figure A.26.

From the magnetic flux distribution analysis from FEMM, shown in Figure A.29, cross-saturation is significant when saturation already occurs on D-axis by injecting high Q-axis current. It might be possible that the separated implementation of D-axis saturation and Q-axis saturation would improve the CHA accuracy. However, the Q-axis saturation of the generator in the lab is not easy accessible, therefore, only D-axis
saturation that representing the total magnetic flux saturation is implemented in this project.

Figure A.29 Rms of magnetic flux density along the airgap contour with different D-axis and Q-axis stator current for 15-phase fully-pitched (left) and short-pitched (right) from FEA, where the rotor is aligned with stator $\alpha$-axis position.

The single saturation factor represents the saturation trends on the D-axis and Q-axis separately well, however, the cross-coupling saturation effect is not considered. In Figure A.29, when D-axis saturates, increasing Q-axis current increases the total flux level, while this is not obvious when Q-axis saturates. Tip saturation on both the stator teeth and rotor arc also affect the waveform.
A.12. Model Application Plots

A.12.1. Single diode leg fault phase current variation plots for series stacked GRS.

Figure A.30 Phase current variation under single diode open-circuit condition for $3 \times 5\varphi$ series stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

Figure A.31 Diode current variation under single diode open-circuit condition for $3 \times 5\varphi$ series stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).
Figure A.32 Phase current variation under single diode open-circuit condition for $5 \times 3\phi$ series stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

Figure A.33 Diode current variation under single diode open-circuit condition for $5 \times 3\phi$ series stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).
A.12.2. Single diode leg fault phase current variation plots for parallel stacked GRS.

Figure A.34 Phase current variation under single diode open-circuit condition for 3×5φ parallel stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

Figure A.35 Diode current variation under single diode open-circuit condition for 3×5φ parallel stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).
Figure A.36 Phase current variation under single diode open-circuit condition for $5\times3\phi$ parallel stacked with polygon connection (top two) and star connection (bottom two), as in fully-pitched (left column) and short-pitched (right column).

Figure A.37 Diode current variation under single diode open-circuit condition for $5\times3\phi$ parallel stacked with polygon connection, as in fully-pitched (left column) and short-pitched (right column).