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Statistical analysis of vortical structures in turbulent boundary layer over directional grooved surface pattern with spanwise heterogeneity

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We examine the turbulent boundary layers developing over convergent-divergent riblets (C-D riblets) with three different heights ($h^+ = 8, 14$ and 20) at $Re_\theta = 723$ using particle image velocimetry. It is observed that although a logarithmic region presents in the velocity profiles over the converging and diverging line, Townsend's outer-layer similarity hypothesis is invalid. Compared to the smooth-wall case, C-D riblets with a height of 2.4% of the smooth-wall boundary layer thickness can cause a significant increase in the turbulence production activities over the converging region, as evidenced by a more than 50% increase in the turbulent shear stress and in the population of prograde and retrograde spanwise vortices. In contrast, the impact of riblets on the diverging region is much smaller. The slope of vortex packets becomes steeper and they are more streamwise stretched in the outer layer over the diverging region, whereas their shape and orientation is less affected over the converging region. Furthermore, the number of uniform momentum zones across the boundary layer increases over the converging region, causing a reduction in the thickness of uniform momentum zones in the outer part of the boundary layer. Overall, while an increased riblet height affects a large portion of the boundary layer away from the wall over the converging region, the impact on the diverging region is largely confined within the near-wall region. Such distinct differences in the response of the boundary layer over the diverging and converging region is attributed to the opposite local secondary flow motion induced by C-D riblets.

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NOMENCLATURE

$x (m)$	streamwise coordinate
$y (m)$	wall-normal coordinate
$z (m)$	spanwise coordinate
$u (ms^{-1})$	streamwise velocity
$v (ms^{-1})$	wall-normal velocity
$U_{\infty} (ms^{-1})$	freestream velocity
$U_{\tau} (ms^{-1})$	friction velocity
$\Delta U (ms^{-1})$	difference in time-averaged streamwise velocity
$\nu (m^2 s^{-1})$	kinematic viscosity
$\delta (m)$	99% U_{∞} boundary layer thickness
$\theta (m)$	boundary layer momentum thickness
$Re_{\theta} (1)$	momentum thickness Reynolds number
$Re_{\tau} (1)$	friction Reynolds number
$K (1)$	acceleration parameter
$\Lambda (m)$	riblet wavelength
$h (m)$	riblet height
$s (m)$	riblet spacing
$\gamma (1)$	riblet yaw angle
$w_c (m)$	riblet crest width
$w_r (m)$	riblet root width
$A_g (m^2)$	groove cross section
$U_c (ms^{-1})$	convection velocity
$W_x (m)$	streamwise width
$H_y (m)$	wall-normal height
$\kappa (1)$	von Kármán constant
$B (1)$	log-law constant
$y_d (m)$	zero-plane displacement
$S (m)$	spanwise spacing
$\lambda_{ci} (s^{-1})$	swirling strength

$\langle \lambda_{ci} \rangle (s^{-1})$	signed swirling strength
$\omega_z (s^{-1})$	vorticity in $x - y$ plane
$\Pi (1)$	population density of vortices
$R (1)$	spatial correlation function
$\sigma (1)$	standard deviation
$L_x (m)$	streamwise length scale
$\alpha (1)$	inclination angle
$N_{UMZ} (1)$	number of uniform momentum zones
$t_{UMZ} (m)$	thickness of uniform momentum zones
$ \cdot $	absolute value
$\rightarrow$	vector
$-$	ensemble average

Superscript

+	normalization in wall units
'	fluctuating component

Subscript

s	smooth wall
p	prograde vortex
r	retrograde vortex
rms	root mean square
ref	reference point of correlation origin

Abbreviation

C-D riblets	convergent-divergent riblets
CL	converging line
DL	diverging line
PIV	particle image velocimetry
UMZ	uniform momentum zone
TNTI	turbulent/non-turbulent interface
p.d.f.	probability density function

FIG. 1. Schematic diagram of convergent-divergent riblets (C-D riblets) with definitions of diverging line (DL), converging line (CL), riblet wavelength (Λ) and yaw angle (γ). The time-averaged roll mode is shown in the cross-stream plane.

I. INTRODUCTION

Convergent-divergent riblets (C-D riblets) are composed of a spanwise array of sections of
 10 left-yawed and right-yawed microgrooves arranged in an alternating manner, see Fig. 1. Due to
 their potential for reducing surface friction drag¹ and suppressing flow separation,^{2,3} C-D riblets
 have attracted considerable research attentions in the recent few years.⁴⁻⁹ Similar to the patterns
 of spanwise alternating rough/smooth strips^{10,11} or longitudinal elevated/lower surfaces,¹²⁻¹⁴ C-D
 riblets are a type of surface patterns with spanwise heterogeneity, which are capable of generating
 15 a secondary flow in the boundary layers. A summary of the published work on C-D riblets can be
 found in Xu et al.⁷

As a result of the directional orientation of their microgrooves, C-D riblets generate inherently
 different flow structures in a boundary layer from those produced by longitudinal riblets.¹⁵⁻²¹ While
 the effect of longitudinal riblets is known to be confined in the vicinity of the riblet surface, C-D
 20 riblets are capable of producing a profound modification to the entire boundary layer.⁴⁻⁷ Over the
 diverging region, the streamwise velocity in the near-wall region is increased compared with that
 in the smooth-wall case, leading to a reduced boundary layer thickness, and the opposite occurs
 over the converging region.^{5,7,8} Such a modification to the velocity field has been attributed to the
 presence of a weak time-averaged secondary flow motion (roll mode) in the cross-stream plane in
 25 the boundary layer, whereby an upwelling (downwelling) over the converging (diverging) region

is created (see Fig. 1).^{6,7} The roll mode is essentially caused by the directional orientation of the yawed microgrooves which results in a surface flow directing from the diverging region to the converging region, as evidenced by the dye visualization experiment conducted by Xu et al.⁷

The secondary flow motions created by C-D riblets are also found to radically modify both the
30 time-averaged turbulence characteristics of the boundary layer and the coherent structures. While reduced near-wall turbulence intensity is observed over the diverging region, increased near-wall turbulence intensity is found over the converging region. This is in line with a reduction in the skin friction coefficient over the diverging region and an increase over the converging region as estimated using the modified Clauser technique.⁵ Based on single hot-wire measurements across
35 the boundary layer and PIV measurements in the wall-parallel plane located in the logarithmic region, the roll mode seems to have gathered the low-speed fluid and relocated the very-large-scale streamwise structures and features with high vortical activities over the converging region of the C-D riblets.^{5,6,22,23} With a large field of view PIV measurements in all orthogonal planes, Kevin et al.⁹ confirmed that C-D riblets result in formation of large-scale coherent structures over
40 the converging region, which dictate the instantaneous behavior of the roll mode and share some characteristics, such as meandering and breaking, displayed by the superstructures observed in canonical smooth-wall flows.

It is known that a turbulent boundary layer at low Reynolds numbers is populated by horse-shoe/hairpin vortices, which are typically composed of either one or two streamwise-oriented legs
45 connected to a spanwise-oriented head whose rotation is of the same sense as the mean shear.^{24,25} Hairpin vortices are prevalent in the transitional flow and they may not retain a complete hairpin shape in a turbulent boundary layer at a high Reynolds number.^{26,27} In the longitudinal (streamwise wall-normal) plane, the presence of these vortices is manifested by the appearance of ‘prograde’ spanwise vortices and ‘retrograde’ spanwise vortices. Prograde vortices, whose rotation is in the
50 same sense as the mean shear, are often associated with strong ejection and sweep events occurring in the lower upstream and upper downstream regions of their vortex centers, respectively.²⁸ Retrograde spanwise vortices, whose rotations are opposite to the mean shear direction, are believed to be the spatial signatures of bent necks of horseshoe/hairpin vortices.²⁹ The horseshoe/hairpin vortices are often grouped together in an orderly manner to form vortex packets, resulting in δ -scale
55 coherent structures which are inclined downstream.²⁴ The superstructures, which have a streamwise length scale substantially larger than the local boundary layer thickness, are observed in the outer region, and their motion is a self-sustained process without drawing energy from the hairpin

vortices in the near-wall region.^{30,31} As such, a hierarchy of vortical structures of different scales are observed in a turbulent boundary layer.

60 Based on the existing studies, as a type of spanwise heterogeneous surface pattern C-D riblets generate a secondary flow motion due to directional arrangement of their microgrooves. This secondary flow subsequently results in a profound difference in the time-averaged flow field in the converging and diverging region and a redistribution of turbulent activities in the spanwise direction. Therefore, the way the coherent structures organize themselves in the converging and
65 diverging region is expected to be distinctly different. So far, a fair amount of experimental data has become available enabling a good understanding of the time-averaged behavior of the flow field developing over C-D riblets. However, a more detailed description of the activities of coherent structures of different scales is still lacking. Hence further studies are required to gain a new insight in this regard.

70 Some statistical techniques have been applied to analyze the characteristics of coherent structures in the boundary layer developing along smooth- or rough-wall surfaces.^{32–34} A vortex identification technique was first proposed by Wu & Christensen²⁸ and Natrajan et al.,²⁹ to study the population of prograde and retrograde spanwise vortices across a boundary layer. Spatial correlation maps have been widely used to extract the time-averaged shape of hairpin packets. The
75 concept of uniform momentum zones (UMZs) in a turbulent boundary layer was put forwarded by Meinhart & Adrian³⁵ to infer the organized spatial distribution of hairpin vortices. The boundaries of a uniform momentum zone are found to pass through the heads of hairpin vortices,³⁶ and hence they are associated with step changes in the instantaneous streamwise velocity profile across the boundary layer. However, these analytical techniques have not been applied to examine the
80 turbulent boundary layer flow developing over C-D riblets or other types of surface patterns with spanwise heterogeneity.

Therefore, in this paper we perform a statistical analysis of the instantaneous velocity vector fields, focusing on the effect of C-D riblets on the characteristics of vortical structures over the diverging and the converging region. Particle image velocimetry (PIV) measurements are con-
85 ducted on the turbulent boundary layer over C-D riblets with three different heights. The vortex identification technique is improved in efficiency and applied to study the population of prograde and retrograde spanwise vortices across the boundary layer. Spatial correlation maps are examined to extract the information about the streamwise coherence length and orientation of vortex packets. The number and thickness of uniform momentum zones in the boundary layer are also

90 investigated. The effect of riblet height on these properties is summarized.

II. EXPERIMENTAL SET-UP

A. Water flume and flat plate

The present experiment was conducted in a low-speed water flume at the University of Manchester. The water flume has a test section measuring $305\text{mm} \times 305\text{mm}$ in cross-section. Four metal
95 screens arranged in a decreasing order of mesh sizes were installed at the inlet of the flume to ensure a low level of background turbulence in the test section. Further details of the water flume can be found in our previous work.⁷

To facilitate the experimental study, a 1350mm -long flat plate with its test section facing upward was designed, see Fig. 2. It has a super-elliptical leading edge to avoid flow separation from
100 the leading edge. To minimize plate bending along the streamwise direction, the flat plate is strengthened by two angle irons attached to its bottom side. The inclination angle of the flat plate can be finely adjusted using the threads on the four steel legs mounted on the bottom floor of the water flume. There is a small gap between the flat plate and each side wall of the water flume, which restrains the growth of the side wall boundary layers along the flat plate. A cavity measuring
105 $500\text{mm} \times 250\text{mm} \times 6\text{mm}$ (in length, width and depth respectively) starting at 550mm downstream of the leading edge was machined on the test plate. Rectangular tiles with either a smooth surface or rough pattern as the upper surfaces can be inserted into the cavity of the flat plate.

As shown in Fig. 2, axes x , y and z represent the streamwise, wall-normal and spanwise directions respectively, while u , v and w denote the corresponding velocity components. The ori-
110 gin of the coordinate system is located at the beginning of the riblet section, and it levels with the smooth wall and coincides with the center line of the test plate. The instantaneous velocity $\vec{u} = (u, v, w)$ is composed of the time-averaged component $\vec{U} = (U, V, W)$ and the fluctuating component $\vec{u}' = (u', v', w')$, i.e., $\vec{u} = \vec{U} + \vec{u}'$.

B. Parameters of convergent-divergent riblets

115 In the present experiment, C-D riblets were machined on rectangular tiles using 3D (three-dimensional) printing technique, as shown in Fig. 3a. The layer thickness and particle diameter were set as $25\mu\text{m}$, and black resin was used to reduce the surface reflection of the laser light. These

FIG. 2. Schematic diagram of the mono-PIV system in the longitudinal plane. The riblet spacing and the riblet height are not drawn to scale.

riblet tiles can be inserted into the cavity of the flat plate to form the surface pattern. The riblet section is 500mm in length and 250mm in width. The riblet wavelength (Λ) is 83.33mm , which is 2.55 times of the turbulent boundary layer thickness over the smooth wall (δ_s) at the measurement station. Across the span of the cavity, three riblet wavelengths were accommodated. The yaw angle (γ), which is defined as the angle between the yawed grooves and the freestream direction, is selected as 30° based on the optimal value proposed by Chen et al.¹ In our experiment, C-D riblets with three different heights (h), i.e., 0.8mm , 1.4mm and 2mm , and the same riblet spacing (s) of 2mm were tested. In all cases, the smooth wall is leveled with the mid height of the riblets. The blockage ratio, i.e., the riblet height to the boundary layer thickness of the smooth wall (h/δ_s), is 2.4%, 4.3% and 6.2% respectively at the measurement station. Over a homogeneous rough-wall surface, the outer-layer similarity is expected to hold for all these three height values (h/δ_s).^{37,38} A flat instead of curved riblet surface was chosen, since the former is easier to machine in future engineering applications. As indicated in Fig. 3b, to ensure durability, the riblets have trapezoidal teeth with a width of 0.4mm at the crest (w_c) and 0.8mm at the root (w_r).

The riblet height can be expressed in wall units using the local friction velocity (U_τ) and the kinematic viscosity (ν) as $h^+ = hU_\tau/\nu$. To ensure consistency, the friction velocity over the smooth wall ($U_{\tau s}$) is applied in the normalization. Along the streamwise length of the riblet section, the normalized riblet height (h^+) in wall unit varies by less than 6.3%. At the measurement station, the riblet height of 0.8mm , 1.4mm and 2.0mm corresponds to approximately 8, 14 and 20 in wall units respectively, and the riblet spacing in wall units is $s^+ = sU_\tau/\nu = 20$. Hereafter, we refer

FIG. 3. (a) Perspective view of a tile with C-D riblets on its upper surface. A £1 coin on its side is displayed for visual comparison. (b) Sectional view of C-D riblets along A – A, showing the trapezoidal profile of riblet teeth.

to these C-D riblets as $h^+ = 8$, $h^+ = 14$ and $h^+ = 20$ riblets respectively. García-Mayoral & Jiménez³⁹ proposed to use the groove cross section (A_g) to characterize the drag reduction effect of longitudinal riblets. The values of $A_g^{+1/2} = \sqrt{A_g \nu^2 / U_\tau^2}$ for the trapezoidal riblets tested in our experiments are 10.6, 14 and 16.7 respectively. Unlike the $h^+ = 8$ or $h^+ = 14$ riblets which stay within the viscous sublayer and the buffer layer, the riblets with $h^+ = 20$ approach the lower bound of the log-law region (see Fig. 4).

Note that the present study and the work by Nugroho et al.⁵ investigate the effect of riblet height from two complementary aspects. In our study, the freestream velocity was kept the same (the same baseline turbulent boundary layer), and the change in h^+ was obtained by changing the physical height (h) of C-D riblets. Since the riblet spacing remained the same, the ratio h/s varied from 0.4 to 1.0. By contrast, in the work by Nugroho et al.,⁵ the height of riblets was kept constant, and the riblet height in wall units (h^+) was changed by varying the freestream velocity. In their study, the baseline turbulent boundary layers were different, whilst the ratio h/s remained constant.⁵

C. Particle image velocimetry setup

The mono-particle image velocimetry (PIV) system used in our experiment is composed of a light-guiding arm, a New Wave Solo-PIV 120 laser generator, a laser pulse synchronizer (Model 610036), and a 4-mega pixel charge-coupled device (CCD) camera with 12-bit intensity. The maximum image-capturing frequency of the PIV system is 7.5Hz. The resolution of the CCD camera is 2352×1768 pixels. The hollow glass seeding particles have a mean diameter of $10\mu\text{m}$

and a density of $1.1 \times 10^3 \text{ kg/m}^3$, which were estimated to have a settling velocity of approximately $5 \times 10^{-6} \text{ m/s}$.

160 Mono-PIV measurements were performed in the longitudinal (x - y) plane, located in the middle of the spanwise direction. The longitudinal measurement plane covers the region located between $12\delta_s$ and $15\delta_s$ downstream of the start of the riblet section, where δ_s is the boundary layer thickness over the smooth wall at $x = 450 \text{ mm}$ ($\delta_s = 32.5 \text{ mm}$). The aperture of the lens was set at $f/4.0$. The time interval between two laser pulses was set at 2 ms . The width of the laser light sheet is around
165 1 mm . A total of 6000 image pairs (snapshots) were captured to ensure statistical convergence. A low sampling frequency of 0.5 Hz , which corresponds to 12.3 times the turnover time scale in the smooth-wall boundary layer (δ_s/U_∞), was used to minimize the statistical dependence between adjacent snapshots.

The PIV image pairs were processed using the cross-correlation algorithm in the InSight 4GTM
170 software. The first interrogation area is 64×32 pixels, and the final interrogation area is 24×24 pixels with a 50% overlap. The field of view is around $100 \text{ mm} \times 75 \text{ mm}$, which corresponds to a spatial resolution of $0.51 \text{ mm} \times 0.51 \text{ mm}$, or approximately 5×5 in wall units. The streamwise length of the PIV field is approximately 1000 in wall units. The average particle image diameter is 2 – 3 pixels, which ensures the sub-pixel accuracy. The image deformation algorithm is applied
175 considering the high disparity in the displacements of particles at different wall-normal distances. The measurement uncertainties in velocity components are estimated using the Peak Ratio (PR) method incorporated in the InSight 4GTM software. The expanded uncertainty in velocity components, i.e., 95.4% possibility of having the true value within the uncertainty bound, is estimated to be $0.47\% U_\infty$ for the instantaneous flow field and $0.03\% U_\infty$ for the time-averaged flow field.

180 **D. Characterization of the turbulent boundary layer over the smooth wall**

The baseline turbulent boundary layer was tripped by a 4 mm diameter rod installed 75 mm downstream of the leading edge of the flat plate. The inclination angle of the flat plate was adjusted to zero degree to minimize the pressure gradient in the freestream direction. The freestream velocity (U_∞) is 0.2 m/s . The turbulence intensity in the freestream region is around 0.7%, which
185 is acceptable considering the low Reynolds number of the flow. The acceleration parameter ($K = \nu(dU_\infty/dx)/U_\infty^2$) is approximately 5×10^{-8} in the measurement section, which ensures a streamwise quasi zero-pressure-gradient condition. At the measurement station of $x = 450 \text{ mm}$,

Parameter	U_∞	K	δ_s	$U_{\tau s}$	Re_θ	Re_τ
Value	$0.2m/s$	5×10^{-8}	$32.5mm$	$9.75 \times 10^{-3}m/s$	723	320

TABLE I. Experimental parameters of the turbulent boundary layer.

the boundary layer thickness (δ_s) is $32.5mm$ and the friction velocity ($U_{\tau s}$) is $9.75 \times 10^{-3}m/s$. The momentum thickness Reynolds number ($Re_\theta = U_\infty \theta / \nu$) is 723 and the friction Reynolds number
190 ($Re_\tau = U_\tau \delta / \nu$) is 320. Table I lists experimental parameters of the baseline turbulent boundary layer.

Figure 4 shows the time-averaged streamwise velocity profile (U^+) of the smooth-wall turbulent boundary layer. For clarity of presentation, not all the data points in the high y^+ region are presented. The friction velocity U_τ is estimated using the Clauser method with $\kappa = 0.4$ and
195 $B = 5.0$.⁴⁰ There exist a number of limitations in the Clauser method, including the selection of κ and B values and the subjectivity involved in selecting the data points in the logarithmic region.⁴¹ The profiles of wind tunnel datasets at $Re_\theta = 617$ (Erm et al.⁴²) and at $Re_\theta = 697$ (Erm et al.⁴³) are included for comparison. DNS datasets at $Re_\theta = 670$ (Spalart⁴⁴) and at $Re_\theta = 800$ (Wu & Moin⁴⁵) are also used as references. It can be seen that the measured mean velocity profile ob-
200 tained from our experiment follows the log-law in the region of $30 < y^+ < 120$. While there is a good consistency between our data and the references in the inner layer, some differences exist among these datasets in the wake region of $y^+ > 300$. Besides the minor difference in the Reynolds number, some other factors, e.g., differences in the numerical method used to introduce turbulence, numerical discretization scheme and resolution in simulations or the tripping device
205 used to trigger laminar-turbulent transition in the experiments,⁴³ may also contribute to the discrepancies observed. Overall, the baseline streamwise velocity profile is in good agreement with the reference profiles.

Figure 5 compares our profiles of turbulence intensities of u'_{rms}^+ and v'_{rms}^+ (rms refers to the root mean square) with four reference datasets at similar Reynolds numbers. The outer coordinate is
210 applied here to be consistent with reference datasets. Overall, the profiles of turbulence intensities in the present experiment exhibit a very similar trend as the reference profiles. The peak value and its vertical position are similar among these datasets. In the experiments of Erm et al.,^{42,43} the lower near-wall peak of u'_{rms}^+ could be attributed to the large value of the hot-wire etched length.⁴⁶ Interestingly, the present turbulence intensity profiles are closer to the published experimental

FIG. 4. Time-averaged streamwise velocity profile U^+ as a function of inner coordinate y^+ of the turbulent boundary layer.

FIG. 5. Turbulence intensity of (a) streamwise component $u'_{rms}+$, (b) wall-normal component $v'_{rms}+$, as a function of outer coordinate y/δ of the turbulent boundary layer.

215 data^{42,43} than the numerical simulation results.^{44,45} In the region outside the boundary layer, turbulence intensities of $u'_{rms}+$ and $v'_{rms}+$ in the present experiment are slightly larger than those in the other studies. We attribute this discrepancy to the slightly higher level of background freestream turbulence in our water flume.

Figure 6 compares profiles of the Reynolds shear stress ($-\overline{u'v'}+$) of the turbulent boundary layer 220 with four reference datasets.^{42–45} Despite the general agreement with these reference datasets in the profile shape, the peak value of $-\overline{u'v'}+$ is slightly lower in the present experiment. At a low Reynolds number, Erm et al.⁴³ revealed that the Reynolds shear stress profile is sensitive to the way the tripping device is installed. Thus, the minor variation of $-\overline{u'v'}+$ may be attributed to the

FIG. 6. Reynolds shear stress $-\overline{u'v'}$ as a function of outer coordinate y/δ of the turbulent boundary layer.

size and location of the tripping device used.

225 III. STATISTICAL TECHNIQUES

In this section, the statistical techniques used for extracting the characteristics of vortical structures are outlined. To identify spanwise vortices from PIV data, an efficient vortex recognition algorithm based on the queue data structure is utilized, which enables the population density of prograde and retrograde vortices to be computed. The spatial correlation of instantaneous velocity
230 fields is applied to reveal the streamwise extent and inclination angle of vortex packets. The analytical technique to extract the features of uniform momentum zones in a turbulent boundary layer is also introduced.

A. Recognition and distribution statistics of spanwise vortices

Prograde and retrograde vortices are spanwise vortices which possess negative and positive
235 values of the spanwise fluctuating vorticity, respectively. The signed swirling strength ($\langle \lambda_{ci} \rangle$), as proposed by Zhou et al.,⁴⁷ can be applied to identify the instantaneous spanwise vortices by extracting local swirling motion from the mean shear flow. In the longitudinal plane, the definition of the signed swirling strength is given by

$$\langle \lambda_{ci} \rangle = \lambda_{ci} \frac{\omega_z}{|\omega_z|}, \quad (1)$$

where λ_{ci} is the imaginary part of the complex conjugate eigenvalue of the in-plane velocity gradient tensor, ω_z is the in-plane vorticity, and the operator $|\cdot|$ denotes the absolute value. The sign
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FIG. 7. Sample contour of the instantaneous signed swirling strength $\langle \lambda_{ci} \rangle$ and in-plane velocity vectors $(u - U_c, v)$ revealing the distribution of prograde (blue) and retrograde (red) vortices.

of the in-plane vorticity $(\omega_z/|\omega_z|)$ indicates the rotational direction of the local swirling motion.

Figure 7 shows the distribution of spanwise vortices in a randomly selected instantaneous field obtained from the present study over the smooth wall. Due to the direction of the mean shear, there are more prograde vortices than retrograde vortices. The in-plane velocity vectors in Fig. 7 are obtained using Galilean decomposition by subtracting a convection velocity of $U_c = 0.85U_\infty$. The vortical structures which have the same convection velocity can be identified by the closed stream-line patterns, and the positions of these structures are aligned with the areas of high magnitude of $\langle \lambda_{ci} \rangle$.

The regions in the longitudinal plane constituting individual prograde/retrograde vortices can be identified by specifying predetermined thresholds of value and size. The vortex identification technique was first proposed by Wu & Christensen²⁸ and Natrajan et al.²⁹ In order to further improve computational efficiency and reduce memory usage, a (breadth-first) vortex recognition algorithm based on a queue data structure is proposed. Compared with the brute force method, this algorithm improves efficiency and reduces computer memory usage by using dynamic data structures.^{48,49} Details of this algorithm are listed below.

- S1. Scan each point in the instantaneous field to mark the points which exceed the threshold value(s) in terms of the signed swirling strength.
- S2. Find a marked point in the field (e.g., the point with the largest $\langle \lambda_{ci} \rangle$), then enqueue (i.e., insert the point item at the back of the queue) and unmark it.
- S3. While the queue is not empty, dequeue (i.e., remove) the first point. Then enqueue and

unmark its adjacent marked points (four candidates are the points at the top, bottom, left and right). Repeat this operation until the queue is empty.

S4. The points dequeued in S3 constitute a connected area. Determine whether or not that area can be considered as a prograde/retrograde vortex by evaluating its spatial range.

265 S5. Repeat S2-S4 until there is no marked point left in the field.

To extract the centers of spanwise vortices found with the vortex recognition algorithm, the weighted average of the points constituting a prograde/retrograde vortex is calculated, with the local swirling strength λ_{ci} as the weighting coefficient. At a given wall-normal station, the threshold value of the vortex intensity is selected to be proportional to the root-mean-square of the local
270 signed swirling strength, which accounts for the non-homogeneity of the flow in the wall-normal direction.^{28,50} In the following analysis, the threshold of the vortex intensity is selected as 1.5 times of the root-mean-square of $\langle \lambda_{ci}(y) \rangle$ over the smooth-wall case at the same location. In accordance to the choice of 22 wall units (three grid points) in Wu & Christensen,²⁸ the threshold of vortex size is set as four grid points in the present study, which corresponds to approximately 20 wall
275 units over the smooth wall. By using the same thresholds obtained from the smooth-wall case, we can quantitatively compare the population of prograde/retrograde vortices over C-D riblets with different riblet heights.

In our study, the population trends of prograde and retrograde vortices are presented as a function of the wall-normal distance. The population density of prograde (retrograde) vortices is defined as
280

$$\Pi_{p(r)} = \frac{N_{p(r)}}{H_y \cdot W_x / \delta_s^2}, \quad (2)$$

where $N_{p(r)}(y)$ is the ensemble-averaged number of prograde (retrograde) vortices whose centers are located in a rectangular area of $H_y \times W_x$. H_y is the wall-normal height which is selected as $0.1\delta_s$ (which equals 33 in wall units), and W_x is the streamwise width of the field of view of the PIV images. The center of the rectangular area is located at a wall-normal station of y . The definition of
285 population density in Eq. (2) is similar to the one proposed by Wu & Christensen²⁸ Thus, $\Pi_{p(r)}(y)$ indicates the normalized population of prograde (retrograde) vortices within an area of $\delta_s \times \delta_s$ in the longitudinal plane.

B. Extraction of spatial correlation features

The vortex packet (hairpin packet), which is composed of a hierarchy of hairpin vortices arranged in an organized manner, can be analyzed using spatial correlations of the instantaneous velocity fields in the longitudinal plane. The two-dimensional spatial correlation function of streamwise fluctuating velocity fields is defined as

$$R_{u'u'}(\Delta x, y; y_{ref}) = \frac{\overline{u'(x, y_{ref})u'(x + \Delta x, y)}}{\sigma_{u'(y_{ref})}\sigma_{u'(y)}}, \quad (3)$$

where Δx is the streamwise separation, y_{ref} is the wall-normal distance at which the correlation is calculated, σ is the standard deviation, and the overline denotes an ensemble average of all PIV snapshots.

To quantitatively analyze the features of vortex packets, the streamwise length and the inclination angle can be extracted from the spatial correlation contour. With a reference to Christensen & Wu,³² the streamwise length scale (L_x) can be defined as the streamwise distance between the upstream location and the downstream location of the $R_{u'u'} = 0.5$ contour line at y_{ref} .

The inclination angle (α) is obtained by best fitting the $R_{u'u'}$ contour of a specific value with an ellipse which is centered at the location where the correlation is taken. The angle of the major axis of the ellipse forms with the streamwise direction is taken as the inclination angle.⁵¹ In the present study, the overall inclination angle of the vortex packet is estimated by averaging the inclination angles of the $R_{u'u'} = 0.4, 0.5$ and 0.6 contours. The method of finding the inclination angle using the contour lines of intermediate values of correlation coefficients described above is found to yield more robust results than using the contour lines of low values of correlation coefficients. This is because this method is less susceptible to the noise in the correlation field. Christensen & Wu³² reported that the inclination angle α of the $R_{u'u'}$ contour gives a good indication of the inclination angle of the vortex packet. The inclination angle is largely invariant over a wide range of Reynolds numbers.⁵²

C. Identification of uniform momentum zones

The presence of uniform momentum zones (UMZs) in the turbulent boundary layer was first revealed by Meinhart & Adrian³⁵ and further investigated by Adrian et al.,³⁶ Kim & Adrian,⁵³ de Silva et al.^{34,54} etc. Within such a zone there is a relatively small momentum variation, and

a significant momentum change only occurs across the boundary between adjacent zones. The UMZs are prevalent across a turbulent boundary layer, and they can persist for long distances along the streamwise direction.³⁵

Figure 8a illustrates the UMZs in a randomly selected instantaneous field obtained from the present study over the smooth wall. The solid black lines delimit UMZs, and the solid red line indicates the interface between the turbulent regime and the non-turbulent regime. The region between solid lines has an approximately uniform streamwise momentum. The turbulent/non-turbulent interface (TNTI) is identified using the kinematic energy criterion in the frame of reference moving with the freestream, as proposed by de Silva et al.⁵⁴

Figure 8b compares the instantaneous and time-averaged streamwise velocity profiles along the vertical dotted line in Fig. 8a. There exists a significant change in the instantaneous streamwise velocity across the boundary of UMZs. The horizontal dotted lines delineate the boundaries of UMZs, while the upper dotted line indicates the TNTI. In Fig. 8c, the probability distribution function (p.d.f.) of the instantaneous streamwise velocity is applied to demarcate UMZs. The vertical dotted lines indicate the velocities which delimit UMZs, while the closed pentagrams indicate modal velocities. The entire field of view of the PIV snapshot in Fig. 8a is accounted to make the probability distribution as smooth as possible. PIV data points in the non-turbulent region are excluded to avoid their dominance in p.d.f. statistics. In the histogram, the value at each peak point (modal velocity) delineates the characteristic velocity within a UMZ, whereas the value at each valley point corresponds to the boundary between adjacent UMZs.

In the present study, the entire PIV field of view which corresponds to $L^+ \approx 1000$, is taken into account to analyzed features of UMZs, and the non-turbulent regime above the TNTI is excluded. Our parameter study using all PIV snapshots reveals that, when $L > \delta_s$ (i.e., $L^+ > 320$), the value of L has a negligible effect on the statistical characteristics of UMZs. This result is consistent with the finding of de Silva et al.,³⁴ which indicates that a selection of L^+ greater than 300 does not affect the p.d.f. of N_{UMZ} .

IV. RESULTS AND ANALYSIS

In this section, after briefly introducing the profiles of velocity and Reynolds shear stress, we reveal the effect of C-D riblets on the vortical structures in three inter-related aspects. The population densities of prograde/retrograde spanwise vortices are compared over C-D riblets and the

FIG. 8. (a) Sample contour of the instantaneous streamwise velocity u revealing the presence of uniform momentum zones. (b) Profiles of the instantaneous (—) and time-averaged (- - -) streamwise velocity along the vertical dotted line. (c) P.d.f. of the instantaneous streamwise velocity normalized by U_∞ .

smooth wall. The features of vortex packets, including the streamwise extent and the inclination angle, are extracted and compared. The effect of the newly created vortical structures on the uniform momentum zones is also analyzed. The riblet height effect on these aspects is revealed by comparing three sets of C-D riblets with different heights.

A. Velocity and Reynolds shear stress

The effect of C-D riblets is first revealed by the time-averaged streamwise velocity profiles over the CL and the DL in Fig. 9a. The converging line (CL) and the diverging line (DL) are defined by the C-D riblets geometry relative to the crossflow, see Fig. 1. Compared to the smooth-wall case, the local streamwise velocity decreases over the CL and increases over the DL. Over the CL, a decrease in the local streamwise velocity is observed in the entire boundary layer. By contrast,

over the DL, the increase in the local streamwise velocity is only significant in the near-wall region. Figure 9b shows the profiles in the time-averaged streamwise velocity difference ($\Delta U = U - U_s$) between C-D riblets and the smooth wall at the same wall distance. Over the CL, the magnitude of ΔU is larger (more negative) as the C-D riblets become higher. Over the DL, an increase in the riblet height primarily affects the near-wall region of $y/\delta_s < 0.1$. At a higher position of $y/\delta_s > 0.1$, the magnitude of ΔU is only slightly beyond zero. The difference between the affected regions, i.e., the entire flow field over the CL while only the near-wall region over the DL, is consistent with the previous observations in the laminar boundary layer⁷ and the turbulent boundary layer.^{5,6} The mechanism can be attributed to the direction of vertical flow and the influence of wall surface.⁷

Figure 9c compares profiles of the time-averaged wall-normal velocity (V) over the CL and the DL of C-D riblets and the smooth wall. Compared with the smooth-wall case, a strong upwelling occurs over the CL and an intense downwelling is seen over the DL in the time-averaged sense. It is found that the downwelling over the DL is more intense than the upwelling over the CL in the near-wall region, which is aligned with the previous observation in the laminar boundary layer.⁷ Over the DL, as C-D riblets become higher, the magnitude of V increases continuously, while the vertical peak position remains essentially unchanged. An increased magnitude of V can be attributed to the increased cross-sectional area of the yawed grooves as the C-D riblets become higher. Over the CL, as the riblet height h^+ increases, the magnitude of V becomes larger, i.e., the upwelling turns more intense. The peak position of V over the CL shifts away from the wall as the riblet height increases, which is attributed to the more intense upwelling over this region.

Figure 9d compares profiles of the Reynolds shear stress ($-\overline{u'v'}$) over the CL and the DL of C-D riblets and the smooth wall. The Reynolds shear stress $-\overline{u'v'}$ indicates the wall-normal momentum transfer due to turbulent events.⁵⁵ Over the CL, $-\overline{u'v'}$ increases dramatically compared to that over the smooth wall, with a larger riblet height leading to a higher Reynolds shear stress. In a spanwise-periodic in-phase sinusoidal channel, ejection events are preferentially located near the peaks where there present upwelling secondary flows.⁵⁶ These ejection events serve as the main contributor to the high $-\overline{u'v'}$ observed at the peaks. With an increased riblet height, the vertical position of the Reynolds shear stress peak over the CL shifts away from the wall. The vertical shift of the peak may be attributed to the increased magnitude of the wall-normal velocity over the CL (see Fig. 9c). Over the DL, a less intense Reynolds shear stress than that over the smooth wall is observed, and the lowest riblet height $h^+ = 8$ (open triangle Δ) corresponds to the lowest magnitude of $-\overline{u'v'}$. Since the region away from the wall has a lower magnitude of Reynolds shear

FIG. 9. Profiles of (a) time-averaged streamwise velocity U , (b) time-averaged streamwise velocity difference $U - U_s$, (c) time-averaged wall-normal velocity V , (d) the Reynolds shear stress $-\overline{u'v'}$, as a function of outer coordinate y/δ_s .

stress, the downwelling over the DL transfers the fluid with low $-\overline{u'v'}$ downward, making the local $-\overline{u'v'}$ smaller than that of the smooth wall. The secondary flow in the turbulent boundary layer becomes more intense as the riblet height increases, which contributes to more fluctuations over the diverging region. Thus, as the riblet height becomes larger, the Reynolds shear stress over the DL slightly increases. A change in the riblet height influences the distribution of $-\overline{u'v'}$ in the entire boundary layer over the CL and affects only the region of $y/\delta_s < 0.5$ over the DL.

In summary, C-D riblets lead to an increased local streamwise velocity, a downwelling and a reduced Reynolds shear stress over the DL, and the opposite applies to the CL. The secondary flow becomes more intense as the riblet height increases. For the streamwise velocity and the Reynolds shear stress, the impact of a changing riblet height is mainly reflected in the near-wall area over the DL and in the whole boundary layer over the CL. As the riblet height increases, the cross-sectional area of the yawed microgrooves becomes larger, resulting in greater flow capability and

more intense secondary flow.

400 B. Wall-shear stress and outer-layer similarity

To gain an insight on the local wall-shear stress, we analyze further profiles of the streamwise velocity over the CL/DL of C-D riblets and the smooth wall. Figure 10a compares profiles of the time-averaged streamwise velocity, wherein the friction velocity over the smooth wall ($U_{\tau s}$) is applied in the normalization. It is clearly seen that the near-wall velocity increases over the DL and decreases over the CL, and a quasi-logarithmic region exists in all velocity profiles over the CL/DL. The slope of the quasi-logarithmic region is larger over the CL and smaller over the DL, and a larger riblet height enhances such a trend. The existence of the log-law region makes it possible to estimate the wall-shear stress using modified Clauser method. Since the time-averaged spanwise velocity is nominally zero over the CL/DL, such analysis could be regarded valid.

410 As revealed by Clauser,⁴⁰ the surface pattern of uniformly distributed sand grits leads to an increased wall-shear stress and a downward shift of the mean flow in the logarithmic region of the boundary layer. In the modified Clauser method,^{5,57,58} the following equation is applied to the logarithmic law region of the streamwise velocity to estimate the wall-shear stress,

$$\frac{U}{U_{\tau}} = \frac{1}{\kappa} \ln \frac{(y - y_d)U_{\tau}}{\nu} + B - \Delta U^+, \quad (4)$$

where y_d is the zero-plane displacement, ΔU^+ is the roughness function which represents the vertical shift of the logarithmic region. The friction velocity can be determined by fitting the logarithmic law with three floating variables of U_{τ} , y_d and ΔU^+ . A positive (negative) ΔU^+ represents a higher (lower) value of U_{τ} , and thus an increase (decrease) in wall-shear stress. The value of ΔU^+ depends on various parameters including the shape and configuration of the surface roughness. To eliminate the term ΔU^+ , as proposed by Nugroho et al.,⁵ Eq. (4) is differentiated with respect to y , which leads to

$$\frac{\Delta U}{\Delta y} = \frac{U_{\tau}}{\kappa} \frac{1}{y - y_d}. \quad (5)$$

The friction velocity U_{τ} , along with the zero-plane displacement y_d , can be estimated by best fitting the time-averaged streamwise velocity with Eq. (5) in the logarithmic region.

The logarithmic region is manually selected using the linear section in Fig. 10a. Figure 10b shows the same profiles of the time-averaged streamwise velocity presented in Fig. 10a, with the local friction velocity applied in the normalization. The most apparent feature is the downward

425

h^+	8	14	20
CL	+50%	+58%	+63%
DL	-21%	-24%	-29%

TABLE II. Variation of wall-shear stress over the CL/DL compared to the smooth wall.

shift of the velocity profile over the CL and the upward shift over the DL, which indicate an increased drag over the CL and a decreased drag over the DL respectively. The estimated changes of the wall-shear stress ($\tau_w = \rho U_\tau^2$) are listed in Table II. The changing trend of the local wall-shear stress, i.e., an increase over the CL and a decrease over the DL, is consistent with the previous
430 estimations.^{5,6} The overall wall-shear stress over C-D riblets is not available in the present experiment. For longitudinal riblets with the same height of $h^+ = 8, 14$ and 20 (the same trapezoidal riblet teeth with $s^+ = 20$), the variation of the wall-shear stress is estimated to be -6% , -5% and -1% , respectively.³⁹ Compared to the longitudinal riblets that exhibit spanwise homogeneity, C-D riblets result in a spanwise variation of the wall-shear stress. The spanwise variations in wall-shear
435 stress over C-D riblets may be attributed to the flow topology in the yawed microgrooves observed by dye visualization.⁷ Over the diverging region, the fluid from the upper region is diverted away along the yawed microgrooves rather than directly impinging on the wall surface, resulting in a decrease in wall-shear stress. In contrast, over the converging region, the fluid being transported from both sides collides with each other there, and the wall-shear stress is increased correspond-
440 ingly. As the riblet height increases, the drag variations compared to the smooth-wall case become more pronounced. With an increased riblet height, the drag increase over the converging region is more significant than the drag reduction over the diverging region. It should be mentioned that, as noted by Nugroho et al.,⁵ the estimation accuracy using the modified Clauser method is limited.

Over C-D riblets, the spanwise direction of the secondary flow is from the low wall-shear stress
445 region (diverging region) to the high wall-shear stress region (converging region). Due to the low resistance, the fluid tends to flow in the direction parallel to the yawed microgrooves.^{7,59,60} This contributes to the formation of the secondary flow in order to satisfy the continuity requirement. The spanwise flow induced by the mean flow skewing has a contribution to the large-scale secondary flow over C-D riblets disregard of the state of boundary layer. It should also be mentioned
450 that the turbulence contribution in generating the secondary flow cannot be ignored, especially in the boundary layers at high Reynolds numbers.

FIG. 10. (a) Time-averaged streamwise velocity profile U/U_{τ_s} as a function of inner coordinate yU_{τ_s}/ν , where U_{τ_s} is the friction velocity over the smooth wall. (b) Time-averaged streamwise velocity profile U/U_{τ} as a function of inner coordinate yU_{τ}/ν , where U_{τ} is the local friction velocity.

Townsend's outer-layer similarity hypothesis over C-D riblets, which states

$$\frac{U_{\infty} - U}{U_{\tau}} = f\left(\frac{y}{\delta}\right), \quad (6)$$

where $(U_{\infty} - U)/U_{\tau}$ is the normalized velocity decay, $f(\cdot)$ is assumed universal in the outer layer regardless of the surface pattern. Figure 11 shows the profiles of velocity decay in the outer region over C-D riblets. It can be clearly seen that the velocity profiles do not overlap in the outer region, which indicates that Townsend's outer-layer similarity is not valid over C-D riblets in the present study. The deviation from the baseline (smooth-wall case) is more severe over the CL than that over the DL. This phenomenon can be attributed to the upwelling over the converging region,

FIG. 11. Velocity decay profile $(U_\infty - U)/U_\tau$ as a function of outer coordinate y/δ . The local friction velocity U_τ and boundary layer thickness δ are applied in the normalization.

which affects the velocity profile in the entire boundary layer. As the riblet height increases, the velocity decay profile over the CL moves upward and hence deviates further from the smooth-wall case. Even with a tentative adjustment of the local friction velocity, an overlap of the velocity decay profiles in the outer region cannot be obtained. Nugroho et al.⁵ observed non-overlapping of velocity decay profiles over C-D riblets at a high yaw angle of $\gamma = 30^\circ$. The friction velocity (wall-shear stress) may not be estimated directly using Eq. (6) by assuming Townsend's outer-layer similarity. Furthermore, it was found that Townsend's outer-layer similarity hypothesis does not hold for other types of spanwise heterogeneous surface patterns, including spanwise alternating roughness strips⁶¹ or surface elevation.⁵⁸ Thus, it is likely that the spanwise flow generated over a surface pattern with spanwise heterogeneity undermines the similarity in the outer layer.

In summary, C-D riblets lead to a significant change in the wall-shear stress, with an increase over the CL and a decrease over the DL. The drag variations compared to the smooth-wall case become more pronounced as the riblet height increases. Townsend's outer-layer similarity hypothesis is not valid over C-D riblets in the present study.

C. Population density of spanwise vortices

The population distribution of spanwise vortices is obtained by the recognition and statistical techniques described in Section III A. Figure 12 shows the population density of prograde vortices (Π_p) and retrograde vortices (Π_r). The population density of prograde vortices is larger than that of retrograde vortices, which can be attributed to the direction of the mean shear. In the tur-

bulent boundary layer more prograde vortices which follow the direction of the mean shear are
 accounted.^{24,28} In general, an increased population density of prograde/retrograde vortices is ob-
 480 served (except for the DL with $h^+ = 8$), with a more significant increase over the CL than the
 DL. The greater population of spanwise vortices over the CL was also qualitatively observed by
 Kevin et al.⁹ Over the CL, the population density of spanwise vortices in the boundary layer in-
 creases by more than 50% (more than two times in most of the regions for $h^+ = 20$) as compared
 with the smooth-wall case, implying a significant increase in the turbulence production activities.
 485 Although the identified structures may be affected by the particular choice of structure and iden-
 tification method, it is clear that the intensity of coherent motions over the converging region is
 significant increased. The significant increase over the CL confirms that there exists a creation of
 new coherent structures over the converging region, as proposed by Kevin et al.⁹ The change in the
 riblet height affects the whole boundary layer for prograde vortices and only the near-wall region
 490 of $y/\delta_s < 0.5$ for retrograde vortices, and the population density of the vortices presents a positive
 correlation with the riblet height. Over the CL, the vertical position with the maximum population
 density of retrograde vortices (Π_r) shifts away from the wall as the riblet height increases, while
 a vertical shift of the peak for prograde vortices (Π_p) is not apparent. The vertical shift of the
 Π_r peak over the CL can be attributed to the more intense upwelling secondary flow, which are
 495 essentially caused by the greater capacity of the spanwise flow with an increased riblet height.
 Over the DL, the population density of prograde and retrograde spanwise vortices remains almost
 unchanged in a large portion of the boundary layer except for some small changes in the near-wall
 area. We attribute these phenomena to the downwelling secondary flow, which transports the up-
 per undisturbed fluid to the near-wall region. The population density of these spanwise vortices is
 500 not sensitivity to an increase in the riblet height over the DL.

In summary, the population and distribution of prograde and retrograde spanwise vortices are
 compared to analyze the features of vortical structures over C-D riblets. The creation of new
 coherent structures can be confirmed by the significant modification of the population density of
 spanwise vortices over C-D riblets. Over the CL, the number of spanwise vortices is significantly
 505 increased by more than 50% over even the lowest C-D riblets. Over the DL, the change of the
 population density of spanwise vortices is far less significant. A height increase of C-D riblets
 leads to a larger population of spanwise vortices over both the CL and the DL. Over the DL, a
 variation in the riblet height is primarily reflected in the near-wall region.

FIG. 12. Population density of (a) prograde vortices Π_p , (b) retrograde vortices Π_r , as a function of outer coordinate y/δ_s .

D. Spatial correlation

The features of vortex packets are obtained by the spatial correlation analysis in the longitudinal plane described in Section III B. Figure 13 shows six representative contours of $R_{u'u'}$ over the CL/DL of $h^+ = 14$ C-D riblets and the smooth-wall case at two wall-normal stations of $y/\delta_s = 0.2$ and $y/\delta_s = 0.7$. Contour lines are between 0.3 and 0.8 with a spacing of 0.1. The central dots indicate the correlation origins, the solid line indicates the streamwise length scale L_x , and the dashed line shows the inclination angle α . Only the case of $h^+ = 14$ is provided in Fig. 13, and the similar cases of $h^+ = 8$ and $h^+ = 20$ are not shown here for the sake of brevity. The boundary layer thickness over the smooth wall (δ_s) is applied to normalize the horizontal and vertical coordinates, which ensures a consistent physical station of correlation origins. If the local boundary layer thickness was applied to normalization, a comparison of the length scales would be misleading. These contours indicate a distinct structure which is inclined at a shallow angle in the streamwise direction.³² For the correlation origin at $y/\delta_s = 0.2$, the contour over the C-D riblets extends to a shorter streamwise distance in comparison with that over the smooth wall, and inclines at a steeper (shallower) angle over the CL (DL), as indicated in Fig. 13c (Fig. 13e). The contour shape with the correlation origin at $y/\delta_s = 0.7$ is indicative of the spatial arrangement of vortex packets found in the outer region.^{36,62} The differences in the contour shapes over the CL in Fig. 13d and the smooth wall in Fig. 13b are not significant. Over the DL, the contour in Fig. 13f extends to a shorter streamwise distance and inclines at a steeper angle.

To quantitatively extract the features of vortex packets, the preceding contours are analyzed in

FIG. 13. Correlation contours of the streamwise fluctuating velocity in the longitudinal plane along (a) smooth wall at $y/\delta_s = 0.2$, (b) smooth wall at $y/\delta_s = 0.7$, (c) $h^+ = 14$ - CL at $y/\delta_s = 0.2$, (d) $h^+ = 14$ - CL at $y/\delta_s = 0.7$, (e) $h^+ = 14$ - DL at $y/\delta_s = 0.2$, (f) $h^+ = 14$ - DL at $y/\delta_s = 0.7$.

terms of the streamwise length and the inclination angle. Figure 14a compares the streamwise
 530 length scale (L_x) extracted at the contour level of $R_{u'u'} = 0.5$. Over the smooth wall, L_x decreases
 rapidly in the region of $y/\delta_s < 0.2$, and then presents a slight increase followed by a mild decrease,
 with an approximate platform at $L_x \approx 0.65\delta_s$ in the region of $0.4 < y/\delta_s < 0.8$. Near the edge of
 the boundary layer, L_x decreases rapidly as a consequence of the low freestream turbulence. Both
 the changing trend and the platform value of L_x , including the rapid drop in the near-wall region,
 535 agree well with the result by Volino et al.³³ For C-D riblets, over the CL, L_x is significantly smaller
 in $y/\delta_s < 0.5$ and slight larger in $y/\delta_s > 0.65$ than that over the smooth wall, and the platform
 area does not exist. A lower (higher) value of L_x indicates a reduced (increased) streamwise spacing
 between consecutive vortices in the vortex packets.⁶³ The smaller L_x in the region of $y/\delta_s < 0.5$
 over the CL can be attributed to the larger population of spanwise vortices as revealed in Fig. 12,
 540 i.e., the creation of new coherent structures. Similarly, over the rough wall made of a woven
 mesh surface, Volino et al.³³ also observed a lower L_x in the near-wall region compared with the

smooth wall. However, the mechanism of the rough wall is attributed to the destruction of the near-wall streaks, since no upwelling secondary flow occurs in the time-averaged sense. Due to the upwelling over the converging region, the area with a lower value of L_x over the CL of C-D
545 riblets ($y/\delta_s < 0.5$) is much larger than that of the rough wall ($y/\delta_s < 0.1$). The riblet height has little effect on L_x over the CL, with an increased riblet height leading to a slightly larger value of the streamwise length in the outer region. Over the DL, a narrower streamwise length L_x of the $R_{u'u'}$ contour than that over the smooth wall is observed in the entire boundary layer, with an increased riblet height leading to a shorter streamwise length of vortex packets. The platform region of L_x
550 exists over the DL, but does not present over the CL. A change in the riblet height corresponds to a significant variation of L_x over the DL. Volino et al.³³ showed that a homogeneous rough surface only modifies L_x in the near-wall area, whereas the feature of L_x in the outer flow remains essentially unchanged. In contrast, the C-D riblets profoundly modify the spatial distribution of coherent structures by introducing a large-scale secondary flow, leading to a significant change in
555 L_x across the entire boundary layer.

Figure 14b shows the wall-normal variation of the inclination angle (α) over the C-D riblets and the smooth wall. Over the smooth wall, the inclination angle is approximately 10.5° for $0.2 < y/\delta_s < 0.7$ in the present study, which is close to the results obtained in other studies (e.g., 10° by Krogstad & Antonia,⁶⁴ 12° by Adrian et al.,³⁶ $12^\circ - 13^\circ$ by Christensen & Adrian,⁶² 11°
560 by Christensen & Wu,³² $10.7^\circ - 15.7^\circ$ by Volino et al.³³). Compared with the smooth wall, the inclination angle is larger (smaller) over the CL (DL) in the near-wall region of $y/\delta_s < 0.25$. The larger (smaller) inclination angle over the CL (DL) can be tentatively attributed to the upwelling (downwelling) in the near-wall region. Similarly, a larger inclination is observed over the upwelling (elevated) region over an elevation heterogeneity, which is attributed to the enhanced
565 mixing within the high momentum pathways.⁶⁵ Away from the wall, the inclination angle in the region of $y/\delta_s > 0.25$ is larger over the DL and slightly smaller over the CL compared with the smooth wall. The steeper hairpin packet in the outer region over the DL may be attributed to the smaller number of spanwise vortices, i.e., fewer hairpin vortices in the boundary layer. The riblet height has a negligible effect on the inclination angle in the region between $0.3\delta_s$ and $0.7\delta_s$. The
570 rough wall is observed to lead to an inclination angle 1° to 3° larger than that over the smooth-wall case.^{33,66} For the C-D riblets, the changing trend of the inclination angle is inconsistent in the boundary layer, e.g., an increased α occurs only in the near-wall region over the CL and only in the outer region over the DL.

FIG. 14. Profiles of (a) streamwise length scale L_x , (b) inclination angle α , of the streamwise velocity correlation contour as a function of outer coordinate y/δ_s .

FIG. 15. Schematic diagram of vortex packet in the outer region over (a) smooth wall, (b) CL of C-D riblets, (c) DL of C-D riblets, indicating its streamwise length scale and inclination angle. The brown dots indicate the heads of hairpin vortices.

In summary, the spatial correlation characteristics in the longitudinal plane suggest that C-D
 575 riblets lead to a significantly modified spatial arrangement of vortex packets. By synthesizing
 Figs. 14a and 14b, schematics of the shapes of vortex packets in the outer region are shown in
 Fig. 15. In comparison with the smooth-wall case, while the δ -scale vortex packets are steeper and
 less stretched in the outer region over the diverging region, they are less affected (slightly flatter
 and a bit more stretched) over the converging region. An increase in the riblet height results in a
 580 further shortening of these packets in the streamwise direction over the diverging region.

E. Uniform momentum zones

The uniform momentum zones (UMZs) are identified by the technique described in Sec-
 tion III C. Figure 16a compares the p.d.f. of the number of UMZs (N_{UMZ}) over the C-D riblets and
 the smooth wall. The dominant number of UMZs, i.e., N_{UMZ} with the highest probability, changes

FIG. 16. (a) P.d.f. of the number of UMZs N_{UMZ} . (b) Mean number of UMZs $\bar{N}_{UMZ}$ as a function of friction Reynolds number Re_τ .

from $N_{UMZ} = 2$ over the smooth wall or the DL to $N_{UMZ} = 3$ over the CL. The average number of UMZs ($\bar{N}_{UMZ}$) can be calculated by the probability weighted summation of N_{UMZ} in Fig. 16a. As stated by de Silva et al.,³⁴ $\bar{N}_{UMZ}$ presents a log-linear increase with increasing friction Reynolds number (Re_τ) at medium to high Reynolds numbers. The value of $\bar{N}_{UMZ}$ over the smooth wall at the present Reynolds number is 2.371, which also follows this log-linear relationship as indicated in Fig. 16b. The consistency here provides further support to the hierarchical length scale distribution of vortical structures within the boundary layer.⁶⁷ $\bar{N}_{UMZ}$ is larger over the CL than the smooth wall, and the riblet height has a positive correlation with $\bar{N}_{UMZ}$ ($\bar{N}_{UMZ} = 2.567/2.608/2.658$ for C-D riblets with $h^+ = 8/14/20$ respectively). Over the CL, an increased riblet height leads to a more intense common-flow upwelling and a higher population density of hairpin vortices, which directly contribute to more UMZs. A similarity exists over the CL between the $h^+ = 8$ and $h^+ = 14$ cases, followed by a large difference to the $h^+ = 20$ case. On the contrary, $\bar{N}_{UMZ}$ is lower over the DL in comparison with the smooth wall ($\bar{N}_{UMZ} = 2.295/2.290/2.260$ for C-D riblets with $h^+ = 8/14/20$ respectively), while the riblet height has a negligible effect on the distribution of N_{UMZ} . The insensitivity of $\bar{N}_{UMZ}$ to the riblet height over the DL may be attributed to the time-averaged downwelling. The downwelling secondary flow, which transports the fluid from the upper region to the riblet surface, inhibits its direct influence on the spatial distribution of the upper vortical structures.⁷

Figure 17a compares the p.d.f. of the mean thickness of UMZs ($\bar{t}_{UMZ}$) over the C-D riblets and the smooth wall. The calculation of the UMZ thickness at a certain streamwise station is indicated in Fig. 8b, and its averaged value in the streamwise direction is defined as the mean

FIG. 17. (a) P.d.f. of the mean thickness of UMZs $\bar{t}_{UMZ}$ normalized by δ_s . (b) Mean thickness of UMZs $\bar{t}_{UMZ}$ as a function of outer coordinate y/δ_s .

UMZ thickness. Over the CL, the p.d.f. of the UMZ with a thickness of $\bar{t}_{UMZ}/\delta_s \approx 0.2$ increases dramatically. An increase in the riblet height leads to a higher probability of $\bar{t}_{UMZ}/\delta_s \approx 0.2$. The significant change over the CL can be primarily attributed to the rearrangement of the newly created coherent structures over the converging region.⁹ The p.d.f. of the UMZ thickness over the

DL remains basically unchanged compared with that over the smooth wall.

Figure 17b shows the wall-normal variation of the mean thickness of UMZs ($\bar{t}_{UMZ}$) over C-D riblets and the smooth wall. The wall-normal station of a UMZ in a snapshot is represented by the y coordinate of its geometric center. For all cases, the normalized UMZ thickness ($\bar{t}_{UMZ}/\delta_s$) increases as its wall-normal station moves away from the wall. Over the CL, the mean UMZ thickness is significantly reduced in the region of $y/\delta_s > 0.2$, while in the near-wall region of $y/\delta_s < 0.2$ the difference from the smooth-wall case is negligible. The slope of the curves over the CL is significantly lower than that over the smooth wall or the DL. The reduced mean UMZ thickness over the CL in the region of $0.2 < y/\delta_s < 0.4$ helps explain the significantly increased probability of $\bar{t}_{UMZ}/\delta_s \approx 0.2$ as indicated in Fig. 17a. An increase in the riblet height leads to a more significant decrease in the thickness of UMZs in the region away from the wall over the CL. As validated in Fig. 12, due to the creation of new coherent structures, the population density of spanwise vortices increases significantly over the CL. The reduced $\bar{t}_{UMZ}/\delta_s$ over the CL is directly related to the reorganization of these coherent structures. Over the DL, C-D riblets lead to a slight increase in the mean thickness of UMZs compared with the smooth-wall case, which is aligned with the slightly decreased value of $\bar{N}_{UMZ}$.

In summary, for the uniform momentum zones, C-D riblets result in a larger ensemble-averaged

number over the CL and a slightly smaller number over the DL, compared with the smooth-wall case. The thickness of UMZs increases as the station moves away from the wall. Compared with the smooth-wall case, UMZs are thinner over CL especially in the area far above the wall, which is
630 attributed to the upwelling and the spatial rearrangement of the newly created coherent structures. A greater riblet height results in a more profound modification to the UMZ features over the CL. Over the DL, the effect of C-D riblet on the thickness of UMZs is not significant.

V. CONCLUSIONS

A particle image velocimetry (PIV) study of the turbulent boundary layers developing over
635 convergent-divergent riblets (C-D riblets) of three different heights ($h^+ = 8, 14$ and 20) is undertaken at $Re_\theta = 723$. The purposes of this study are firstly to examine the effect of C-D riblets on the behaviors of turbulent structures in comparison to the smooth-wall case and the differences in such behaviors between the converging region and the diverging region in a more quantitative manner, and secondly to establish a better understanding of how riblet height affects such behaviors.

It is found that in comparison to the baseline case, an increase in the time-averaged streamwise
640 velocity and a reduction in the time-averaged Reynolds shear stress are observed in the near-wall region over the diverging line. In contrast, the opposite occurs over the converging line and the impact extends across almost the entire boundary layer. An increase in riblet height enhances such a trend and affects the profiles of velocity and Reynolds shear stress more profoundly over
645 the converging line. Furthermore, it is observed that although a logarithmic law region is present in the velocity profiles over the converging and diverging line, Townsend's outer-layer similarity hypothesis is invalid.

With a large sample size of PIV snapshots available, the population of prograde and retrograde
spanwise vortices are investigated. It is found that with a riblet height of $h^+ = 8$ (2.4% of the
650 baseline boundary layer thickness), the population densities of prograde and retrograde spanwise vortices are increased by more than 50% over the converging region compared to the smooth-wall case, implying a significant increase in the turbulence production activities. An increase in the height of C-D riblets results in a substantial increase in the population of both prograde and retrograde spanwise vortices. Over the diverging region, except for the small changes in the near-
655 wall region, the population density of prograde and retrograde spanwise vortices remains almost unaffected in a large portion of the boundary layer, and it appears to be much less sensitive to an

increase in the riblet height.

The inclination angle and streamwise extent of vortex packets are also extracted from two-point correlations. In comparison with the smooth-wall case, while the δ -scale vortex packets are
660 steeper and less stretched in the outer region over the diverging region, they are less affected over the converging region. The effect of an increasing riblet height is mainly manifested as a further shortening of these packets in the streamwise direction over the diverging region.

Finally, the zonal behavior of uniform streamwise momentum in the boundary layer is examined. It is observed that, in comparison with the smooth-wall case, over the converging region
665 while the probability of having three or four uniform momentum zones increases, the probability of having one or two uniform momentum zones decreases at the Reynolds number investigated in the present study. Correspondingly, the uniform momentum zones in the outer part of the boundary layer over the converging region become thinner. Again, an increase in the riblet height enhances such a trend.

670 Overall, it is observed that while an increase in the riblet height affects a large portion of the boundary layer away from the wall over the converging region, the impact on the diverging region is largely confined within the near-wall region. Such distinct differences in the response of the boundary layer over the diverging and converging region is attributed to the opposite local secondary flow motion induced by the riblets.

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