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DOI:

[10.1016/j.compfluid.2019.02.005](https://doi.org/10.1016/j.compfluid.2019.02.005)

Document Version

Submitted manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Utyuzhnikov, S., & Wang, C. (2019). Exact non-overlapping domain decomposition for near-wall turbulence modeling. *Computers and Fluids*, 181, 283-291. <https://doi.org/10.1016/j.compfluid.2019.02.005>

Published in:

Computers and Fluids

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On an exact domain decomposition method for near-wall turbulence modeling

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Abstract

The near-wall domain decomposition (NDD) method based on non-overlapping domain decomposition is proved to be very efficient for near-wall turbulence modeling. It entitles users to adjust the trade-off between accuracy and computational time. The NDD method is suggested as a promising way for turbulence modeling. It can be divided into the approximate NDD method and the exact NDD method. The approximate NDD method has successfully been applied to the Reynolds-averaged Navier-Stokes models. In this paper, the exact NDD method is implemented in a low-Reynolds-number model for the first time to simulate a 1D channel flow. The results demonstrate that the exact NDD approach is much better than the approximate NDD method and it can provide almost perfectly accurate results with only 8 % of original simulation time.

Keywords: Near-wall turbulence; Domain decomposition; Interface boundary condition; Low-Reynolds-number model.

1. Introduction

Near-wall turbulent flows are prevalent in many industrial problems. Although the near-wall region is very thin, its resolution has significant influence on the reliability of the result in the entire flow. An accurate enough resolution of the near-wall region requires a very thin mesh, which takes up to 90% of the total computational time [1]. Furthermore, the thickness of the turbulent boundary layer shrinks with the increase of Reynolds number, which indicates that the resolution of the near-wall region requires more computational time with higher Reynolds numbers. Thus, it can be seen that the near-wall region of turbulent flows is extremely complex for numerical computation. As a result, mathematical modeling of near-wall flows has been a research hotspot in the past few decades [2, 3].

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By discretising Navier-Stokes (N-S) equations in the entire computational domain, Direct Numerical Simulation (DNS) can provide the most accurate prediction for a flow. However, the iteration number of DNS is proportion to Re^3 . This means that DNS requires large computing resources and it is so far applicable only at flows with relatively low Reynolds numbers. In Large Eddy Simulation (LES), the motions of large scales are numerically resolved whilst the small scales are modeled. However, it is extremely expensive to apply LES to near-wall turbulent flows [4]. As demonstrated in [5], the computational cost of a wall-resolved LES is proportional to $Re^{2.4}$.

With a good compromise between accuracy and computational cost, Reynolds-averaged Navier-Stokes (RANS) models are still a good choice in the industrial community. RANS models merely solve the mean velocity field. There are two types of models in RANS to solve problem of the near-wall turbulence modeling: low-Reynolds-number (LRN) models and high-Reynolds-number (HRN) models. By involving near-wall correction functions, LRN models can resolve the structure of the entire turbulent boundary layer. However, a dense mesh is required for the near-wall region to capture the rapid changes of turbulence within the thin turbulent boundary layer. Therefore, the applications of LRN models are often unappealing for industrially-relevant simulations because of its prohibitively expensive computational cost.

Considering the high computing expense of near-wall turbulence simulation, HRN models are widely used in the industrial community. In HRN models, the near-wall region is represented by a semi-empirical Dirichlet boundary condition called wall function. The standard wall function was proposed by making an assumption of logarithmic law for the velocity in the turbulent boundary layer [6]. This has been widely used especially in engineering field for its relatively short simulation time and reasonably acceptable accuracy. This wall function requires the size of near-wall cell within some range. However, this requirement usually cannot be fulfilled in advance and a set of meshes should be compared to obtain a proper mesh. Moreover, it is very difficult to satisfy this condition for complicated flows. For example, there is no logarithmic region in the boundary layer for separated flows. As a result, many modifications were made to improve the validity of the standard wall function. In [7], scalable wall functions were established to improve the validity of the logarithmic law by assuming the first cell locates at the viscous sublayer edge if it is within the viscous sublayer. Because the requirement of the first cell position has been weakened, the wall function has been widely used as an improvement of the standard wall function. By analytically integrating simplified boundary layer equations and using a piecewise-linear profile of turbulent viscosity in the near-wall region, the analytical wall function can reduce the dependency on the size of near-wall cells and make it possible to simulate mixed and force-convection flows with HRN models [8]. The numerical wall function solves the boundary layer equations within the near-wall cells and obtains a first-rate agreement with the data predicted by the LRN models with the computational time less than an order of magnitude against the standard LRN models [9]. The numerical wall function has then been improved by a few researchers [10, 11, 12, 13]. Although this wall function is more accurate than analytical wall function, it suffers from stability problems [14]. The adaptive wall function [15] uses look-up tables, generated by simplified boundary layer equations without

consideration of source terms, to evaluate friction velocity and enhance numerical stability [16]. The compound wall function [17] blends the results of the near-wall region and log-law region together to ensure it is valid in any point of the turbulent boundary layer. Despite the fact that this wall function is simple for application and robust for industrial simulations, this method is unstable for complicated flows. Because all the wall functions are locally one-dimensional, their implementation to complicated geometries is always difficult for the lack of nonlocal boundary conditions.

The near-wall domain decomposition (NDD) method is a promising way to solve the problem of near-wall turbulence modeling [18]. In this approach, the computational domain is divided into two types of regions: the inner region and the outer region. The interface boundary conditions (IBCs) are obtained between inner and outer regions by transferring a Dirichlet boundary condition from the wall to the interface boundary. The interface boundary conditions are proved to be of Robin-type and represented in a mesh independent form. In [19], it is illustrated that the Robin-type interface boundary conditions can also be derived by the theory of the Caldern-Ryabenkii potentials [20, 21]. Recently, the method has been implemented to the RANS equations. It has successfully been applied to both HRN and LRN models. The results [22, 23, 24, 25, 26] show that the solution is not very sensitive to the interface boundary position. Compared with LRN models, the NDD method can reduce the simulation time by one order of magnitude with only about 1% drop in the accuracy. In [27, 28], the IBCs have been modified to take account of nonlocal effects in both the space and time. Recently, the method has successfully been used to model laminar-turbulent transition in the free wave of the Prandtl boundary layer model [29].

However, the current NDD method is the approximate NDD method, which is based on a simplified boundary layer type model. Because the IBCs are derived based on turbulent viscosity profiles set in the inner region, the trade-off between accuracy and computational time depends on interface boundary position. It cannot provide accurate prediction for complicated flows. Moreover, it is not good enough for laminar-turbulent transition modeling. Therefore, as an enhanced approach of the approximate NDD, the exact NDD method is proposed in this paper. In the exact NDD method, the Robin-type interface boundary condition is totally the same with the approximate NDD approach. The turbulent viscosity solved in the inner region will be used for the calculation of Robin-type boundary conditions instead of using turbulent viscosity profiles in approximate NDD method. In the paper, the exact NDD method is for the first time tested in a 1D channel flow with LRN $k-\varepsilon$ models. In section 2, the methodology of the exact NDD approach is described in detail. And then, the test results of the method are provided. The results show that the exact NDD method can provide nearly perfect accordance with the LRN results with minimum 8% of original computational time.

2. Methodology

The NDD method based on non-overlapping domain decomposition works by dividing the computational domain into an outer region and one or more inner regions. For a RANS equation for a function Ψ defined in $\Omega = [0, y_e]$:

$$\frac{\partial(\rho\Psi)}{\partial t} + \rho \frac{\partial(U_i\Psi)}{\partial x_i} = \frac{\partial}{\partial x_i}(\Gamma_\Psi \frac{\partial\Psi}{\partial x_i}) + F_\Psi \quad (1)$$

where

$$\Gamma_\Psi = \frac{\mu}{\sigma_\Psi} + \frac{\mu_t}{\sigma_{t,\Psi}} \quad (2)$$

F_Ψ includes all source terms and pressure gradients, μ and μ_t represent molecular viscosity and turbulent viscosity, respectively, σ_Ψ and $\sigma_{t,\Psi}$ represent Prandtl number and turbulent Prandtl number, respectively.

Now, let us split the domain into an inner region: $\Omega = [0, y^*]$ and an outer region: $\Omega = [y^*, y_e]$, where y^* is the interface boundary.

The governing equation in the inner region is reduced as:

$$\frac{\partial}{\partial y}(\Gamma_\Psi \frac{\partial\Psi}{\partial y}) = R_\Psi \quad (3)$$

R_Ψ contains wall-parallel diffusion, the convection terms and all source terms.

According to previous papers, the IBCs can be written as [23, 25]:

$$\Psi^* = f_1 \frac{\partial\Psi}{\partial y}(y^*) + f_2 \quad (4)$$

where

$$f_1 = \int_0^{y^*} \frac{\Gamma_\Psi(y^*)}{\Gamma_\Psi(y)} dy \quad (5)$$

$$f_2 = - \int_0^{y^*} \frac{\int_y^{y^*} R_\Psi d\xi}{\Gamma_\Psi(y)} dy + \Psi_w \quad (6)$$

Here, $y = 0$ is the wall, $y = y^*$ is the interface boundary, Ψ_w is the Dirichlet boundary condition at the wall.

It can be seen that equation (4) is a Robin-type boundary condition that can be used to transfer the boundary conditions from the wall to the interface boundary. In the exact NDD method, the Robin-type interface boundary condition is totally the same with the approximate NDD approach. The only difference is that the inner region is solved together with the outer region with the Dirichlet boundary condition:

$$u_{1|\Gamma}^k = \lambda^k \quad (7)$$

where

$$\lambda^k = \theta u_{2|\Gamma}^k + (1 - \theta) \lambda^{k-1} \quad (8)$$

λ^0 is taken from an initial value of approximate NDD method.

Therefore, the turbulent viscosity solved in the inner region will be used for the cal-

ulation of Robin-type boundary conditions instead of using turbulent viscosity profile in approximate NDD method.

3. Test case

In this paper, the LRN k- ε model proposed by Chien is chosen [30]:

$$\frac{\partial}{\partial y}[(\nu + \nu_t)\frac{\partial u}{\partial y}] = \frac{P_x}{\rho} \quad (9)$$

$$\nu_t(\frac{\partial u}{\partial y})^2 - \varepsilon + \frac{\partial}{\partial y}[(\nu + \frac{\nu_t}{Pr_k})\frac{\partial k}{\partial y}] = 0 \quad (10)$$

$$C_{\varepsilon 1} \frac{\tilde{\varepsilon}}{k} \nu_t (\frac{\partial u}{\partial y})^2 - C_{\varepsilon 2} f_2 \frac{\tilde{\varepsilon}}{k} + E + \frac{\partial}{\partial y}[(\nu + \frac{\nu_t}{Pr_\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial y}] = 0 \quad (11)$$

where

$$\nu_t = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}, f_\mu = 1 - e^{-0.0115y^+}, \varepsilon = \varepsilon_0 + \tilde{\varepsilon}$$

$$\varepsilon_0 = 2\nu \frac{k}{y^2}, f_2 = 1 - 0.22e^{(\frac{-Re_t}{6})^2}, Re_t = \frac{k^2}{\tilde{\varepsilon}\nu}$$

$$C_{\varepsilon 1} = 1.35, C_{\varepsilon 2} = 1.8, E = -2\nu \frac{\tilde{\varepsilon}}{y^2} e^{-y^+/2}$$

The 1D channel flow is chosen as a test case for the exact NDD approach. The simplicity of the 1D channel flow helps to save much computation time for the sake of testing the applicability of the method. For a 1D channel flow with channel half-height h , the relationship between friction velocity and pressure gradient is as follows:

$$u_\tau = \sqrt{h|-\partial_x P|/\rho} \quad (12)$$

Similar to [32], two cases have been simulated: $Re_\tau=395$ and $Re_\tau=3950$. The friction Reynolds number Re_τ is:

$$Re_\tau = \frac{u_\tau h}{\nu} \quad (13)$$

The dimensionless velocity is calculated as:

$$U^+ = \frac{U}{u_\tau} \quad (14)$$

3.1. 1D channel flow at $Re_\tau=395$

The mesh of the 1D channel flow at $Re_\tau=395$ is non-uniformed with 338 cells, which follows a geometric progression defined by the first space step of and the constant ratio of two successive space steps.

According to equation (4), for the Robin type interface boundary conditions of u , k and ε , the source terms are:

$$R_U = \frac{\partial P}{\partial x} \quad (15)$$

$$R_k = \varepsilon - \nu_t \left(\frac{\partial u}{\partial y} \right)^2 \quad (16)$$

$$R_\varepsilon = C_{\varepsilon 2} f_2 \frac{\tilde{\varepsilon}^2}{k} - C_{\varepsilon 1} \frac{\tilde{\varepsilon}}{k} \nu_t \left(\frac{\partial u}{\partial y} \right)^2 + E \quad (17)$$

In order to improve the accuracy of ε , the integral of R_ε is splitted into two parts: $\Omega_1 = [0, y^{**}]$ and $\Omega_2 = [y^{**}, y^*]$. y^{**+} is set as 5 in this paper.

In Ω_1 , all terms from R_ε can be neglected but E . This integral can be approximated with a good accuracy as follows:

$$\int_0^{y^{**+}} R_\varepsilon = \nu \frac{\tilde{\varepsilon}(y^{**})}{y^{**}} \quad (18)$$

As illustrated from Figure 1 to Figure 4, U^+ is very close to the result of original LRN k- ε model with a large variation of y^{*+} from 10 to 200. The same case has been simulated with approximate NDD method [31], the results show that the discrepancy of velocity profile with the LRN results increases with the interface boundary position being far from the wall. Similar results are found from Figure 5 to Figure 12 for k and ε . Therefore, it can be concluded that the exact NDD method is more accurate than approximate NDD method.

In order to compare the computational cost between exact NDD method and original LRN k- ε model, the time step of the inner region of the exact NDD method is set the same with that of LRN case. The outer region time step is increased to an optimum value to reduce the simulation time. The results of the LRN k- ε model have been validated by the DNS data [32]. In Figure 13, all the iteration numbers of the exact NDD method has been divided by the iteration number of the LRN case. It can be seen that the computation time can be reduced at least 70% compared with that of LRN. The simulation cost increases with the rising of interface boundary position. For the case of $y^{*+}=10$, the relative iteration number is only 8% of the LRN case.

3.2. 1D channel flow at $Re_\tau=3950$

In order to further test the validity of Exact NDD, the case of $Re_\tau=3950$ for 1D channel flow is carried out. Similar to the setting of the $Re_\tau=395$ case, the mesh at $Re_\tau=3950$ has 604 cells.

From Figure 14 to Figure 25, it is demonstrated that for y^{*+} increases from 10 to 200, Similar to the case of $Re_\tau=395$, U^+ , k and ε are almost perfectly matched with the LRN case. As mentioned in section 3.1, the results of approximate NDD method at $Re_\tau=3950$ also depends on the position of the interface boundary [32]. This further confirms that, in a wide changing range of interface face positions, the exact NDD method can accurately

predict the near-wall turbulent flow.

Figure 26 shows that when the interface position y^{*+} is under 150, the computation cost is under 15% of LRN case. The value for $y^{*+}=160-200$ is relative high, this is because the interface boundary is too far away from the wall. Therefore, it is recommended to set interface boundary position y^{*+} under 150 for the exact NDD method.

4. Conclusions

In this paper, the exact NDD method has been implemented in a LRN $k-\varepsilon$ model to simulate a test case of 1D channel flow at $Re_\tau=395$ and $Re_\tau=3950$. The results show that the exact NDD method predicts perfectly accurate results with one order of magnitude of LRN simulation time. The Exact NDD method is better than the approximate NDD method. In the future, the exact NDD method will be tested by more complex cases.

5. Acknowledgements

The first author is financially supported by the China Scholarship Council - The University of Manchester Joint Scholarship.

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figure 14

Figure .1: Velocity comparison with LRN for $y^{*+}=10\sim 50$ ($Re_\tau=395$).

Figure .2: Velocity comparison with LRN for $y^{*+}=60\sim 100$ ($Re_\tau=395$).

Figure .3: Velocity comparison with LRN for $y^{*+}=110\sim 150$ ($Re_\tau=395$).

Figure .4: Velocity comparison with LRN for $y^{*+}=160\sim 200$ ($Re_\tau=395$).

Figure .5: k comparison with LRN for $y^{*+}=10\sim 50$ ($Re_\tau=395$).

Figure .6: k comparison with LRN for $y^{*+}=60\sim 100$ ($Re_\tau=395$).

Figure .7: k comparison with LRN for $y^{*+}=110\sim 150$ ($\text{Re}_\tau=395$).

Figure .8: k comparison with LRN for $y^{*+}=160\sim 200$ ($\text{Re}_\tau=395$).

Figure .9: ε comparison with LRN for $y^{*+}=10\sim 50$ ($Re_\tau=395$).

Figure .10: ε comparison with LRN for $y^{*+}=60\sim 100$ ($Re_\tau=395$).

Figure .11: ε comparison with LRN for $y^{*+}=110\sim 150$ ($Re_\tau=395$).

Figure .12: ε comparison with LRN for $y^{*+}=160\sim 200$ ($Re_\tau=395$).

Figure .13: Relative iteration numbers with different interface positions ($Re_\tau=395$).

Figure .14: Velocity comparison with LRN for $y^{*+}=10\sim 50$ ($Re_\tau=3950$).

Figure .15: Velocity comparison with LRN for $y^{*+}=60\sim 100$ ($Re_\tau=3950$).

Figure .16: Velocity comparison with LRN for $y^{*+}=110\sim 150$ ($Re_\tau=3950$).

Figure .17: Velocity comparison with LRN for $y^{*+}=160\sim 200$ ($Re_\tau=3950$).

Figure .18: k comparison with LRN for $y^{*+}=10\sim 50$ ($Re_\tau=3950$).

Figure .19: k comparison with LRN for $y^{*+}=60\sim 100$ ($Re_\tau=3950$).

Figure .20: k comparison with LRN for $y^{*+}=110\sim 150$ ($Re_\tau=3950$).

Figure .21: k comparison with LRN for $y^{*+}=160\sim 200$ ($Re_\tau=3950$).

Figure .22: ε comparison with LRN for $y^{*+}=10\sim 50$ ($Re_\tau=3950$).

Figure .23: ε comparison with LRN for $y^{*+}=60\sim 100$ ($Re_\tau=3950$).

Figure .24: ε comparison with LRN for $y^{*+}=110\sim 150$ ($Re_\tau=3950$).

Figure .25: ε comparison with LRN for $y^{*+}=160\sim 200$ ($Re_\tau=3950$).

Figure .26: Relative iteration numbers with different interface positions ($Re_\tau=3950$).