Eddy Current Based Non-Destructive Testing of the Advanced Gas-Cooled Reactor Core

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Science and Engineering

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<table>
<thead>
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<th>Description</th>
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<tr>
<td>AGR</td>
<td>Advanced Gas-cooled Reactor</td>
</tr>
<tr>
<td>HPC</td>
<td>Hinkley Point C</td>
</tr>
<tr>
<td>EngD</td>
<td>Engineering Doctorate</td>
</tr>
<tr>
<td>NDT</td>
<td>Non-Destructive Testing</td>
</tr>
<tr>
<td>EC</td>
<td>Eddy Current</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>RGN</td>
<td>Regularised Gauss Newton</td>
</tr>
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<td>RLM</td>
<td>Regularised Levenberg Marquardt</td>
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<tr>
<td>SSE</td>
<td>Scottish and Southern Energy</td>
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<tr>
<td>OFGEM</td>
<td>Office of Gas and Electricity Markets</td>
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<td>NPG</td>
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<td>WPD</td>
<td>Western Power Distribution</td>
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<td>GTN</td>
<td>Gas Transmission Networks</td>
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<td>PWR</td>
<td>Pressurised Water Reactor</td>
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<tr>
<td>ALARP</td>
<td>As Low As Reasonably Practicable</td>
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<tr>
<td>ONR</td>
<td>Office for Nuclear Regulation</td>
</tr>
<tr>
<td>CBMNU</td>
<td>Channel Bore Measurement Unit</td>
</tr>
<tr>
<td>EIM</td>
<td>EDF-Energy Integrated Methodology</td>
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<tr>
<td>PGA</td>
<td>Pile Grade A</td>
</tr>
<tr>
<td>PGB</td>
<td>Pile Grade B</td>
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<td>Prototype Eddy Current Inspection Tool</td>
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<td>GTT</td>
<td>Graphite Trepanning Tool</td>
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<td>NICE</td>
<td>New In-Core Inspection Equipment</td>
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<td>PEEK</td>
<td>Polyether Ether Ketone</td>
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<td>ECIT</td>
<td>Eddy Current Inspection Tool</td>
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<td>PC</td>
<td>Pickup Coil</td>
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<tr>
<td>BC</td>
<td>Backing-off Coil</td>
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<td>Abbreviation</td>
<td>Description</td>
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</tr>
<tr>
<td>Tx1</td>
<td>Transmitter Coil</td>
</tr>
<tr>
<td>MIT</td>
<td>Magnetic Induction Tomography</td>
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<tr>
<td>IS</td>
<td>Inductance Spectroscopy</td>
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<td>GD</td>
<td>Gradient Descent</td>
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<td>GN</td>
<td>Gauss Newton</td>
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<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
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<td>TV</td>
<td>Total Variation</td>
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<tr>
<td>GSVD</td>
<td>Generalised Singular Value Decomposition</td>
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<td>TSVD</td>
<td>Truncated Singular Value Decomposition</td>
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<tr>
<td>DF</td>
<td>Difference Operator</td>
</tr>
<tr>
<td>NOSER</td>
<td>Newton One Step Error Reconstructor</td>
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<tr>
<td>LM</td>
<td>Levenberg Marquardt</td>
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<tr>
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Abstract

This thesis presents research towards the realisation of new eddy current-based multi-frequency techniques for non-destructive testing of the graphite bricks within the core of an advanced gas-cooled nuclear reactor (AGR). The graphite core suffers from two main ageing mechanisms (density reduction and cracking) and both were investigated in this thesis. There is a close relationship between the density of graphite and its electrical conductivity, which allows density changes to be monitored using eddy current techniques.

A novel eddy current sensor configuration was specifically optimised for inspection of the graphite bricks, which contain the AGR fuel elements. The new sensor was designed to operate within the fuel channel bricks and its performance was validated using both Finite Element modelling and experimental studies. Results show that the new sensor offers an approximately 43% improvement in sensitivity with depth, compared with the existing eddy current sensor currently used for routine reactor core inspections.

The thesis also considers the novel solutions to the problem of determining the depth profile of the electrical conductivity of the bricks. The depth profile of electrical conductivity could allow the density profile to be inferred. This study has focused on the formulation of a suitable non-linear inversion algorithm that can accurately estimate the depth profile of the electrical conductivity based on the measurements collected from a real reactor core, through the implementation of a new forward model calibration and constraining techniques.

Two different Tikhonov based inversion algorithms have been studied, namely regularised Gauss Newton and regularised Levenberg Marquardt. The former algorithm was already implemented for this application in a previous PhD project. This research has now extended the algorithm with new constraining techniques to improve its performance. The results from the reconstructed profiles using data from a representative laboratory sample show that the graphite cross-sectional profile can be reproduced with 98.7% accuracy when the new constraining technique is applied within the algorithm.

A new application of the regularised Levenberg Marquardt algorithm was adapted for the conductivity profiling as part of this work, and the performance of this algorithm was compared with the regularised Gauss Newton method. The two algorithms were compared based on the reactor data inverse solutions. Slightly faster convergence rate was observed from the regularised Gauss Newton algorithm, but it suffers from inaccuracies with increasing depth from the graphite bore. The regularised Levenberg Marquardt algorithm tends to produce much more accurate profiles, although it takes longer to arrive at the final solution. Comparisons between the reconstructed profiles of the fuel channel bricks using RLM and the measurements from the trepanned sample also showed reasonable agreements between one another, with mean profile errors ranging between 1.5% and 16.9%. This is the first time that such good agreement has been obtained from reactor data.

Experimental studies concerning with realistic sub-surface crack have been carried out using the new sensor. During these studies an attempt was made to extend the existing method of determining crack location. The results from these studies show that the new sensor along with the extended data processing method allows a detection of realistic crack that was 34% of the entire graphite wall, whereas the existing sensor was only detected a crack that is 48% of the entire graphite wall.
Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.
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Chapter 1

1 Introduction

1.1 Project Motivation

Currently in the UK there are fourteen Advanced Gas-cooled Reactors (AGRs) in operation, which account for approximately 19% of the total electrical energy supply. The oldest AGRs in the UK started operation in around 1976, and were followed by other similar type reactors entering the energy industry between 1983 and 1989 [1]. The AGRs were originally designed to operate for up to 25 full power years, which also means that most of them are already approaching their originally planned lifetime whilst some of them already exceed it with the approval of an appropriate life-extension programme [2].

From the energy security point of view, closure of the AGRs, prior to building new plant, could lead to electrical energy shortages within the UK. This is mainly because the country’s electrical energy is dependent in part on the existing fourteen AGRs. Besides energy security there are other economic reasons influencing the continued operation of the AGRs. The closure of the existing AGRs would result in a significant loss of revenue for the operator EDF Energy; in fact an outage of a single reactor during planned routine core inspections costs EDF Energy around £500k per day [2]. Based on this figure, it is clear that the annual revenue loss over a year when decommissioning a plant with twin reactors will be around £365 million. Furthermore, the closure of the existing reactors means that a significant investment needs to be made to build replacement reactors to fill the gap between the supply and demand sides of the UK electricity sector. This investment cost is currently estimated to be around £18 billion (based on the Hinkley Point C design) [2].

One of the significant issues with the AGR design is graphite ageing. Due to the high radiation dose and the coolant chemistry within the reactors, the graphite bricks that make up the AGR core are prone to degradation. The AGR fuel channel bricks are in a direct proximity of the fuel elements and therefore tend to suffer the highest radiation flux. As the result, these bricks suffer from fast neutron irradiation.
and radiolytic oxidation during the course of their lifetime [3-4]. Radiolytic oxidation of the graphite bricks are mainly caused by a chemical decomposition of the carbon dioxide coolant into carbon monoxide and oxidizing species when exposed to gamma radiation, which then attacks the graphite structure to increase the size of the pores, and hence resulting in a graphite weight loss (density loss) [3-5].

Fast neutron irradiation causes the AGR graphite bricks to undergo dimensional change, the rate of which is dependent upon their radiation dose [3-4]. The parts of the brick in the bore, which are nearest the fuel elements, are exposed to the highest levels of radiation and this gives rise to a gradient in the irradiation levels over the cross-section of the brick. This irradiation gradient introduces differential stresses between the graphite bore and periphery, and eventually leads to cracking. The tensile stress at the bore is much higher than the periphery in early life of the reactor, and hence crack growth tends to occur form the bore outward to the periphery. As the reactors get older, which is the current state of most AGRs in the UK, the stresses reverse so that cracks are expected to initiate from the graphite key-way corners and grow towards the bore.

The possible safety implications of these two graphite ageing mechanisms are: brick distortion that could reduce the amount of control over the chain reaction if the movement of the control rods is restricted, and reduction in graphite strength, which could affect the entire core structural integrity if the safety margins are exceeded. Strictly speaking, the graphite bricks in the reactor core are irreplaceable, and hence the general operational and safety function of the AGRs depends significantly upon the conditions of the graphite bricks. Therefore, the ability to monitor the effects of graphite ageing mechanisms is essential in assessing the condition of the bricks in the core, which is in term essential for supporting the safety case for continued operation of the reactors.

Currently, EDF Energy is working on a life-extension programme for the existing reactors. This programme includes EDF Energy, as well as Engineers and Researchers across different industrial and academic groups, who are carrying out extensive research that allows them to understand the behaviour of the graphite core under irradiation and radiolytic oxidation. Some of this research involves the
development of different techniques that can accurately and efficiently determine the current state and future behavior of the graphite core.

The work in this thesis has particular relevance to the AGRs’ life-extension programme set out by EDF Energy, and is mainly focused around the development of techniques and sensor system by which the amount of graphite density loss and key-way root cracking can be estimated.

An earlier EngD project in this area has been completed with a successful outcome. The work in this thesis is a continuation of this previous project, and aims to advance the inspection techniques further to help realise and deploy the best possible Non-Destructive Testing (NDT) technique for the inspection of the graphite bricks that make up the AGR core. The work in this thesis aims to achieve this through the development of a new EC based multi-frequency NDT system, along with a graphite conductivity/resistivity profiling algorithm that allows to estimate the graphite conductivity/resistivity profile as a function of depth. On metals, EC system could not be used at the thicknesses (=95 mm) of interest in the graphite bricks. The reason that EC method is suitable for nuclear graphite inspection is because graphite has a much lower electrical conductivity (=100 kS/m assuming virgin graphite), so the EC at any given frequency penetrate further. With an application of multi-frequency EC system, it is possible to probe the graphite brick to different depths and collect enough data to assess the presence of key-root cracks and determine the graphite electrical conductivity, which is closely related to a graphite density or weight loss.

1.2 Project Aim and Objectives

1.2.1 Project Aim

The general aim of this project is to help realise and deploy the best possible NDT techniques for inspection of the graphite bricks in AGR core. This aim can be divided into three major sections:

- Research and development of a sensor system optimised for AGR core inspection
• Research and development of a multi-frequency EC-based conductivity/ 
  resistivity profiling system for the graphite bricks in AGR core.

• Study of the sensor response to realistic sub-surface graphite key-way root 
  cracks

The development of a new sensor system in this project has a particular relevance in 
improving the detection capability of changes in the graphite conductivity with 
distance from the bore of the brick. This task is performed through a structured 
design and optimisation process, including FE studies of various sensor 
configurations and experimental validations.

Feasibility studies of the conductivity profiling system have been previously 
reported in [6-7], which demonstrated the practicality of this method for the 
inspection of the graphite bricks in the AGR core. In the current project, the primary 
focus is to advance the conductivity profiling technique through the implementation 
of an alternative conductivity-profiling algorithm whilst undertaking studies of the 
main parameters affecting the accuracy of the reconstructed profiles.

The primary focus of the study concerning with sub-surface graphite key-way 
root cracks was to assess the capability of the developed sensor system in locating 
realistic key-way root cracks. Furthermore, this task has investigated the feasibility of 
detecting realistic graphite cracks using EC method in general. A study concerning 
with an improvements of the data processing method for a cracked brick was carried 
out. These studies have particular relevance to the current state of AGR core, as key-
way root cracks are predicted to occur around the present age of the reactors.

1.2.2 Project Objectives

The project aim described above in Section 1.2.1 was realised through the 
following objectives:

• To familiarise with previous work related to AGR core inspection and general 
  NDT literature relevant to the project.
• To investigate, design, optimise and experimentally validate a new EC probe with better depth sensitivity in the graphite bricks.

• To develop FE models of the graphite bricks and sensing system in order to study and analyse the sensor response to changes in graphite properties and sensor parameters.

• To study and formulate a complete non-linear conductivity profiling algorithm that can be used to estimate the graphite internal property.

• To investigate the effect of measurement errors on the conductivity profiling algorithm through artificially generated errors.

• To study the effect of constraints on the reconstructed solutions.

• To investigate and analyse the errors in the reconstructed conductivity profiles.

• To validate the reconstruction process using measured EC data from reactor channels, which have been subsequently trepanned and the density/resistivity gradient measured.

• To investigate and analyse the sensor response to different sizes of slots and realistic key-way root cracks using both FE models and experimental work.

• To identify the limitations of the developed system, and propose potential improvements.

1.3 Achievements and Publications

Publications


Presentations and Proceedings


1.4 Thesis Organisation

Chapter 1 of this thesis has given a description of the motivations behind this project and the project aim and objectives.

Chapter 2 will give a brief introduction to the general industrial context of this thesis with a description of the UK energy market and EDF Energy, including its reactor life-extension programme strategy. This is then followed by a brief explanation of the relationship between the reactor safety-case and the work in this project. For completeness, this chapter also describes briefly the general NDT market, focusing on EC NDT.

Chapter 3 begins with an introduction to the various components of the AGR core, followed by descriptions of the nuclear graphite properties and its main ageing mechanisms when exposed to radiation under operational conditions. The AGR core inspection tools and techniques that are currently used by EDF Energy, and the information they seek from the in-core inspections, are briefly described.

Chapter 4 starts with the theory and a literature review of EC-based NDT systems. This is followed by a description of the numerical implementation of the
graphite forward model and sensitivity matrix. The remaining sections of this chapter mostly focus on the design, optimisation and experimental validation of the new EC sensor designed as part of this project for inspection of the graphite core in AGR.

Chapter 5 begins by introducing the background theory and the challenges associated with a typical EC inverse problem along with descriptions of the fundamental inversion algorithms. This chapter also discusses different regularisation techniques applied to general EC inverse problems, but with a particular reference to the graphite conductivity-profiling problem. An introduction of the non-linear inversion algorithm formulated for the graphite conductivity reconstruction problem is presented.

Chapter 6 presents model-based studies of the graphite conductivity reconstruction problem carried out to investigate the effects of applying different regularisation parameters, operator matrices, and data errors on the accuracy of the converged solutions. Results from reconstruction of experimental data measured from a representative graphite sample with and without new constraining techniques are presented, and compared in terms of profile errors.

Chapter 7 presents conductivity profiling procedures applied to the reactor data measured from two different reactors during inspection outages. The graphite brick conductivity profiles in this chapter are reconstructed using both the previously developed RGN algorithm, and the RLM algorithm formulated as part of this project. In this chapter, measurements from trepanning sample were used as a means of validation for the reconstructed profiles.

Chapter 8 presents a study of graphite sub-surface cracks and slots using both FE models and experimental work. Some of the studies in this chapter are limited to central key-way slots, whereas the others are mostly focused on realistic key-way root cracks similar to those expected in an AGR core.

Finally, chapter 9 concludes the thesis with a summary of the major results and suggestions for potential improvement and future work.
Chapter 2

2 Industrial Context of this Thesis

2.1 The UK Energy Market and Key Players

The UK energy market is operated by private companies, and categorised into three sectors: (i) Electricity Generation, (ii) Transportation of Electricity and Gas, and (iii) Electricity and Gas Supply. These three sectors operate independently from one another, but they all form a complete system in delivering energy from the source to consumers. UK energy consumption is also categorised into three groups: industrial, commercial (including transport sectors) and domestic (see Table 2.1 for UK electricity consumption).

There are many companies operating within the UK energy market, although large companies such as EDF Energy, npower, Centrica, E.ON, and SSE generally dominate the market. Some of these companies operate in all three sectors whilst others are limited to the generation or supply sectors.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Consumption in 2015</th>
<th>Consumption in 2016</th>
<th>Market share in 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial</td>
<td>23 TWh</td>
<td>23.2 TWh</td>
<td>33.6 %</td>
</tr>
<tr>
<td>Domestic</td>
<td>25 TWh</td>
<td>24.2 TWh</td>
<td>35 %</td>
</tr>
<tr>
<td>Commercial</td>
<td>22.2 TWh</td>
<td>21.6 TWh</td>
<td>31.3 %</td>
</tr>
</tbody>
</table>

*Table 2.1: UK electrical energy consumption trends between 2015 and 2016, reproduced from [8].*

Besides these companies actively operating in the energy market there is an independent national regulatory authority known as the Office of Gas and Electricity Markets (OFGEM), which ensures competition and fair trade between the energy companies and consumers. The UK government also has a large involvement within the energy market, in particular in the electricity sector with the aim of reducing the carbon footprint whilst addressing future energy demand [1].
2.1.1 Electricity Generation and Wholesale Market

The vast majority of UK electricity is generated by large-scale power stations that operate at fixed locations, and is transported through the national transmission network, although there are some distributed generators, which can be fed directly into the distribution networks. The large-scale power stations in the UK are made up of different generation units, including nuclear, coal and gas-fired generation units. However, the need for low-carbon electricity has influenced the production of electricity from these generation units resulting in a reduced output of coal-fired generation from 17 % to 4 % of the total generation between 2015 and 2016. At the same time the electricity outputs from nuclear and gas-fired generators have increased from 21 % to 25 % and 35 % to 44 % between 2015 and 2016 respectively [8-9]. Similarly, the output from renewable generators, which are the ideal candidates for future low-carbon generation, increased from 24 % to 25 % between 2015 and 2016, whilst imported electricity and that generated using oil-fired generators remained around 3 % [8].

Eight electricity generation companies dominate the UK wholesale market (Table 2.2), each with a wholesale market share ranging between 4 % and 24 %. According to a report published by OFGEM [10], EDF Energy achieved the largest market share in 2016 (24 % of the wholesale market share), followed by npower and Centrica with 16 % and 8 % market share respectively.

2.1.2 Electricity and Gas Transportation

Currently, there are three transmission companies that operate within the UK. National Grid plc is one of the three operators of the transmission network in England and Wales, and is responsible for secure and smooth operation of the entire UK transmission system [11]. Scottish Power Transmission and Scottish Hydro-Electric Transmission are the other two transmission network operators, which operate some portion of the transmission networks within their own distinct onshore transmission areas [11]. In addition to the transmission networks, there are distribution networks, which take electricity from the national transmission
networks and distribute it to industrial, commercial and domestic users. The distribution network in the UK is owned by six companies: NPG, SSE, ENW, SPEN, UKPN, and WPD.

In addition to electricity transmission, National Grid operates the high-pressure gas transmission networks in the UK. Currently, four companies (Cadent Gas, Northern Gas Network, Wales and West Utilities, and SGN) own and manage the eight main gas distribution networks, which cover different geographical areas within the UK.

2.1.3 Electricity and Gas Supply

As discussed previously, companies such as EDF Energy, Centrica, E.ON, SSE, Scottish Power and npower dominate the electricity and gas market in the UK. The combined domestic market share of these companies is around 85% and 82% for the electricity and gas markets respectively [8]. A report in [8] shows that Centrica leads the domestic market share in both electricity and gas supply, followed by SSE, E.ON and EDF Energy, whereas the electricity wholesale market is highly dominated by EDF Energy (Table 2.2).

From the consumer side, the demand for electricity and gas is highly driven by weather conditions and the time of the day, although advances in technology play a part in this due to an increased efficiency of energy usage (through smart meters). However, from the supplier side the demand for electricity and gas is driven by both weather conditions as well as the price they are selling energy to their consumers, since consumers are now more price-conscious than before, and able to switch between different suppliers to get a better deal [8,12]. Nevertheless, the general UK consumer spending on gas and electricity is estimated to have increased by 25.5% from £32.9 billion to £41.3 billion between 2013 and 2018 [12].
<table>
<thead>
<tr>
<th>Company Name</th>
<th>Market share in the UK by the end of 2016</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Electricity Wholesale</td>
<td>Electricity supply (Domestic)</td>
</tr>
<tr>
<td>EDF Energy</td>
<td>24 %</td>
<td>12 %</td>
</tr>
<tr>
<td>npower</td>
<td>16 %</td>
<td>10 %</td>
</tr>
<tr>
<td>Centrica</td>
<td>8 %</td>
<td>23 %</td>
</tr>
<tr>
<td>E.ON</td>
<td>--</td>
<td>14 %</td>
</tr>
<tr>
<td>SSE</td>
<td>7 %</td>
<td>15 %</td>
</tr>
<tr>
<td>Drax</td>
<td>7 %</td>
<td>--</td>
</tr>
<tr>
<td>Scottish Power</td>
<td>4 %</td>
<td>11 %</td>
</tr>
<tr>
<td>InterGen</td>
<td>5 %</td>
<td>--</td>
</tr>
<tr>
<td>Others</td>
<td>23 %</td>
<td>15 %</td>
</tr>
</tbody>
</table>

Table 2. 2: The UK energy market share by company, reproduced from [8]. Note: the companies with a market share of less than 4 % in the wholesale column, and less than 10 % and 8 % in the domestic gas and electricity columns, are included with all other companies in the table.

2.1.4 Market Regulators

As stated previously the UK government and OFGEM continuously regulate the UK energy market. OFGEM is an independent national regulatory authority that works to protect the interests of electricity and gas consumers [13]. In addition to protecting the consumer’s interests, OFGEM is also responsible for monitoring and investigating activities, which may undermine competition, and for making sure that the system operator National Grid is supplied with sufficient electrical energy to balance the supply and demand sides in a continuous manner.

Advances in technology and the need to produce low-carbon electrical energy means the potential replacement of existing marketing strategies and systems. According to a report published by OFGEM [13], the UK needs £ 100 billion capital investment to meet future demand and replacement of the old system. OFGEM has an important role in achieving these as the UK heads towards a greener and smarter energy market [13]. In addition, the UK government closely monitors the energy market with the aim of reducing total carbon emissions by 57 % to meet a 2030 target. The government aims to achieve this by introducing several market reforms such as the energy bill introduced in December 2013 that aims to reform the UK energy market to help stabilise the revenues for low-carbon energy
investors, and tax those using fossil-fired generators according to their carbon emissions [1-2].

2.2 Overview of EDF Energy

EDF Energy is one of the largest active players in the UK energy industry, and involved in both the generation and supply sides of the energy market with total annual revenue of approximately £ 8.6 billion as of 2015 [2]. The company has a vision to become an “efficient, responsible electricity company and champion of low-carbon growth” [14].

The general business structure of EDF Energy is divided into two sectors: generation and supply sectors. The generation sector generates and sells electricity onto the wholesale market, whereas the supply sector buys electricity and gas from the wholesale market and sells them back to its customers.

EDF Energy is the only operator of nuclear power plants, and is the biggest supplier of electricity in the UK, with a total nuclear generation capacity of 8883 MWe (see Table 2.3). The company supplies around 5 million businesses and residential houses in the UK, and produces approximately one-fifth of the total electricity [14]. Currently EDF Energy operates eight nuclear power plants located in different parts of the UK. Seven of the eight nuclear power plants consist of two AGR reactors each, whilst the remaining one is based on a Pressurised Water Reactor (PWR) design. These power plants contribute around 64.3 % to EDF Energy’s total electricity output. In addition to the nuclear power plants, the company operates two coal and one gas-fired conventional power plants, with a total electricity output of around 14.5 % and 8.6 % respectively. The remaining 12.6 % of EDF Energy’s electricity output is generated using 31 wind farms [14].

As can be seen in Table 2.3, eight of the fifteen reactors will be shut down within the next five to six years (between 2023 and 2024). This could possibly lead to a gap between the supply and demand sides of the UK electricity market. However, EDF Energy is continuously investing in strategic programmes, which aim to secure lifetime extension of the existing reactors through structured safety cases [2, 15]. In parallel with this, the company is investing in building new nuclear power plants with
the aim of securing continuous electrical energy supply to the UK. In 2016, EDF Energy, in partnership with China General Nuclear Corporation, was given the “go-ahead” by the UK government to build a new nuclear power station at HPC in Somerset, and is planning to build another two stations (Sizewell in Suffolk and Bradwell B in Essex) [2]. The HPC construction is due to be completed in 2025, its output is estimated to cover 7% of the UK energy supply (3200 MWe), and it is expected to have a 60-year lifespan [1].

<table>
<thead>
<tr>
<th>Power Plant</th>
<th>Type</th>
<th>Capacity (MWe)</th>
<th>First Generation</th>
<th>Expected Shutdown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dungeness B1 &amp; B2</td>
<td>AGR</td>
<td>2 x 520</td>
<td>1983 &amp; 1985</td>
<td>2028</td>
</tr>
<tr>
<td>Hartlepool 1 &amp; 2</td>
<td>AGR</td>
<td>595, 585</td>
<td>1983 &amp; 1984</td>
<td>2024</td>
</tr>
<tr>
<td>Heysham I 1 &amp; 2</td>
<td>AGR</td>
<td>580, 575</td>
<td>1983 &amp; 1984</td>
<td>2024</td>
</tr>
<tr>
<td>Heysham II 1 &amp; 2</td>
<td>AGR</td>
<td>2 x 610</td>
<td>1988</td>
<td>2030</td>
</tr>
<tr>
<td>Hinkley Point B 1 &amp; 2</td>
<td>AGR</td>
<td>475, 470</td>
<td>1976</td>
<td>2023</td>
</tr>
<tr>
<td>Torness 1 &amp; 2</td>
<td>AGR</td>
<td>590 &amp; 595</td>
<td>1988 &amp; 1989</td>
<td>2030</td>
</tr>
<tr>
<td>Sizewell B</td>
<td>PWR</td>
<td>1198</td>
<td>1995</td>
<td>2035</td>
</tr>
<tr>
<td>Total: 15 units</td>
<td></td>
<td>8883 MWe</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. 3: UK nuclear power plants operated by EDF Energy, reproduced from [1].

2.3 Factors Influencing AGR Life-Extension

2.3.1 Economic and Energy Security

The decisions to extend the operational lifetimes of the AGRs are driven by many factors including economic, energy security, operational experience and assessments of the operational safety margins through well-structured safety case procedures [2, 15].

The main economic motivation for continued operation of the AGRs is loss of income. As mentioned previously this cost can be estimated through the assessment of the loss of revenue during the planned core inspection outage of a single reactor, which costs EDF Energy around £ 500k per day [2]. Therefore, the annual cost of a closure of an AGR power station with twin reactors will be around £365 million, this
is of course without taking into account the decommissioning costs. Closure of the existing reactors also means that a significant investment needs to be made to build replacement reactors to fill the gap between the supply and demand sides of the UK electricity sector. This has an estimated cost of around £18 billion per power station [2].

Closure of the majority of the AGRs within the next five to six years prior to building a complete replacement plants, could also lead to electrical energy shortages within the UK. This is mainly due to the UK’s electrical energy dependency (19 % the total supply) on the existing fourteen AGRs. One could argue that the possible shortages in electrical energy could be met through conventional fossil-fuel generation units, but with the consequence of undermining the low-carbon target set out by the UK government.

Therefore, maintaining the existing reactors in operating condition, both safe and commercially sustainable, is essential for both the operator EDF Energy and the UK as a nation.

2.3.2 Safety and Regulatory Requirements

The majority of the AGR core is made up of stacked cylindrical graphite bricks. These bricks serve three main functions: to moderate the fast moving neutrons, maintain the lattice spacing between the fuel elements and control rods, and provide pathway to direct the flow of the coolant gas (carbon dioxide) to cool the core structure and transfer thermal energy to the boilers [3-4]. As mentioned previously, the graphite bricks in the AGR core suffer from irradiation and radiolytic oxidation, which could result in a brick distortion and lose of graphite strength [5, 19]. In the worst-case scenario these could affect the entire core structural integrity [3]. The graphite bricks in the AGR core cannot be replaced. Consequently, the conditions of the graphite core play a significant part on the assessment of the residual operational lifetime of the reactors, although other components in the reactors, such as the conditions of the boilers, also play part in this [2].
Therefore, in order to maintain the AGRs in operating condition, the operator EDF Energy must fulfil its regulatory obligations by demonstrating that the reactors are safe to operate through the implementation of structured safety cases for all activities that might affect safety on site [16]. This typically means that EDF Energy must provide a safety case, which demonstrates that the graphite core is free from defects that could impair its safety functions, or that the safety functions are tolerant to any known defects [15-16]. The safety case takes different aspects of the graphite properties into consideration, including those that have indirect impact on the fundamental safety requirements of the AGR core [15-16]. The fundamental safety requirements of the AGRs core relevant to this project can be categorised into three sections [15]:

- The graphite core must allow unimpeded movements of the control rods and fuel
- The core must be able to direct the flow of coolant gas to ensure sufficient cooling of the core structure, in normal and fault conditions
- Limits on the allowable core weight loss by radiolytic oxidation, which also means that the core must have sufficient level of neutron moderation, as well as the structure of the core must remain intact at least over the next 2 to 3 year operating period

The Office for Nuclear Regulation (ONR) is responsible for setting out assessment strategies for the safety cases in order to decide whether the safety cases are adequate and the risks are As Low as Reasonably Practicable (ALARP) [16]. The ONR has powers that allow it to take several actions against the operator including revoking the operating licence, which could result in the closure of the reactors that are believed to be unsafe, in particular if they are not satisfied with the actions taken to understand and monitor the graphite aging mechanisms in the AGR core.

2.3.3 EDF Energy’s Strategy
EDF Energy has continuously demonstrated that the reactors are safe to operate through the implementation of structured safety cases for all activities that might affect safety on site. As a result the life of the AGRs has been extended by an average of eight years during its ownership [12]. The company is continuously investing in strategic research programmes, which support the life-extension of the existing reactors through well-structured safety cases [2, 15]. This includes major projects such as project Blackstone and project ACCENT [17, 18]. These projects run by EDF Energy and its strategic partners to understand, assess and predict the behaviour of the graphite core under irradiation and radiolytic oxidation, as well as to gather data, which can be directly fed into the reactor safety case.

In addition, EDF Energy uses several specialised reactor inspection tools and techniques to predict the behaviour of the graphite bricks in the AGR core (see Chapter 3). These in-core inspections are currently carried out every two to three years for each reactor, but it is expected to increase in frequency as the reactors approach the end of their lifetimes [19]. The output data from the in-core inspections perform a significant role in the entire safety case, providing a means of validation and input data for the models, particularly whole core and stress analysis models, currently being used to predict the current state and future behaviour of the graphite cores [16].

In order to allow unimpeded movements of the control rods and fuel elements (one of the safety requirements of the graphite cores) within the AGR core, the graphite bricks must maintain their original structures. EDF Energy obtains in-core structural information for the graphite bricks using a tool known as Channel Bore Measurement Unit (CBMU), which measures the bore diameter and the relative tilt of the fuel bricks, as well as providing visual inspection images. The volumetric inspection of the graphite core is carried out by extraction of small samples from core using a tool known as a trepanning tool. Trepanned samples are used to assess the graphite density as well material property information. The most recent addition to the inspection capability is an eddy current (EC) based in-core inspection system used by EDF Energy to inspect for near surface crack formation and density mapping [20].
2.3.4 How this Project Relates to the AGRs Safety Case

The volumetric inspection system for the graphite bricks in the core relies primarily on the trepanned cylindrical graphite samples from the core, which then analysed under laboratory conditions. The data from the samples subsequently used as an input to various models, which aim to determine the behaviour of the graphite bricks under irradiation and radiolytic oxidation.

Although this method is proven to be valuable source of information about the individual graphite brick, the cylindrical samples are small in size (nominal Ø19 mm x 70 mm) compared to both the circumferential and radial sizes of the graphite brick being analysed. Due to the localised effects of the radiolytic oxidation within the graphite bricks this could lead to some level of uncertainty in the amount of volumetric information captured about the graphite brick. Therefore, the volumetric inspections through sample extraction could benefit from another means of validation.

Around 36 samples are trepanned at every statutory outage currently, and these numbers are likely to increase as the reactors approach the end of their lifespans. It should be noted that the samples are small in size, and hence they will not have an effect on the on the structural integrity of the core. On average it takes between 12 and 18 hours to trepan 6 samples, which also means that the reactors need to remain shut-down for the duration of the process. This process in general costs EDF Energy between £1.5 million and £1.25 million to obtain 36 trepanning samples (assuming this is the only outage task), although other activities are normally carried out simultaneously. Therefore, an alternative NDT method could significantly reduce the cost and the time of the inspections.

Multi-frequency EC-based NDT with an appropriate conductivity profiling technique could be used to provide conductivity versus depth information for the individual graphite brick, and provide a means of validation against the trepanning data. The low electrical conductivity nature of the graphite compared with other electrically conductive materials means that the EC is able to penetrate deeper within the volumes of the graphite bricks at any given frequency. Therefore, it is possible to probe the graphite brick to different depths and collect enough data to
reconstruct the conductivity profile over the brick cross-section. This system could support the trepanning technique and offer an advantage in terms of significant cost reduction and capturing a larger proportion of volumetric information relative to the graphite brick size. Furthermore, the conductivity distribution of the graphite brick could then be directly related to other properties of the graphite bricks through the EDF Energy process known as the EDF Energy Integrated Methodology, supporting the implementation of an adequate safety case.

A review of the AGR core assessment methodology has identified several improvements that need to be addressed, including reductions in the uncertainties through the development of a research programme to help understand the behaviour of the graphite bricks in the core [15]. In line with this, particular suggestions were made to seek further enhancements of the examinations, inspections, surveillance, sampling and testing programmes [15]. The research works in both this and previous projects can be considered as a stepping-stone towards achieving this goal, particularly in improving the NDT inspection technique with the implementation of multi-frequency EC inspection system along with a conductivity-profiling algorithm.

2.4 Overview of the NDT Market

Although the NDT of the AGR core is the main interest of this project, this section aims to briefly introduce the wider applications and market drivers of NDT in many industrial sectors.

NDT equipment and services play important roles in many industrial sectors, in particular in sectors where safety is a vital part of the industrial process. Government regulations and stringent maintenance are generally the main demand drivers for NDT equipment and services. In addition, the ever-Increasing global population means that industries such as the energy and transportation sectors must operate to the maximum to fulfil everyone’s need. This has a positive impact on the market for new and effective NDT equipment and services, as the more that these industries operate the more frequently the assets need to be inspected to maintain
continuous operation. The power generation sector, in particular the renewable energy sector including nuclear, is expected to increase the global NDT market by the year 2020 [21]. The forecasted revenue of the NDT market derived from renewable energy industries is $179 million, and 70% of it is expected to be from the nuclear industry.

A recent NDT market report [21] shows a decline in the demand for NDT within the oil and gas industries due to the fall in oil and gas prices. However, in contrast, the report also forecasts increasing revenue for NDT services and equipment in transportation industries such as the aerospace, rail and automotive industries [22].

According to a report published by Frost & Sullivan in 2016 [23], the main drivers for the continued demand for NDT services and equipment in the next five years are: adherence to comply with safety regulations, transition from conventional to advanced NDT systems and the need to run assets at higher capacities. The first of these is the most influential market driver in the Europe region, including within the nuclear industry.

The overall market size for NDT services in Europe is forecasted to grow to €2315.7 million, with expected market growth rate of 1.6% by 2020 [23]. The UK is the fourth biggest NDT market in Europe with a 12.7% share of the overall NDT service market, and has revenue of around €285.3 million, with expected growth rate of 3.3% by 2020 [23].

Although the above NDT market covers a wide range of services that use different types of energy for inspection ranging from EC to Gamma based inspections, EC systems are strong candidates within the future NDT industries [24]. Another report published by Frost & Sullivan [24] estimated the global EC equipment market as $315 million in 2016 with 7.5% annual growth, with much of it expected to come from the emergence of new EC systems.
Chapter 3

3 Nuclear Power Stations in the United Kingdom

The UK civil nuclear power industry initially started in 1946 followed by the establishment of one of the world’s first nuclear power plants in 1956 [25-26]. This commercial scale reactor was built in southwest Scotland and given the name “MAGNOX” due to the fuel cladding (magnesium non-oxidising alloy) used to contain the fuels. The MAGNOX reactor used natural uranium metal fuel to generate thermal energy while using graphite bricks to moderate the fast-moving neutrons within the core. Two grades of graphite bricks were used in the MAGNOX design; these were Pile Grade A (PGA) and Pile Grade B (PGB). The former was used as a neutron moderator due to its high density and lower neutron absorption, whereas the latter was mostly used as a reflector [25]. The mechanism used to transfer heat energy and cool the reactor core was to blow carbon dioxide ($CO_2$) gas at about 20 bar into the core. The inlet gas temperature blown into the core was around 225°C, which then primarily passed through the core increasing its temperature to about 370°C. This was then used to convert the water in the boilers into steam, and subsequently drive the turbines [25]. In total twenty-six MAGNOX reactors were built in the UK between 1956 and 1972, with the later design having radiation shields, and the boilers enclosed within the concrete vessel [25].

However, the MAGNOX reactors had low steam cycle efficiency of around 31%. This was mainly due to the low melting point of magnesium fuel cladding in the MAGNOX reactor that limited the peak fuel temperature to approximately 660 °C [27]. This in turn resulted in a low outlet temperature of approximately 370°C, and hence reduced turbine output of around 300 MW rather than the standard 660MW turbine output. These limitations of the MAGNOX reactors led to an improved AGR design with the aim of enhancing the efficiency and making the generating system more cost-effective [4, 25-27]. The first AGR started operation in the UK around 1976 followed by other similar types of reactors entering the energy industry between 1983 and 1989. All of the new reactor designs were fully based on the AGRs; therefore, no more MAGNOX reactors were built after 1972; in fact the last
operational MAGNOX reactor was closed down in late 2015, which ended the operation of this type of reactor in the world.

Currently, there are fifteen operating reactors in the UK with a combined total output of around 8883 MWe, and one under construction [2]. Fourteen of the fifteen operating reactors are AGRs and located at seven different power stations, making the AGRs the dominant nuclear reactor design in the UK. The remaining two reactors (including the one under construction) are based on the PWR design.

### 3.1 Advanced Gas-Cooled Reactors

AGRs are the second generation of UK nuclear power plants, and initially designed to improve the steam cycle efficiency of the early reactor design (MAGNOX). This type of reactor exists only in the UK and is currently operated by EDF Energy (formerly British Energy). Since the AGRs’ first operation in 1976, they have become the UK’s dominant reactor after showing an increased economic benefit and steam cycle efficiency of approximately 42%, compared with a MAGNOX reactor, which had approximately 31% efficiency [27]. AGRs are capable of producing peak fuel temperatures of approximately 800 °C and a coolant gas temperature of approximately 650 °C at 40 bar. The steam generated by the AGRs can reach up to 540 °C at 165 bar, which is capable of driving standard 660 MW turbines.

Some of the main differences between the early MAGNOX reactors and AGRs that have key impacts on the reactor operational efficiency include: the fuel with which they operate, fuel cladding used to contain the fuels and the arrangement of the core structure [27]. As mentioned previously, and also discussed in [2, 4, 25-28], the MAGNOX reactors use natural uranium metal fuel and a magnesium alloy fuel cladding for the fuel and fuel assembly arrangement. The core structure of these reactors consists of stacked graphite bricks with square array fuel channels.

In contrast, the AGRs use enriched uranium oxide fuel and stainless steel as fuel cladding, which allows an increased operational temperature. Like the early reactor designs, AGRs are gas-cooled and use multiple but cylindrical graphite bricks as a moderator and main core structural component.
3.2 Advanced Gas-Cooled Reactor Core

The AGR core consists of several components that ensure a safe operation of the reactor. Starting from the inner part of the core, a large proportion of the AGR core comprises multiple hollow cylindrical graphite fuel channel bricks (Figure 3.1) stacked on top of one another to make up between 10-15 layers. Each graphite layer is approximately 850-900 mm high depending on the design of the reactors [4, 28-29].

These graphite bricks consist of different parts including; fuel channel, filler and interstitial bricks, and interlocked with neighbouring bricks using loose and integral keys to achieve robust structural integrity, accommodate core movements and seismic loading (see Figures 3.1 and 3.2). The fuel channel bricks in the AGR are used to contain the fuel assembly, whereas the interstitial and the filler bricks are used for inserting the control rods into the core and maintaining the critical lattice spacing between the fuel and control rods. In total, there are 332 fuel channels and 252 interstitial channels (including the fillers) in a typical AGR core [4, 28-29]. In addition to these bricks, the core contains several layers of reflector bricks located around its edge (Figure 3.2). The reflector bricks are mainly used as a shielding mechanism to contain the neutrons within the core.

The entire graphite core in the AGR is covered with a stainless steel gas-baffle to provide a route for the flow of coolant gas. The gas-baffle contains some steel guided tubes on the top to allow the passage of fuel assemblies and control rods [4]. The control rods contain a large amount of boron, which allows them to absorb neutrons and provide control over the fission chain reaction in the core.

Twelve boilers, arranged into four quadrants, surround the gas-baffle and allow energy conversion. The thermal energy from the core is transferred to the $CO_2$ coolants, which is pumped into the core using gas-circulators around the gas-baffle and the boilers. The boilers surrounding the gas-baffle are used as a means of energy transferring mechanism from the $CO_2$ coolants to generate steam, which eventually drives the generator turbines. The entire AGR core is enclosed within a concrete pressure vessel, which has a thickness of around 5 metres.
Figure 3.1: A typical AGR core brick arrangement, reproduced from [5]. Note: the fuel channel is hollow cylindrical graphite brick and it is ≈ 900 mm long each.

Figure 3.2: A quarter of the entire AGR core that shows some of the key components within the core, reproduced from [28]

3.2.1 Graphite Properties

Graphite is one of the allotropes of carbon in which the carbon atoms form layered parallel planes. Each graphite plane consists of hexagonally interlocked
The hexagonally joined structures of the graphite atoms are formed as the result of covalent bonds between each atom and its three neighbouring atoms that are separated by 120° and located at 0.142 nm apart [3]. These atoms maintain the covalent bonds using three of their four electrons, leaving one to make a weak bond in between the planes. There are no covalent bonds between the atoms located in different planes, and therefore the ABAB... structure of the graphite planes are maintained only by the weak bond created by the remaining electron. This electron moves freely in between the graphite planes and allows the graphite to be electrically conducting. The force that keeps the graphite atomic layer structure in place is known as the Van Der Waal force [30]. The diagram in Figure 3.3 shows the typical graphite atomic structure.

There are two types of graphite: natural and synthetic graphite. The natural graphite, as the name suggests occur naturally (such as vein and flake), and their availability is normally limited. This type of graphite is generally regarded as anisotropic and suffers from impurities to some degree. In fact, this is one of the reasons that synthetic graphite is used in many industrial applications, including the nuclear industry. Synthetic graphite is highly purified in terms of its carbon content, and usually manufactured using pitch coke.

Nuclear graphite can be considered as an example of synthetic graphite. This type of graphite has a high thermal conductivity ($\approx 78\, W/m^{-1}K^{-1}$) and low coefficient of thermal expansion ($\approx 4 \times 10^{-6}K^{-1}$). Furthermore, nuclear graphite has a low neutron absorption cross-section, which makes it an ideal moderator in civil nuclear applications. Despite these advantages, this type of graphite exhibits
some form of anisotropy, in particular PGA grade graphite [30]. The anisotropic nature can be attributed to the difference in graphite properties between in-plane and cross-plane directions relative to the alignment of its atomic structure. This property of graphite causes a significant difference in thermal expansion between the in-plane and cross-plane directions, resulting in an expansion of the distance between the planes whilst introducing some shrinkage in the basal planes. This in turn changes the graphite properties in different ways, including its dimensions [30].

The grade of graphite used in the AGR core is known as Gilsocarbon, and manufactured from Gilsonite pitch coke. Gilsocarbon graphite has near-isotropic structure, and is specifically manufactured to accommodate the effects of high temperature and radiation flux within the nuclear reactors [30-31]. Furthermore, the AGR graphite is purified to reduce some of the neutron-absorbing impurities to allow it to act as a very good moderator.

3.2.2 Functions of Nuclear Graphite in the AGR Core

Nuclear reactors are classified into two main categories, those that use fast-moving neutrons (known as fast neutron reactors) and those that use moderated neutrons to induce fission (known as thermal reactors) [32]. The fast-neutron reactors use relatively more enriched fuel than thermal reactors to allow them to sustain a fission reaction using fast-moving neutrons, and hence they do not require a moderator [33]. Fission in thermal reactors is only induced when fissile nuclei absorb a neutron with sufficiently low thermal energy. This means that the fast-moving neutrons must be adequately moderated to allow fission to occur.

The AGR is a type of thermal reactor that requires approximately 0.025eV thermal energy neutrons to sustain a fission chain reaction within the core. The energy produced by nuclear fission of an individual uranium atom ranges between a fraction of an eV and 10 MeV, with an average energy of approximately 2 MeV [3, 34]. The average energy is much larger than the energy required to sustain a fission chain reaction, therefore graphite bricks are used to moderate the fast-moving neutrons, slow them to thermal energies and sustain a fission chain reaction in the
core. Although the primary function of the graphite bricks in the AGR core is to moderate the fast-moving neutrons, the graphite core structure is arranged to allow unimpeded movement of the control rods and to direct the flow of coolant gas to ensure sufficient cooling of the fuel and the core as a whole.

Graphite has a low neutron absorption cross-section, meaning that the fast-moving neutrons will be scattered by the graphite carbon atoms when the two come into contact, which in turn increases the probability of a fission chain reaction. However, this process can result in a displacement of the graphite carbon atoms, whilst the gamma products in combination with the CO₂ coolants increase the rate of radiolytic oxidation [3, 34], see Section 3.3 for more detail. These in general degrade the graphite’s physical, chemical and mechanical properties, and eventually reduce its moderation capacity. This effect is normally expected particularly in the fuel channel bricks due to their proximity to the fuel elements.

The rates of radiation damage in the nuclear graphite bricks are even greater when the graphite properties are anisotropic [28]. For this reason, the nuclear graphite brick goes through various steps during the manufacturing process, mainly to make them near-isotropic and to increase their purity.

### 3.2.3 Nuclear Graphite Fabrication

The need to manufacture synthetic nuclear graphite arises for two reasons. The first is to allow the graphite to accommodate the effects of high temperature and radiation dose within the reactor core, whilst increasing its purity for moderation purposes [35]. The second and the most general reason is the limited availability of naturally-occurring graphite. For these reasons, nuclear grade graphite is manufactured artificially from carefully-selected raw materials [35-36].

Nuclear graphite has a polycrystalline structure, which has petroleum coke filler (Gilsonite pitch coke filler in the case of AGR bricks) particles that are bound together by a coal-tar pitch binder [35-36]. The two carbon products are combined using a well-defined quantity ratio of 30 parts of binder for every 100 parts of filler [36]. If the binder product is much higher than the specified ratio it will result in a
poor properties of the final product. On the other hand, having too little binder cause cracks in the final product due to a high strain inside the filler [36].

During the manufacturing process the base material petroleum cokes are initially heated to around 1300°C to remove any impurities. After the first heat treatment the petroleum cokes are milled and crushed to form small particles. A mixture of fillers and binders is added to the crushed petroleum cokes, and mixed under heating conditions. The mixed product is then used to form the product using moulding or extrusion techniques.

The moulding and extrusion processes along with the types of grain used determine the property of the final product. The extrusion process with a mix that has needle like filler grains causes the grains to align on one common axis (the extrusion direction). Therefore results in an anisotropic material [30]. An example of extruded nuclear grade graphite product is PGA graphite brick, which was used within MAGNOX reactor cores. On the other hand, the packing process of graphite through the moulding technique using spherical grains results in more isotropic graphite properties. The Gilsocarbon graphite bricks in an AGR core are manufactured using spherical grains long with a moulding technique. Therefore, they tend to have near-isotropic properties [30].

Following the moulding/extrusion process, the material will form what is known as a green article. The green article is then baked and impregnated with liquid pitch several times. The later process is mainly used to reduce the pores created during the baking process and increases the material’s density and mechanical strength. Finally, the baked article is graphitized using heat ranging between 2600°C and 3000°C to increase the crystal growth and maximise the graphite atomic order [35-36]. This process also helps purify the graphite in terms of its carbon content and unwanted neutron-absorbing substances. The final nuclear grade graphite product is then machined into cylindrical blocks and keys, which lock to form the entire reactor core. The flowchart in Figure 3.4 shows the entire manufacturing process of nuclear grade graphite.
Figure 3.4: Flowchart that shows the graphite production process. The grey blocks in the flowchart indicate the materials, whereas the light green blocks show the production process, reproduced from [36].
3.3 Nuclear Graphite Ageing Mechanisms

As described in previous sections the AGR graphite bricks are manufactured from Gilsonite pitch coke, to form what is known as Gilsocarbon graphite. This type of graphite has near-isotropic properties, and is preferred for moderator design due to its low neutron absorption cross-section and high thermal conductivity.

Even though the Gilsocarbon graphite is optimised for moderation and thermal conduction requirements of nuclear reactors, it is not completely immune to the high-dose radiation and fast-moving neutrons generated during the fission reaction in the reactor core. In fact, virgin Gilsocarbon graphite contains some level of porosity, which accounts for approximately 19 % of its entire structure [37]. Approximately 40 % of the total pores are closed [37].

As mentioned in the earlier sections, the graphite bricks that make up the AGR core are affected by two main types of ageing mechanisms: irradiation and radiolytic oxidation [3, 34, 36]. Sections 3.3.1 and 3.3.2 will briefly introduce these two graphite-ageing mechanisms.

3.3.1 Irradiation

Irradiation damage in nuclear graphite is caused by the interaction between the fast-moving neutrons generated from the fission reaction and the graphite carbon atoms. The fission reaction is essentially a reaction caused when a uranium nucleus absorbs a neutron and splits to form multiple nuclei. This reaction creates (on average) three neutrons each having a mean energy of approximately 2 MeV [3, 34]. When these neutrons are scattered after a reaction, they will come into contact with graphite carbon atoms. The energy contents of the neutrons are much higher than the binding energy of carbon atoms in the lattice, which is \(\approx 7\) eV [3]. Therefore, as soon as they come into contact with the graphite they start displacing the carbon atoms from the lattice, and the displaced atom is termed as primary knock-on atom [3, 34]. This process transfers a fraction of the neutron energy to the primary knock-on atom. The energy content of the primary knock-on atom will still be very high, and some of its energy will transfer to secondary knock-on atoms during subsequent
collisions, resulting in random cascade of displacements of the graphite atoms until the energy exhausted [3, 34].

The mean free path travelled by the primary knock-on atom is usually larger than the interatomic spacing of the graphite, suggesting that the primary knock-on atom displace more than one atom with a single strike. As the result, the displaced atoms will have multiple localised groups of atoms each containing between 5 and 10 atoms [3,34]. Further collision in the displacement groups can be considered as an independent event until the energy reduces to approximately 100 eV. Once the energy reaches 100 eV, the displacement due to collision is comparable to the interatomic spacing. Some of the displaced atoms will recombine with the graphite vacancies in the lattice while the others move in between the graphite planes increasing the probability of further collision with an incoming primary knock-on atom [3]. In general, a single neutron strike on a graphite carbon atom produces up to $10^8$ atom displacements [34-35].

As the result of the neutron strikes, the graphite planes will have fewer hexagonally interlocked atoms than they had prior to irradiation. Consequently, they tend to shrink in the in-plane axis relative to the alignments of their atomic structure. On the other hand, the displaced atoms located in between the graphite planes tend to expand the graphite in the cross-plane axis.

However, the expansion in the cross-plane axis remain small due to the porous nature of the graphite, the net result in the bulk polycrystalline graphite is overall shrinkage, provided that the graphite is exposed to a relatively low dose or at the early stage of its operational lifetime. Further increase in radiation dose means more atoms are being displaced from their lattice, and positioned in between the planes. Therefore, the shrinkage in the in-plane axis will become dominated by the expansion in the cross-plane axis, resulting in overall expansion of the bulk polycrystalline graphite.

The above phenomenon is known as neutron irradiation damage, and the point at which the graphite changes from overall shrinkage to overall expansion is termed the turnaround point.

In general, the graphite bricks in the AGR core can develop differential stresses between the bore and the periphery when they are irradiated. This is due to
the bore being exposed to a higher irradiation dose from fast-moving neutrons than the periphery, resulting in different expansion and shrinkage rates. This mis-match in dimensional change may eventually lead to cracking and brick distortion [5, 19].

3.3.2 Radiolytic Oxidation

There are two types of oxidation that affect the graphite properties in the presence of heat and \( CO_2 \): thermal oxidation and radiolytic oxidation. The effect of thermal oxidation in graphite is usually significant at the temperatures above 800 °C, which is higher than the temperatures within the AGR core. Therefore, the effect that thermal oxidation in \( CO_2 \) has on the AGR bricks is negligible. The majority of the oxidation within the AGR core is caused by radiolytic oxidation.

Radiolytic oxidation in the graphite is caused by a chemical reaction between the gamma-activated \( CO_2 \) coolants and the graphite carbon atoms C [3, 34]. During the operation of the AGR, the deposition of energy in high-pressure \( CO_2 \) coolant from the gamma radiation decomposes the \( CO_2 \), and generates approximately three active species for every 100 eV absorbed [34]. When the active species \( O^* \) comes into contact with the graphite surface some of them will diffuse through the surface-breaking pores in the graphite and remove the carbon atoms by forming carbon monoxide \( CO \). These lead to increase in graphite porosity, and hence reduction in a graphite density. The chemical reaction that describes this process is shown in equation 3.1 and 3.2:

\[
\begin{align*}
CO_2 \text{ coolant} & \xrightarrow{\text{irradiation}} CO + O^* \text{ activation} \\
C \text{ moderator} + O^* & \rightarrow CO \text{ oxidation}
\end{align*}
\]  
(Eq 3.1)

(Eq 3.2)

Equation 3.1 shows the decomposition process of the \( CO_2 \) coolant after being exposed to gamma radiation, whereas equation 3.2 shows the process of removing a graphite carbon atom.
The oxidation rate in the AGR is normally reduced by the addition of methane $CH_4$ into the reactor coolants, which then flow through the methane holes in the fuel channel bricks. The methane inhibitor acts to de-activate the active species by radiolytically forming carbon-depositing species, which has an effect of removing the active species [34]. However, the radiolytic oxidation rate could also increase with increasing operational lifetime of the reactors, mainly due to the increased porosity of the graphite bricks allowing the active species to diffuse through the brick interior. Eventually, these could result in increasing porosity, and hence in further graphite weight/density loss.

Radiolytic oxidation generally causes exponential decays in Young’s modulus and strength of the graphite bricks [4]. These in turn cause some economic and safety implications, including loss of moderation capability and ability to function as a core structural component.

Currently, multiple techniques are used by EDF Energy to estimate the density loss of graphite bricks (see Section 3.4). Many academics, engineers and scientists are also working in this area to understand graphite behaviour under irradiation and radiolytic oxidation [3, 20, 34, 38-41]. This includes the study carried out by Hacker et. al [41], which showed a direct relationship between the nuclear grade graphite samples density/weight loss and their electrical resistivity.

The AGR graphite density loss is the primary interest of this project. The work in this thesis will examine and assess the capability of the EC-based NDT system in determining the graphite density loss via the estimation of the fuel channel bricks volumetric resistivity profiles.

### 3.4 Reactor Core Inspection Systems

As stated in Section 2.3.2 some of the major requirements of the reactor safety case are: the AGR core must allow unimpeded movement of the control rods and the fuel assemblies, and must be able to direct the flow of coolant gas [15]. In addition, the weight loss must be determined and kept within a safety margin to
maintain core structural integrity. The individual graphite brick weight loss is also related to the loss of moderation.

For these reasons, the AGRs are subjected to regular in-core inspections. EDF Energy uses several specialised in-core inspection tools, which are inserted into vacated fuel channel bricks to assess the condition of the reactor core. Figure 3.5 illustrates the general arrangement of the inspections, with reference to the EC inspection tool known as PECIT. Due to the fuel channel brick proximity to the fuel elements, most of the in-core inspections are carried out to assess the conditions of these bricks. During the inspection process the tool is lowered into the reactor core using chains and an umbilical cable. The speed, locations and other inspection functions of the tool are set from the control console on the pile cap.

![Diagram of AGR core inspections with reference to PECIT.](image)

*Figure 3.5: The general arrangement of the AGR core inspections with reference to PECIT.*

### 3.4.1 Trepanning Tool

The Trepanning Tool (TT) is used to obtain the fuel channel brick volumetric information through the extraction of small cylindrical samples from the core by trepanning, and analysed in a laboratory environment. As illustrated in Figure 3.5,
the TT is deployed into the core using a hoist, and its functions are set from the control console. The tool has a cylindrical stainless steel casing and uses a cutting head driven by a compressed air motor to cut into the brick and remove a cylindrical graphite sample from the core, which is then transferred into a flask for transportation. The stability of the TT during trepanning process is maintained by attaching pneumatic jacks fitted on the tool to the graphite bore. A section of the TT is shown in the Figure 3.6.

![Cutting head](image)

Figure 3.6: Section of Trepanning Tool that shows the cutting head.

### 3.4.2 Channel Bore Measurement Unit

The Channel Bore Measurement Unit (CBMU) measures the distortion and alignment of the fuel channels bricks in the reactor core. Like the TT, the CBMU is enclosed within a cylindrical stainless steel casing, and deployed using a hoist on the top of the pile cap. The tool contains two sets of four wheels on the top and bottom of feelers. The feelers measure the diameter and tilt information about the fuel channel bricks, while the wheels maintain the tool at the channel centre during the inspection. During the inspection process, the CBMU is initially lowered into the reactor core until it reaches the lowest brick layers, and the measurements are taken by deploying the feelers to contact the graphite bore as the tool rises back to the top.
3.4.4 New In-Core Inspection Equipment

The In-Core Inspection Equipment (NICIE) is operated from the pile cap and consists of a set of stabilising arms, a rotating radiation-tolerant camera and a mirror, which enable side-ways viewing of the channel wall. The second generation, i.e. Mark II NICE allows even more flexibility and functionality with the incorporation of a channel bore measurement system, which gives both television images, and channel profiles and tilt. The Mark II NICE contains a pressure interface when connected to the reactor, and can be deployed into the core while the core is in $CO_2$.

3.4.5 Prototype Eddy Current Inspection Tool

Prototype Eddy Current Inspection Tool (PECIT) is an EC-based inspection tool that consists of two different types of EC sensors (differential probe, and two gradiometers) separated circumferentially round the tool by 120°. The EC sensors in the tool are arranged to inspect the graphite brick over its radial cross section as well as to measure the density variation around the brick bore axially and circumferentially [20].

![Sensor head](image)

Figure 3.7: Section of the PECIT that shows the location of the sensor.

The tool has a rotating heads deployed from cylindrical stainless steel body and uses scissor-type arms made from Polyether Ether Ketone (PEEK) to deploy the EC sensors to the graphite bore.
Currently a new tool known as Eddy Current Inspection Tool (ECIT) is being developed, and expected to offer an improved inspection capability for both radial and circumferential properties of the fuel channel bricks. Figure 3.7 shows a section of the PECIT tool.
Chapter 4

4.1 Non-Destructive Testing

As the name suggests, Non-Destructive Testing (NDT) is essentially a technique used to ensure a material or a component is fit for a purpose, without altering its properties. Generally speaking, this means that the NDT is performed without affecting the integrity of the component, such as its internal properties and dimensions, although in some cases materials are disassembled and inspected using various types of NDT methods. These features of NDTs have made them the most popular and widely used inspection techniques within many industrial sectors including: energy, manufacturing, quality control, asset management and transportation [20, 42-44].

There are wide range of NDT systems that use different forms of energies (such as: X-ray, ultrasonic, EC and many more), and techniques to inspect a material of interest [42-45]. The suitability and effectiveness of each NDT method depend upon many conditions including, the properties of the material to be inspected and the environment in which the test need to be carried out.

In the case of this work, the regions of interest within the AGR core are the fuel channel bricks, mainly due to their proximity to the fuel elements. Generally speaking, the AGR core has a very complex structure; therefore inspection access is only limited through the bore of the fuel channel bricks. In addition, the AGR core contains a high level of radiation, which makes it difficult to deploy a sensor head containing electronic components, unless it is protected by some kind of radiation shield [46-47]. Shielding electronic components for use in hostile environments such as the AGR core also adds further technical complexity, and may not be the natural choice when it comes to NDT applications.

Of particular concern to the inspection of the AGR core integrity are the through-wall density/electrical resistivity profiles of the graphite bricks. This inspection cannot currently be carried out using almost all of the existing NDT methods due to the complex structure and hostile nature of the core, although the through-wall information is currently obtained destructively via trepanning. Up to
the time of writing this thesis, the most suitable NDT method for AGR core inspection is based on EC methods. This is because EC inspection system can be optimised to adapt the graphite complex structure increase the inspection capability, and often operated using different excitation techniques to inspect a material at a required depth. Therefore it can be used to obtain the data related to the through-wall resistivity profiles of the nuclear graphite bricks [6, 7, 48, 50]. The added advantage of the EC system is that the sensors can be optimised to conform to the application by altering the design parameters such as sensor geometry and number of turns to achieve the highest possible sensitivity to the material being inspected [48, 56-57]. Finally, EC sensors can be made purely from coils of conducting wires and radiation-resistant former, so the effects that radiation has on them is minimal compared to a sensor containing active electronic components.

The work in this thesis is mainly focused around an EC system for inspection of the graphite bricks that make up the AGR core, and this method will be further discussed in the remaining part of this Chapter.

4.1.1 Eddy Current Testing

EC-based NDT is one of the most widely-used methods for assessing the properties of conducting or magnetic materials, particularly metallic components. This method has widespread use in industrial applications, ranging from quality control in manufacturing industries to asset management and safety assessment in the energy and transportation sectors [20, 24, 49-51]. Many EC-based material characterisation systems have been developed in the past by different researchers and engineers, each employing different techniques such as different sensor configurations, test arrangements and data processing methods designed for specific application, see for example [52-54].

An EC inspection system operates on the principle of electromagnetic induction. Excitation of a coil with an alternating current generates a time-harmonic primary field around the coil. Placing the coil next to a sample of electrically conducting material allows the primary field to penetrate into the sample and induce
closed-loop EC (Figure 4.1). The induced EC in turn generate a secondary field, which can be detected by a pickup coil.

![Graphical illustration of the eddy current flows in a conducting plate, reproduced from [45].](image)

Figure 4.1: Graphical illustration of the eddy current flows in a conducting plate, reproduced from [45].

Change in the material’s electrical conductivity, magnetic permeability or a defect inside its volume will alter the flow of the EC, and hence affect the signal detected by a pickup coil. This information can therefore be used (in principle) to determine the electrical conductivity distribution or the presence of a defect within a given sample volume.

Beside the variation in material properties, the EC sensor output signal can also be influenced with the factors including the coil configuration, frequency and type of the excitation signal [45, 55]. Nevertheless, except for those related to the material properties (magnetic permeability, electrical conductivity, sample geometry), all of these factors could be optimised to meet the requirements of specific application [56-57].

The outputs from the EC sensors can be represented in different formats such as a time versus amplitude plot and a complex impedance plane (Eq 4.1 and 4.2) [55].

\[ A(t) = A \sin(2\pi ft + \theta) \]  
(Eq 4.1)

\[ Z = R + j2\pi fL \]  
(Eq 4.2)
where \( A(t) \) is the time-varying amplitude of the measured signal, \( f \) is the excitation signal frequency, \( \theta \) is the phase shift of the signal due to the EC, \( Z \) is the coil impedance, \( R \) is the coil resistance and \( L \) is the coil self-inductance.

For the cases where the EC system involved inductively coupled pickup coils (i.e. a gradiometer sensor), the output data can be represented either by a complex differential induced voltage or complex mutual inductance between the two coils (Eq 4.3 and Eq 4.4).

\[
V_{\text{ind}} = j2\pi f M I_0 \quad \text{(Eq 4.3)}
\]

\[
M = \Re\left(\frac{V_{\text{ind}}}{2\pi f I_0}\right) + \Im\left(\frac{V_{\text{ind}}}{2\pi f I_0}\right) \quad \text{(Eq 4.4)}
\]

where \( V_{\text{ind}} \) is the induced complex voltage, \( M \) is the mutual inductance, and \( I_0 \) is the excitation current.

Equations 4.2, 4.3 and 4.4 show the most common representation of the EC measurements; mainly because of the convenience they offer when it comes to analysing the measured data. By contrast, the time versus amplitude representation in equation 4.1 requires further data processing to represent the EC signal in a more useful format for interpretation of the test data.

In practice, an additional term exists in equation 4.2 for the stray capacitance between the coil windings. However, this term can only contribute at very high frequencies. The frequency at which the capacitive effect becomes dominant is known as the resonant frequency, and given by equation 4.5.

\[
f_{\text{reson}} = \frac{1}{2\pi \sqrt{LC}} \quad \text{(Eq 4.5)}
\]

where \( C \) is the stray capacitance generated between the pair windings. Equation 4.5 shows the maximum threshold frequency at which the EC inspection can be made reliably. In most cases, the EC inspections are carried out well below the resonance frequency where the effect of stray capacitance on the measured signal is negligible.
4.1.2 Techniques for Eddy Current Inspection

The sensitivity of EC NDT to sub-surface information is highly dependent upon the penetration depth of the field into the material. The penetration depth in turn depends upon the material’s electrical conductivity, magnetic permeability and the frequency of the excitation signal. Detailed descriptions about the dependencies of the penetration depth on the mentioned three variables can be found [45, 59]. Assuming an EC probe is placed above the surface of a large defect-free plane object that has magnetic permeability $\mu$, electrical conductivity $\sigma$ and a thickness of $x$, the distribution of the EC within the object when the probe is injected by AC signal of frequency $f$ can be described using equation 4.6.

$$J_x = J_0 e^{-x(\pi f \mu \sigma)^{1/2}}$$  \hspace{1cm} (Eq 4.6)

where: $J_x$ is the EC in the $x$ direction normal to the coils, and $J_0$ is the initial current density at the surface of the object.

Equation 4.6 essentially describes how the EC varies with depth from the surface of the material. In other words, when an excitation coil is placed on the top of a large conducting plane material, the concentration of the induced EC will be higher at the surface ($J_0$) and decrease exponentially (by 1/e or to 37% of its initial value at standard penetration depth $\delta$). At double standard penetration depth, the EC will reduce to $(1/e)^2 \approx 13\%$ of its initial value. This phenomenon is known as the skin effect, and the mathematical relationship between the material parameters, excitation frequency and the standard penetration depth is given in equation 4.7.

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$  \hspace{1cm} (Eq 4.7)

where $\delta$ is the standard penetration depth. It should be noted that the exponential nature of the EC depth dependence is only true for uniform plane geometries; nevertheless it gives a good overall indication of the skin effect.

The reduction in EC density with distance away from the material surface can be viewed in two different ways. When the conductivity of an isotropic test object is
very high at some fixed frequency, the EC density will be higher at the surface of the material and acts as a shield preventing or weakening the penetration of the primary field into the test object. This leads to a reduction of the EC in exponential fashion away from the object surface. These phenomena can also be described in terms of opposing primary and secondary fields generated by the excite signal and the EC respectively.

Assuming an isotropic test material with a constant conductivity, the penetration depth depends only on the frequency of the excitation signal. This means that varying the frequency of the AC signal allows the penetration depth to be adjusted. To illustrate the EC variations in a conducting material as a function of excitation frequency, a cylindrical graphite brick model that has a symmetrical gradiometer sensor placed next to its bore was created (Figure 4.2). The model was created using layered cylindrical graphite, and contains key-ways and methane holes. The cylindrical structure of the graphite model was reduced by exploiting its quarter symmetry (only shows one quarter of the entire geometry), and simulated with a constant electrical conductivity, whilst sweeping the frequency of the excitation signal between 10 Hz and 100 kHz.

As can be seen in Figure 4.2 at higher frequencies the penetration depth is lower but the magnitude of the EC at the surface region is higher, meaning that the changes in the material properties closer to the sensing coil can be probed more accurately. As the frequency of the AC signal reduces, the EC extend further into the graphite volume resulting in a larger penetration depth. Therefore, combining the information obtained from measurements at each frequency in principle allows us to acquire information about the graphite volumetric conductivity variations.

Graphite is a weak electrical conductor compared with most metals, having a maximum electrical conductivity of approximately 100 kS/m. This is much lower than the electrical conductivity of typical conducting materials, for instance the electrical conductivity of Stainless Steel is approximately 1.35 MS/m. This property of the graphite offers a significant increase in the penetration depth, which allows the inspection of the entire radial thickness of the brick provided that the excitation frequency is chosen appropriately.
Some authors have suggested that in order to accurately obtain the sub-surface information of conducting material with a certain thickness, the frequency of the excitation signal must be chosen such that it gives a penetration depth of approximately one-third of the material thickness [45, 58]. Nevertheless, this is mainly useful when the EC system is operated in a single frequency mode.

The single frequency excitation method is the simplest form of EC testing, but suffers from a limitation on the amount of information it can obtain. This is because the penetration depth of the field generated by the single frequency is fixed, so the
sample can only be probed at a fixed depth. However, there are circumstances in which a single frequency excitation becomes more useful, in particular when inspecting for surface-breaking defects or a defect with known approximate depth location.

An EC system that operates in a multi-frequency mode overcomes some of the limitations of a single frequency system. In multi-frequency system, a test material is interrogated by sweeping the frequency with a wide spectrum, allowing more information about the test material to be obtained at different depths (Figure 4.2). The added advantage of this method is that the sensor’s response in the frequency domain can be directly used to infer some information about the material under test [7, 53, 58, 60].

A closely related system known as pulsed EC has also been used in many applications to inspect for sub-surface defects and measure the thickness of a material under test [61-63]. Unlike the sinusoidal excitation technique discussed above, the pulsed EC system uses a train of rectangular pulses. The duty cycle ratio of the pulse train depends upon the requirement of the test frequency; the shorter the pulses the wider the frequency range. In general, this method allows information about the sample to be acquired over a wide range of frequencies at once, increasing the speeds at which the inspection can be carried out. However, this system requires more complex post measurement data processing hardware or software to process and analyse the measured data [58, 61-63].

The multi-frequency technique is the one adopted in this thesis for the graphite conductivity reconstruction problem (see Chapters 6 and 7). This is mainly due to its simplicity, improved signal to noise performance and the advantages it offers in gathering information at different radial depth in the cylindrical AGR graphite bricks. Figure 4.3 illustrates the excitation signals and the corresponding frequency spectra of the single frequency, multi-frequency and pulsed EC excitations.
4.1.3 Eddy Current Sensors

The EC sensors can be configured in variety of ways (see for example [45,64-65]) with each having different design to adapt specific test arrangement. However, the configuration of most EC probes are derived from one of the four common EC sensors: absolute, differential, reflection and gradiometer probes.

The simplest form of EC sensor is known as the absolute probe. This probe is relatively easy to construct, and requires less post-measurement processing. This type of probe is made from a single coil that operates in dual function mode, acting as both exciter and pickup coil. The probe usually requires a reference coil with an identical inductance value, and it is mainly used for detection of relatively large defects, but is generally regarded as sensitive to background objects and temperature variations.

A differential probe is a closely related probe that operates almost on the same principle as the absolute probe, except in this case the probe is created using a pair of identical coils with a small gap between them. In this system, the two coils are wound in series opposition so that they cancel each other in the absence of
defects within the material of interest. The main advantage of this probe is its ability to cancel background noise and its increased sensitivity to smaller defects. However, differential probes are usually unable to detect defects bigger than the gap between the two coils.

Another popular EC sensor configuration is the reflection probe, which can be made from two independently wound coils. In this configuration the two coils operate as exciter and receiver coils. This typically means that one of the coils operates to generate the primary field, while the other is used to detect the EC signal. These types of probes offer the advantage that the two coils can be optimised independently to achieve better measurement sensitivity.

A gradiometer probe is another EC sensor, which is commonly used in various NDT applications. The connectivity of this probe is derived in part from reflection and differential probes. The gradiometer probe is one of those used within the PECIT for AGR inspection, and also the one adopted for the work involved in this project. Gradiometer sensors are made up of three coils, exciter, pick-up and backing-off coils that are arranged either along the same axis, as in the case of an axial gradiometer or in the same plane in the case of planar gradiometer. In the case of the axial gradiometer the pick-up and backing-off coils are normally placed on opposite side of the exciter coil and wound in series opposition to allow cancelation of the background field in the absence of nearby conducting materials. The three coils are independent from one another, except in the case of a symmetrical gradiometer, which has identical pickup and backing-off coils. This means the parameters for each coil can be optimised to gain better measurement sensitivity. The arrangements of the gradiometer probes are usually preferred when detecting deeper flaws with relatively small sizes or small conductivity variations in the subsurface region.

4.2 Forward Problem

The inspection problem in this thesis was approached by initially developing a forward model that represents the physical and electromagnetic properties of the nuclear graphite bricks and the EC measurement system. By definition a typical
Forward problem is the “prediction of the electric fields that arise, and consequently the boundary values, due to the electrical excitation to specific distribution of the material” [49]. Following the forward problem definition it is apparent that the development of a forward model requires the user to have a complete knowledge about the material of interest and the sensing system parameters. Assuming all of the above parameters are known, and then a complete model can be created, and solved using the FE method, in this case using the edge FE method [66-71].

4.2.1 Theoretical Background

The FE method is a well-established numerical technique for solving many engineering problems. In this method the given problem domain is discretised into a mesh consisting of multiple mesh elements. The response of each mesh can be approximated by algebraic equations according to the connectivity of the mesh elements. The problem is then solved using appropriate boundary conditions and the physics governing the given problem.

The EC phenomena discussed in Section 4.1.1 can be fully described using Maxwell’s equations. Starting from the differential form of Maxwell’s equation, the main equations describing the graphite forward problem can be expressed as follows:

\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \text{(Faraday’s Law)} \]  
\[ \nabla \times H = J_T + \frac{\partial D}{\partial t} \quad \text{(Ampere’s Law)} \]  
\[ \nabla \cdot D = \rho \quad \text{(Gauss’ Law)} \]  
\[ \nabla \cdot B = 0 \quad \text{(Continuity of magnetic flux)} \]

The relationships between the graphite model electromagnetic properties can be represented using constitutive equations:

\[ B = \mu H \]  
\[ D = \varepsilon E \]
\( J_{in} = \sigma E \) \hspace{1cm} (Eq 4.14)
\( J_T = J_{in} + J_e \) \hspace{1cm} (Eq 4.15)

Here \( E \) is the electric field, \( H \) is the magnetic field, \( B \) is the magnetic flux density, \( D \) is the electric displacement field, \( \rho \) is the free charge density, \( J_T \) is the total current density, \( \varepsilon \) is the permittivity, \( \mu \) is the magnetic permeability, \( \sigma \) is the electrical conductivity, and \( J_{in} \) and \( J_e \) are the internal and external current densities.

Helmholtz’s theorem states that a vector field is only uniquely defined when its divergence and curl are given [72]. Hence, setting a Coulomb Gauge condition on the magnetic vector potential (\( \nabla \cdot A = 0 \)), which specifies the divergence free vector field [73-75], and defining the magnetic flux density (\( B \)) in terms of the curl of the magnetic vector potential (\( A \)) yields:

\( \nabla \times A = B \) \hspace{1cm} (Eq 4.16)

Since the problem is based on a time-varying excitation signal, the equations need to be represented as time-harmonic Maxwell’s equation. Hence, representing equation 4.8 and 4.9 in the time-harmonic Maxwell’s form yields:

\( \nabla \times E = -j\omega B \) \hspace{1cm} (Eq 4.17)
\( \nabla \times H = J_T + j\omega D \) \hspace{1cm} (Eq 4.18)

where, \( \omega \) is the angular frequency of the excitation current (\( \omega = 2\pi f \)).

Substituting equation 4.16 in 4.17, and equation 4.12, 4.13, 4.14 and 4.15 in equation 4.18 respectively yields:

\( E = -j\omega A \) \hspace{1cm} (Eq 4.19)
\( \nabla \times \frac{1}{\mu} B = \sigma E + J_e + \omega^2 \varepsilon A \) \hspace{1cm} (Eq 4.20)

Once again substituting equation 4.16 and 4.19 in 4.20 and rearranging the equation then gives the Ampere’s law equation that can be solved by FE software for a given
problem domain, in this case for each sub-domain or layer of the graphite brick model.

\[ \nabla \times \left( \frac{1}{\mu} \nabla \times A \right) + (j \omega \sigma - \omega^2 \varepsilon)A = J_e \]  \hspace{1cm} \text{(Eq 4.21)}

In the edge FE method the problem domain is initially discretised into multiple tetrahedral elements. Each tetrahedral element is then approximated by an algebraic equation that corresponds to the tangential components of the \( A \)-field (the \( A \)-fields along the edges of each tetrahedral element). In this method the vector field is represented using a basis of vector valued function \( N_{ij} \) associated with the edge between node \( i \) and \( j \).

\[ N_{ij} = L_i \nabla L_j - L_j \nabla L_i \]  \hspace{1cm} \text{(Eq 4.22)}

where \( L_{ij} \) is the nodal basis function, which is a low order piecewise polynomial of the coordinates \( x, y \) and \( z \).

After representing the current density in the region of the excite coil \( J_e \) by the current vector potential \( T_e \) so that the current density becomes divergence-free [67, 70, 76].

\[ \nabla \times T_e = J_e \]  \hspace{1cm} \text{(Eq 4.23)}

\[ \nabla . J_e = 0 \]  \hspace{1cm} \text{(Eq 4.24)}

The Ampere's law equation (Eq 4.21) can then be approximated using Galerkin's method as follows [50, 76]:

\[ \int_{\Omega} \left( \nabla \times N \frac{1}{\mu} \nabla \times A \right) dv + \int_{\Omega_e} (j \omega \sigma - \omega^2 \varepsilon)N.A dv = \int_{\Omega_c} (\nabla \times N \cdot T_e) dv \]  \hspace{1cm} \text{(Eq 4.25)}

where \( \Omega \) is the entire problem domain, \( \Omega_e \) is the domains where the EC exist, \( \Omega_c \) is the coil region and \( N \) is any linear combination of edge basis functions.

The induced voltage \( V \) in the receiver coils can then be calculated using equation 4.26.
\[ V = j\omega \int_{\Omega_c} A J_0 \, dv \]  
(Eq 4.26)

where \( J_0 \) is a virtual unit current density applied to the receiver coils.

The integrals in equation 4.25 corresponding to the different regions of the problem domain are evaluated element-wise; hence there are simultaneous equations equivalent to the number of elements within the entire problem domain. These local equations are then arranged in a global matrix known as the stiffness matrix and solved using appropriate solvers and boundary conditions.

For the forward problem considered in this thesis the Magnetic Field solver within COMSOL Multiphysics software package, along with an iterative approach known as Flexible Generalised Minimum Residual (FGMRES) has been used [77]. To increase the convergence rate of this solver a preconditioning Geometric Multigrid (GMG) was applied into the iterative solver, which aims to reduce the condition number of the stiffness matrix [77]. The boundary condition applied for this problem is magnetic insulation boundary condition, which fixes the field variables being solved for (in this case \( A \)-field) to be zero on the selected boundary [155]. This boundary condition also ensures that the magnetic field is tangential to the boundary, and defines the symmetry condition, which has a mirror effect of the magnetic field.

4.3 Construction of the Sensitivity Matrix

The sensitivity of the EC probe to the test material is critical for a successful characterisation of the material’s internal properties. The sensitivity of a given EC sensor to an electrically conducting material can be determined using both experimental and numerical methods through the evaluation of the sensitivity matrix (also known as the Jacobian matrix). The Jacobian matrix in this context describes how sensitive a measurement is to a change in the material electrical conductivity, or links small change in the boundary measurements to a change in the material electrical conductivity. Mathematically, this can be represented as the first
order partial derivative of the induced voltage with respect to the material conductivity as shown in equation 4.27.

\[ J(\sigma) = \left[ \frac{\partial V}{\partial \sigma_i} \right] \]  

(Eq 4.27)

Several authors in the electromagnetic NDT field have proposed different techniques for mapping the sensitivity of the material under test [78-82]. These include the work by Yin and Peyton [79], which used the perturbation technique to map the change in coil inductance with respect to a small change in the conductivity of a test material. In the perturbation method the sensitivity matrix is constructed by first perturbing the system’s electrical conductivity and measuring the change in the boundary measurements such that: \( J = \frac{\Delta V}{\Delta \sigma} \). This is equivalent to evaluating the sensitivity of each element independently to acquire a complete sensitivity matrix of the given system. If the problem domain contains a layered conductor as in the case of the graphite brick model, then the perturbation can be made in each layer one at a time to obtain a full set of data for the Jacobian. Assuming \( n \) layers and \( m \) frequency points, the total number of forward solutions required to construct the full Jacobian matrix using the perturbation technique is \( n \times m \).

A more computationally efficient sensitivity formulation that can be applied for general electromagnetic problems was presented by Yin and Peyton, and Dyck et al [80-81]. This method is known as the “E.E” formulation, and it was derived based on the reciprocity theorem, which allows the calculation of the changes in the electromagnetic fields due to changes in the material properties and the source current. In order to compute a complete sensitivity matrix of a given problem using this method, an adjoint system needs to be defined and solved. Calculation of the sensitivity matrix then requires evaluation of both the forward and adjoint problems. Once the problems are solved then the Jacobian matrix can be computed using the dot product of the E-fields giving similar result as the perturbation technique, but in a much more computationally efficient way.

In this thesis the E.E method is adopted to carry out the study of the EC probe sensitivity to a conducting graphite brick, which is subsequently used to solve the
graphite conductivity reconstruction problem. Section 4.3.1 presents the mathematical derivation of the E.E method as presented in [80], but in this case neglecting the velocity term from the original formulation.

4.3.1 Theoretical Background

Starting from the relationship between the electromagnetic variables and the measured parameters of a physical system, and assuming a change in the measured parameters due to small changes in the variables, the following equations can be written to describe these relationships (Eq 4.28 and 4.29).

\[ D = \int_V f(E, H) dv \]  \hspace{1cm} (Eq 4.28)

\[ \delta D = \int_V (\nabla_E f(E, H) \cdot \delta E + \nabla_H f(E, H) \cdot \delta H) dv \]  \hspace{1cm} (Eq 4.29)

where \( D \) is a measurement from a physical system that is a function of the magnetic field \( (H) \) and electric field \( (E) \), \( \delta E \) and \( \delta H \) are the perturbations in electric and magnetic fields that causes a change in the measured value \( \delta D \) and \( v \) is the volume of the physical system over which the magnetic and electric fields are integrated.

Changing the system parameters such as \( \sigma, \mu, \varepsilon \) and \( I \), will result in a change in the electromagnetic properties \( (E \) and \( H) \), and hence in the measured quantities. Therefore equation 4.29 can be expressed using the material parameters:

\[ \delta D = \int_V (S_\sigma \delta \sigma + S_\varepsilon \delta \varepsilon + S_\mu \delta \mu + S_J \delta J_e) dv \]  \hspace{1cm} (Eq 4.30)

where \( S \) is the sensitivity of \( D \) in response of the concerned parameters respectively.

Defining a quasi-poynting vector \( (S_{xy} = E_x \times H_y) \) between two systems \( x \) and \( y \) with a possible difference in material parameters and excitation current, taking the volume integral of the divergence of these fields, and applying the divergence theorem, yields:

\[ \int_V \nabla \cdot S_{xy} dv = \int_V \nabla \cdot (E_x \times H_y) dv = \oint_S (E_x \times H_y) \cdot n ds \]  \hspace{1cm} (Eq 4.31)
where $s$ is the surface bounding $v$, and $n$ is the unit vector normal to the surface pointing out of the surface. Equation 4.31 can be simplified further by making use of the vector identity $\nabla \cdot (a \times b) = b \cdot \nabla \times a - a \cdot \nabla \times b$.

$$\int_v (H_y \cdot \nabla \times E_x - E_x \cdot \nabla \times H_y) \, dv = \oint_s (E_x \times H_y) \cdot n \, ds$$  \hspace{1cm} (Eq 4.32)

Representing the curl of $E$ and $H$ fields in the time-harmonic form:

$$\nabla \times E = -j\omega \mu H$$  \hspace{1cm} (Eq 4.33)

$$\nabla \times H = (\sigma + j\omega \varepsilon)E + J_e$$  \hspace{1cm} (Eq 4.34)

and substituting equation 4.33 and 3.34 into equation 4.32 gives:

$$\int_v \left\{ j\omega \mu_x H_x \cdot H_y + \left[ (\sigma_y + j\omega \varepsilon_y)E_y + J_{ey} \right] \cdot E_x \right\} \, dv = -\oint_s (E_x \times H_y) \cdot n \, ds$$  \hspace{1cm} (Eq 4.35)

Exchanging the two systems (with $x$ and $y$ exchanged) will give the same result as shown in equation 4.36.

$$\int_v \left\{ j\omega \mu_y H_y \cdot H_x + \left[ (\sigma_x + j\omega \varepsilon_x)E_x + J_{ex} \right] \cdot E_y \right\} \, dv = -\oint_s (E_y \times H_x) \cdot n \, ds$$  \hspace{1cm} (Eq 4.36)

We replace one of the systems (in this case system $y$) with fixed material properties ($\mu_y \to \mu$, $\sigma_y \to \sigma$ and $\varepsilon_y \to \varepsilon$), and the properties of system $x$ with small variations in material properties:

$$\mu_x \to \mu + \delta\mu$$  \hspace{1cm} (Eq 4.37)

$$\sigma_x \to \sigma + \delta\sigma$$  \hspace{1cm} (Eq 4.38)

$$\varepsilon_x \to \varepsilon + \delta\varepsilon$$  \hspace{1cm} (Eq 4.39)

The above changes in the material properties of the system $x$ will also change the fields in system $x$:

$$E_x \to E_x + \delta E_x$$  \hspace{1cm} (Eq 4.40)
\[ H_x \rightarrow H_x + \delta H_x \]  \hspace{1cm} (Eq 4.41)

\[ J_{e,x} \rightarrow J_{e,x} + \delta J_{e,x} \]  \hspace{1cm} (Eq 4.42)

Assuming now the two systems operate at an identical frequency (equal angular frequency) and substituting the above changes into equation 4.35 and 4.36 yields:

\[
\int_v \left\{ j\omega (\mu + \delta\mu). (H_x + \delta H_x). H_y + \left[ (\sigma + j\omega\varepsilon)E_y + J_{e,y} \right] . (E_x + \delta E_x) \right\} dv
\]

\[ = - \mathbf{\phi}_s \cdot ((E_x + \delta E_x) \times H_y) \cdot \mathbf{n} \cdot ds \]  \hspace{1cm} (Eq 4.43)

\[
\int_v \left\{ j\omega\mu (H_x + \delta H_x). H_y + \left[ (\sigma + \delta\sigma) + j\omega(\varepsilon + \delta\varepsilon) \right] . (E_x + \delta E_x) + (J_{e,x} + \delta J_{e,x}) . E_y \right\} dv
\]

\[ = - \mathbf{\phi}_s \cdot (E_y \times (H_x + \delta H_x)) \cdot \mathbf{n} \cdot ds \]  \hspace{1cm} (Eq 4.44)

Subtracting equation 4.35 from 4.43, and equation 4.36 from 4.44 respectively, while ignoring the higher order terms \((\delta^2)\) to limit the system to a small perturbations result in:

\[
\int_v \left\{ j\omega (\delta\mu H_x + \mu \delta H_x). H_y + \left[ (\sigma + j\omega\varepsilon)E_y + J_{e,y} \right] . \delta E_x \right\} dv
\]

\[ = - \mathbf{\phi}_s \cdot (\delta E_x \times H_y) \cdot \mathbf{n} \cdot ds \]  \hspace{1cm} (Eq 4.45)

\[
\int_v \left\{ j\omega\mu \delta H_x . H_y + \left[ (\delta\sigma + j\omega\delta\varepsilon)E_x + (\sigma + j\omega\varepsilon) \right] . \delta E_x + \delta J_{e,x} . E_y \right\} dv
\]

\[ = - \mathbf{\phi}_s \cdot (E_y \times \delta H_x) \cdot \mathbf{n} \cdot ds \]  \hspace{1cm} (Eq 4.46)

Subtracting equation 4.45 from 4.46 then gives the final sensitivity formulation:

\[
\int_v J_{e,y} . \delta E_x . dv = \int_v \left( E_x . E_y \delta\sigma + j\omega E_x . E_y \delta\varepsilon - j\omega H_x . H_y \delta\mu + E_y . \delta J_{e,x} \right) dv
\]

\[ + \mathbf{\phi}_s \cdot (H_y \times \delta E_x + E_y \times \delta H_x) \cdot \mathbf{n} \cdot ds \]  \hspace{1cm} (Eq 4.47)
Representing the left-hand side of equation 4.47 in the form of the vector potential yields:

\[
j_\omega \int V \delta A_x J_{ey} \cdot dv = \int V (E_x \cdot E_y \delta \sigma + j_\omega E_x \cdot E_y \delta \varepsilon - j_\omega H_x H_y \delta \mu + E_y \cdot \delta J_{ex}) \cdot dv \\
+ \oint (H_y \times \delta E_x + E_y \times \delta H_x) \cdot n \, ds \quad \text{(Eq 4.48)}
\]

The external current density \((J_{e_y})\) in equation 4.48 only exists in the coil region, therefore the volume integral on the left-hand side of equation 4.48 becomes just a volume integral in the coil region. As has been demonstrated previously in equation 4.26, the left-hand sides of equation 4.47 and 4.48 correspond to a change in induced voltage in the receiver coil due to a change in the material parameters in the excite system. Therefore, in the case of inductively coupled coils, which are the case in a gradiometer sensor, system \(y\) can be considered as receiver coils and system \(x\) as an exciter coil. The surface integral in equation 4.48 can also be neglected provided that the boundary of the system is taken in such a way that the field decays to zero at the boundary, as in the method adopted in this thesis. Therefore, the right-hand side of equation 4.48 becomes:

\[
\int V (E_{ex} \cdot E_{rec} \delta \sigma + j_\omega E_{ex} \cdot E_{rec} \delta \varepsilon - j_\omega H_{ex} H_{rec} \delta \mu + E_{rec} \cdot \delta J_{e,ex}) \cdot dv \quad \text{(Eq 4.49)}
\]

where \(e_{ex}\) and \(e_{rec}\) are the subscripts used to represent the exciter coil driven and receiver coil driven systems respectively. Comparing equation 4.49 with equation 4.30 gives, the sensitivity equation for each perturbation as shown below:

\[
S_\sigma = E_{ex} \cdot E_{rec} \quad \text{(Eq 4.50)}
\]

\[
S_\varepsilon = j_\omega E_{ex} \cdot E_{rec} \quad \text{(Eq 4.51)}
\]

\[
S_\mu = -j_\omega H_{ex} \cdot H_{rec} \quad \text{(Eq 4.52)}
\]

\[
S_J = E_{rec} \quad \text{(Eq 4.53)}
\]

As has been mentioned previously, this project is mainly concerned with the sensitivity formulation that maps a small change in the material conductivity to a
change in the induced voltage in the receiver coils. Therefore, only equation 4.50 is relevant to the work presented in this thesis. This is based on the assumption that the changes in other properties of the material (\(\delta\mu\) and \(\delta\varepsilon\)) are negligible. However, for other parameter perturbations such as change in material permeability and permittivity the corresponding formulation (\(-j\omega H_{\text{ex}} \cdot H_{\text{rec}}\) and \(j\omega E_{\text{ex}} \cdot E_{\text{rec}}\)) can be used to formulate the sensitivity matrix.

To determine the adjoint model for computation of the sensitivity values, the induced voltage in the receiver coil due to the perturbations in material parameters when an exciter coils is injected with a current, is represented in terms of the electric field and the current flowing in the receiver coil:

\[
V_{\text{rec}} = \int \frac{E_{\text{ex}} J_{\text{rec}}}{l_{\text{rec}}} \, dv
\]  
(Eq 4.54)

where \(E_{\text{ex}}\) is the electric field generated by the source current injected into the exciter coil, and \(J_{\text{rec}}\) and \(l_{\text{rec}}\) are the current density and the current flowing in the receiver coil. Note that \(\frac{J_{\text{rec}}}{l_{\text{rec}}}\) is a unit vector, hence equation 4.54 is essentially equivalent to the left-hand side of equation 4.47, with \(J_{\text{ey}} = \frac{J_{\text{rec}}}{l_{\text{rec}}}\) [80]. Since equation 4.54 also has a form of equation 4.28, the left hand side of equation 4.29 can be used as a source current of the receiver coil to compute the sensitivity [81].

\[
J_{\text{ey}} = \nabla_E f(E, H) = \nabla_E \left( \frac{E_{\text{ex}} J_{\text{rec}}}{l_{\text{rec}}} \right)
\]  
(Eq 4.55)

Since \(J_{\text{ey}} = \frac{J_{\text{rec}}}{l_{\text{rec}}}\) and the field is a linear function of the source current, the sensitivity in the receiver coil due to a conductivity perturbation is given by equation 4.56 [81].

\[
S_\sigma = \frac{E_{\text{ex}} F_{\text{rec}}}{l_{\text{rec}}}
\]  
(Eq 4.56)

The extension of equation 4.56 to a symmetrical gradiometer configuration was presented in [6]. In this thesis equation 4.56 is slightly modified to an asymmetrical gradiometer configuration. Assuming the coils are modelled as current-carrying filaments (which is the modelling method adopted in this thesis), the pickup and
backing-off coils will have different coil parameters for an asymmetrical gradiometer configuration. In this case the induced differential output voltage between the pickup (p) and the backing-off (b) coils due to the excitation current in the exciter coil can be derived from equation 4.54.

\[ V_{p,ex} - V_{b,ex} = \int \frac{E_{ex}I_p}{l_p} \, dv - k \int \frac{E_{ex}I_b}{l_b} \, dv \]  

(Eq 4.57)

Where \( k \) is the turn-ratio between the two receiver coils. In this case the backing-off coil is assumed to be smaller than the pickup coil, hence it needs to bescaled with the turn-ratio to balance the gradiometer. Therefore, the source current for the adjoint system is then given by:

\[ J_{s,adj} = \frac{I_p}{l_p} - k \frac{I_b}{l_b} \]  

(Eq 4.58)

Equation 4.58 shows that the source in the adjoint system must have opposite polarities, and scaled according to the turn-ratio between the pickup and backing off coils. The sensitivity of the asymmetric gradiometer is then simply computed using equation 4.50.

4.4 Custom-Designed Eddy Current Sensor for AGR Core Inspection

EC probes can often be custom-designed to meet the requirements of a specific inspection system. These requirements could arise due to conditions such as inspection access, probe sensitivity, sample geometry and many more. In general, custom-made EC probes can be found in various sectors of industrial NDT applications, and have also been proposed by many authors in the past, including [56-57, 84-85]. For instance, the authors in [56] have carried out a study of custom-made EC probes for inspection of complex shapes, comprising turbine blades and generator bodies. Another custom-made EC probe with different probe arrangements has been proposed in [57]. In this study, a sensor array was used to inspect the surface-breaking bore defects of a cylindrical conducting sample instead of the usual single probe. The authors in [84, 85] also designed flexible EC probes,
which could be used to inspect materials with complex geometrical profiles. The remaining part of this chapter describes the design and optimisation process for a custom-designed EC probe specifically optimised to enhance the capability of the EC-based inspections of the graphite bricks in an AGR core.

4.4.1 Theoretical Background of Asymmetric Gradiometer Sensor

An asymmetric gradiometer sensor is a type of magnetic field sensor that measures the magnetic field gradient generated from external source in the form of differential output voltages between the pickup (PC) and backing-off (BC) coils connected in series opposition. Assuming a general case where external magnetic field generated from a coil excited by an alternating current (primary field source), and two axial receiver coils (Pickup and Backing-off coils) connected in series opposition (i.e. first order asymmetric gradiometer as shown in Figure 4.4b), then the magnetic flux (\(\phi\)) passing through each coil can be represented by the surface integral of the flux density (\(B\)) over the coil area.

\[
\phi_{pc} = \int_{S_{pc}} B_{pc} \cdot dS \\
\phi_{bc} = \int_{S_{bc}} B_{bc} \cdot dS
\]  
(Eq 4.59  Eq 4.60)

where \(B_{pc}\) and \(B_{bc}\) are the flux densities in the PC and BC coils; \(\phi_{pc}\) and \(\phi_{bc}\) are the flux lines passing through the PC and BC coils; and \(S_{pc}\) and \(S_{bc}\) are the surface area enclosed by the coils respectively. Faraday’s law states that a time-varying flux linking two coils induces voltages in the coils proportional to the rate of change of the flux. Hence, the induced voltage in the PC and BC coils can be represented as in equation 4.61 and 4.62.

\[
V_{pc}(t) = -N_{pc} \frac{d}{dt} \phi_{pc} \\
V_{bc}(t) = -N_{bc} \frac{d}{dt} \phi_{bc}
\]  
(Eq 4.61  Eq 4.62)
Therefore, rewriting equation 4.61 and 4.62 in terms of flux density yields:

\[ V_{pc}(t) = -N_{pc} \frac{d}{dt} \int_{S_{pc}} B_{pc} \cdot dS \]  
\[ V_{bc}(t) = -N_{bc} \frac{d}{dt} \int_{S_{bc}} B_{bc} \cdot dS \]  

(Eq 4.63)

(Eq 4.64)

where \( V_{pc} \) and \( V_{bc} \) are the induced voltage in the PC and BC coils, and \( N_{pc} \) and \( N_{bc} \) are the number of turns in the PC and BC coils respectively. Since the PC and BC coils are connected in series opposition, the gradiometer output signal can then be expressed as the differential induced voltage between the two coils as in equation 4.65.

\[ V_{gra} = V_{pc}(t) - V_{bc}(t) \]  

(Eq 4.65)

In this case, the net signal received by the gradiometer due to the primary field is zero provided that the two coils are perfectly balanced. This is because in the induced complex voltage in the two coils will have equal magnitude and opposite polarity resulting in cancellation. This phenomenon is known as magnetic balancing.

In the presence of the primary field, placing the gradiometer near to an electrically conducting object result in the generation of EC within the conductor. This in turn generates a secondary field, which has an opposing effect on the primary field. In this situation, the signal received by the PC coil closer to the sample will exhibit the highest change in both its amplitude and phase. The change in the received voltage by the pickup coil mainly depends upon the conductivity distribution within the volume of the conducting object. If the conducting object contains a defect within its volume then the path in which the induced ECs flow will be altered, resulting in a different amplitude and phase compared with the case where the conducting object contains no defect. In either case the received complex voltage in the pickup coil can be represented by the phasor diagram as shown in Figure 4.4a.
In the above phasor diagram $V_0$ is the induced voltage due to the primary field, $\delta V$ is the change caused by the opposing effect of the secondary field, and $\delta \theta$ is the phase angle between the primary and secondary fields. Since the two coils are assumed to be perfectly balanced the net voltage detected by the gradiometer is now the induced voltage due to the secondary field (represented by $\delta V$ in the phasor diagram). This is the simple mechanism by which a gradiometer cancels the background signal, and as a result it offers a reasonable signal-to-noise ratio in applications where the EC signals are much smaller than the background signals.

4.5 Review of Eddy Current Based AGR Core Inspection

As discussed in Chapters 2 and 3, the AGR operators and their research partners have developed various systems and techniques to inspect the condition of the graphite bricks in the reactor core [6,7, 20, 48, 50, 89, 90]. Some of these include: a visual in-core inspection, individual fuel channel bore measurement and EC-based NDT systems.

The first EC systems for AGR core inspections were carried out four decade ago. These preliminary studies were reviewed by Fletcher et al [6], and for completeness the reference for each study are provided here [86-88]. According to the review by [6], the studies were mainly concerned with the feasibility of the EC method for inspection of possible graphite key-way root cracks and used relatively large probe, which was designed to be co-axial with the graphite brick. The main limitation of this system was that the probe was not able to localise the radial defect within the graphite due its co-axial configuration, although surface and sub-surface
graphite defects were differentiated. This review also identified that further studies by [88] showed a better detection capability using a new 70 mm axial probe by detecting a graphite slot that was 11% of the graphite through-wall extent. 

Bloodworth et al. [20] has further developed a complete EC inspection system along with a 70 mm axial gradiometer probe, which is now being used for routine AGR core inspections.

Feasibility studies of multi-frequency EC system to profile the graphite radial conductivity were carried out in the past [6-7]. Dekdouk et al. [7] adopted an optimisation technique to carry out a feasibility study of the graphite conductivity reconstruction problem using a custom-designed EC probe. Although this study was made on flat graphite sections, it provided confidence for further development and refinement of this technique. A study concerned with the detectability of graphite sub-surface defects was also carried out by Fletcher et al. [50]. The work in [50] has investigated the effects of different key-way originated slots on the response of a symmetrical gradiometer sensor, and proposed a novel technique by which the positions and the sizes of the slots could be estimated. Later this study was extended to reconstruct the electrical conductivity profiles of a cylindrical graphite bricks [6], demonstrating the practical feasibility of the optimisation method in estimating the graphite conductivity as a function of radial depth.

Currently, the AGR operators use the EC inspection system discussed in [20] during the routine inspections of the core, and these probes provide valuable information about the reactor core. However, as these probes were initially designed for near-surface defect detection and graphite bulk density mapping, they may lack the sub-surface sensitivity at greater depths required for reconstruction of the through-wall electrical conductivity profile of individual graphite bricks. To reconstruct the graphite conductivity profile accurately, the probes must be sensitive to conductivity through the entire radial thickness of the graphite brick. Furthermore, increased probe sensitivity throughout the entire radial extent of the graphite brick also increases the likelihood of detecting closed sub-surface cracks. This is particularly useful when inspecting for key-way root cracks, which are normally expected to initiate from the brick key-way corners and to grow inwards towards the bore. Although the probes discussed above operate with the required
sensitivities for the applications for which they were initially designed, each suffers from constraints that might affect the maximum sensitivity they can achieve. In particular, if one tries to optimise some of the sensor parameters to enhance the sensitivity at larger depths or increase the probe sensitivity with depth. For instance, increasing the probe diameter in [6-7,20] would in principle increase its sensitivity to changes in graphite properties, but at the same time the curvature of the graphite brick would constrain the amount by which the sensor diameter could be increased, and hence the improvement in the sensitivity will be limited. In the case of the sensor designed by Dekdouk et al [7], the diameter of the exciter coil is too large to be used for inspection of a realistic AGR graphite brick unless the lift-off distance is increased. Increasing the lift-off distance would degrade the probe sensitivity, as the probe is essentially moved away from the graphite bore.

The following section presents an attempt made in this thesis to overcome the existing EC probe limitations with a design of custom EC sensor optimised for AGR core inspection.

4.6 Sensor Design

The gradiometer probe gives a greater advantage when it comes to cancelation of the background signals on the EC measurements. The removal of the background signal is generally achieved by balancing the PC and BC coils that are normally located on the opposite sides of an exciter coil. The new EC sensor operates on the same principle as the conventional gradiometer sensors, but in this case the background signals are cancelled by balancing the PC and BC coils using the coil’s turn-ratio.

4.6.1 Sensor Description

In order to increase the depth sensitivity the probe needs to be as large as possible, and the lift-off distance must be kept to minimum. Given the geometry of the PECIT and the graphite brick, an increase in the probe size whilst maintaining constant lift-off can only achieved when the probe is designed with a curved profile.
Furthermore, the stainless steel casing of the PECIT has ovular aperture, which suggest that non-circular coil is best suited. This consideration leads us to the development of a novel sensor configuration consisting of three curved elliptical coils. The probe is configured as an asymmetrical gradiometer probe, consisting of PC, exciter (Tx1) and BC coils. The PC coil serves to detect the EC signal whereas the BC is responsible for cancelling the background field. The difference in sizes between the PC and BC coils was compensated for by using a 1:6 turn-ratio, and connected in series opposition to null out the background field. The Tx1 coil is configured to be enclosed within the PC coil at an identical lift-off from the graphite bore, and curved to adopt the graphite geometry and maintain constant lift-off from the bore. These in turn allowed an increase in the overall probe size and improved the probe’s sensitivity with depth.

Figure 4.5: The arrangement of the new custom designed asymmetric gradiometer sensor. (a) cross-sectional view of the sensor along with inspection tool components and graphite brick (b) the EC sensor and the tool side view, (c) the EC sensor and the tool top view.
<table>
<thead>
<tr>
<th></th>
<th>Mean size (mm)</th>
<th>Mean Lift-off (mm)</th>
<th>Mean Separation (mm)</th>
<th>Wire Diameter (mm)</th>
<th>Number of turns</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>Z=35, X=30</td>
<td>25.2</td>
<td>17.7</td>
<td>0.315</td>
<td>840</td>
</tr>
<tr>
<td>PC</td>
<td>Z=75, X=60</td>
<td>7.5</td>
<td></td>
<td>0.315</td>
<td>140</td>
</tr>
<tr>
<td>Tx1</td>
<td>Z=70, X=55</td>
<td>7.5</td>
<td></td>
<td>0.5</td>
<td>92</td>
</tr>
</tbody>
</table>

Table 4.1: The specification for the new eddy current sensor. Note: the Z and X in the above table represent the major and minor sizes of the elliptically configured coils.

Table 4.1 shows the mean values of the proposed probe specifications, whereas Figure 4.5 shows the general arrangement of the probe and the PECIT. It should be noted that the letters Z and X in Table 4.1 represent the major and minor sizes of the elliptically configured coils. The complete drawing of the proposed prototype sensor former can be found in Appendix C.

### 4.6.2 Design Specification

The main considerations and specifications used as a guideline for the new sensor design are listed below:

- The sensor must achieve an increase in overall sensitivity for the given AGR brick, and significantly better sensitivity than the existing gradiometer sensor.
- The sensor dimensions must be maximum 120 mm by 150 mm to fit within the PECIT/ECIT aperture.
- The sensor must not be adversely affected by the PECIT/ECIT stainless steel casing.
- Suitable material for the sensor former must be considered.
- The capacitance of the umbilical cable must be considered when selecting the number of turns, to limit the resonance effect.
- The overall speed of the measurement system must be considered, as the EC inspection is given two hours slot during the outage.
It should be noted that the overall speed of the measurement system is mostly related to that of the complete inspection system rather than the EC probe. Therefore, it is not considered during design of the EC probe.

**4.6.3 Two-Dimensional Axisymmetric Model**

The preliminary work on the sensor was to develop a simplified axisymmetric FE model to determine whether the proposed sensor gave the expected results, particularly an increase in sensitivity with increasing coil diameter. The two-dimensional axisymmetric model was mainly used to speed up the process of obtaining a representative range of model parameters. In addition, the model was used to study the stainless steel casing effects on the probe sensitivity. In this initial study, an absolute probe was used instead of full set of gradiometer sensor by assuming that the behaviour of a single excite/receive sensor was representative of a typical gradiometer sensor in terms of the change in coil sensitivity due to a change in coil parameters.

During the implementation of the FE model the (≈100 mm) graphite were represented by ten rectangular plates (each 400 mm long and 10 mm high). The sensor was placed above the graphite plates (at 7.5 mm lift-off distance). The PECIT casing in this model was represented by two stainless steel sheets with a separation equal to the PECIT casing inner diameter (220 mm) and a 5 mm thickness.

![Figure 4.6: 2D axisymmetric model for the layered graphite plates and the approximated PECIT casing](image)
The model was run using 10 Hz frequency to simulate the lowest inspection frequency. The probe sensitivity was calculated using the E-field calculated within the graphite volume (each graphite sections), as the coil radius varied between 15 mm and 130 mm. Two different cases were simulated, one containing the stainless steel sheets and the other without it. The sensitivity values calculated from the two simulations were then compared to see the effects of the PECIT casing on the probe sensitivity. Figure 4.6 shows a screenshot for the 2D axisymmetric model arrangement.

4.6.4 Results and Discussion

Figures 4.7a and 4.7b show the calculated sensitivity as a function of coil radius and depth from the top of the graphite plates when the model was run with and without the stainless steel casing, whereas Figures 4.7c shows the difference insensitivity between the two cases. These results show two important features, one of which is the increased sensitivity with increasing coil radius. The increased sensitivity with increasing coil radius is to be expected as the field distribution expands with increasing coil radius; in fact one of the main reasons for implementing the 2D axisymmetric model was to prove this concept while at the same time assessing the interaction between electromagnetic fields and the stainless steel casing with increasing probe diameter.

The other important feature of the 2D modelling result is the effect of the casing on the sensitivity of the sensor. This effect is not very recognisable from the first two plots due to the small difference between the two data sets. However, looking into the difference between the two datasets (Figure 4.7c) clearly shows increased casing effect with increasing coil radius. This is what should be expected in practice. As the radius of the probe increases, the closer it becomes to the casing and the more the casing interacts with both primary and secondary fields resulting in reduced sensitivity of the sensor to the given sensor parameter.
Figure 4.7: The sensitivity for the absolute probe as a function of coil radius and distance from the top of the graphite plates at 10 Hz. (a) With casing, (b) Without casing and (c) The difference in sensitivity between (a) and (b).
4.6.5 Three-Dimensional Model

After proving the design concept and determining the stainless steel casing impact on the probe sensitivity, the next stage was to implement a more realistic graphite brick model with an actual coil arrangement. A pre-design discussion with EDF Energy indicated that the maximum allowable sensor dimension that could fit onto the existing PECIT/ECIT was approximately 120 mm by 150 mm. Therefore, the maximum sensor radius was initially limited to 60 mm (in the case of circular coils). Later the design was modified and the coils were modelled with an elliptical shape (with major and minor sizes in Z and X axis) to fully utilise the available space on the PECIT/ECIT head. These configurations of the sensor have improved the overall sensitivity due to the increased coil area. The elliptic coils were curved on the sensor design to adapt to the curvature of the graphite bricks. This feature of the sensor has given the advantage of maintaining constant lift-off across the entire sensor region. The mean pickup (PC) and the exciter (Tx1) coil sizes were then fixed to 75 mm by 60 mm and 70 mm by 55 mm respectively, leaving a 5 mm gap between the two. The backing-of (BC) coil parameters were then varied to try and find the optimum value that gave the highest possible depth sensitivity. The parameters varied during the optimisation process include: separation distance, coil radius, and the number of coil turns (with an integer turn-ratio between PC and BC coils).

Since the EC problem involves time-harmonic signals, the sensitivity study of the EC probes considered in this Chapter was performed using the frequency domain solver and the magnetic field formulation available in the AC/DC module within the COMSOL Multiphysics version 5.0 software package. The cylindrical graphite domain in the model (Figure 4.8) was discretised into 10 annular layers each with 9.62 mm thickness, and a separate Ampère's law node was assigned to each. This has allowed the computation of the sensitivity values independently for each layer throughout the modelling process.

The edge element technique discussed in Section 4.2.1 was used to formulate the problem. The multi-turn winding of the real probe was approximated as a filament for all three coils assuming uniform current density over the cross-section of the physical coils, and positioned at the location where it would be in the PECIT/ECIT
if the probe were to be fitted in the tool. In order to reduce the computational time, the model was simulated using the one-quarter symmetry of the brick and sensor geometry.

![Heysham 1 brick without Methane holes and part of the PECIT model](image)

![HPB brick without Methane holes and part of the PECIT model](image)

**Figure 4.8:** Screen shoot of the two different model geometries used to simulate the AGR bricks during the sensor optimisation process.

The differential output voltages between the PC and BC coils were then calculated using equation 4.66 after injecting Tx1 with a unit current and performing a parametric sweep of frequency over the range between 10 Hz and 100 kHz with 5 points per decade, giving a total of 21 complex data points over the five frequency decades:

\[
V_{PC} - V_{BC} = j\omega \oint A_{PC} \cdot dl - k j\omega \oint A_{BC} \cdot dl
\]

(Eq 4.66)

where \(k\) is the scaling factor of the detection coil turn-ratio (1:6), and \(V_{PC}\) and \(V_{BC}\) are the induced complex voltages in the PC and BC respectively.

The probe sensitivity was calculated using the E.E method discussed in Section 4.3. The electric field \(E_{Tx}\) were extracted at predefined points within each discrete graphite layer after injecting Tx1 with a unit current and solving a forward model. Similarly, the electric field matrix \(E_{Rx}\) was extracted using the same technique but in this case injecting the PC with a unit current and the BC with a
current that had opposite polarity and adjusted for the turn-ratio (see Section 4.3).

Once the values of the electric fields were evaluated, the sensitivity matrix was then simply computed using equation 4.51, and integrated over each graphite layer with respect to each frequency point as illustrated in equation 4.67 below.

\[
\mathbf{J}^{m \times n} = \nu_L \begin{bmatrix}
\sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_1 f_1} & \sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_2 f_1} & \ldots & \sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_n f_1} \\
\sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_1 f_2} & \sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_2 f_2} & \ldots & \sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_n f_2} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_1 f_m} & \sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_2 f_m} & \ldots & \sum_{i=1}^p (\mathbf{E}_{TX} \cdot \mathbf{E}_{RX})_{\sigma_n f_m}
\end{bmatrix}
\]

(Eq 4.67)

In the above equation, each element of the single column sensitivity matrix \( \mathbf{J} \) corresponds to the integrated value of the dot products of the field vectors \( \mathbf{E}_{TX} \cdot \mathbf{E}_{RX} \) over the graphite layer volume \( \nu_L \), which has a conductivity \( \sigma_n \) for all frequency points \( f_m \); where \( p \) is the number of points at which the electric fields are evaluated. It should be noted that equation 4.67 is equivalent to the partial derivatives of the voltages with respect to the conductivities of each graphite layer.

The two different brick geometries shown in Figure 4.8 were used during the optimisation process, each containing the PECIT/ECIT casing. The first model has brick geometry identical to Heysham 1 bricks, whereas the second model is created using the HPB brick design, but in both cases the methane holes were omitted from the model to ease the computational demand. In addition, two different types of coil arrangement were used on the models. The first type consisted of a curved elliptical configuration for all three coils, whereas the second model contained a straight elliptical configuration for the BC coil and curved elliptical coils for the PC and Tx1 coils. The second configuration was mainly used to ease the difficulty associated with the practical implementation (winding) of the BC coil, as it would have a much larger number of turns than the other two coils.

### 4.6.6 Results and Discussion

Table 4.2 and Figure 4.9 show some of the simulated coil parameter combinations and the resultant mutual inductance responses after fixing the PC and Tx1 sizes to the maximum values where the effects of the PECIT casing is negligible.
These results show several features that assisted the developments of the new probe. The first is that the probe sensitivity mainly depend upon the size of the PC and Tx1, and since these two parameters are fixed to the maximum values (based on constraint imposed by the tool and the brick geometry), variation in the BC coil parameters only introduced fractional change in sensitivity at the maximum depth (≈87.4 mm) from the graphite bore. Nevertheless, any gain in sensitivity should be considered as an improvement, and hence these fractional changes in sensitivity were taken into account during the optimisation process.

Secondly, an increase in the separation distance with some fixed coil size reduces the probe sensitivity whilst increasing the balancing turn-ratio. From a practical point of view having a coil with a large turn-ratio is not the ideal choice, as it would limit the range of the frequencies by which the probe can be operated (as it would increase the coil stray capacitance). For this reason, the combinations of coil parameters that resulted in a large turn-ratio (i.e. case 3 and 4) were ruled out from the selection of the optimum probe parameters, although the parameters in case 3 show reasonable response. Increasing the BC coil size at some fixed separation distance, could reduce the balancing turn-ratio, but this leads to increased interaction between the fields and the PECIT casing, which results in reduced mutual inductance response gain. This can be clearly seen in figure 4.9 where probe’s mutual inductance exhibits significant reduction for the cases (i.e. case 6, 7, 8 and 9) where the BC coil contains slightly larger size and separation distance.

These results also show that the highest sensitivity at the brick periphery is achieved for parameters in case 5 (≈ 8 % of the maximum sensitivity) and in case 10 (≈ 8.1% of the maximum sensitivity). In both cases the BC coil was set to 35 mm by 30 mm for the major and minor sizes respectively, but in case 10 the BC is left straight for the reasons mentioned earlier. A closer look into case 5 and case 10 also shows that the change in the BC coil configuration from curved to straight is compensated by increasing the initial separation distance by approximately 2.01 mm. This change was mainly made to maintain a constant turn-ratio between the BC and PC coils.

In general, the coil parameters are dependent between one another, meaning that changing one will requires a change to other in order to maintain
balance between the BC and PC coils. For instance, increasing the BC separation distance whilst maintaining fixed coil size results in increased coil turn-ratio and vice versa. This also applies to a change in coil size whilst maintaining fixed separation distance. Therefore, in order to achieve the optimum sensor parameters all four variables of the BC coils need to be optimised simultaneously through successive FE simulations, which is what it has been done throughout the optimisation process.

The asymmetrical gradiometer with circular configuration (case 2) was simulated with the maximum of 72 mm diameter for the PC, mainly due to the mechanical constraints of the brick geometry and the PECIT. In this configuration, the probe sensitivity to the graphite brick at a depth approximately 87.4 mm was only twice of the existing probe. Therefore, on the bases of probe sensitivity, the turn-ratio and consideration of practical implementation the sensor parameters in the case 10 were chosen as optimum parameters and a prototype have been developed for further test (see Section 4.7).

<table>
<thead>
<tr>
<th>Simulated cases</th>
<th>BC mean size (mm)</th>
<th>PC mean size (mm)</th>
<th>Tx1 mean size (mm)</th>
<th>Mean Lift-off (mm)</th>
<th>Mean Separation (mm)</th>
<th>Sensitivity at the Periphery</th>
<th>Turn Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 (Existing sensor)</td>
<td>25.75</td>
<td>25.75</td>
<td>28.5</td>
<td>PC=7.5</td>
<td>Tx1=13.5</td>
<td>BC=6</td>
<td>1.64%</td>
</tr>
<tr>
<td>Case 2 (circular)</td>
<td>18</td>
<td>36</td>
<td>27</td>
<td>7.5</td>
<td>4.5</td>
<td>3.54%</td>
<td>2:1</td>
</tr>
<tr>
<td>Case 3 (elliptic curved)</td>
<td>Z=25</td>
<td>Z=75</td>
<td>Z=70</td>
<td>7.5</td>
<td>21.35</td>
<td>7.99%</td>
<td>14:1</td>
</tr>
<tr>
<td>Case 4 (elliptic curved)</td>
<td>X=20</td>
<td>X=60</td>
<td>X=55</td>
<td>7.5</td>
<td>25.93</td>
<td>7.9%</td>
<td>15:1</td>
</tr>
<tr>
<td>Case 5 (elliptic curved)</td>
<td>Z=35</td>
<td>Z=75</td>
<td>Z=70</td>
<td>7.5</td>
<td>15.645</td>
<td>8.0%</td>
<td>6:1</td>
</tr>
<tr>
<td>Case 6 (elliptic curved)</td>
<td>X=30</td>
<td>X=60</td>
<td>X=55</td>
<td>7.5</td>
<td>24.762</td>
<td>7.85%</td>
<td>7:1</td>
</tr>
<tr>
<td>Case 7 (elliptic curved)</td>
<td>Z=35</td>
<td>Z=75</td>
<td>Z=70</td>
<td>7.5</td>
<td>30.77</td>
<td>7.77%</td>
<td>8:1</td>
</tr>
<tr>
<td>Case 8 (elliptic curved)</td>
<td>X=30</td>
<td>X=60</td>
<td>X=55</td>
<td>7.5</td>
<td>22.262</td>
<td>7.83%</td>
<td>5:1</td>
</tr>
<tr>
<td>Case 9 (elliptic curved)</td>
<td>Z=40</td>
<td>Z=75</td>
<td>Z=70</td>
<td>7.5</td>
<td>29.7122</td>
<td>7.69%</td>
<td>6:1</td>
</tr>
<tr>
<td>Case 10 (elliptic, straight BC, and curved PC and Tx1)</td>
<td>Z=35</td>
<td>Z=75</td>
<td>Z=70</td>
<td>7.5</td>
<td>17.7</td>
<td>8.1%</td>
<td>6:1</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of the selected parameter combinations, which were tested during the optimisation process and the associated sensitivity of each. Note: the complete drawing for the existing and the proposed sensor can be found in Appendix C and D.
Figure 4.10 shows normalised sensitivity plots for the existing gradiometer sensor (case1), a circular asymmetric gradiometer (case 2) and for the two optimum sensor parameters (case 5 and 10), while Figure 4.11 shows the effect of the casing on the finalised sensor parameters (case 10) as a function of the graphite brick radial dimension. It should be noted that the sensitivity values plotted in Figure 4.10 are normalised to the sensitivity value calculated in the graphite layer closest to the probes for each configuration (the first layer from the bore), such that, $S_{\sigma, \text{norm}} = S_{\sigma,n}/S_{\sigma,1}$, where $S_{\sigma, \text{norm}}$ is the normalised sensitivity value, and $S_{\sigma,n}$ and $S_{\sigma,1}$ are the $n^{th}$ and $1^{st}$ graphite layer respectively. This allowed us to visualise the reductions of the probe sensitivity as a function of depth from the measurement boundary, and helped us to compare the probe performance calculated using different probe parameters.
The result in Figure 4.10 clearly shows the advantage of the new sensor configuration over the existing sensor in terms of coil sensitivity. The existing sensor’s sensitivity falls to 13.51% of its maximum value at 32.41 mm away from the brick bore whereas the sensitivity for the new sensor falls to 35.46% of its maximum value at the same distance away from the brick bore giving 21.95% improvement in sensitivity over the existing sensor (Figure 4.10). Furthermore, the sensitivities for each sensor at the brick periphery are: 1.64% and 8.1% for the existing (case 1) and the new sensor (case 10) respectively giving around a five-fold improvement in sensitivity at the maximum depth from the graphite bore (around the brick edge). It
is also clear to see from Figure 4.11 that the effect of the casing on the sensitivity of the new sensor is insignificant. The change in sensitivity due to the presence of the casing accounts for much less than 1% of the maximum sensitivity, so in principle this should not affect sensor response for a given graphite brick.

Figure 4.12 shows the normalised sensitivity values of the new and the existing sensors plotted as a function of depth at four different frequencies. For the lowest operational frequency considered in this thesis, the new EC sensor offers an average of 43 % improvement in sensitivity. This shows the direct effect of the magnitude of the EC density generated by a larger Tx1 and the ability of the PC to capture a greater flux due to its increased coil diameter and curved geometry.

![Figure 4.12: The normalised probe sensitivities of the new and existing EC probes.](image)

This is one of the most important features of the new probe, which may improve characterisation of the graphite resistivity profiles and detection of sub-surface cracks located away from the graphite bore.

When the probe operational frequency increases the difference in sensitivity reduction rate between the two sensors was reduced. This is because at higher
frequencies the EC tend to concentrate around the graphite bore surface, and hence both sensors tend to exhibit comparable sensitivity changes with depth.

4.7 Sensor Construction and Experimental Validation

After selecting the optimum sensor parameters the next stage was to validate the simulation results through experimental work. A prototype former was designed using the parameters extracted from the modelling work and 3D-printed. The former for each winding was printed separately to ease the winding process and to correct any mis-balance caused by the mechanical structure of the elliptical coils. The coils were wound in such a way that they closely aligned with the curvature of the sensor former. This was achieved using adhesives to attach the wires to the former following each turn.

4.7.1 Preliminary Test in Free Space

As explained in Section 3.4, AGR core inspections are made from the pile cap at the top of the core by lowering the inspection tool into the fuel channels. Therefore, it was necessary to take into account the capacitive effect of the long umbilical cable (≈ 60 metre) on the quality of the measured signal as well as to test the frequency response of the probe itself.

During the experiment, each coil was first connected to a Solartron 1260 Impedance Analyser. A current was injected into the coils, the complex impedance response of each coil was measured in air space multiple times for frequencies ranging between 10 Hz to 100 kHz and averaged over each frequency point. The experiment was repeated again after connecting 3.3 nF capacitors across the PC and BC coil terminals to represent the capacitive effect of the umbilical cable.

4.7.2 Result and Discussion

The plots in Figure 4.13 show the impedance magnitude of both the BC and PC coils, and the differential output of the gradiometer probe with and without the
shunt capacitance connected on the coil terminals. These results show reductions in the frequency resonance when a shunt capacitor is connected across the coil terminals. This was expected as the coils overall stray capacitance is increased by 3.3 nF.

Figure 4.13: Plots showing the resonance frequencies of the backing-off coil, pickup coil and the differential output calculated using the mean value of each response.
The BC coil showed an approximate resonance frequency of 50 kHz before the shunt capacitance was connected to it. Similarly, the PC starts to resonate at approximately 63 kHz prior to connecting its terminal to 3.3 nF shunt capacitor. Connecting the coils with the shunt 3.3 nF capacitor (to represent the PECIT umbilical cable) reduces the resonance frequencies of the BC and PC coils to 12 kHz and 44.7 kHz respectively. The gradiometer differential output resonance frequency was reduced to approximately 12 kHz when a shunt capacitor is connected between the receiver coils.

The volumetric graphite inspections are normally made using relatively low frequencies, up to 10 kHz, and hence the quality of the data that can be collected below the resonance frequency of the new probe is sufficient to gather relevant information to assess the graphite radial conductivity variations and key-way root sub-surface cracking. Therefore, the results from the above test gave confidence on the chosen coil’s number of turns. In cases where the requirements of higher frequencies test arise in the future, then the resonance frequency can be increased with reduction of the number of turns, whilst maintaining the specified turn-ratio.

### 4.7.3 Experiment using Drilled Graphite Brick Sample

A reference brick for the AGR with a known electrical conductivity distribution was not available at the time of this experiment. Therefore, it was necessary to validate the new sensor using the existing graphite sample that was used on the previous work [6]. The graphite sample has semi-cylindrical geometry with 250 mm height, and contains radially drilled holes. The drilled holes are 4 mm in diameter, and separated by 14.3 mm and 23.83 mm in the bore and periphery regions respectively (Figure 4.14a). The holes across the radial thickness of the brick were drilled to qualitatively replicate the density variation nature of a real oxidised AGR brick, so that the density at the bore region was lowest (holes closest together) and increased towards the periphery. Consequently, the through-wall electrical conductivity of this sample would be expected to vary between the bore and periphery in an approximately linear fashion. Fletcher et. al [6] has approximated the bore and periphery conductivities of the drilled graphite brick experimentally as 79.3
kS/m and 102 kS/m respectively. Based on these estimates a model was created by increasing the conductivity value of each graphite model layer from bore to periphery by an increment of ≈ 2.52 kS/m up to 102 kS/m so that the model conductivity replicates the physical graphite brick through-wall electrical conductivities allowing a direct comparison between the model and measured data (Figure 4.1).

During this experiment, the prototype probe was aligned within the graphite bore using non-conducting supports situated at 125 mm from the top and bottom of the brick. The gradiometer input and output were then connected to a Solartron 1260 Impedance Analyser. A current was injected into the excite coil, and the probe complex inductance response was measured multiple times giving multiple sets of complex inductance data for frequencies ranging between 10 Hz and 10 kHz with 5 points per decade, in accord with the FE model.

Similarly, the probe was placed in air and the same set of measurements was repeated. The measured data were then averaged over each frequency point to minimise random measurement errors. Finally, the free space data were subtracted from the graphite data to eliminate any residual background noise. Finally, the simulated data were then scaled with the practical coil number of turns assuming uniform current density over the cross-section of the prototype probe, and with the experimental data as shown in Figure 4.15.

Figure 4. 14: Experimental setup and the FE model geometry of the graphite brick used for sensor design validation. (a) Experimental setup and (b) FE model geometry of the graphite brick.
4.7.4 Result and Discussion

Figure 4.15 shows the real and imaginary parts of the mutual inductance response of the probe for both model and experimental data. The frequency points in Figure 4.15 were deliberately kept below to 10 kHz to allow a direct comparison between the model and the experimental data prior to the occurrence of resonance. It is clear to see that the model and experimental curves agree reasonably well with each other, except for the small differences seen at the high end of the frequency range.

These differences may be attributed to the approximation of the multi-turn coils, which have a finite cross section, by filaments in the FE model. Therefore, the model assumes that the experimental probe exhibits a uniform current density over its cross-section, which becomes less accurate at higher frequencies due to the skin effect. A possible correction for this is to calibrate the model and the measurement system to take into account the differences between the model and physical probe.
parameters. Alternatively, the probe could be modelled using cylindrical geometries, but with the penalty of increased computational cost as this technique requires a finer mesh within the coil volume.

4.8 Summary

This chapter has presented the fundamentals of the EC-based NDT systems with reference to both practical application and the numerical implementations. The design, optimisation and experimental validation of a novel EC probe designed for inspection of the fuel channel brick in AGR core have been described in detail. From the studies presented in this chapter the following conclusions can be made.

- The sensitivity of the asymmetric gradiometer sensor depends mainly upon the parameters of the exciter and pickup coils rather than the backing-off coil
- For asymmetrically configured sensor, the coil parameters are dependent between one another, meaning that changing one will requires a change to other in order to maintain balance between the pickup and backing-off coils
- The effects of the PECIT stainless steel casing on the gradiometer response increase with increasing coil diameter
- At the lowest operational frequency considered in this chapter the new sensor showed 43% improvement in sensitivity, and reduces with increasing frequency.
- The modelling approach in this chapter agrees with the experimental data, and hence can be used as a forward model for reconstruction problem considered in the following chapter.
Chapter 5

5 Review of Inverse Eddy Current Methods

Following the discussion in Chapter 4 it is now clear that EC-based object characterisation and defect detection systems can be used in various sectors including: transport, energy and other industrial applications [91, 92-96]. One of these object characterisation systems is known as “Magnetic Induction Tomography (MIT)” which can be implemented to create an image showing an internal property of an object from EC measurements taken around its boundary [49, 69, 97-100]. Another closely related system that can be used to inspect materials within industrial applications, and also relevant to the work presented in this thesis, is known as Inductance Spectroscopy (IS) [7, 50, 54, 57, 79, 101]. As discussed in Chapter 4, the EC-based NDT system uses coils and AC signals to generate EC within a material of interest. Variations in material properties or the presence of defects in the material alter the flow of the EC, which leads to a change in the amplitude and phase of the detected signal. This information can be used to profile the material internal properties (such as electrical conductivity and magnetic permeability) or to locate a defect within the material under test.

The inspection system implemented in this thesis is based on the principles of the IS system. The IS system uses the frequency dependence of the probe response due to the object being inspected, in contrast the closely related MIT system uses multiple sensor arrays located around the test object, and often operated in a single frequency mode. In the case of the work in this thesis, the EC probe is placed at a fixed location within the graphite brick bore, and measurements are obtained using multi-frequency excitations to measure the variations in the graphite conductivity as a function of depth.

The way in which the EC signals are processed to determine the material internal property depends upon the type and complexity of the inspection system. For instance, to identify the location of a defect inside the volume of an object, the impedance plane technique or other empirical techniques such as the method presented by Fletcher et. al [50] can be used. For a more complex inspection system
including the identification of defects with small sizes and material property characterisation (in this case graphite brick conductivity profiling), one needs to solve a series of both forward problems (which are typically FE predictions, see for instance Section 4.3) and inverse problems to recover the material internal properties from the measured data [7, 53, 60, 70, 102-107]. Equations 5.1 and 5.2 show simplified mathematical representations of the forward and inverse problems respectively.

\[ V = F(\sigma) \]  
\[ \sigma = F^{-1}(V) \]

(Eq 5.1)  
(Eq 5.2)

where: \( V \) is the voltage at the boundary, \( \sigma \) is the conductivity distribution within the object, and \( F \) and \( F^{-1} \) are the forward and the inverse operators respectively. The forward problem in equation 5.1 is a straightforward problem to solve provided all the required model inputs are available (i.e. brick geometry, location and parameters of the sensor, and excitation current and frequency). On the other hand, the inverse problem adds an additional complexity to the problem since it aims to recover material parameters from the measured values, and hence the governing forward equation needs to be inverted. In addition, the field distribution and the material conductivity are at the same time unknown. This along with electromagnetic skin-effect makes the problem highly non-linear and ill-posed. For these reasons, the EC inverse problem is often solved using iterative inversion algorithms. A detailed discussion of the forward problem, along with the approaches adopted to implement the forward models for the AGR graphite bricks were presented in Section 4.2. The following sections of this chapter focus more on the fundamental theory of inverse EC problems and the common mathematical tools used to analyse and solve non-linear and ill-posed inverse problems.
5.1 Theoretical Background

An inverse problem may be thought as a problem that seeks to determine a solution about an unknown cause based on the known information about the incidence. In more practical terms relevant to an EC-based inverse problem, the unknown cause can be related to the property of the material being reconstructed and the incidence to the boundary measurements obtained through an experiment. Therefore, the inverse EC problem in this work can be described as a mathematical model that seeks to estimate the conductivity profile of the graphite bricks based on the impedance measurements obtained at the boundary (Figure 5.1).

![Figure 5.1: A simplified block diagram that shows the relationship between the graphite forward and inverse problems.](image)

5.1.1 Overview of the Inverse Eddy Current Problem

The EC-based inverse problem is normally regarded as highly non-linear. The non-linearity is mainly caused by the scattered fields, which affect the measured signals on the receiver coils differently whenever there is a variation in electrical conductivity anywhere within the volume of interest and in the background. The problem may also be regarded as an ill-posed problem. By definition, a mathematical model of a practical system is said to be ill-posed if it does not fulfil all of the Hadamard conditions, that is: for all admissible data a solution must exist, the solution must be unique, and the solution must be stable or must depend continuously on the data [91]. The first two conditions can be achieved by imposing
some constraints on the solution; therefore the third condition is the one that has a dominant effect on the inverse EC solution. This is because the EC inside the conducting object decay in an approximately exponential fashion due to the skin effect, resulting in the Jacobian matrix becoming badly conditioned. As the inverse problem involves inversion of the Jacobian matrix, this will make the solution unstable and very sensitive to modelling error and measurement noise. Consequently, a small measurement or modelling error could result in a very large change in the reconstructed conductivity profile. Implementation of regularisation techniques that involve an additional penalty term and smoothing parameter to penalise the undesired variation on the solution usually mitigate this problem [108].

Various techniques exist for obtaining the inverse solution of a given EC problem, but the most popular and commonly used methods are based on deterministic approaches [7, 53, 56, 58, 108-111]. In a deterministic approach the objective is to find a model that best fits the experimental data. This typically involves the formulation of a discrete optimisation problem with the aim to minimising the difference between the model and the observed data in a least-square sense. This technique offers an advantage in terms of simplicity over probabilistic approaches that characterise the unknown solution as a set of random variables with a probability density to estimate the unknown parameter [112].

A large amount of work and effort has been made in the past by many researchers and academics to extend the capability of EC systems for industrial and medical applications. For instance, Norton et.al [82] presented the detailed theory of a non-linear EC inverse problem, which was based on a least-square reconstruction technique. This work demonstrated mathematically the capability of the least-square inversion method for use in industrial NDT applications. A related work presented by Bowler et.al [53] showed another example of the methods used to solve an inverse EC problem. In this study, the authors used a multi-frequency measurement system and an algorithm known as gradient descent to reconstruct the conductivity profile and thickness of a layered conductor. Nair and Rose [102] proposed another reconstruction technique for EC-based NDT of conducting material using a direct inversion method. In this method, the Born approximation was applied to linearize the system integral equations and solve the given reconstruction problem. Further
works related to inverse EC problems were presented by [49, 69-70, 97]. In these works, the authors implemented one of the first non-linear imaging systems (the MIT systems) to assess the production process in the steel manufacturing industry. Inverse EC problems in NDT applications use similar principles to the MIT system developed by [49, 69-70, 97]. The main difference between these systems is that inverse EC in NDT applications is mainly interested in recovering some form of discontinuity or variation in the properties within the material of interest, whereas the MIT system seeks to image the shape of an object often using a coil array located around the object, and usually operated in a single frequency mode. For these reasons, most of the methods used in the MIT system can be directly applied to NDT applications and vice versa.

Other authors also published further works in the context of NDT. These include the work carried out by Yin et. al [101], which used an IS system that operated in a multi-frequency excitation mode to reconstruct the permeability distribution of layered conductors. This work explored the capability of the Newton-Raphson method in recovering the unknown permeability distribution of the conductors. Specific to the graphite inverse problem, a feasibility study of graphite sample reconstruction was carried out by Dekdouk et. al [7]. In this work a similar excitation techniques, but different sensing system to that in [101] was used to obtain the relevant inductance data. The material of interest in this work was layered graphite plate which had a total thickness of 100 mm, and the aim was to recover the conductivity gradient of the sample as a function of depth using one step, semi-linear and non-linear Regularised Gauss Newton (RGN) inversion algorithms. Although this study was made on flat graphite sections, it provided confidence for further development and refinement of these techniques. Fletcher et. al [6] extended the initial study carried out by [7], to reconstruct a more realistic AGR graphite brick conductivity profile using the RGN algorithm and a similar probe configuration to that in [7, 20]. The work in [6] fully demonstrated the practical feasibility of the method in profiling the AGR brick electrical conductivity. Similar studies concerning inverse EC problems for other NDT applications have been published by many authors including the authors of [71, 79, 113, 68-82].
5.1.2 Optimisation Problem

Following the introduction section it is now clear that in order to solve the inverse EC problem one needs to transform the problem into an optimisation problem that continuously seeks to find a solution that minimises the given objective function (Equation 5.5). Two ways exist for formulating optimisation problems: discrete and continuous optimisation problems (see for example [128-129]). Mathematical models of EC inverse problems are usually formulated as discrete optimisation problems. One of the reasons for this is the non-locality of the EC problem. In other words, the inverse EC problems are normally affected by the scattered fields within the volume of the test material, meaning that the boundary measurements from the EC sensor depend upon the material parameters of the entire test object. This may lead to non-unique solutions as perturbations at different locations in the volume of interest could result in the same boundary measurements. For this reason it is often sensible to discretise the problem domain whilst making sure that the numbers of unknowns are smaller than the number of data points to prevent the system from being underdetermined. In this way, one can calculate every discrete unknown with respect to the residuals (the difference between the model and measured data) at every data point in the least-square sense. To do this, first the residuals between the forward model and measured data are arranged in the form of a residual vector $r(\sigma)$ and the minimisation problem is formulated as follows:

$$r(\sigma) = (r_1(\sigma), r_2(\sigma) \ldots \ldots r_m(\sigma))^T$$  \hspace{1cm} (Eq 5.3)

$$\sigma^* = \text{argmin}_\sigma \{g(\sigma)\}$$  \hspace{1cm} (Eq 5.4)

$$g(\sigma) = \frac{1}{2} \|r(\sigma)\|^2 = \frac{1}{2} \sum_{j=1}^{m} r_j^2(\sigma)$$  \hspace{1cm} (Eq 5.5)

where $\sigma^* \in \mathbb{R}^n$ are the discrete solutions for the unknown conductivity vector $(\sigma^* = (\sigma_1, \sigma_2, \sigma_3 \ldots \ldots \sigma_n)^T)$ that minimise the objective function $g(\sigma)$, and $r(\sigma) \in \mathbb{R}^m$ is the residual vector between the modelled and measured data.
The first-order partial derivatives of the residuals can then be used to express the Jacobian matrix of the problem, which maps the change in residual with respect to a change in the conductivity as shown in equation 5.6.

\[
\mathbf{J}(\sigma) = \begin{bmatrix}
\frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \cdots & \frac{\partial r_1}{\partial \sigma_n} \\
\frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \cdots & \frac{\partial r_2}{\partial \sigma_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial r_m}{\partial \sigma_1} & \frac{\partial r_m}{\partial \sigma_2} & \cdots & \frac{\partial r_m}{\partial \sigma_n}
\end{bmatrix}
\]  

(Eq 5.6)

where \( j \) is the index of a measurement point (\( j = 1,2,3 \ldots m \)), and \( i \) is the index of the discrete unknown conductivities (\( i = 1,2,3 \ldots n \)), making up an \( m \times n \) matrix. Representing the Jacobian matrix in terms of the gradients of the residuals then yields:

\[
\mathbf{J}(\sigma) = \begin{bmatrix}
\nabla r_1(\sigma)^T \\
\nabla r_2(\sigma)^T \\
\vdots \\
\nabla r_m(\sigma)^T
\end{bmatrix}
\]  

(Eq 5.7)

The term \( \nabla r_j(\sigma) \) in equation 5.7 or each row in equation 5.6 is the gradient of the residuals \( r_j \) for \( j = 1,2,3 \ldots m \).

The gradient and the Hessian of the objective function \( g(\sigma) \) required to solve the optimisation problem are normally obtained by computing the first and second order partial derivatives of \( g(\sigma) \) respectively. However, since it is possible to formulate the Jacobian by computing the first-order partial derivatives of the residuals, this information can be simply used to express the gradient and the Hessian in terms of the Jacobian as shown in equations 5.8 and 5.9 respectively.

\[
\nabla g(\sigma) = g(\sigma)' = \sum_{j=1}^m r_j(\sigma) \nabla r_j(\sigma)
\]

\[ = \mathbf{J}^T r(\sigma) \]  

(Eq 5.8)
\[ \nabla^2 g(\sigma) = g^{\prime\prime}(\sigma) = \sum_{j=1}^{m} r_j(\sigma) \nabla r_j(\sigma)^T + \sum_{j=1}^{m} r_j(\sigma) \nabla^2 r_j(\sigma) \]

\[ = J^T J + \sum_{j=1}^{m} r_j(\sigma) \nabla^2 r_j(\sigma) \]  

(Eq 5.9)

In the case of a non-linear inverse problem, which the EC inverse problem falls into, the problem is normally solved using an iterative scheme that generates a series of conductivity vectors \( \{\sigma_1, \sigma_2, \sigma_3, ..., \sigma^*\} \) that eventually minimise the objective function \( g(\sigma) \). The iterative solution updates during the iterations until the solution converges to the expected true profile. If the problem is formulated as a fully non-linear problem then the Jacobian may also need to be recomputed during the iterations. This also means that two forward models need to be evaluated per iteration (to compute the sensitivity matrix) depending upon the type of the algorithm used. This process in general comes at high computational cost and may take some time before the final solution is obtained, in particular for large optimisation problems.

The inverse problem in its simplest form can be formulated as a linearized discrete problem. The linearization is achieved by approximating the residual vector as shown in equation 5.10, where the residual is given as the difference between the approximated forward model and the measured data. However, this type of formulation is mostly used for problems that require the reconstruction of relatively simple conductivity distributions such as small conductivity variations in a given problem.

\[ r(\sigma) = J\sigma - V \]  

(Eq 5.10)

where \( J \) is the Jacobian matrix usually calculated using constant reference conductivity value, and \( V \) is the vector of the measured data.

The above approximation works fairly well for problems involving simple conductivity distributions, which was proven in the medical imaging work carried out by [130-132]. In this work, the authors used a single-step differential imaging method to reconstruct the shape of an object. The results from this work showed
reasonable reconstruction of the object’s shape but suffered from image blurring. From the NDT point of view the linearization technique works to some extent as demonstrated in [133], where the authors reconstructed the shapes and the directions of cracks in conducting material. In general, this method plays a vital role in the NDT applications where inspection speed is more important than the quality of the results.

Although the above method has been proven to give a reasonable solution for medical and some industrial applications, one of the main reasons that this method will not be suitable for the inverse problem considered in this thesis is due to the ill-posed nature of the graphite inverse problem. In other words, the Jacobian matrix for the graphite inverse problem is usually close to becoming singular and unstable against measurement noise and modelling error. These may result in the generation of poor conductivity profiles if this method is adopted to solve the problem considered in this thesis. Furthermore, the primary concern of this work is not the speed of the reconstruction algorithm, instead the accuracy of the reconstructed profiles are much more important. For this reason, more robust non-linear inversion techniques are adopted in this thesis (see Section 5.4).

5.1.3 Local and Global Minima of the Objective Function

It is apparent from Section 5.1.2 that the optimisation problem seeks to find a solution that minimizes the given objective function of the system or the conductivity vector that approaches the expected true profile \( \sigma \rightarrow \sigma^* \). However, the challenge in many optimisation problems is finding a global minimum. If the problem is parabolic, then the local minimum is also the global minimum of the solution, and a reasonable estimate to the exact solution can be obtained using the simple optimisation problem discussed above. However, in most practical systems such as the one related to inverse EC, the problem may contain multiple local minima (see for example Figure 5.2). In these situations there is high probability that the algorithm might become trapped within the local minima and never reaches the solution that minimises the objective function globally.
By definition the solution $\sigma^*$ is said to be a local minimum if there is a neighborhood $\mathcal{N}$ of $\sigma^*$ such that $g(\sigma^*) < g(\sigma)$ for $\sigma \in \mathcal{N}$ with $\sigma \neq \sigma^*$. On the other hand the solution is said to be a global minimum if $g(\sigma^*) < g(\sigma)$ for all values of $\sigma$. However, it is often difficult to identify whether the computed solution is local or global minimum. In some special cases where the objective function is parabolic and twice differentiable the identification process eases through examination of the gradient ($\nabla g(\sigma)$) and the Hessian ($\nabla^2 g(\sigma)$) of the objective function. The detailed description of global and local minima in the context of optimisation problem can be found in [129].

To address the issue associated with local minima on the graphite inverse problem, a constraining technique was applied within part of the reconstruction algorithm in this thesis (see Chapter 6). This technique was adapted with the aim of preventing the solution from becoming trapped in any local minima of the objective function whilst reducing the profile error with respect to the expected conductivity profile.

5.1.4 Constrained and Unconstrained Optimisation Problems

In most cases a practical optimisation problem contains some form of prior information to stabilise and allow the convergence of the solution to a level at which it can best describe the practical system. A priori information applied to the optimisation problem can take different forms depending upon the physical system,
the type of problem and the availability of a priori information. Once the type of a priori information is decided then it may be incorporated into the formulation of the problem in the form of constraints or penalty terms. A constrained optimisation problem normally includes explicit conditions on the variables which the solution must satisfy. The conditions forced on the problem could be upper and lower bounds, non-negativity, monotonicity or even takes a non-linear form such as quadratic constraints [129]. An example of a constrained inverse problem can be seen in equation 5.11 where the optimisation problem of a linearized system

\[ g(\sigma^*) = \frac{1}{2}\|J\sigma - V\|^2 + \frac{1}{2}\lambda\|L\sigma\|^2 \]

is formulated using two inequality constraints.

\[
\sigma^* = \arg \min_{\sigma} \{ g(\sigma) \} \quad \text{Subject to} \quad \begin{cases} 
\sigma \geq 0 \\
L\sigma \geq 0
\end{cases}
\]

(Eq 5.11)

where \(L\) is a matrix consisting monotonically arranged diagonal elements.

The two inequality constraints in equation 5.11 are known as non-negativity and monotonicity constraints respectively. These types of constraints normally arise when the inverse solution is expected to have non-negative and monotonically increasing or decreasing values. An example of this condition could be the conductivity values estimated from physically measured data using the drilled graphite brick (see Chapter 6).

The main advantage that an explicit constraint gives over an unconstrained problem is that it is able to force the solution towards a global minimum provided sufficient a priori information is transferred to the minimisation problem through the inequality constraint.

In a general unconstrained optimisation problem the solutions are normally unrestricted by the conditions which the algorithm must satisfy. In other words, the convergence of the solution near the global minimum is not guaranteed and may or may not represent the physical system. Another special form of unconstrained optimisation problem can be described as the reformulation of the constrained problem by means of replacing the explicit constraints with a penalty term. These constraints are normally incorporated within the objective function of the unconstrained problem.
5.2 Methods for Solving Eddy Current Inverse Problems

Various algorithms exist for solving both linear and non-linear inverse problems. The effectiveness of each method depends upon the type of the problem to be solved. For instance, in EC inverse problems, the most appropriate algorithm is based on non-linear methods due to the non-linearity and ill-posedness of the problem. As this work is a continuation of the preliminary studies carried out by [6-7], which reconstructed the electrical conductivity profiles of the graphite plate samples and a cylindrical brick as a function of depth from the measurement boundary, similar non-linear approaches are adopted here. The work carried out by [6] has also investigated different approaches by which the Jacobian matrix could be arranged throughout the reconstruction process. The outcome of this study suggested that using both real and imaginary parts of the Jacobian (the real and imaginary inductance data) gives better estimates of the graphite conductivity profiles. This conclusion is reasonable, since using both real and imaginary parts of the data gives more information (data points) about the graphite being inspected. In contrast, the work carried out by [7] estimated graphite plate conductivity profiles using only the real part of the impedance data (imaginary part of the inductance data). Both studies produced reasonable estimates of the true conductivity profiles under investigation, with the profile error ranging between 10% and 43% respectively. However, the profile of errors in [6] clearly shows that using both parts of the data have gave a better estimate of the true conductivity profile of the graphite brick.

Following the findings in [6], the Jacobian matrix in this thesis was constructed using both real and imaginary parts of the data. This method of constructing the Jacobian matrix allows all of the available inductance data to be utilised with the aim of increasing the accuracy of the inverted profiles.

An improvement in the sensitivity of the new EC probe over that of the existing gradiometer probe used by [6] means that the new sensor should in principle reconstruct more accurate profiles when used in conjunction with an appropriate reconstruction algorithm [48]. For this reason, the reconstruction work initially starts by extending the conventional RGN algorithm that was used by [6-7]
through the inclusion of constraints. The constraints in this work are set based the priori information of the expected conductivity profile. This process is followed by a development of a fully automatic non-linear algorithm known as Regularised Levenberg Marquardt algorithm (RLM), which is then used to invert real reactor data. The RLM algorithm has been used previously by other authors to reconstruct object shapes within EIT and MIT applications [71, 134-136]. However, to the author’s knowledge this is the first time that this algorithm has been formulated to reconstruct graphite conductivity profiles. Detailed descriptions of both RGN and RLM algorithms are presented in Section 5.4.

As a starting point, the remaining part of Section 5.2 will discuss some of the fundamental techniques normally used to solve a typical inverse problem.

5.2.1 Gradient Descent Method

The Gradient Descent (GD) method is one of the fundamental optimisation methods that can be implemented by only computing the first-order partial derivatives of the objective function of the problem. Following the discussion in Section 5.1.2 or given the objective function \( g(\sigma) \), the first order Taylor series expansion of the objective function around \( \sigma \) gives:

\[
g(\sigma + \alpha d) = g(\sigma) + \alpha d^T \nabla g(\sigma) + O(\alpha^2) \tag{Eq 5.12}
\]

For a sufficiently small step length \( \alpha \to 0 \) the third term in equation 5.12 can be ignored and the Taylor series expansion of the objective function can be approximated as:

\[
g(\sigma + \alpha d) \approx g(\sigma) + \alpha d^T \nabla g(\sigma) \tag{Eq 5.13}
\]

where \( \nabla g(\sigma) = \partial g(\sigma) / \partial \sigma_i \) is the gradient, \( \alpha \) is the step length and \( d \) is the step direction.
This method works on the basis that a differentiable objective function $g(\sigma)$ defined at $\sigma$ decreases fastest in the neighborhood of $\sigma$ in a particular direction. Setting the condition $g(\sigma + \alpha d) < g(\sigma)$ so that the term $d$ is negative descent direction gives:

$$d^T \nabla g(\sigma) < 0$$  \hspace{1cm} (Eq 5.14)

The relative reduction in the objective function (Eq 5.13) for the calculated step $(\alpha d)$ is given by:

$$\lim_{\alpha \to 0} \frac{g(\sigma) - g(\sigma + \alpha d)}{\alpha \|d\|} = - \frac{d^T \nabla g(\sigma)}{\|d\|} = - \|\nabla g(\sigma)\| \cos(\theta)$$  \hspace{1cm} (Eq 5.15)

where $\theta$ is the angle between the gradient of the objective function $\nabla g(\sigma)$ and the step vector $d$. By observation it is clear to see that the largest reduction in $\nabla g(\sigma)$ is achieved when $\theta = \pi$. Therefore, the gradient descent direction is obtained when $\theta = \pi$ which also corresponds to:

$$d_{GD} = -\nabla g(\sigma)$$  \hspace{1cm} (Eq 5.16)

Based on equation 5.16, the conductivity vector $\sigma$ can now be updated using equation 5.17.

$$\sigma_{k+1} = \sigma_k - \alpha \nabla g(\sigma)$$  \hspace{1cm} (Eq 5.17)

The positive scalar step length $\alpha$ in equation 5.17 determines the amount by which the algorithm moves in the direction of $d$ to minimise the objective function, and is calculated using the line-search method [129]. Various ways of calculating the optimum step length exist for a given problem (see for example in [129]). However, in all cases there will be a tradeoff between finding the step length that gives the highest reduction in the objective function and the number of objective function evaluations. The GD method is the simplest form of optimisation algorithm, as it only requires the evaluation of first-order partial derivatives of the objective function.
However, the downside of the GD method is that it lacks higher-order information about the objective function, which often leads to a slow convergence rate of the objective function compared to other methods (e.g. Newton’s Method).

### 5.2.2 Newton Method

The Newton approach is another fundamental optimisation algorithm that has been used to solve various inverse problems. The main difference between the Newton and GD methods is that the Newton method uses second-order partial derivatives of the objective function, whereas the GD method only uses first-order partial derivatives. Furthermore, this algorithm uses a step length of one ($\alpha = 1$), meaning that unlike the GD method the Newton method eliminates the need for calculating the step length $\alpha$ during every iteration. Since the Newton approach uses the second-order derivatives of the objective function and $\alpha = 1$, it is the obvious choice for the application that requires a fast convergence rate during the computation of the inverse problem, at least compared to the GD method. As in the case of the GD approach, the Newton method is derived first by approximating the objective function by its Taylor series expansion, but in this case using the second-order Taylor series expansion (Eq 5.18).

$$g(\sigma + d) = g(\sigma) + d^T \nabla g(\sigma) + \frac{1}{2} d^T \nabla^2 g(\sigma) d + O(||d||^3) \quad \text{(Eq 5.18)}$$

assuming a small $||d||$, which makes the higher-order term trivial, then equation 5.18 can be approximated to have a quadratic form as shown in equation 5.19.

$$g(\sigma + d) \approx g(\sigma) + d^T \nabla g(\sigma) + \frac{1}{2} d^T \nabla^2 g(\sigma) d \quad \text{(Eq 5.19)}$$

The difference between the original (Eq 5.18) and approximated (Eq 5.19) equations for the objective functions must be small for this approximation to hold and obtain a quadratic convergence rate. Provided that the difference is small, the formulation of
the Newton optimisation problem is arranged by equating the derivative of the objective function to zero as in equation 5.20.

\[ \nabla g(\sigma + d) = \nabla g(\sigma) + \nabla^2 g(\sigma)d = 0 \]  

(Eq 5.20)

where \( \nabla g(\sigma) \) is the gradient \((\nabla g(\sigma) = \partial g(\sigma)/\partial \sigma_i)\) and \( \nabla^2 g(\sigma) \) is the Hessian \((\nabla^2 g(\sigma) = \partial^2 g(\sigma)/\partial \sigma_i\partial \sigma_j)\).

Assuming an invertible Hessian and rearranging equation 5.20 gives the final form of the Newton algorithm:

\[ \nabla^2 g(\sigma)d_N = -\nabla g(\sigma) \]  

(Eq 5.21)

and hence,

\[ d_N = -\nabla^2 g(\sigma)^{-1} \nabla g(\sigma) \]  

(Eq 5.22)

Equation 5.22 gives the Newton step direction, which corresponds to the calculated changes in the conductivity vector. The conductivity vector can then be updated using equation 5.23 up to the point where the user-defined exit conditions are met.

\[ \sigma_{k+1} = \sigma_k + d_N = \sigma_k - (\nabla^2 g(\sigma))^{-1} \nabla g(\sigma) \]  

(Eq 5.23)

The descent direction in this case is achieved when the Hessian is positive definite, in other words, if the Hessian is positive definite it will not become singular when multiplied by any non-zero vectors \((\nu^T \nabla^2 g(\sigma) \nu > 0)\). Hence, multiplying both sides of equation 5.21 with \(d_N\) demonstrates that when the Hessian is positive definite the Newton direction is descent.

\[ d_N^T \nabla^2 g(\sigma)d_N = -d_N^T \nabla g(\sigma) > 0 \]  

(Eq 5.24)
One of the main advantages of the Newton method over GD is that it uses the information about the second-order derivatives of the objective function, which gives it a fast convergence rate. If the Hessian is positive definite, a quadratic convergence to the minimum can be achieved. However, due to the ill-posed nature of the inverse problem the Hessian might not always be positive definite, which also means that it might get close to becoming singular. In this situation, the inverse of $\nabla^2 g(\sigma)^{-1}$ might not even exist or become poorly conditioned. Furthermore, the evaluations of the second-order term $\nabla^2 g(\sigma)$ are often error-prone and computationally expansive requiring $mn (n + 1)/2$ or in our case 770 second order derivatives to be computed during every iteration. For these reasons, a more robust and less computationally demanding non-linear inversion technique is adopted for the work presented in this thesis.

### 5.2.3 Gauss-Newton Method

The Gauss-Newton (GN) method is similar to the Newton method discussed in Section 5.2.2 except that the formulation is modified by approximating the second-order partial derivatives of the objective function (the Hessian) with the squared Jacobian matrix (Eq 5.26). Following equations 5.6 and 5.7 in Section 5.1.2, which show the formulation of the Jacobian using the residual matrix in a discrete optimization problem, the gradient and the Hessian in the GN method can be expressed as:

$$\nabla g(\sigma) = J^T r(\sigma)$$  \hspace{1cm} (Eq 5.25)

$$\nabla^2 g(\sigma) = J^T J + \sum_{j=1}^{m} r_j(\sigma) \nabla^2 r_j(\sigma)$$

$$\nabla^2 g(\sigma) \approx J^T J$$  \hspace{1cm} (Eq 5.26)

This approximation of the Hessian simplifies the way in which the optimization problem is solved. In other words, once the gradient is computed in terms of the
Jacobian, then the Hessian can be simply approximated using this information avoiding the need to evaluate the term $\sum_{j=1}^{m} r_j(\sigma)\nabla^2 r_j(\sigma)$ or the second derivatives of the residuals. This modification of the Hessian is the main property of the GN method that distinguishes it from the standard Newton method. The modification in the GN method should be seen as an advantage over the standard Newton method, as it offers a significant reduction in computational demand.

Based on this approximation, the final GN equation can then be expressed as:

$$d_{\text{GN}} = - (J^T J)^{-1} J^T r(\sigma)$$

(Eq 5.27)

Following equation 5.27 the new conductivity vector can then be updated using:

$$\sigma_{k+1} = \sigma_k + d_{\text{GN}}$$

(Eq 5.28)

In order for the above approximation to hold and obtain a quadratic convergence rate, the residual vector elements $r_j$ in equation 5.9 need to be sufficiently small. This is normally the case for the problems considered in this thesis. The above approximation will not hold for the problems involving large residuals and in these situations the term $\nabla^2 r_j$ should not be ignored during the computation of the Hessian. Alternatively quasi-Newton method can be used to solve a problem involving large residuals [125]. This method attains the quadratic convergence property of the GN method but in this case the second-order derivatives of the objective function ($\nabla^2 g(\sigma)$) is approximated using information about the change in the gradient $\nabla g(\sigma)$ along the search direction. There are two main techniques (Davidson-Fletcher-Powell and Boyden-Fletcher-Goldfarb-Shanno) by which this approximation can be made, described in [138] and [137, 139, 140] respectively.

The general EC and graphite inverse problems are ill-posed and highly non-linear. The ill-posedness comes from the solution not being continuously dependent upon the measured data. This means that small changes in the measured data may lead to large undesired changes in the reconstructed profile leading to a poor solution. On the other hand, the non-linearity, in particular for the graphite inverse
problem may arise because we are attempting to reconstruct the graphite brick conductivity profile based on the impedance data of the sensing coils measured at the brick boundary, and developing a relationship between the physical and electromagnetic properties of the graphite brick. Clearly this relationship is non-linear due to the soft-field effect. Furthermore, in reality it is almost impossible to avoid small modeling and measurement errors, which may also lead the Jacobian to become badly conditioned. The approximation of the Hessian by $J^TJ$ also means doubling the condition number (the ratio between largest and smallest singular values). In these situations the solution to the inverse problem becomes unstable and possibly diverges from the minimum. This problem can be overcome by incorporating some form of regularization techniques in the formulation of the inverse problem. Sections 5.3 will briefly discuss some of the most common techniques often used to analyze the ill-posedness of the inverse problems and the methods adapted to address it.

5.3 Regularisation Techniques

Reductions to undesired step changes in the inverse solutions are normally achieved by implementing a regularisation technique within the reconstruction algorithm. The application of a regularisation technique along with the selection of appropriate regularisation parameter and operator matrix is an active research area in the field of image reconstruction and some NDT applications.

Many authors working in this field have proposed various techniques by which ill-posed inverse problems may be regularised so that a reasonable estimate of the solution can be made. These include the authors of [141], who developed a new method of selecting a regularisation operator matrix for an ill-posed problem with the aim to close the gaps between Truncated Singular Value Decomposition (see Section 5.3.3) and the general Tikhonov schemes (see Section 5.3.4). The results from this study show less error in the inverted solution compared with the standard identity matrix operator. Other techniques of implementing the regularisation by means of adding a penalty term to the original least square formulations (such as
Total Variation (TV) to preserve edges in the inverted profile and Tikhonov to smooth any undesired step changes) have been published by many authors including [59, 142-143]. However, as the work in this thesis is mainly concerned with the relatively smooth electrical conductivity profile of the AGR bricks, only the smoothing techniques and the theory behind them will be described here. Detailed descriptions of the fundamental theory of the most common types of regularisation techniques applied in discrete ill-posed problems can be found in [142, 144-146].

5.3.1 Singular Value Decomposition

The Singular Value Decomposition (SVD) method gives a way of assessing how ill-conditioned the Jacobian matrix is for a given inverse problem [145]. Given a linear system \( J\sigma = V \), the SVD of the Jacobian matrix \( J \in \mathbb{R}^{m \times n} \) (in our case the dot product of the E-fields inside the graphite calculated using the forward and adjoint graphite model \( J = E_{Fwd} \cdot E_{Adj} \)) is given by:

\[
J = U S V^T = \sum_{i=1}^{m} u_i s_i v_i^T \quad \text{(Eq 5.29)}
\]

where \( U = (u_1, u_2, \ldots, u_m) \in \mathbb{R}^{m \times m} \) and \( V = (v_1, v_2, \ldots, v_n) \in \mathbb{R}^{n \times n} \) are the matrices which have orthogonal column vectors (i.e. \( U^T U = V^T V = I_n \)), and \( S = \text{diag}(s_1, s_2, \ldots, s_n) \in \mathbb{R}^{m \times n} \) is a matrix of singular values, which has elements arranged in non-increasing order such that: \( s_1 \geq s_2 \geq \ldots \geq s_n \).

Assuming the Jacobian is invertible, the inverse of \( J \) using the SVD can be computed as:

\[
J^{-1} = \sum_{i=1}^{n} v_i s_i^{-1} u_i^T \quad \text{(Eq 5.30)}
\]

The solution to the system \( J\sigma = V \Rightarrow \sigma = J^{-1}V \) can then be obtained by:

\[
\sigma = J^{-1}V = \sum_{i=1}^{n} s_i^{-1}(u_i^T V)v_i \quad \text{(Eq 5.31)}
\]
However, due to the reasons discussed in Section 5.2.3, the Jacobian matrix of the inverse EC problem cannot be inverted using equation 5.30, instead a more general approach are usually adapted. That is using a Moore-Penrose Generalized Inverse approach [145].

$$J^\dagger = \sum_{i=1}^{\text{rank}(J)} v_i s_i^{-1} u_i^T$$  \hspace{1cm} (Eq 5.32)

As in equation 5.31 the solution to the system can then be obtained by:

$$\sigma = J^\dagger V = \sum_{i=1}^{\text{rank}(J)} s_i^{-1} (u_i^T V) v_i$$  \hspace{1cm} (Eq 5.33)

In an ill-posed problem such as an EC problem, the singular value of the Jacobian gradually decays towards zero and the singular vectors tend to change sign more frequently as the index $i$ increases. Since the Jacobian is decomposed into $v_i, s_i$ and $u_i$, the solution to the system $\sigma = J^{-1}V$ involves the contributions of all three terms. As $s_i$ decreases the contributions of the $v_i$ and $u_i$ to the solution increase, and may affect the solution even in the presence of sufficiently small modelling or rounding error. The condition number of the singular value for an ill-posed problem (normally calculated as the ratio between the largest and the smallest singular value $s_1/s_n$) will be large. This condition number in SVD can also provide additional information on whether the problem is rank-deficient.

The inverse problem is said to be rank-deficient if there is a clear distinct gap in the collection of singular values (a noticeable gap between the zero and non-zero singular values) such that: $\text{rank}(J) < n$. The rank of the Jacobian is normally determined by inspection of the singular values up to the point where this gap occurs on the spectrum. However, in most practical problems the singular values will not show an obvious gap in the spectrum or will not have exactly zero values. Instead they will have small positive values caused by the modelling and rounding errors in the computation. This could lead to a large error in the inverse solution as the minimisation problem involves inverting the Jacobian matrix. In this situation, the Jacobian need to be truncated (i.e. using the Truncated Singular Value
Decomposition technique) and reformulated so that the small singular values are nulled out.

To illustrate the ill-posed nature of the graphite inverse problem and the oscillatory behaviour of the singular vector (in this case \(v_i\)), a Jacobian matrix was constructed using the dot product of the electric fields calculated in the graphite brick volume when the coils were excited using a multi-frequency signal. The Jacobian matrix was then decomposed into three components \(v_i\), \(u_i\) and \(s_i\) and plotted to illustrate how it behaves. It should be noted that the labels C1-C10 in Figure 5.3 correspond to the column vectors of \(v_i\). Figure 5.4 clearly shows the gradually decaying behaviour of the singular values as a function of index number. On the other hand, the \(v_i\) vectors exhibit oscillatory behaviour as the index number increases. The oscillations in the \(v_i\) normally increases as the singular value decreases, in other words the introduction of a small error in either \(J\) and \(V\) or attempting to invert an ill-posed problem directly without any preconditioning is likely to make the solution unstable.
Figure 5. 3: The plots that show the oscillatory behaviour of the singular vector $v_i$ as the index number increases. The X-axis is the index number and the Y-axis is magnitude of the vector.

Figure 5. 4: The normalised singular value of the Jacobian calculated using homogenous conductivity.
5.3.2 Generalised Singular Value Decomposition

The GSVD method is similar to SVD in the sense of how it is used to study the severity of the ill-conditioning of the Jacobian. In addition, the GSVD can also be used to study the regularisation technique applied to treat the ill-conditioning of the Jacobian [145]. The decomposition of matrix pair \((J, L)\) is used in this method, so that the pair \((J, L)\) is the square root of the generalised eigenvalues of the matrix pair \((J^T J, L^T L)\), where \(J \in \mathbb{R}^{m \times n}, L \in \mathbb{R}^{p \times n}\), and the matrix dimensions are arranged to satisfy \(m \geq n \geq p\). Hence, decomposition of \(J\) and \(L\) in GSVD are given by:

\[
J = U \begin{pmatrix} \Sigma & 0 \\ 0 & I_{n-p} \end{pmatrix} X^{-1} \quad \text{(Eq 5.34)}
\]

\[
L = V(M \ 0) X^{-1} \quad \text{(Eq 5.35)}
\]

where \(I_{n-p} \in \mathbb{R}^{(n-p) \times (n-p)}\) is the identity matrix, \(U \in \mathbb{R}^{m \times n}\) and \(V \in \mathbb{R}^{p \times p}\) are orthogonal, \(\Sigma \in \mathbb{R}^{p \times p}\) and \(M \in \mathbb{R}^{p \times p}\) are diagonal matrices with \(\Sigma = \text{diag}(s_1, s_2, ..., s_p)\) and \(M = \text{diag}(\mu_1, \mu_2, ..., \mu_p)\), and \(X \in \mathbb{R}^{n \times n}\) is a non-singular matrix.

The diagonal matrices \(\Sigma\) and \(M\) are arranged such that:

\[
0 \leq s_1 \leq s_2, \leq \cdots \leq s_p \quad \text{(Eq 5.36)}
\]

\[
1 \geq \mu_1 \geq \mu_2, \geq \cdots \geq \mu_p \quad \text{(Eq 5.37)}
\]

and normalised to unity using:

\[
s_i^2 + \mu_i^2 = 1 \quad \text{for } i = 1, 2, ..., p \quad \text{(Eq 5.38)}
\]

The generalised singular values \((\gamma_i)\) of the matrix pair \((J, L)\) are then given by the ratio between the diagonal elements of \(\Sigma\) and \(M\), which are arranged in non-decreasing order unlike standard SVD.

\[
\gamma_i = \frac{s_i}{\mu_i} \quad \text{(Eq 5.39)}
\]
5.3.3 Truncated Singular Value Decomposition

The Truncated Singular Value Decomposition (TSVD) is one of the techniques that can be used to deal with the rank-deficient problem discussed in Section 5.3.1 [145]. This method deals with the rank-deficient problem by explicitly truncating the small singular values at the point just before distinct gap between the singular values or ignoring the last \((n - k)\) of the SVD or GSVD decomposition of \(\mathbf{J}\), such that

\[ k=\text{rank}(\mathbf{J}). \]

The truncated inverse of \(\mathbf{J}\) is then expressed as:

\[
\mathbf{J}^\dagger = \sum_{i=1}^{k} \mathbf{v}_i s_i^{-1} \mathbf{u}_i^T
\]

(Eq 5.40)

Hence, the truncated solution to the system \(\mathbf{\sigma} = \mathbf{J}^\dagger \mathbf{V}\) is obtained using:

\[
\mathbf{\sigma}_{TSVD} = \sum_{i=1}^{k} f_i \frac{\mathbf{u}_i^T \mathbf{V}}{s_i} \mathbf{v}_i
\]

(Eq 5.41)

where \(f_i\) is a filter factor, which has the form of:

\[
f_i = \begin{cases} 
1, & i \leq k \\
0, & i > k 
\end{cases}
\]

(Eq 5.42)

The quality of the truncated solution in equation 5.41 will not be affected by truncation process, since most of the information required for solving the inverse problem accurately is normally contained within the first ranks of \(k\). When \(k\) increases, then the quality of the information degrades as the result of noise contamination and modelling error. However, as partially demonstrated in Figure 5.3, the singular spectrum of the Jacobian \(\mathbf{J}\) related to the inverse EC problem does not show a distinct gap between the singular values. In these situations it is difficult to decide at which point the truncation needs to be applied to the solution using the TSVD technique. Therefore, another technique must be considered to deal with the ill-posed nature of the inverse EC problem related to this work. The technique
adopted in this work is known as Tikhonov Regularisation and this method will be
discussed in the following Section 5.3.4.

5.3.4 Tikhonov Regularisation

The Tikhonov regularisation technique is one of the most commonly used
regularisation method for solving discrete ill-posed problems and the one related to
EC reconstruction problems [6, 7, 141, 145-148]. This technique regularises an ill-
posed inverse problem by introducing a smoothing term within a discrete least
square formulation. This helps to smooth out any undesired step changes and
extreme values in the solution which may be caused by small singular values of \( J \).
The smoothing on the solutions is normally made with the respect to some initial
estimate of the solution (\( \sigma_0 \)) so that any extreme deviations in the solutions from
the initial estimate can be penalised. However, in most practical problems it is
difficult to determine an initial estimate of the exact solution, so instead a constant
value is often used as an initial estimate.

The regularised formulation of the linearized Tikhonov based minimisation
problem is given by:

\[
g(\sigma) = \arg \min_{\sigma} \left\{ \frac{1}{2} \| J \sigma - V \|^2 + \frac{1}{2} \lambda \| L (\sigma - \sigma_0) \|^2 \right\}
\]  
(Eq 5.43)

It should be noted that the above equation is similar to the least-square equation
presented in Section 5.1.2 (Eq 5.5), except that in this case an additional penalty
term \( \frac{1}{2} \lambda \| L (\sigma - \sigma_0) \|^2 \) is introduced to reflect the penalty on the solution. From
equation 5.43 it is clear to see that the solution now depends upon both the norm of
the residual vector and norm of the difference between the initial estimate and the
calculated solution. Therefore, the task in this formulation is to minimise both
simultaneously or to find a solution that satisfies both conditions. The term \( L \) in
equation 5.43 is known as the regularisation operator matrix and usually contains a
priori information about the smoothness of the solution. Different arrangements of
the regularisation operator matrix exist, but the choice depends upon the type of the
problem to be solved. The most common choices of \( \mathbf{L} \) for ill-posed inverse problems are either the identity matrix, a scaled diagonal matrix of the Jacobian also known as “NOSER” or a discrete approximation to the first or the second order derivative operator also known as the “Difference operator”. The term \( \lambda \) in equation 5.43 known as the “regularisation parameter” and controls the amount of the smoothing applied to the least-square solution. As \( \lambda \) increases, the smoothness of the solution will also increase and vice versa.

In order to see how the Tikhonov method achieves the regularisation of the solution associated with small singular values of the Jacobian, it is often convenient to represent the solution of the regularised equation in terms of both SVD and GSVD.

Starting from the SVD of the Jacobian matrix, the solution to the regularised problem presented in equation 5.43 can be represented as:

\[
\sigma_{\text{Reg}} = \sum_{i=1}^{k} \left( f_i \frac{u_i^T v}{s_i} + (1 - f_i) v_i^T \sigma_0 \right) v_i
\]

(Eq 5.44)

Where \( f_i \) is the filter factor given by:

\[
f_i = \frac{s_i^2}{(s_i^2 + \lambda^2)} \quad \text{for} \quad \mathbf{L} = \mathbf{I}_n
\]

(Eq 5.45)

Representing the same equation in terms of the GSVD gives:

\[
\sigma_{\text{Reg}} = \sum_{i=1}^{p} \left( f_i \frac{u_i^T v}{s_i} + (1 - f_i) Y_i^T \sigma_0 \right) x_i + \sum_{i=p+1}^{n} u_i^T V x_i
\]

(Eq 5.46)

where \( Y_i \) is the \( i^{th} \) row of \( \mathbf{X}^{-1} \) in the equations 5.37 and 5.38, and \( f_i \) is a filter factor calculated using the singular value of GSVD \( \gamma_i \) and the regularisation parameter \( \lambda \) as in equation 5.47.

\[
f_i = \frac{\gamma_i^2}{(\gamma_i^2 + \lambda^2)} \quad \text{for} \quad \mathbf{L} \neq \mathbf{I}_n
\]

(Eq 5.47)
It should be noted that when \( L = I_n \) the \( u_i \) and \( v_i \) components in SVD are similar to those of GSVD, and the singular values of GSVD \((J, L)\) are equal to the singular values of SVD, but ordered differently, hence the equation reduces to the one shown in equation 5.44.

A closer look at the filter factor \( f_i \) in equations 5.45 and 5.47 show that the amount of the filter factor applied to equation 5.44 and 5.46 depends upon the value of \( \lambda \). This also means that \( \lambda \) controls the amount of filtering applied on the solution. The main properties that the filter factor must have in order to deal with an ill-posed problem are as follows:

I. As the singular values \((\gamma_i \text{ or } s_i)\) decrease \( f_i \) must tend to zero \((f_i \rightarrow 0)\) and

II. When the singular values are large \((\gamma_p \text{ or } s_1)\) then \( f_i \) must tend to one \((f_i \rightarrow 1)\).

These conditions also mean that the value of \( \lambda \) must be chosen such that \( \gamma_i < \lambda \) or \( s_i < \lambda \) and \( \lambda < \gamma_p \) or \( \lambda < s_1 \) to satisfy the above filtering conditions.

These features of the filter factor allow it to filter-out the solutions associated with small singular values while maintaining a solution corresponding to the large singular values provided an appropriate value of \( \lambda \) is chosen.

To analyse the impact of \( \lambda \) on the regularised solution it is more convenient to first assume some fixed singular value. It can now be seen that for a large value of \( \lambda \) the denominator in the equations 5.45 and 5.47 will be dominated by the value of \( \lambda \), and the filter factor becomes effectively zero. The regularized solution of equation 5.46 in this particular case can be expressed as:

\[
\sigma_{\text{Reg}} \approx \sum_{i=1}^{p} Y_i^T \sigma_0 x_i + \sum_{i=p+1}^{n} u_i^T V x_i
\]

(Eq 5.48)

By contrast, a small value of \( \lambda \) gives a filter factor \( f_i \) close to one and the the regularised solution in equation 5.46 become:

\[
\sigma_{\text{Reg}} \approx \sum_{i=1}^{p} \frac{u_i^T V}{s_i} x_i + \sum_{i=p+1}^{n} u_i^T V x_i
\]

(Eq 5.49)
It is clear to see from equation 5.48 that the approximate regularized solution will be dominated by the initial estimate \( \sigma_0 \) when the value of \( \lambda \) is chosen to be large. In other words, the larger the value of \( \lambda \) the more smoothing is applied to the solution, and hence the solution approaches the initial estimate \( \sigma_0 \). On the other hand, equation 5.49 shows that for a small \( \lambda \) value the approximate regularized solution will be dominated by the contributions of the first and second terms in equation 5.46, which consist of the measured value of \( V \) or in more practical terms the solution mostly depends on the measured data of the vector \( V \).

Ignoring our assumption of fixed singular values we now see that when the value of \( \lambda \) is chosen to be smaller than the smallest singular value, the contribution of singular values to the filter factor \( f_i \) will increase, and hence \( f_i \approx 1 \). These lead to the solution that is highly contaminated with the contributions of the small singular values and possible measurement error or noise in the data \( V \) provided the value of \( \lambda \) is chosen such that \( \lambda < \gamma_i \) or \( \lambda < s_i \). On the other hand, when \( \lambda \) is greater than the largest singular value \( (\gamma_p \text{ or } s_1) \) it will effectively make the solution under-regularised as the information contained in \( \gamma_p \text{ or } s_1 \) will also be filtered out, and the solution tend towards the initial estimate. The aim of the filter factor is to keep the solutions associated with large singular values \( (\gamma_p \text{ or } s_1) \) and filter-out those related to smaller singular values \( (\gamma_i \text{ or } s_i) \). Therefore, the selection of the regularisation parameter \( \lambda \) must consider the singular values of the Jacobian such that \( \gamma_i < \lambda \) and \( \lambda < \gamma_p \) where \( \gamma_i \) and \( \gamma_p \) are the smallest and the largest singular values of GSVD.

Clearly from the above discussions we know that the selection of the regularisation parameter must balance the tradeoff between the contributions of the initial estimate and the measured data to the solution by means of finding the so-called optimum value of \( \lambda \). Section 5.3.5 will briefly describe the common techniques used to find a near optimum value of \( \lambda \) for an ill-posed inverse problem.

### 5.3.5 Selection of Regularisation Parameter

In the above section, it was shown the regularisation parameter \( \lambda \) has a key role in the solution of discrete ill-posed problems. However, the challenge is to find
an appropriate value of $\lambda$ that gives a reasonable estimate of the exact solution by maintaining the balance between the contribution of the residuals norm and the initial estimate on the regularised solution. If some additional a priori information is known about the amount of noise on the measured data, then the value of $\lambda$ can be easily determined in a trial and error fashion or using other methods such as the discrepancy principle and Generalised Cross Validation (GVD) methods [149-150]. However, in most practical applications there is no such information available and the methods that depend on the information about noise level on the measured data cannot be used to determine the optimum value of $\lambda$. The most practical technique normally used to determine the value of $\lambda$ in such cases is based on the technique called L-curve method [145]. This is because the L-curve method does not require a priori knowledge about the amount of noise in the measured data. The following part of this section will briefly describe the theory behind the L-curve technique and its implementation in establishing the optimum value of $\lambda$ that determines the weight of the smoothness applied in the penalty term of a discrete ill-posed minimisation problem.

The L-curve method can be best described as a graphical tool for the analysis of a discrete ill-posed problem. The analysis is simply made by plotting the norm of the regularised solutions $\|L\sigma_{Reg}\|$ against the norm of the residuals $\|J\sigma_{Reg} - V\|$ for all valid values of $\lambda$ on a log-log scale. The curve will normally have an L-shape that displays the trade-off between the solution norm and the residuals norm. In an ideal case it contains a distinct corner that corresponds to the optimum value of $\lambda$.

To understand how the L-curve method works it is often useful to represent the discrete ill-posed problem in terms of two components that correspond to regularisation and the perturbation errors. This is because in reality the measured data $V$ will always contain some type of errors caused by noise, rounding or modelling errors.

Given the system $J\sigma = V$ and data $V = \bar{V} + \varepsilon$, where $\bar{V}$ is the error free component and $\varepsilon$ is the error in the data $V$, the regularised and the exact solution to the system can be written as:
\[ \sigma_{\text{Reg}} = J^{\text{Reg}}(\bar{V} + \varepsilon) \]  
\[ \bar{\sigma} = J^\dagger \bar{V} \]  
(Eq 5.50)

where \( \sigma_{\text{Reg}} \) is the regularised solution, \( J^{\text{Reg}} \) is the regularised inverse of the Jacobian (\( J^{\text{Reg}} = (J^T + \lambda L^T L)^{-1} J^T \)), \( \bar{\sigma} \) is the solution corresponding to error free data and \( J^\dagger \) is the Moore-Penrose Generalised inverse of the Jacobian (\( J^\dagger = (J^T J)^{-1} J^T \)).

The error between the regularised and the exact solution can then be represented as:

\[ \sigma_{\text{Reg}} - \bar{\sigma} = J^{\text{Reg}}(\bar{V} + \varepsilon) - J^\dagger \bar{V} \]

\[ = J^{\text{Reg}} \bar{V} - J^\dagger \bar{V} + J^{\text{Reg}} \varepsilon \]  
(Eq 5.52)

It is clear to see from equation 5.52 that the solution to the discrete problem consist of two components, namely the perturbation error caused by the errors in the measured voltage (\( J^{\text{Reg}} \varepsilon \)) and the regularisation error caused by regularisation of the error free solution (\( J^{\text{Reg}} \bar{V} \)). When the amount of regularisation applied to the solution is large (large value of \( \lambda \)) then the error will be dominated by regularisation error leading to an over-damped solution, and the term \( \| J\sigma_{\text{Reg}} - \bar{V} \| \) in the L-curve become more sensitive to small changes in the regularisation parameter \( \lambda \). On the other hand, for a small amount of regularisation (small value of \( \lambda \)), the error will now be dominated by the perturbation error leading to an under-damped solution. In this case the term \( \| L\sigma_{\text{Reg}} \| \) will become more sensitive to small change in the regularisation parameter \( \lambda \).

The characteristics of the L-curve is better understood when the errors are presented in terms GSVD along with filter factors [145], such that:

\[ \sigma_{\text{Reg}} - \bar{\sigma} = \left( \sum_{i=1}^{n_p} f_i \frac{u_i^T e}{\gamma_i} x_i + \sum_{i=p+1}^{n} (u_i^T e) x_i \right) + \sum_{i=1}^{n_p} (f_i - 1) \frac{u_i^T \bar{V}}{\gamma_i} x_i \]  
(Eq 5.53)

The first term in the equation 5.53 corresponds to the perturbation error caused by \( \varepsilon \) on the measured data, whereas the second term corresponds to the regularization error.
error caused by the regularization of the error free solution. Given the filter factor presented in equation 5.47, it is clear to see that the filtering on the solution depends strongly upon the value of $\lambda$.

For large value of $\lambda$ such that $\lambda \rightarrow \lambda \gg \gamma_i$ the filter factor $f_i$ is effectively zero ($f_i \approx 0$) and the solution errors will be dominated by the regularization errors (the second term in equation 5.53). Therefore, the term $\|L\sigma_{Reg}\|$ on the L-curve becomes almost constant (less sensitive to small changes in the value of $\lambda$) up to the point where the value of $\lambda$ is less than or equal to the largest singular value ($\lambda \leq \gamma_p$) whilst making $\|J\sigma_{Reg} - V\|$ sensitive to small changes in the value of $\lambda$. On the other hand, for a filter factor almost one ($f_i \approx 1$), that is when the value of $\lambda$ is chosen such that $\lambda \rightarrow \lambda \ll \gamma_i$, the solution error will be dominated by the perturbation errors (the first term in equation 5.53), resulting the term $\|L\sigma_{Reg}\|$ becoming more significant on the L-curve whilst making the term $\|J\sigma_{Reg} - V\|$ nearly constant for small changes in the value of $\lambda$. These characteristics of the L-curve with changing values of $\lambda$ can be seen on the simplified plot in Figure 5.5. It should be noted that the labels less filtering and more filtering on the Figure 5.5 corresponds to decreasing and increasing values of $\lambda$ respectively. Detailed analysis of the L-curve plot and the associated discrete ill-posed problem can be found in [145,151].

![Figure 5.5: A Plot showing the typical shape of the L-curve, reproduced from [145].](image)
There are other ways of selecting the near optimal or initial regularisation parameter using information about the Jacobian matrix such that $\lambda_0 = k \ast \max(diag(J_0^T J_0))$, where $k$ is a user-defined scaling factor used to scale the value of $\lambda$. The scaling factor $k$ can be updated depending on the implementation of the optimisation algorithm so that the regularisation parameter $\lambda_k$ updates throughout the iterations. For a constant value regularisation parameter as in the case of the Tikhonov scheme adopted here, the scaling factor is equal to one ($k = 1$) which also corresponds to $\lambda_0 = \lambda$. This technique is often used as an initial estimate for the value of $\lambda$ within the most practical inversion algorithms, in particular in inverse EC problems (see for example [71, 76]).

5.3.6 Selection of Regularisation Operator Matrix

As it has been briefly mentioned in Section 5.3.4, the main purpose of the regularisation operator matrix is to apply a penalty term to the standard discreet least square formulation. In the case of a Tikhonov scheme, which assumes a relatively smooth solution, the operator matrix carries a priori information about the smoothness of the solution with respect to some initial estimate (if the information is available). The operator matrix may either be the identity matrix, scaled diagonal values of the sensitivity matrix (also referred as NOSER), or first or second order derivative operators depending on the type of the problem to be solved.

The following parts of this section will briefly describe some of the most common arrangements for the different regularisation operator matrices along with the advantages and disadvantages of each.

**Difference Operator matrix**

The Difference operator (DF) is one of the regularisation operator matrices often used within reconstruction algorithms. The application of this matrix within the algorithm is made on the assumption that the solution to the problem is relatively smooth. This means that during the computation of the solution the matrix
transfers the information about the assumed smoothness of the computed solution by means of the penalty term [6, 71, 76].

The DF is essentially a difference operator matrix between neighbouring unknowns or in our case the conductivity values of neighbouring graphite layers. This matrix can be constructed so that it can have smaller or higher order depending on the problem to be solved, and the conditions by which the matrix construction is made shown below:

$$L_{ij} = \begin{cases} \text{diag}(u_i) & \text{for } i = j \\ -1 & \text{for adjacent } i \text{ and } j \\ 0 & \text{otherwise} \end{cases} \quad (\text{Eq 5.54})$$

where $u_i$ is the order of the discrete derivative operator.

Assuming the second order operator matrix and substituting this into equation 5.43 to make it act as a penalty term of the solution, it can imminently be seen that the value of $\lambda$ needed along with this matrix to regularise a given problem will be much smaller than the value needed for the other operators mentioned previously. The main reason for this is that this matrix damps the solution much more than the other operators (i.e. the NOSER operator and the identity matrix). This also means that the rate at which the solution moves towards the initial estimate for a large value of $\lambda$ is faster compared to other types of operator matrices. The main advantage of using this operator matrix as a smoothing priori is its ability to smooth the solution vector element-wise whilst ensuring a smooth transition between neighbouring elements. In addition, the DF is able to remove the instability in the computed solution even with less optimal value of $\lambda$ provided the expected solution is relatively smooth.

**Newton One-Step Error Reconstructor Operator matrix**

The Newton One-Step Reconstructor (NOSER) itself is a Newton-based one-step reconstruction algorithm, which is why the operator matrix in this method is known as NOSER. The NOSER operator matrix is constructed by scaling the uniform weight of the identity matrix with the diagonal elements of the squared Jacobian matrix (Eq 5.55). This technique can allow the solutions to the inverse problem to be
regularised according to the corresponding Jacobian matrix elements [126,152]. In the case of the graphite inverse problem the estimated electrical conductivity values of the graphite brick can be regularised according to their corresponding sensitivity values such that:

\[ \mathbf{L} = \text{diag}(\mathbf{J}^T \mathbf{J})^p \]  

(Eq 5.55)

Where \( \mathbf{L} \in \mathbb{R}^{n \times n} \) is the NOSER operator matrix and \( p \) is a scaling factor with a value between 0 and 1. When \( p = 0 \) the NOSER operator become equal to an identity matrix (\( \mathbf{L} = \mathbf{I} \)) and for \( p = 1 \) the operator matrix will take the full weight of \( \mathbf{L} = \text{diag}(\mathbf{J}^T \mathbf{J}) \). The choice of the scaling factor in the problem considered in this thesis can be seen as a compromise in smoothing the conductivity solution between the graphite periphery and the bore region. For \( p \approx 1 \) the solutions near to the bore are most likely to be over-smoothed whereas for \( p \approx 0 \) the smoothing weight on the solution will be uniform and hence, the solutions near to the periphery will be smoothed more as the sensitivity in that region is much smaller than in the bore region. Therefore, the selection of the scaling factor is a compromise between the solutions at the bore and the periphery. For the work presented in this thesis the value of \( p \) is chosen as 0.25. The selection of the \( p \) value was mainly made based on trial-and-error investigations of the one-step solutions.

The main advantage of this method is that it takes into the account the field distribution within the problem domain. In other words, during the regularisation process, each diagonal element of the operator matrix corresponds to the computed electric fields of the discretised problem domain. Hence the smoothness of the solution is controlled accordingly.

### 5.4 Non-linear Inversion Techniques

Non-linear inversion algorithms are mainly used to overcome the problems associated with ill-posed and non-linear systems (such as the inverse EC problems). The main property of non-linear inversion algorithms is that they determine a
solution to the given problem iteratively. This means that the solution to a non-linear inverse problem is found by applying an iterative scheme that finds a series of solutions for every discrete unknown until the given minimisation condition is satisfied. The condition by which the iteration terminates could be set differently, but the most common termination condition is normally set through the objective function.

Various formulations exist that are capable of estimating the solution to non-linear and ill-posed inverse problems. These include the formulations adopted by [49, 69, 148] to implement image reconstruction algorithms, which were used along with MIT system to reconstruct low and high conductivity images. Works by [6, 7, 70, 79, 101, 123, 127] also demonstrated various successful inversion techniques that were used to reconstruct the conductivity profiles and image shapes for MIT and IS systems respectively. The following Sections 5.4.1 and 5.4.2 present the non-linear inversion algorithms adopted in this thesis for reconstruction of graphite conductivity profiles.

5.4.1 Regularised Gauss-Newton Method

The RGN is one of the fundamental iterative inversion techniques normally used to solve non-linear inverse problems, and is one of the algorithms adopted in this thesis. The RGN method can be seen as an extension of the GN method with an additional penalty term and smoothing parameter. The idea of extending GN to RGN comes from the fact that most practical inverse problems are non-linear and they all exhibit ill-posed behaviour to some extent (the Jacobian may be poorly conditioned). In these situations, approximating the second order partial derivatives of the objective function as in the case of GN method \((\nabla^2 g(\sigma) \approx J^T J)\) means that we may be amplifying the small singular values that are responsible for the problem becoming ill-conditioned. Therefore, the solution to a given problem could become increasingly unstable and may not even be unique.

To address these problems, the GN method in equation 5.27 is modified in RGN as shown in equation 5.56:
\[ d_{RGN} = - (J^T J + \lambda L^T L)^{-1} J^T r(\sigma) \]  
\text{(Eq 5.56)}

Considering the Generalised Tikhonov regularisation technique presented Section 5.3.4, which incorporates a penalty term and smoothing parameter to smooth out any instability in the computed solutions, the RGN can then be modified as Tikhonov based RGN minimisation problem:

\[ \sigma^* = \arg\min \left\{ \frac{1}{2} \|r(\sigma)\|^2 + \frac{1}{2} \lambda \|L(\sigma - \sigma_0)\|^2 \right\} \]  
\text{(Eq 5.57)}

For the problem considered in this thesis the residual vector \( r(\sigma) \) is given by the difference between the forward model \( F(\sigma) \) and the measured data \( M \).

\[ r(\sigma) = F(\sigma) - M \]  
\text{(Eq 5.58)}

Substituting equation 5.58 into equation 5.56 then gives the solution to the search direction of the iterative RGN method with the Tikhonov regularisation scheme.

\[ d_{RGN} = - (J_k^T J_k + \lambda L_k^T L)^{-1} J_k^T \left( (F(\sigma_k) - M) + \lambda L_k^T (\sigma_k - \sigma_0) \right) \]  
\text{(Eq 5.59)}

The conductivity profile of the problem domain can then be updated as:

\[ \sigma_{k+1} = \sigma_k + d_{RGN} \]  
\text{(Eq 5.60)}

where \( k \) is the \( k \)th iteration, \( d_{RGN} \) is the search direction of the solution at the \( k \)th iteration, \( \sigma_k \) is the value of conductivity computed at the \( k \)th iteration and \( J_k \) is the Jacobian calculated using the conductivity values estimated at the \( k \)th iteration.

For the RGN algorithm adopted in this thesis the final solution to the problem is found through the condition at which the iteration of the RGN must terminate. That is: the objective function of the next iteration must be smaller than the previous iteration \( (g(\sigma_{k+1}) < g(\sigma_k)) \). In some applications this type of termination may contain some tolerance, which the difference between the two objective function values must satisfy. If the difference between the objective function is
below a given tolerance then the algorithm will be terminated, which also means that the solution found at the present iteration will be the solution of the problem. The latter technique gives the advantage of reducing computational time by ignoring very small changes in the objective function. However, for the problem considered in this project, the former exit condition was used, at least for the RGN method.

5.4.2 Regularised Levenberg Marquardt Method

It has been shown in Section 5.4.1 how the RGN algorithm is formulated to effectively deal with the ill-posed and non-linear behaviour of an inverse EC problem. This section introduces another non-linear algorithm capable of dealing with inverse EC problems, particularly the graphite reconstruction problem considered in this thesis.

Given the Taylor series expansion of the original and the approximated objective functions in equation 5.18 and 5.19, it is apparent that the difference between the two is the higher order term $O(||d||)^3$. Now we assume that $M(d)$ is the quadratic model that approximates the behaviour of the original objective function $g(\sigma + d)$ in the neighbourhood of current iteration.

$$M(d) = g(\sigma) + d^T \nabla g(\sigma) + \frac{1}{2} d^T \nabla^2 g(\sigma) d \quad \text{(Eq 5.61)}$$

For a small value of $d$ the model will be a good approximation to the original objective function $g(\sigma + d)$.

The original Levenberg Marquardt (LM) algorithm [153-154] uses this information to implement the damping technique. The LM method can be thought as an extension of the GN method discussed in Section 5.3.3. The main difference between the two algorithms is that the LM incorporates an additional damping term $\gamma I$ that can be updated through the iterations depending on how well the model represents the actual system.

$$d_{LM} = -(J^T_k J_k + \gamma I)^{-1} J^T_k (F(\sigma_k) - M) \quad \text{(Eq 5.62)}$$
where $\gamma$ is known as a damping parameter that can be used to apply a damping weight to the diagonal elements of the Hessian. This damping parameter gives the LM algorithm an additional flexibility to switch between the GD and GN methods throughout the iterations.

For a very large value of $\gamma$ ($\gamma \gg \text{diag}(J^T J)$), the term $\gamma I$ in equation 5.62 dominates $J^T J$, resulting in $d_{LM} \approx -\frac{1}{\gamma} J^T (F(\sigma) - M)$, which is equivalent to the GD method with step size $\alpha = \frac{1}{\gamma}$. On the other hand, when $\gamma$ is very small ($\gamma \ll \text{diag}(J^T J)$), the term $\gamma I$ becomes insignificant, and hence: $d_{LM} \approx -(J^T)^{-1} J^T (F(\sigma) - M)$, which is the same as the GN method. In this way the LM algorithm switches the method by which it solves the solution for the given optimisation problem. When the solution at the current iteration is far from the actual solution, the LM switches to the GD method by increasing the value of the damping parameter $\gamma$. As the solution at the current iteration approaches the final solution then the LM switches to the GN method [134-135].

Another form of LM algorithm known as the Regularised Levenberg Marquardt (RLM) was suggested by [71,134] for the application of image reconstruction in MIT and EIT systems respectively. The suggestion was to apply the LM damping technique directly into RGN so that the algorithm contains a priori information about the smoothness of the solution through the term $L^T L$, while having all the features of LM. This modification leads to the formulation of the RLM algorithm (Eq 5.63).

$$d_{RLM} = -(J^T_k J_k + \gamma I + \lambda L^T L)^{-1} J^T_k \left( (F(\sigma_k) - M) + \lambda L^T L(\sigma_k - \sigma_0) \right) \quad \text{(Eq 5.63)}$$

$$\sigma_{k+1} = \sigma_k + d_{RLM} \quad \text{(Eq 5.64)}$$

The value of the regularisation parameter $\lambda$ in the RLM algorithm is kept constant as in the RGN method, but the damping parameter $\gamma$ updates throughout the inversion process. The way in which the damping parameter $\gamma$ updates depends upon how good or bad the reductions in the quadratic model $M(d)$ from equation 5.61
approximates the reductions in the actual objective function. In this algorithm the updating criterion is normally set through the calculation of the gain factor [134-135].

\[
\rho = \frac{g(\sigma_k) - g(\sigma_k + d)}{M(0) - M(d)}
\]  

(Eq 5.65)

where \( \rho \) is the gain factor that governs the update of the damping parameter.

Note: in equation 5.65 the numerator is the difference in the Tikhonov objective function (Eq 5.57), which corresponds to the reductions in the system objective functions, whereas the denominator is the difference in reductions between the model objective functions as shown in equation 5.66.

\[
M(0) - M(d) = -d^T \nabla g(\sigma) - \frac{1}{2} d^T \nabla^2 g(\sigma) d
\]  

(Eq 5.66)

When \( \rho \) is large this implies that the model \( M(d) \) is a good approximation to \( g(\sigma_k + d) \), and hence the value of the damping parameter \( \gamma \) can be reduced to make the RLM step closer to RGN (less damping of the solution). When the value of \( \rho \) is much smaller but positive, \( M(d) \) at the current iteration is said to be a poor approximation to the \( g(\sigma_k + d) \), and this means that the value of \( \gamma \) must be increased to make RLM closer to GD so that the step length can be reduced (more damping on the solution). On the other hand having \( \rho < 0 \), or a negative gain factor, implies that the numerator is negative suggesting that the solution is not in descent direction. In this situation the step \( d \) must be rejected with the use of a constraining technique and the iteration process is repeated with the aim of making the solution descend.

The mechanism by which the value of the damping parameter \( \gamma \) is updated for the work presented in this thesis is directly adopted from [135-136] and it has the following form:

\[
\text{if } \rho > 0; \quad \gamma = \gamma \times \max(0.5, 1 - (2\rho - 1)^3);
\]
\[
\eta = 2;
\]

\textit{else}
\[
\gamma = \gamma \times \eta;
\eta = \eta \times 2;
\]

\textit{if} \quad \eta = 32;
\textit{exit};

It should be noted in the above algorithm the value of the damping parameter for a single iteration is only tried five times. If the value of \(\rho\) is still negative after the fifth trial the algorithm terminates through the conditions in above algorithm.

Another termination criterion also exists, which can be used along with the RLM algorithm. This condition is slightly different to that of the RGN method. In the RGN method if the objective function does not show the descent direction \(g(\sigma_{k+1}) < g(\sigma_k)\), then it terminates automatically. Unlike RGN, the RLM stopping criterion adopted in this thesis uses a step-tolerance [136] and function-tolerance in which the algorithm must satisfy:

\[
\|\sigma_{k+1} - \sigma_k\| < \varepsilon_1(\|\sigma_k\| + \varepsilon_1) \quad \text{(Eq 5.67)}
\]

\[
\|g(\sigma_{k+1}) - g(\sigma_k)\| < \varepsilon_2(1 + \|g(\sigma_k)\|) \quad \text{(Eq 5.69)}
\]

where \(\varepsilon_1\) and \(\varepsilon_2\) are user defined positive values. The termination criterion in equation 5.67 ensures that a very small change in \(\sigma_{k+1} - \sigma_k\) relative to \(\varepsilon_1(\|\sigma_k\| + \varepsilon_1)\) is rejected, which in turn prevents unnecessary reductions in the calculated conductivity values. If the condition in equation 5.67 is not enforced the algorithm could try to continuously find the best fit between the model and the actual objective functions resulting in a continuous reduction in step \(d\). By contrast, the condition in equation 5.68 rejects small changes in the objective function \(g(\sigma_{k+1}) - g(\sigma_k)\) relative to \(\varepsilon_2(1 + \|g(\sigma_k)\|)\). This condition is mainly set to prevent the algorithm from becoming trapped in a local minimum. In addition, the conditions in equations 5.67 and 5.68 also helps to reduce the computational demand for a given problem, as small changes in both the step size and the objective function are rejected and the solution at the current iteration is assumed to be the final solution.
Chapter 6

6 Graphite Sample Conductivity Profiling

This Chapter presents detailed studies of the graphite sample conductivity profiling techniques adopted in this thesis. The studies in this Chapter have a particular relevance to the AGR brick reconstruction problem, and form the basis for reconstruction of real reactor data, which will be described in the following Chapter.

Following the construction of the EC sensor described in Chapter 4, the extended version of the RGN algorithm has been used throughout the reconstruction process in this Chapter. The RGN algorithm adopted in this thesis is similar to the algorithm used by [6-7], but extended further to make it easier to incorporate various types of constraints. The graphite forward model (discussed in Chapter 4) was calibrated with the measurement system prior to the reconstruction process. This was mainly necessary to reduce the discrepancy between the model and the measurement system so that the model fully represented the physical and electromagnetic properties of the graphite brick and the measurement system.

Throughout this work the reconstruction algorithm was initially tested using synthetic data. These data were generated from the FE forward model, but deliberately contaminated with different levels of artificial errors. After successful operation with the synthetic data, inversion of the experimental data was attempted. The experimental data were measured on a representative graphite brick that had a known electrical conductivity profile (see section 4.7.3). The former test was mainly carried out in order to assess the effects of data errors on the accuracy of the reconstructed profiles. The remaining parts of this Chapter present a detailed description of the methodology and the results of the reconstruction problems for both the simulated and experimental data.

6.1 Methodology

For the reconstruction problem considered in this thesis, identical forward modelling and experimental procedures to those presented in Chapter 4 were used,
but in this case the measured data were initially scaled by the number of coil-turns, so that the model and the measured data represented the values of the induced voltage in a single coil turn. The graphite domain in the forward model was discretised into ten sub-domains and assigned a separate Ampère's law node for each, and was enclosed within a spherical boundary to ensure a continuous electromagnetic field around the inner boundary [77,155]. The discretisation of the graphite model was mainly applied to calculate the electrical conductivity of each sub-domain independently throughout the inversion process.

After implementing the forward model that represented the physical and electromagnetic properties of the graphite brick and measurement system, further refinement of the forward model was made by calibrating the model with measurements taken from a brick with known electrical conductivity profile.

6.1.1 Model Calibration

A high-accuracy forward model was required to solve the graphite conductivity reconstruction problem. Therefore, it was necessary to calibrate the graphite brick forward model with the physical system prior to attempting the reconstruction procedure. In this case, the need for calibration arose because the forward model approximated the multi-turn asymmetric gradiometer with filaments assuming constant current density over the cross-section of the practical coil. The physical system consisted of multiple windings that were bound together. Therefore, the physical probe parameters will have small variations in diameter and positions, which may lead to variation in induced voltage, even though the model was created using the mean values of these parameters. For this reason, a model calibration procedure was essential to account for these differences in the coil diameter and position, as well as any other differences that were not accounted for in the model.

Since the probe had three independent elliptically shaped multi-turn windings (Tx1, PC and BC coils) with different parameters, the calibration process involved adjusting eight parameters, namely the three major and three minor axes of the three coils, the lift-off and the separation distance. However, the initial study during the sensor design had indicated that the sensor response was more sensitive
to a change in coil diameter (major and minor sizes) than the liftoff or the separation distance (see Chapter 4). Based on this study, the model calibration variables were reduced to six parameters, namely the three major and three minor axes of the coils.

The model calibration was performed by varying the six variables up to the point where the difference between the measured and model data was minimized, while maintaining these parameters within the range of the practical coil sizes. Since the sensor response also depends upon the graphite sample geometry, its electrical properties and the experimental setup, two different samples and experimental setups were studied during the calibration process. The two graphite samples used during the calibration process were a virgin Gilson carbon graphite brick with known homogenous electrical conductivity approximately 91 kS/m (based on the 4-point measurements at the bore and periphery, and assuming homogenous conductivity over the brick radial thickness) and a half-graphite sample with radially drilled holes, which had maximum and minimum electrical conductivities of 79.3 kS/m and 102 kS/m respectively (see Section 4.7.3 for a details of the drilled brick).

6.1.2 Measurement Systems

As mentioned above, two different measurement setups were used throughout the calibration process. During the first experimental setup the drilled graphite brick, 250 mm high, was used, and the asymmetric gradiometer was aligned with the brick bore using a non-conducting support at the centre (125 mm from both ends) of the sample brick (Figure 6.1). The gradiometer input and output were then connected to a Solartron 1260 Impedance Analyser. A current was injected into the exciter coil and the complex mutual inductance responses of the sensor were measured multiple times giving multiple sets of complex mutual inductance data for frequencies ranging between 10 Hz and 4 kHz, with 5 points per decade. These gave a total of 14 data points. Similarly, the sensor was placed in air (after removing the sample) and the same set of measurements was repeated. The measured data were then averaged over each frequency point to minimise random measurement errors and finally, the air data were subtracted from the graphite data to reduce any residual background noise.
The second experiment was carried out using a larger graphite brick sample that was 350 mm high, and a scanner that operated in the same fashion as the PECIT. During this experiment the asymmetric gradiometer was fitted onto a scanner and connected to a solation Solartron 1260 Impedance Analyser. Similar processes as in the first experiment were repeated to obtain the required data, but in this case the air data were obtained by attaching the probe to a cylindrical nylon bore to maintain support for the scanner (Figure 6.2). The resultant data from these experiments were then scaled by the number of coil turns and compared against the model data.

Throughout the calibration process the measurements from the drilled graphite brick were treated as the main calibration data, and the FE model was simulated using various combinations of the six coil parameters mentioned above. These simulations were mainly carried out with the aim of extracting the calibration parameters that gave the least error between the model and the measured data, such that the combination of the coil parameters that gave the least error between the measured and simulated mutual inductance data became the final calibration parameters. Once model calibration parameters were extracted using the drilled graphite brick data, another model for the 350 mm high graphite brick was simulated using the calibration coil parameters, and compared against the second set of experimental data. Figure 6.1 and 6.2 show the experimental setups for the two systems, while Figure 6.3a and 6.3b show comparisons between the calibrated model and measured data from the two systems.
Figure 6. 1: The first experimental setup for the graphite brick using the asymmetric gradiometer.

Figure 6. 2: The second experimental setup for the graphite brick using the asymmetric gradiometer. It should be noted that during the measurement of the graphite brick, the nylon was replaced by the identically-sized graphite brick.
6.1.3 Results and Discussion

Table 6.1 shows the calibrated coil parameters that gave the least error between the measured and simulated data, while Figure 6.3 shows comparisons of the mutual inductance responses between the calibrated model and the measured data for the two measurement systems.

<table>
<thead>
<tr>
<th>Coils</th>
<th>Coil size major axis (mm)</th>
<th>Coil size minor axis (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pickup coil</td>
<td>67</td>
<td>52</td>
</tr>
<tr>
<td>Exciter coil</td>
<td>62</td>
<td>47</td>
</tr>
<tr>
<td>Backing-off coil</td>
<td>30.9</td>
<td>25.9</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of the combinations of the coil parameters for the calibrated forward model.

As can be seen in Figure 6.3a and 6.3b, the calibration process was mostly successful with the real and imaginary mutual inductance error norms of 3.2 % and 2.8 % respectively for the drilled brick, and 10.6 % and 5.36 % for the brick with homogenous bulk conductivity. Further reductions of the error norms were attempted by adjusting the combination of the coil parameters. However, due to the asymmetric nature of the probe it was not possible to reduce the error norm below this level whilst maintaining the balance (keeping the coil turn-ratio to 6:1). This limitation is mainly caused by the geometry of the asymmetric gradiometer, which has different sizes for all three coils (including major and minor diameters). Nevertheless, the accuracy of the current calibration should in principle be sufficient for the inverse problem to converge.

A closer look at the plots in Figure 6.3b reveals larger calibration error norms compared to those in Figure 6.3a. There may be different factors contributing to this, including sample geometry and the differences between the measurement systems and setup in which the two datasets were collected. However, the most obvious reason is the use of calibration parameters extracted from a model, using a measurement collected from a sample that had 250 mm height to model a brick sample with 350 mm height. The EC flows in the samples are dependent upon the coil parameters and the sample geometries, so therefore small differences like these are to be expected.
Furthermore, the two datasets were collected from two different experimental setups. In fact, one of the main reasons for this comparison was to see how the data simulated using the calibrated parameters differed from the data collected using...
different experimental setups. Although the variations caused by the differences in sample geometry and experimental setup were not investigated independently, it is clear to see that the calibration parameters were indeed affected, leading to an increased error norm when a different experimental setups and sample geometry were used. This effect can be directly related to the AGR in-core measurements as this also requires calibration data measured from a brick with known electrical conductivity value, ideally using an identical measurement system and setup to that used to obtain the in-core mutual inductance data.

6.2 Synthetic Data-Based Inversion

The error norm differences between the two measurements in Section 6.1.3 reveal additional information about how the EC data were affected by varying measurement setup and sample geometry. These insights about the variation in calibration error due to the difference in experimental system and sample geometry are particularly useful in situation where a laboratory-based measurement is to be used for calibrating the AGR brick forward model. This also means that a model calibrated using laboratory-based experimental data may not fully represent the real situations as a different measurement system (a system that does not replicate the measurement tool) is used to collect laboratory data.

In order to overcome these uncertainties, and assess the effects that these data variations have on the accuracy of the reconstructed solution, a preliminary study was carried out to reconstruct the conductivity profiles of the graphite brick using artificially generated synthetic data. This was then followed by artificially adding different error levels on the original data to replicate the variations caused by different levels of calibration errors to see how the reconstruction algorithm responds to these errors.

Due to the non-linear and ill-posed nature of the inverse EC problem, the iterative approach presented in Section 5.4.1 was adopted throughout the reconstruction process. There were two main reasons for adopting this approach: the first was that this method uses a natural step size ($\alpha = 1$), and hence did not require the computation of the step size during the reconstruction process, which in
turn eased the computational demand. The second and the main reason was that this technique had proven to be a robust non-linear inversion algorithm for most discrete ill-posed problems, including the problem related to the graphite bricks [6, 7, 69-71].

6.2.1 Simulated Data without Errors

The simulated data reconstruction problem was solved using the RGN algorithm with the DF and NOSER regularisation operator matrices discussed in Section 5.3.6. The main purpose of this inversion was to see how the two-regularisation operator matrices compare in terms of reconstructed profile error and convergence rates. In addition, the results from these reconstruction problems were used to assess how well the algorithm converged to the expected true profile with the new asymmetric gradiometer sensor parameters when the data were free from any error.

To solve these problems, first the forward model was simulated using the pre-determined bore and periphery electrical conductivity values of the drilled graphite brick, and was interpolated linearly to obtain a vector that represented the brick’s radial conductivity values. The simulated data using the conductivity vector were then used as an error-free input data within the inversion algorithm.

During the reconstruction process the initial estimate of the brick bulk conductivity was defined as a 72 kS/m, and two different regularisation parameters that corresponded to DF and NOSER operator matrices were applied. The regularisation parameter in each reconstruction problem was chosen empirically through a single-step reconstruction.

6.2.2 Results and Discussions

The plots in Figure 6.4 show the reconstructed profiles, objective functions and profile errors when the NOSER and DF regularisation operator matrices were applied within the reconstruction algorithm. It should be noted that the values of the regularisation parameters $\lambda$ differed when using these two operator matrices. These
differences were mainly introduced because the DF matrix applies more smoothing to the solution even for very small value of $\lambda$ (for filter factor close to one), and hence the regularisation parameter normally applied to DF was much smaller than for NOSER. On the other hand, the NOSER operator applied smoothing relative to the squared diagonal values of the Jacobian matrix calculated using the initial conductivity estimate, and hence required a much larger regularisation parameter.

(a) The profiles reconstructed using NOSER operator and $\lambda = 1e^{-16}$

(b) The objective function of the reconstructed profile in (a)
Figure 6.4: The reconstructed profiles, objective functions and profile errors for error-free data reconstructed using NOSER and DF operator matrices.
As it can be seen from the results in Figure 6.4 the reconstructed profile using NOSER converged close to the true profile at the 13th iteration, whereas the profile reconstructed using the DF operator matrix converged at the 17th iteration. The initial convergence rates were slightly faster for the NOSER operator than for the DF operator (Figure 6.4e). This could be due to the direct effect of the amount of smoothing applied through the DF matrix on the solution resulting in a slightly damped solution (a smooth transition between neighbouring elements of the conductivity vector), whereas the NOSER matrix applied smoothing relative to the Jacobian matrix. Furthermore, the NOSER operator applied a smoothing to each conductivity vector element independently to prevent a large divergence from the user-defined reference value. On the other hand, the DF operator had similar features to that of NOSER, but it also prevented the divergence between the neighbouring conductivity elements within the reconstructed conductivity vector with additional smoothing effect. These features of the DF operator may have resulted in a stronger smoothing effect than NOSER.

The final global profile error was slightly lower (1.3%) when the DF operator is used as compared with the 1.6% profile error obtained using the NOSER operator. The objective functions of each solution also showed a comparable trend with their corresponding profile errors on the final iteration. It should also be noted that the profile errors (Figure 6.4e) were the direct result of the regularisation error rather than the perturbation errors. This is because identical models with identical mesh distributions were used to obtain the synthetic data and solve the reconstruction problem. Hence, the data is error-free, which also implies that as the number of iterations increases the residual errors should have tended to zero.

From these results it is sensible to conclude that both operators produce reasonable estimates of the exact profile, although the DF operator preforms slightly better than NOSER in terms of profile accuracy. On the other hand, the NOSER operator showed a faster convergence rate than DF. Ideally, the DF operator should be the primary choice for reconstruction of a profile that is expected to have smooth transitions between neighbouring conductivity values (as in the problem considered in his chapter). This is mainly due to the correlations it makes between neighbouring conductivity elements. Further studies of these two operator matrices will be
presented in Section 6.3, where experimental data are used to reconstruct a graphite brick sample profile.

6.2.3 Simulated Data with Added Error

As discussed in the earlier sections, the inverse problem requires an accurate forward model that represents the physical and electromagnetic properties of the material under test as well as the measurement system. These are normally achieved through calibration of the forward model with measurements taken from a sample with known physical and electromagnetic properties (see Section 6.1). However, in some cases it is not possible to achieve 100% calibration (zero error) due to the arrangements of the measurement systems and sample geometry. For these reasons, it was necessary to assess how different levels of calibration errors (in this case the calibration errors seen in Section 6.1.3) affect the accuracy of the reconstructed solutions.

During these studies, initially different levels of error were generated to replicate variations between the calibrated graphite model and the measured data. These errors were then distributed over the original synthetic data (error-free data), and inverted to see to which extent the solutions were affected as the result of these errors. The errors in this part of the study were generated using different percentages of the mean magnitude of the error-free data, and distributed over the original data to replicate the differences between the calibrated model and measured data. Figures 6.5 and 6.6 illustrate the real and imaginary mutual inductance plots of the data contaminated with different levels of error. It should be noted that the plots in Figure 6.6 do not replicate the calibration errors seen in Section 6.1.3, instead these data were used to test how different distributions of the errors affected the reconstructed solutions.
Figure 6.5: The real and imaginary part of the mutual inductance data generated to simulate the variations between the calibrated model and the measured data.

(a) Real part of the replicated mutual inductance data

(b) Imaginary part of the replicated mutual inductance data

(a) The real part of the mutual inductance that contains multi-crossing error distributions
(b) The imaginary part of the mutual inductance that contains multi-crossing error distributions

Figure 6.6: The real and imaginary parts of the mutual inductance data generated to illustrate a constant data error, which crosses the original data at multiple points.

6.2.4 Results and Discussions

A constant regularisation parameter ($\lambda = 1e^{-28}$) and the DF operator matrix were used throughout the reconstruction process. This was mainly to allow the evaluation of only the effects of data errors on the reconstructed profiles by maintaining all other parameters constant. Figure 6.7a shows the reconstructed profiles of the simulated graphite brick using the six replicated data sets each contaminated with different levels of error ranging between 1% and 6%. As would be expected the most representative reconstruction of the true profile was obtained when the smallest amount error was applied to the replicated data (1% error). A similar trend can also be seen in Figure 6.7b, where the objective function settles at the lowest value for the solution corresponding to the data with the least error, and increases according to the amount of error applied to the original synthetic data. This is what should be expected, since the objective function evaluates the norm of the residual errors, which were deliberately set to increase in this case. As the error level applied to the original data was increased, the reconstructed profile diverged away from the exact solution, suggesting that the data were no longer representative of the original data that corresponded to the true profile.

These results indicate that small errors in the data indeed affected the performance of the algorithm, which in turn degraded the accuracy of the reconstructed profiles. This also means that in order to improve the accuracy of the
reconstructed profiles the calibration errors needs to be sufficiently small. However, further investigations of this problem have indicated that the accuracies of the reconstructed profiles depend not only on the amount of error applied to the data, but also upon the distributions of the errors over the data. In particular, it was found that when the data were contaminated with errors which had a feature that allowed them to cross the original data at multiple points (as shown in Figure 6.6a and 6.6b), the algorithm tend to recover much more accurate profiles than the profiles reconstructed using a data with fewer crossing points on the original data. This can be clearly seen in Figure 6.7d where the profiles were reconstructed using the data consisting of 4 % and 5 % multi-crossing errors, which show much more accurate profiles than the profiles reconstructed using the replicated data (Figure 6.7a) with identical level of errors to those presented in Figure 6.7d. One of the reasons that the multi-crossing errors produced better accuracy could be that the constant distributions of the errors at each frequency points may have allowed the algorithm to estimate the data points with better degree of accuracy than the replicated data. Although these types of error are not normally expected within the problem considered in this thesis, these results clearly indicated that the accuracy of the reconstructed profiles depends upon both the levels and distributions of the errors in the data.

The fact that the reconstructed solutions using the replicated data (Figure 6.7a) diverged away from the true profile as the level of data error increased, and still continued to show reduction in objective function, also suggests that the algorithm might have been trapped in a local minimum. This means that additional a priori information may need to be incorporated within the algorithm to constrain the solution around the true profile, in particular in situations where the replicated calibration error is more than 2% of the original data.

Nevertheless, the reconstructed near-bore conductivity solutions using the replicated data show profile errors of less than 1 % for all cases, suggesting that the errors in the data are not affecting the accuracy of the reconstructed bore conductivity values. The level of accuracy seen around the brick bore region is what should be expected in reality, as the sensitivity to a change in conductivity is much greater next to the probe than at periphery. These results are in accord with the
results presented by Fletcher et al [6], which showed reasonable bore conductivity estimates for experimental data.

Further investigations of the reconstructed profile errors with respect to the data errors showed an approximately linear relationship between the two (Figure 6.7c). These suggest that the profile errors are mainly dominated by the perturbation error rather than the regularisation error. This is what should be expected as the error levels are deliberately increased in the simulated data whilst maintaining a constant regularisation parameter for all cases. However, this might not be exactly the case in general EC measurement systems as the errors in the measured data could have random values and distributions, which could result in random errors in the reconstructed profiles. Nevertheless, the results in this section show the case where constant error levels are distributed in different fashions over the original data. The simulated calibration errors in the data in this case roughly indicate how accurately the model needs to be calibrated in order to reconstruct a profile that represents the conductivity distribution of the measured graphite brick.

(a) Reconstructed profiles using the simulated data with different levels of error
(b) The reconstructed profiles objective functions in (a)

(c) The profile errors versus mutual inductance data errors of the profiles in (a)

(d) The reconstructed profiles using 4% and 5% multi-crossing errors
6.3 Experimental Data Based Inversion

The inverse problems in this section were solved using experimentally measured data from the drilled graphite brick (see Section 4.7.3 for more detail about the drilled graphite brick). Two different approaches were adopted throughout the reconstruction process, namely unconstrained and constrained reconstructions using the RGN algorithm. These approaches were briefly described in Section 5.1.3 along with reasons why these techniques need to be applied within a practical inverse problem. The unconstrained inverse problem in this work was solved without the aid of a priori knowledge of the expected through-wall electrical conductivity profile trend in the drilled graphite brick, whereas the constrained reconstruction applied a priori knowledge to constrain the inverted solution.

It has been briefly mentioned in Section 4.7.3 that the bore density of the drilled graphite brick is lower than the density at the periphery. This is because of the arrangement of the radially drilled holes which have increasing gaps with distance from bore towards the periphery proportional to the through-wall depth into the graphite brick (i.e. assumed homogenous bulk conductivity, and hence the change in density is dominated by the distance between the holes). As the gaps between the holes increase the equivalent bulk density of the graphite is also
expected to increase and vice versa. It is apparent that the higher the density of a conducting material the lower its electrical resistivity (the higher its electrical conductivity). Based on this information it is logical to assume that the radial electrical conductivity profile of the drilled graphite brick varies monotonically from the bore to the periphery within the range of the measured bore and periphery conductivity values. This a priori information is the basis of the constrained inverse problem adopted in this thesis to estimate the through-wall electrical conductivity vector of the discretised graphite brick domains under the conditions that each vector elements must a maintain monotonically increasing behaviour.

The following parts of Section 6.3 present the results of the reconstruction problem obtained using the measured data from the drilled graphite sample along with the L-curve method studied to determine the so-called optimum regularisation parameter value.

6.3.1 Regularisation Parameter Selection using the L-Curve Method

In Section 6.2, the regularisation parameter ($\lambda$) was chosen empirically through successive single-step reconstructions. This allowed the value of $\lambda$ that gave the least error between the reconstructed and the true profile to be selected. However, this way of finding a near-optimum regularisation parameter requires a priori knowledge of the exact solution, and hence it will have limited use when dealing with real reactor data where the exact solutions are unknown. Therefore, there needs to be another way of finding the optimum regularisation parameter.

In this section an attempt is made to determine the value of $\lambda$ by means of the L-curve technique discussed in Section 5.3.5. During the construction of the L-curve a sequence of non-linear reconstruction problems were solved using different values of $\lambda$ and the experimentally measured data. The norms of the regularised solutions and norm of the residuals were then plotted against one another on a log-log scale with the aim of determining the distinct corner value of the L-curve that gives the optimum regularisation parameter (Figure 6.8).
The L-curve plot in Figure 6.8 shows the general L-shaped curve, which has the expected features for increasing and decreasing value of $\lambda$. As discussed in Section 5.3.5 increasing the value of $\lambda$ means applying more filtering to the solution, which leads to an over-damped solution. In these situations the term $|| J(\sigma - \sigma_0) - M ||$ becomes more sensitive to small changes in the regularisation parameter $\lambda$ for all values of $\lambda$ that are less than or equal to the largest singular value $\lambda \leq \gamma_p$ (see section 5.3.4 and 5.3.5 for more detail). The largest singular value in this case was calculated as $\gamma_p = 3.98e-24$. Once the value of $\lambda$ becomes greater than $\gamma_p$ the term $|| L(\sigma - \sigma_0) ||$ becomes more sensitive to a small change in the value of $\lambda$ leading to a change in the direction by which the curve moves (in this case towards the vertical direction but moving in a descending direction).

On the other hand, when less filtering is applied on the solution (small values of $\lambda$ or $f_i = 1$), then the solution becomes less damped and the regularised solution $|| L(\sigma - \sigma_0) ||$ becomes more sensitive to small changes in the regularisation parameter, $\lambda$. However, the part of the L-curve in Figure 6.8 that shows the points corresponding the last two smallest values of $\lambda$ did not show the expected constant increase for every reduction in the value of $\lambda$. Instead both the residual and regularised solution norms became sensitive for these particular values of $\lambda$. Furthermore, the L-curve did not produce a distinct corner that corresponded to an
optimum value of $\lambda$ as the theory suggests. Nevertheless, the overall trend of the L-curve shows the range of $\lambda$ values for which the inverse problem can be solved to give a reasonable estimate of the expected true profile. The values that indicated the approximate L-curve corner range between $\lambda = 6e - 28$ and $\lambda = 9e - 29$. These values are generally in agreement with the value of $\lambda$ calculated in the trial-and-error fashion ($\lambda = 1e - 28$).

6.3.2 Unconstrained Inverse Solutions

The unconstrained inverse problems in this section were solved to see how close the RGN algorithm estimate the through-wall conductivity profile of the drilled graphite brick when the data were collected using the new EC sensor. In addition to this, the solutions of the unconstrained inverse problems were also used to determine the types of inequality constraints that could be applied when the unconstrained solution failed to estimate the expected through-wall electrical conductivity profile of the drilled graphite brick.

The inverse problems in this section were solved using two different initial estimates, also referred to as reference profiles throughout this thesis, namely $\sigma_{ref} = 72 kS/m$ and $\sigma_{ref} = 45 kS/m$ along with the NOSER and DF regularisation operators. The two reference values were mainly used to investigate how the solution was affected when the initial estimates $\sigma_{ref}$ was selected to be closer to or away from the expected true profile. The value of $\lambda$ for the DF operator was selected as $\lambda = 1.7e - 28$ (based on the L-curve results), whereas for the NOSER operator $\lambda$ was chosen as $\lambda = 1.7e - 15$ using a single-step inversion.

6.3.3 Results and Discussions

Figure 6.9 shows the unconstrained profiles of the drilled graphite brick along with the objective functions and profile errors. As can be seen in Figure 6.9a the reconstructed profiles do not exactly follow the expected through-wall electrical conductivity of the drilled graphite brick. In fact, they all show some reduction in the conductivity values estimated at 23 mm away from the graphite bore (3rd layer of the
discretised graphite brick domain in the FE model), and beyond before they start increasing for all conductivity values calculated beyond 51 mm (6th layer) from the bore. The trends for these profiles are also in agreement with the profiles reconstructed using synthetic data in Section 6.2.4, although the conductivity values are different from one another. The levels of the reductions away from the exact solution and the overall accuracies seen on the reconstructed profiles slightly vary depending upon the initial estimates as well as the type of operator matrix used within the algorithm. In this particular case, the closer the selection of the initial estimate \( \sigma_{\text{ref}} \) to the expected profile, then the better is the overall accuracy of the reconstructed profile and vice versa.

A closer look at the overall trends of the inverted profiles and the profile errors in Figures 6.9a and 6.9c reveals that the inverted profiles using DF and NOSER operators gave lower global profile errors (11.2 % and 12.5 %) when the initial estimate was chosen as \( \sigma_{\text{ref}} = 72 \, kS/m \) as compared with the profile errors (13.6 % and 16.3 %) when \( \sigma_{\text{ref}} = 45 \, kS/m \). The differences between the global profile errors when using identical operator matrices, but different initial estimates were therefore 2.4 % and 3.8 % for the DF and NOSER operators respectively. In contrast, the profile error differences for identical initial estimates but different operator matrices were 1.3 % and 2.7 % for \( \sigma_{\text{ref}} = 72 \, kS/m \) and \( \sigma_{\text{ref}} = 45 \, kS/m \) respectively.

However, the reconstructed bore conductivity values show the reverse of this effect. In other words, the errors between the reconstructed and actual bore conductivities are much more accurate when reconstructed using \( \sigma_{\text{ref}} = 45 \, kS/m \) than \( \sigma_{\text{ref}} = 72 \, kS/m \), but the differences in bore conductivity errors when using DF and NOSER operators (with an identical initial estimate) are more or less identical (see Table 6.3). The latter is in agreement with the reconstructed profile errors seen in Section 6.2.2, where the profiles were reconstructed using error-free synthetic data with an identical initial estimate, but using the DF and NOSER operators.

Although there are some differences between the reconstructed profile errors using different initial estimates, the overall change in these errors are small, in fact, they are within the range of few a percent. Considering that the profiles were
completely unconstrained and the model contained more than 2.8 % calibration error, these variations in the profiles reconstructed using different initial estimates should be considered as insignificant changes.

In general, the reconstructed bore conductivity values show reasonable estimates of the true bore value in all cases regardless of the regularisation operator matrices and initial estimates with an approximate mean and standard deviation error of 1.6 % and 1.4 % respectively. In contrast, the reconstructed global profile errors have mean and standard deviation errors of approximately 13.4 % and 2.1 % respectively.

In principle, the selection of different initial estimates should not affect the overall accuracies of the reconstructed profiles, except that small initial values may increase the computational time of the solution, provided that identical regularisation parameters and operator matrices are selected. Therefore, the problem in this study may be simply interpreted as a problem containing multiple local minima, and the reconstructed profiles as solutions obtained when the algorithm becomes trapped in one of the multiple local minima during the computation.

For the reconstruction problem involving small initial values (in this case 45 \( kS/m \)) the algorithm might have been trapped within a local minima much earlier than the minima responsible for the solution with the initial values closer to the exact solutions (72 \( kS/m \)). In these situations, the solution with an initial estimate equal to 45 \( kS/m \) may have been prevented from reaching the accuracies obtained when the initial value was chosen as 72 \( kS/m \). Closer examinations of the objective function plots in Figure 6.9b also show nearly constant trends after the 9\(^{th}\), 6\(^{th}\), 8\(^{th}\) and 7\(^{th}\) iterations in each case, suggesting that the algorithm may have become trapped within poor local minima during the computation of the solution. The corresponding profile error reductions after these iterations were also found to be insignificant (Figure 6.9c).
(a) Reconstructed profiles

(b) Objective functions
Based on these results, it can be concluded that the objective functions for unconstrained inverse solutions might contain poor local minima. This statement is made based on the fact that if the function was parabolic then any local minimum would also be the global minimum, and hence the solution should tend to the expected true profile. An optimisation problem with more than one local minima (non-parabolic problem) is one of the main challenges of the inverse problems associated with practical systems as briefly described in Section 5.1.3 and 5.1.4. One of the ways of dealing with this type of problem is to apply a priori knowledge in the form of constraint within the algorithm, and this will be demonstrated in the following section.

6.3.4 Constrained Inverse Solutions

The constrained inverse problem in this section can be seen as an extension of the unconstrained inverse problem discussed in Section 6.3.3. The extension to
the unconstrained inverse problem was made with a condition through which the inverse solutions were forced to tend to the expected true profile. The condition through which the algorithm forced the solution toward the expected profile come from a priori knowledge that we inferred about the drilled graphite brick through-wall density and most importantly from the behaviour of the unconstrained inverse solutions.

At this point it is well known that the drilled graphite brick electrical conductivity varies from the bore to periphery in an approximately linear fashion due to the arrangement of the holes, and also the unconstrained solution can estimate the bore conductivity with a high degree of accuracy. Therefore, we had enough a priori information that can be transferred into the solution using a new constraining technique. Based on this information the optimisation problem given in equation 5.57 was modified as:

\[
g(\sigma) = \arg \min_{\sigma} \left\{ \frac{1}{2} \| r(\sigma) \|_2^2 + \frac{1}{2} \lambda \| L(\sigma - \sigma_{\text{ref}}) \|_2^2 \right\} \quad \text{for } \sigma_{k+1} > \sigma_k \quad \text{(Eq 6.2)}
\]

where the subscript \(_L\) indicates the discretised layers of the graphite brick forward model, which correspond to the local conductivity values through the brick radial thickness.

The condition in equation 6.2 was enforced using two different techniques within the inversion algorithm; these were Linear Constrain (LC), and Monotone Constrain (MON). The MON condition was set based on the difference between the neighboring local conductivity values, whereas the LC required a predetermined constant. The general arrangement of the constrained reconstruction was as follows.

Throughout the reconstruction process first the step direction of the conductivity values at the \(k\)th iteration was calculated using the algorithm given in equation 5.57, and the solution was checked if it satisfied the desired constraint in equation 6.2. If \(\sigma_{k+1} > \sigma_k\) then the unconstrained solution was accepted and the iteration continued until the exit condition was satisfied \((g(\sigma_{k+1}) > g(\sigma_k))\). However, if \(\sigma_{k+1} < \sigma_k\) the monotonicity condition was not maintained, and hence the calculated value was rejected and replaced with either LC or MON to satisfy the
condition $\sigma_{L+1} > \sigma_L$. These values was then used for the next iteration and checked to see if the objective function $g(\sigma_k)$ was minimised. If $g(\sigma_{k+1}) < g(\sigma_k)$ the iteration will continue with the new conductivity values, otherwise the algorithm will terminate. The simple algorithms used to enforce the above conditions are shown in Figure 6.10a and 6.10b for LC and MON respectively along with a simplified block diagram of the entire reconstruction algorithm in Figure 6.10c. It should be noted that both of these constraining techniques enforced monotonicity of the reconstructed profile, and had similar effects to the monotonicity condition discussed in Section 5.1.4.

### Linear Constraint

$n \leftarrow \text{number of discretised domains}$

$d = 1.9e3$

```python
for L = 1:n - 1
    if $\sigma_{k+1}(L + 1,:) \leq \sigma_{k+1}(L,:)$
        $\sigma_{k+1}(L + 1,:)=\sigma_{k+1}(L,:)+d$
    end
end
```

(a) Linear constraint

### Monotonic Constraint

$n \leftarrow \text{number of discretised domains}$

$A = \text{diff}(L_{\sigma})$

```python
for L = 1:n - 1;
    if $\sigma_{k+1}(L + 1,:) \leq \sigma_{k+1}(L,:)$
        $\sigma_{k+1}(L + 1,:)=\sigma_{k+1}(L,:)+abs(A(L,:))$
    end
end
```

(b) Monotone constraint based on the differences between neighbouring graphite layers
(c) Simplified reconstruction algorithm block diagram.

Figure 6.10: The constraining techniques applied in the reconstruction algorithm and the simplified reconstruction algorithm block diagram. Note the blue arrow path in the block diagram indicates a one-step inversion whereas the red arrows indicate the non-linear iterative process.

6.3.5 Results and Discussion

Figure 6.11 shows the constrained solutions of the reconstruction problem along with the profile errors for both LC and MON constraints. The constrained reconstruction problems in this section were solved using the same bulk electrical conductivity initial estimates, regularisation parameters and regularisation operator matrices as in the unconstrained reconstruction problems. However, due to the conditions imposed on the computed solutions, Figure 6.11a shows much more accurate profiles, with the profile errors ranging between 1.32 % and 9.3 %. Furthermore, unlike the unconstrained profiles discussed in Section 6.3.3, each of the constrained profiles show trends which are comparable with the expected true profile for most of the studied cases, although some variations in conductivity around the brick periphery are present when applying the MON constrain that used the NOSER operator within the algorithm.
Figure 6.11: The constrained inverse solutions of the drilled graphite brick reconstructed using Linear (LC) and Monotone (MON) constraints.
Comparisons between the reconstructed profiles using LC and MON constraints show that the LC technique gave less profile error compared with the MON except in one case where the solution was obtained using the DF operator with the initial estimate value equal to 45 kS/m. Nonetheless, the global error difference even in this case was much smaller than 1% (see Table 6.2). The LC gave smaller errors than the MON for all other reconstructed profiles of the drilled graphite brick. This is because the expected profile trend of the drilled graphite brick is linear due to the arrangements of the radially drilled holes through the graphite brick. The reason that the reconstructed conductivity values gave slightly higher errors for LC (in the case where the solution was obtained using the DF operator with an initial estimate $\sigma_{ref} = 45 \text{kS/m}$) lies mainly in the accuracy of the original unconstrained solution. In other words, the constraining techniques discussed in Section 6.3.4 were only enforced when the unconstrained solution lost its monotonicity. Hence, the unconstrained solution was always accepted regardless of its value when the previous local electrical conductivity value was less than the current local electrical conductivity ($\sigma_{L+1} > \sigma_L$). This means that the solution for the constrained inverse solution, in this particular case using the LC, might have inherited the errors from the unconstrained solution (mainly due to constant incremental procedures). On the other hand, the MON constraint used the difference between the neighbouring local conductivity values to maintain monotonicity, which also meant that only the absolute value of the unconstrained step size was used to maintain the monotonicity condition. This in turn allowed the MON constraining technique to avoid the use of user-defined values, and hence this could have influenced its accuracy around the graphite bore in this particular case.

In addition to producing reasonable estimates of the true profile, the constrained solutions have also reduced the number of iterations required to obtain the final solution to a maximum of 4 iterations in the case of LC and 7 iterations in the case of the MON. By contrast, the unconstrained solutions required between 8 and 16 iterations to obtain the final solution. This means that the overall computational cost for the problem is reduced by an average of $\approx 66\%$ and $\approx 53\%$ when applying LC and MON respectively. These reductions in the computational
cost are the direct effect of the constraints imposed on the solutions, which force the algorithm to move towards the exact solution whilst preventing it from becoming trapped in a poor local minimum.

6.4 Summary

This chapter has presented studies of the graphite conductivity reconstruction problem using both simulated and experimental data, along with a new approach adopted to deal with the non-linear and ill-posed nature of the inverse EC problem. Due to the linearity of the true solution in this case (drilled graphite brick), the LC produced the most accurate profiles compared with the MON constraint when reconstructed using different initial estimates and regularisation operator matrices. Table 6.2 shows a summary of the key parameters and the reconstructed profiles of the drilled graphite brick presented in Sections 6.3.3 and 6.3.5.

The constraining technique adopted in this chapter mostly relies on a priori knowledge of the expected profile trend, which also means that the user needs to have some insight into the expected conductivity distribution of the graphite sample under investigation, prior to reconstruction. One of the ways to achieve this is by solving a series of unconstrained problems to initially see how accurately the algorithm estimates the expected solution, and imposing the appropriate constraining technique. In this case the unconstrained bore conductivity of a drilled graphite brick was estimated with a reasonable degree of accuracy in all cases, and it was also known that the sample brick electrical conductivity varied in approximately linear fashion between bore and the periphery. This is why the monotonic constraints were adopted throughout the reconstruction process. The monotonically increasing constraints in this study may be described as the technique that limits the solution space and hence reduces the number of local minimum traps that may lead to errors in the solution.

However, this a priori information is not always available when dealing with real reactor data, and may give rise to additional challenges. Nevertheless, the historic density data collected from trepanned samples taken from in-core graphite
brick could be used as a priori. Therefore, it may be possible to incorporate this information into the reconstruction algorithm through an appropriate constraining technique (see Section 7.1.2).

<table>
<thead>
<tr>
<th>Reconstructed profiles</th>
<th>Reference profiles</th>
<th>Operator Matrix</th>
<th>Regularisation parameters</th>
<th>Bore profile errors</th>
<th>Final number of iterations</th>
<th>Final global profile errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>72 kS/m</td>
<td>DF</td>
<td>1.7e-28</td>
<td>3.2 %</td>
<td>12</td>
<td>11.2 %</td>
</tr>
<tr>
<td>Constrained (L)</td>
<td>72 kS/m</td>
<td>DF</td>
<td>1.7e-28</td>
<td>0.48 %</td>
<td>4</td>
<td>3.01 %</td>
</tr>
<tr>
<td>Constrained (M)</td>
<td>72 kS/m</td>
<td>DF</td>
<td>1.7e-28</td>
<td>1.2 %</td>
<td>4</td>
<td>4.7 %</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>72 kS/m</td>
<td>NOSER</td>
<td>1.7e-15</td>
<td>2.2 %</td>
<td>9</td>
<td>12.5 %</td>
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<tr>
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<td>72 kS/m</td>
<td>NOSER</td>
<td>1.7e-15</td>
<td>2.52 %</td>
<td>3</td>
<td>1.32 %</td>
</tr>
<tr>
<td>Constrained (M)</td>
<td>72 kS/m</td>
<td>NOSER</td>
<td>1.7e-15</td>
<td>2.9 %</td>
<td>3</td>
<td>6.1 %</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>45 kS/m</td>
<td>DF</td>
<td>1.7e-28</td>
<td>0.2 %</td>
<td>16</td>
<td>13.6 %</td>
</tr>
<tr>
<td>Constrained (L)</td>
<td>45 kS/m</td>
<td>DF</td>
<td>1.7e-28</td>
<td>5.2 %</td>
<td>4</td>
<td>4.78 %</td>
</tr>
<tr>
<td>Constrained (M)</td>
<td>45 kS/m</td>
<td>DF</td>
<td>1.7e-28</td>
<td>1.9 %</td>
<td>7</td>
<td>4.76 %</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>45 kS/m</td>
<td>NOSER</td>
<td>1.7e-15</td>
<td>0.6 %</td>
<td>8</td>
<td>16.3 %</td>
</tr>
<tr>
<td>Constrained (L)</td>
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<td>NOSER</td>
<td>1.7e-15</td>
<td>4.02 %</td>
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<td>2.73 %</td>
</tr>
<tr>
<td>Constrained (M)</td>
<td>45 kS/m</td>
<td>NOSER</td>
<td>1.7e-15</td>
<td>3.3 %</td>
<td>6</td>
<td>9.3 %</td>
</tr>
</tbody>
</table>

*Table 6.2: The summary of the reconstructed profiles using experimentally measured data, and the key parameters used within the RGN reconstruction algorithm. Note the letters in the bracket (LC) and (M) indicate the Linear and Monotonic constraints applied within the reconstruction algorithm.*

The general assumptions for the AGR brick radial density/conductivity variations are normally increasing trends between the graphite bore and periphery. This is generally a reasonable assumption as the exposure of the graphite bricks to radiation and the likelihood of the $CO_2$ coolant being activated by the gamma radiation is higher at the brick bore region than at the periphery. However, this assumption is not always guaranteed, and our industrial partners have indicated that there may be limited cases were the density profile (and hence the conductivity profile) may show a non-monotonic trends.
Chapter 7

7 Reactor Bricks Conductivity Profiling

This chapter presents the methodology and results for reconstructed profiles taken from measurements of AGR fuel channel bricks. The radial through-wall profiles of the AGR fuel channel bricks were reconstructed using measurements taken during the in-core inspection of two different reactors. At the time of writing, the new EC probe (discussed in Chapter 4) was still under construction; therefore all of the reactor measurements were obtained using the existing 70 mm diameter symmetrical gradiometer probe.

The reconstructed fuel channel brick profiles were compared to resistivity measurements from trepanned samples taken out of the reactor core at the locations where the mutual inductance data were collected. A further comparison of the reconstructed profiles with measurements from the EC system that is currently used by EDF Energy was also made to see how the reconstructed profiles compared with the bulk resistivity predictions from that system in terms of the top and bottom of the brick resistivity values of individual bricks. The latter comparisons were treated as an additional validation of the reconstructed profiles.

7.1 Methodology

Following the preliminary studies presented in Chapter 6, multiple inductance data have been collected from AGR fuel channels. These data were collected during the 2016 outage at Hinkley Point B (HPB) power station and the 2017 outage at Torness (TOR) power station. In total, three channels’ worth of data were acquired, consisting of 40 sets of multi-frequency data at prescribed trepanning positions at the top and bottom of bricks in each channel.

The two reactors contain bricks that differ slightly in geometry, and therefore two different forward models were developed using the COMSOL Multiphysics software package. The forward models were then calibrated using a new calibration
technique with measurements taken from a calibration brick with known bulk electrical conductivity to make the forward models as accurate as possible.

Throughout the reconstruction process both the RGN and RLM algorithms have been employed to invert identical sets of data. The reason behind this was to investigate and compare the robustness of each algorithm when dealing with actual reactor data through the evaluation of their accuracy. As discussed in Chapter 6, the RGN algorithm is similar to the algorithm used by Fletcher et al [6], but extended further to make it easier to incorporate various types of constraints. On the other hand, the RLM algorithm was formulated specifically for the graphite inverse problem as part of this project. This is not the first time that the RLM algorithm has been used to deal with inverse EC problems. In fact, it has previously been used to reconstruct the shapes of low-conductivity objects, and has shown a good convergence rate and accuracy compared to other reconstruction algorithms [71]. In this thesis, the RLM algorithm has been further formulated to make it suitable for the higher conductivities encountered in this graphite inverse problem and compared with the RGN in terms of profile accuracy.

7.1.1 AGR Graphite Brick Model Calibration

As discussed in Sections 6.1 and 6.2, the accuracy of the forward model is vital in obtaining a good estimate of the graphite conductivity profiles. This means that the model needs to be accurately calibrated to avoid the contributions of calibration errors to the reconstructed profiles, and improve its accuracy.

Due to the variation in test conditions and brick arrangements, the forward model for each reactor brick need to be calibrated using the measurements from a brick with known electrical conductivity measured just before the core inspection. The calibration data for the HPB reactor brick were not available at the time of the reconstruction process. For this reason, the HPB forward model was calibrated using the model calibration parameters extracted from different reactor during previous work [6]. On the other hand, the calibration data for the Torness forward model were available on time, and the profiles were reconstructed using a newly-calibrated forward model.
Unlike the model calibration method discussed in Section 6.1.1, the forward model for the Torness brick was calibrated using a correction factor to reduce the differences between the forward model and the measurements from a brick with 91 kS/m bulk electrical conductivity (the calibration brick located in the hoist (Figure 7.1)). The conductivity value for the calibration was already determined using a four-point measurement method. A Megger DLRO 10X instrument that has a pin spacing of 8 mm was used to measure the bore and periphery conductivity values, on the assumption that the brick will have homogenous conductivity value through the radial wall. Two different model calibration techniques were attempted during the calibration process. These methods are explained in the following paragraphs.

For both calibration methods, a forward model was first simulated using an identical bulk electrical conductivity value to the calibration brick in the PECIT hoist, and the calibration parameters extracted from the previous work [6]. The measured data were then compared to the model data. In the first method, the differences between the model and the measured data were reduced by multiplying the measured imaginary mutual inductance data by a correction factor. The value of the correction factor (in this case 1.26) was chosen in a trial and error fashion until the error between the model and the measured data approached a minimum. Initially, an attempt was made to reduce the differences in both the real and imaginary parts of the mutual inductance data using a single-valued correction factor. However, due to
the distributions of the errors in the measured real mutual inductance data it was not possible to reduce the overall error below 6.5 %. For this reason, the calibration process in the first method was mainly focused on the imaginary part of the mutual inductance. This process gave a significant reduction in error between the model and the measured data, particularly for the imaginary part of the mutual inductance data. Figures 7.2a and 7.2b show comparisons between the model and measured mutual inductance data plots before and after calibration using the first method.

![Diagram](attachment:image1.png)

(a) Data pre-calibration

(b) Data after calibration using a correction factor of 1.26.

Figure 7.2: The mutual inductance plots of the data from the Torness calibration brick against the FE model when calibrated using a single correction factor.
The second method was based on a complex calibration factor, which was a function of frequency, and had different calibration values for each data point. In this method the differences between the model and the measured data from a brick with known electrical conductivity were used to correct the in-core mutual inductance data at each frequency point. This method allowed the differences between the measured and the model data to be reduced to zero.

Following the calibration process, it was necessary to test whether the methods adopted for the calibration yielded the expected constant conductivity profiles for the calibration brick. Figure 7.3 shows reconstructed profiles of the calibration brick after applying the above correction factors on the data, and using these data as an input within the RLM algorithm.

As can be seen in Figure 7.3, the final solution for the reconstructed profile using the partially calibrated model (when a single-valued correction factor is applied in the imaginary part of the data) tends to have a slightly decreasing trend from the true profile, particularly at distances away from the graphite bore. In this case the global profile error for the final solution is 3.2 % compared with the true profile. The presence of the 3.2 % error in the reconstructed profile can be attributed to the residual errors in the data. By contrast, the profile reconstructed using the complex correction factor shows a flat conductivity profile as expected, with a global error of much less than 1 %, giving confidence about the calibration method adopted for reactor data inversion.

![Figure 7.3: The reconstructed profiles for the Torness calibration brick after calibrating the model using both single-valued and complex multi-frequency correction factors. Note: the calibration brick has a constant bulk conductivity of 91 kS/m and the reconstructions were made using the measurements from this brick.](image-url)
The above inverse solutions essentially commit the so-called “inverse crime”. This is because the differences between the model and measured data have already been minimised prior to the reconstruction process. In the case of the first method, a partial inverse crime (on the imaginary part of the data) has been committed, whereas the second method has committed a full inverse crime. Nevertheless, the purpose of this study was only to test how well the calibrated model produced the expected profile, so that the selected correction factor could be applied to the in-core measurements during the reconstruction process. In principle, this should increase the accuracy of the inverse solution, as the model would then be fully calibrated against data collected using the complete measurement system and a brick with a well-defined conductivity value.

7.1.2 AGR Graphite Brick Conductivity Profiling

The AGR graphite inverse problems in this chapter were solved using both the RGN and RLM algorithms. Due to the absence of calibration data for the HPB bricks at the time of the reconstruction process, the HPB brick profiles were reconstructed using the MON constraining technique presented in Figure 6.10b, which ensured monotonically increasing local electrical conductivity values between the graphite bore and periphery.

The exact electrical conductivity profile trend of the AGR brick is unknown. The main reason for implementing this type of constraint within the algorithm was based on an examination of historic density measurements from trepanned samples taken from the graphite bricks, which showed increasing density trends from the bore to periphery in approximately a semi-quadratic fashion. This observation suggested that any reduction in electrical conductivity or increase in electrical resistivity as a function of depth from the graphite bore to the periphery had to be rejected. Therefore, it was necessary to implement the MON constraint to minimise the effect of any abnormalities in the data transferring into the reconstructed profiles. The complete algorithms of RLM and RGN are included in Appendices A and B respectively.
As discussed in Section 5.4, one of the main differences between the RGN and RLM approaches is that the RLM incorporates an additional damping parameter that can be updated during the iterations based on evaluation of the gain factor. Furthermore, unlike the RGN, the RLM algorithm does not terminate immediately when the objective function is not reducing; instead the algorithm updates the damping parameter with the aim of trying to achieve the desired decrease in the objective function, at least five times in this case. On the other hand the RGN uses a fixed regularisation parameter and terminates immediately when the objective function fails to decrease. This allows the RLM algorithm to switch between the RGN and GD method during the computation of the unknown solution (see Section 5.4 for more detail).

During the conductivity reconstruction process, the damping parameter $\gamma$ was initially chosen to have a large value to make the algorithm act as the GD method. This value was then reduced during the iterations, particularly when the estimated solution approached the final solution, and hence made the RLM act more like the RGN. In most cases, at the final stage, when the estimated solution approached a minimum, the damping parameter $\gamma$ was increased to prevent divergence of the estimated solution from the minimum (in this case the conductivity vector that best represented the measured data). This also meant that when the solution approached a minimum or the conductivity vector that best represented the measured data, the RLM could switch back to GD to maintain a small step size. In general, this process led to an increased number of iterations to arrive at the final solution, but expected to yield better accuracy than the RGN approach. The speed of the RLM algorithm in this thesis was improved with an implementation of conditions that dealt with small step changes that were mainly expected to occur during the final stages of computation. The conditions implemented were step and function tolerances, which ensured termination of the algorithm when the changes in the calculated conductivity values and objective functions were below some user-defined values. One of the main advantages of the RLM method is that the value of the regularisation parameter $\lambda$ does not need to be optimal. Instead the lack of optimality (if $\lambda$ is chosen badly) can be compensated through the damping parameter. In addition, the forward model geometries of the
AGR bricks were reduced to a one-quarter symmetry, which contributed to a reduction in the overall computational demand needed to arrive at the final solutions for the graphite conductivity / resistivity profile.
Figure 7.4 shows the one-quarter symmetry of the graphite brick model geometry used during the reconstruction process. It should be noted that for simplicity only one of the two AGR brick model geometries is presented here, although each brick had different methane hole and key-way arrangements.

### 7.2 Results and Discussion for HPB Profiles

Figures 7.5 and 7.6 show the reconstructed profiles of the fuel channel bricks for HPB using both the RGN and RLM algorithms, and also the PECIT C-scan images at the particular frequency of 8 kHz. The PECIT imaging system uses software known as ASPECT, which converts the raw voltage signal from the eddy current instrument into an electrical resistivity value [156].

The resistivity profiles were reconstructed using data measured at the tops and bottoms of six individual bricks within a fuel channel, and compared with the C-scan images and trepanned sample measurements supplied by EDF Energy. It should be noted that the plotted C-scan image measurements in Figures 7.5 and 7.6 were...
acquired from the same fuel channel using an 8 kHz excitation signal, which means that the images mostly represent the near-surface resistivity values of the graphite bricks, and should only be compared with the reconstructed near-bore profiles. Although these C-scan images do not show the resistivity variations through the radial thickness of the graphite bricks, they do give a good indication of the resistivity variations around the circumference of the brick bore including the resistivity variations between the top and bottom of each individual brick. Therefore, a comparison of the reconstructed profiles with the C-scan images in terms of top and bottom resistivity values should give some insight into the performance of the reconstruction algorithms adopted in this thesis.

Each of the C-scan images in Figures 7.5 and 7.6 (right) represents a single graphite brick layer approximately 1 metre tall on the y-axis, while the x-axis represents the circumferential position round the brick which covers a full 360°. The small dashed axial and circumferential ellipses on the C-scan images indicate the approximate locations where the inductance data were acquired for the reconstruction of the brick profiles shown on the left in each Figure. Therefore, when comparing the reconstructed profiles with the C-scan images only the overlapping areas of the two dashed ellipses should be considered.

Qualitative comparisons of the reconstructed profiles with the C-scan images show reasonable agreement in terms of the top and bottom graphite electrical resistivity. This can be clearly seen by comparing the overlapped area of the dashed ellipses at the tops and bottoms of the C-scan images against the reconstructed near-bore profiles (within ≈20 mm distance from the bore). The profiles reconstructed using both RGN and RLM algorithms may not exactly match the actual data from the C-scan images; in fact, the two datasets can only be compared qualitatively, unless the data are extracted from the images. Nevertheless, the trends between the reconstructed top and bottom near-bore profiles completely agree with the C-scan images in all cases, at least qualitatively. These results suggest that both algorithms were operating as they should when a priori constraint (in this case the monotonic constraint shown in Figure 6.10b was applied within the two algorithms.
In terms of the reconstructed AGR brick radial profiles, the RGN seems to produce the largest variations in conductivity values between bore and periphery in all cases, when compared to the profiles reconstructed using the RLM algorithm. These differences should be expected because the RLM algorithm incorporated an additional damping parameter that could be updated during the iterations, whereas the RGN only used a constant regularisation parameter.

A closer look at the profiles reconstructed using RGN (Figure 7.5) also shows that the periphery resistivity values settle around identical values for every top and bottom brick profile in each graphite layer, whereas the reconstructed bore profiles show distinct features between one another. Similar profile trends can also be seen in Figure 7.6 where the profiles were reconstructed using the RLM method. However, in this case the differences between the bore and periphery resistivity values are much smaller than the values reconstructed using the RGN method. For instance, the RLM profile in POS 1 Top has a difference between bore and periphery of approximately $1600\mu\Omega\cdot cm$, compared with $2500\mu\Omega\cdot cm$ in the case of RGN.

Generally speaking the graphite weight/density loss in AGR brick is expected to be higher at the bore than at the periphery; this is mainly due to the higher rate of diffusion of the coolant and the amount of radiation dose in the graphite bore regions. Since the oxidation rates are expected to be lower around the periphery of the graphite brick, higher density (smaller resistivity) values should be expected in these regions. In addition, the profiles around the graphite brick periphery are expected to have small resistivity variations with distance due to the slow oxidation rate. This is indeed seen in the reconstructed profiles in Figures 7.5 and 7.6. In general, the reconstructed profiles using both RGN and RLM broadly agree with the trepanned samples electrical resistivity values. The trends from these profiles suggest that both algorithms have captured the density variation feature of the oxidised AGR graphite brick. Therefore, it is believed that the two algorithms operated as they should when a monotonic constraint is applied to them to reconstruct the electrical resistivity profiles of the bricks from this particular station.
Figure 7.5: The reconstructed resistivity profiles of the fuel channel bricks using data from 2016 HPB reactor outage and constrained RGN algorithm with DF operator matrix, \( \lambda = 1e^{-28} \) and \( \sigma_{ref} = 35 \text{ kS/m} \) (left), and PECIT images at 8 kHz (right).
Figure 7.6: The reconstructed resistivity profiles of the fuel channel bricks using data from 2016 HPB reactor outage and constrained RLM algorithm with DF operator matrix, $\lambda = 1e - 2B$ and $\sigma_{ref} = 35 \, k\Omega\cdot m$ (left), and PECIT images at 8 kHz (right).
In order to quantify the performance of the RGN and RLM algorithms and also to validate the reconstructed profiles, it was necessary to compare some of the reconstructed profiles with resistivity values from trepanned samples supplied by EDF Energy. These resistivity values were deduced from density measurements (using EDF Energy’s power law correlation method, see [156] for more detail) taken on the 75 mm long trepanned cylindrical samples machined out of the same channel locations where the inductance data were acquired.

Figures 7.7a to 7.7d show comparisons between some of the reconstructed profiles and the resistivity measurements from the trepanned samples. Two main features can be seen in these plots.

![Comparison graphs](image)

Figure 7. 7: Comparison between the inverted profiles using the constrained RGN and RLM algorithms and the trepanned sample resistivity measurements from the 2016 HPB rector outage. Note the trepanned samples resistivity values were deduced from the density measurements.

The first is that the monotonic nature of the trepanned resistivity measurements as a function of depth from the graphite bore validates (at least
partially) the suitability of the monotonic constraints applied within the algorithms during the reconstruction process. The second and more important feature is that both of the reconstruction algorithms have produced practical profiles with the correct order of magnitude, which are comparable with the measured resistivity values of the trepanned samples throughout their entire depth profile.

Nevertheless, looking into the details of each plot in Figure 7.7 reveals that the bore resistivity values of the trepanned samples at Position 1 Top (POS 1T) and Position 4 Bottom (POS 4B) are larger than those reconstructed from the inductance data measurements.

The differences in bore resistivity values between the measured and reconstructed profiles may be due to different factors. One of the factors could be the amount of graphite brick volumetric information captured from the trepanned samples. In other words, the trepanned samples covered only a small Ø19 mm by 75 mm cylindrical section of the graphite bricks, whereas the EC measurements interrogated a larger proportion of the graphite volumetric information because of the spatial extent of the eddy current flow inside the brick. Since the radiolytic oxidation of the AGR bricks has a localised effect, the measured bore density/resistivity from the cylindrical trepanned samples might not be exactly identical to the overall bore density/resistivity values. Furthermore, the resistivity values of the trepanned samples are not directly measured; instead these values are calculated from density measurements using a simple power law function. Therefore, this might have contributed some level of error to the estimated resistivity data of the trepanned samples.

On the other hand, the use of historic calibration parameters to calibrate the graphite forward model for the bricks of this particular station might have introduced some errors in the reconstructed profiles. Since the inverse solution is highly dependent upon the accuracy of the forward model, this may have contributed towards the differences between the trepanned samples and the reconstructed resistivity profiles.

Nevertheless, the RGN algorithm has estimated the trepanned sample bore resistivity slightly better than the RLM algorithm, although the trepanned sample bore resistivities have some noticeable differences when compared with the
reconstructed bore profiles using both algorithms, particularly in the case of POS 1T and POS 4B.

As the distance away from the bore increased beyond \(\approx 12\) mm the RLM algorithm estimated the brick resistivity with much higher accuracy than the RGN for all of the cases shown in Figure 7.7. In principle, it would be expected that the algorithm would produce more accurate estimates in the bore region than at the periphery due to the higher sensitivity of the ECs to conductivity variations at the measurement boundary (graphite bore region). However, as mentioned earlier the localised effects of the radiolytic oxidation could have resulted in higher density losses at the locations where the trepanned samples were extracted, and hence led to higher bore resistivity values. The EC measurements covered a larger bore area, which were averaged throughout the reconstruction process, and could have resulted in reduced resistivity values compared to the bore resistivity data of the trepanned samples. As the distance from the graphite bore increases, the rate of radiolytic oxidation reduces, and hence any localised effects (density variations) may be much lower than at the bore. This could lead to near-constant density/resistivity variations at a given distance from the graphite bore over 360 degrees of the graphite brick. This would also mean that the resistivity values of the trepanned samples at a distance away from the bore were now comparable with the reconstructed profiles, which are assumed to have constant resistivity values over each graphite layer.

Tables 7.1 and 7.2 show the comparisons between the mean values of the trepanned sample measurements and the reconstructed profiles using RLM and RGN algorithms respectively. Despite the differences in some of the bore resistivity values, the RLM algorithm has produced reasonable estimates of the trepanned sample resistivity trends, for this particular station. The accuracy achieved when using the RLM algorithm can be clearly seen in Table 7.1 where the mean profile errors range between 1.5 % and 16.9 %. On the other hand, the RGN has produced slightly better estimates of the trepanned bore resistivities, but it suffered from inaccuracy as the distance from the bore increased with the mean profile errors ranging between 19.1 % and 31.8 %. These differences in the reconstructed profiles accuracy clearly indicate the advantage of RLM algorithm over the RGN.
<table>
<thead>
<tr>
<th>Inspected Locations</th>
<th>Mean Profile Value RLM (µΩm)</th>
<th>Mean Trepanning Measurement (µΩm)</th>
<th>Mean Profile Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>POS 1T</td>
<td>2687.50</td>
<td>3132.20</td>
<td>14.20</td>
</tr>
<tr>
<td>POS 3B</td>
<td>2812.50</td>
<td>2855.50</td>
<td>1.50</td>
</tr>
<tr>
<td>POS 4B</td>
<td>2779.40</td>
<td>3343.10</td>
<td>16.90</td>
</tr>
<tr>
<td>POS 6B</td>
<td>2528.80</td>
<td>2951.60</td>
<td>14.30</td>
</tr>
</tbody>
</table>

Table 7.1: Comparisons between the mean values of the trepanned sample measurements and the reconstructed profiles using RLM algorithm.

<table>
<thead>
<tr>
<th>Inspected Locations</th>
<th>Mean Profile value RGN (µΩm)</th>
<th>Mean Trepanning Measurement (µΩm)</th>
<th>Mean Profile Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>POS 1T</td>
<td>2204.00</td>
<td>3132.20</td>
<td>29.60</td>
</tr>
<tr>
<td>POS 3B</td>
<td>2310.00</td>
<td>2855.50</td>
<td>19.10</td>
</tr>
<tr>
<td>POS 4B</td>
<td>2280.60</td>
<td>3343.10</td>
<td>31.80</td>
</tr>
<tr>
<td>POS 6B</td>
<td>2094.60</td>
<td>2951.60</td>
<td>29.00</td>
</tr>
</tbody>
</table>

Table 7.2: Comparisons between the mean values of the trepanned sample measurements and the reconstructed profiles using RGN algorithm

The only drawback of the RLM algorithm is that it takes a slightly longer time than the RGN to arrive at the final solution. This is mainly due to the computations involved in the trials to ensure that the objective function is always reducing through the continual updating of the damping parameter.

### 7.3 Results and Discussion for Torness Profiles

Following the model calibration discussed in Section 7.1.1, the Torness brick resistivity values were reconstructed. In total, two channels (labelled as Channel X and Channel Y) of data measured from the top and bottom of the individual bricks in each channel have been reconstructed. The mutual inductance data measured at each location were corrected using the two calibration factors discussed in Section 7.1.1 during the reconstruction process. These data were then used as an input within the RLM reconstruction algorithm. This process helped to eliminate the need to apply a constraint to the reconstruction problem, as the model was now fully calibrated with measurements from a brick with known bulk conductivity using the complete measurement system (see Figure 7.1).
Figure 7.8 shows the reconstructed resistivity profiles using the two calibration methods on the data measured from the top and bottom of the individual graphite bricks in Channel X, whereas Figure 7.9 shows the reconstructed resistivity profiles of the bricks in Channel Y.

As can be seen in these figures, the reconstructed profiles have the highest resistivity values (highest density losses) in the bore region, and tend towards 1500 $\mu\Omega \cdot \text{cm}$ ($66.7 \text{kS/m}$) around the brick periphery. This behaviour of the reconstructed resistivity profiles is qualitatively in agreement with the generally expected profile from an irradiated AGR brick. However, the variability between the graphite bricks from station to station coupled with the slightly different brick to brick mating arrangement and its influence on the pressure differential through the brick wall, some of the irradiated graphite bricks from this particular station were expected to have non-monotonic density variations.

The reconstructed profiles using the two calibration methods showed the same profile trends for both methods. However, the reconstructed resistivity profiles from the model calibrated with complex multi-frequency correction factors produced small variations between the graphite bore and periphery resistivity values in all cases, although they both settled at approximately identical periphery resistivity values. This also means that the bore resistivity values with the multi-frequency correction factors were smaller than those reconstructed using a constant correction factor on the imaginary part of the reactor data. The differences between these profiles can be attributed to the residual calibration errors in the data when a constant calibration factor was used. The complex multi-frequency calibration factors have zero error in the data, and therefore the reconstructed profiles using this method should be relied on more compared with those reconstructed using a constant correction factor.
(a) Top profiles in channel X reconstructed using constant valued calibration factor

(b) Bottom profiles in channel X reconstructed using constant valued calibration factor

(c) Top profiles in channel X reconstructed using complex multi-frequency calibration factors
(d) Bottom profiles in channel X reconstructed using complex multi-frequency calibration factors

Figure 7.8: Reconstructed profiles of the AGR graphite bricks in Channel X using reactor data measured during the 2017 outage of Torness reactor

(a) Top of the brick profiles within channel Y reconstructed using constant valued calibration factor

(b) Bottom profiles in channel Y reconstructed using constant valued calibration factor
As mentioned in Section 7.2, the RLM algorithm takes more time to arrive at the final solution, but produces better estimates of the expected resistivity profiles. The overall computational speed of the reconstruction process can be improved by simplifying the forward model, which also means removing graphite model features that will not have a significant impact on the overall accuracy of the solution. One of these simplifications could be removing the key-way slots from the graphite forward model through a detailed sensitivity study to determine their effect on the reconstructed solution. Since the EC flow at a circumferential position round the brick 180 degrees opposite to the probe location in the graphite forward model is...
negligible, reducing the graphite brick geometry to a half-brick model could also introduce further simplification to the forward model geometry. This would mean that the semi-cylindrical section of the graphite brick could be further simplified with its quarter symmetry, which would reduce the number of elements by a significant amount. In principle this could greatly increase the reconstruction speed and should be considered in future studies.

The monotonic constraining technique employed in Section 7.2 uses the difference between the estimated neighbouring local conductivities to maintain monotonicity. Although this method is proven to work reasonably, it can only be used in the cases where the solution is expected to have monotonic trend.

The results presented in this chapter showed reasonable agreement with the trepanning measurements and PECIT predictions. However, there are some uncertainties in the approaches adopted to validate the reconstructed profiles. The main limitation of this approach is that the comparison of the Ø19 mm trepanned sample measurements with the reconstructed profiles using multi-frequency EC data. This is because the inspected graphite volume by the two systems is different in size, and could lead to errors when comparing the two. In addition, the trepanned samples extend only up to 75 mm, whereas the actual graphite radial thickness is ≈ 95 mm, hence the reconstructed profiles for the last 20 mm could not be validated using the trepanned sample measurements, even if they were fully comparable.

Nevertheless, there is currently no other means of validation of the reconstructed resistivity profiles, and hence the trepanning sample measurements are the only means of validation for the resistivity profiles studied in this chapter.

7.5 Summary

This chapter has presented studies concerning the AGR fuel channel brick reconstruction problem, along with two different forward model calibration methods. The first calibration method used a single-valued correction factor, but was not able to reduce the errors between the model and the measured data below 4.4 %. The second calibration method was based on complex calibration factors which are a function of frequency, and was able to reduce the data errors to zero.
The graphite inverse problems in this chapter were solved using real data measured from the reactor cores of two different power stations. Two different non-linear reconstruction algorithms (RLM and RGN) were used to reconstruct the in-core graphite brick resistivity profiles. The RLM algorithm tended to produce what are believed to be more accurate profiles than RGN, although it took longer to arrive at the final solutions. Comparisons between the reconstructed profiles of the fuel channel bricks against measurements from trepanned samples and PECIT resistivity predictions also show reasonable agreement. This is the first time that such good agreement has been obtained from reactor data.
Chapter 8

8 Graphite Sub-Surface Crack Detection

This chapter presents a study of graphite sub-surface defects involving both experimental and modelling work. The detectability of graphite sub-surface slots was studied previously by Fletcher et al [6, 50]. The work in [6, 50] investigated the effects of different far-wall slots (i.e. graphite slots machined in the opposite wall to the EC sensor) on the response of a symmetrical gradiometer sensor, and proposed a novel technique by which the positions and the sizes of the far-wall slots could be estimated. In this chapter the aim is to advance this technique by investigating the effects of more realistic graphite cracks (the types of cracks expected to grow within the AGR bricks) using the new asymmetric gradiometer sensor. In this study, firstly simplified graphite models were created using the COMSOL software package to see how the sensor responses changed with different graphite sub-surface defects. This was followed by experimental studies of both graphite sub-surface slots and cracks.

8.1 Eddy Current Based Crack Detection

During the EC inspection procedures, a primary coil is excited by an alternating current to generate a time-harmonic primary magnetic field. This primary field is responsible for inducing EC within the conducting material, which can be detected by a pickup coil. The penetration depth of the EC within the conducting material depends upon the operational frequency; therefore applying a multi-frequency excitation signal allows probing of a material to different depths. If the conducting material contains any defects or cracks within its volume close enough to the EC probe then the paths through which the induced EC flow will be altered, resulting in a different amplitude and phase response of the probe compared with the case where the material is defect-free. This change in the probe response is normally caused when the EC path within the conducting material is interrupted by a void or discontinuity. In principle, this information could possibly be used to
determine the location and size of the defect or in some cases to visualise the shape of any void within the material of interest.

This frequency-dependent principle of EC flow is the main effect explored in this chapter to investigate the influences that different sizes of sub-surface cracks have on the response of the new asymmetric gradiometer sensor.

8.2 Model Based Study of Graphite Sub-Surface Defects

Model-based graphite sub-surface defects were studied using the same COMSOL FE software package as in the reconstruction problem discussed in Chapters 6 and 7. A simplified brick geometry with two different types of defect arrangements were considered in the modelling study; these were: (i) a straight sub-surface slot and (ii) an inclined sub-surface defect that had a zigzag type profile, mainly generated to replicate a real crack while not necessarily being identical. The key-way slots and the methane holes in the graphite brick models were deliberately omitted to reduce the number of elements within the models and the time required to obtain the solutions. As in the graphite model discussed in Chapter 6, the brick geometry was discretised, but in this case into twenty sub-domains each having a thickness of approximately 4.63 mm. The discretisation was made to control the mesh density within each sub-domain independently. However, the initial study of the mesh density in this particular model showed that having a denser mesh around the coil region had a negligible effect on the solution apart from increasing the computational time. For this reason the mesh density was kept constant throughout the brick geometry as shown in Figures 8.1a and 8.1b. The slots in the models were created using the Transition Boundary Condition (TBC) available within the magnetic-field solver in COMSOL Multiphysics. The TBC feature was used to split each of the discretised graphite domains radially to represent the defect by introducing a boundary that limited the EC flow. Various sizes of sub-surface defects were created around 0° with respect to the coil position in this study (see Figure 8.1). The sizes and types of the sub-surface defects studied in this modelling work are summarised in Table 8.1.
Throughout the modelling process, each defect was simulated using frequency and positional sweeping techniques to obtain a set of multi-frequency data at each probe position. The simulations were carried out using a frequency range between 100 Hz and 10 kHz in a logarithmic scale with 5 points per decade, giving a total of 11 complex data points over two decades of frequency at each coil position. The new gradiometer sensor is large in size, and hence fewer positional scans were required to acquire enough information about the sub-surface defects within the graphite brick model. Therefore, the number of positional sweeps was limited to 19 positions over a circumferential angle range of 90° to -90° with an increment of 10°, giving a total data matrix of \( \mathbf{Z} \in \mathbb{R}^{11 \times 19} \) for each phase and amplitude response. In addition to modelled slots, a defect-free model was also simulated to obtain reference data that could be subtracted from the modelled slots to obtain the change in response due to the slot alone and thus hopefully to detect and locate the defect position.

<table>
<thead>
<tr>
<th>Crack type</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclined defect</td>
<td>48 mm</td>
<td>26 mm</td>
<td>19 mm</td>
</tr>
<tr>
<td>Straight defect</td>
<td>40 mm</td>
<td>26 mm</td>
<td>13 mm</td>
</tr>
</tbody>
</table>

Table 8.1 The through-wall (radial) extents of the simulated sub-surface defects

It should be noted that in a real graphite brick the radial thickness equivalent to the first 8 layers of the graphite model (measured from its periphery) is normally occupied by the key-ways (≈ 38 mm from the brick edge). Because of this, the simulated slot sizes given in Table 8.1 are measured from the bottom of the key-way (from the bottom ends of the dashed lines in Figure 8.1).

8.2.1 Results and Discussion

Figures 8.2 and 8.3 show the resultant multi-frequency phase and amplitude responses of the sub-surface slots after subtraction of reference data obtained from a crack-free model. As can be seen in these figures the slot locations can be easily determined by simply looking at the regions where the highest phase and amplitude
changes occur. These results are in agreement with the previous work carried out by Fletcher et al [6, 50], which used different types of EC probe to inspect straight sub-surface slots.

![Diagram of graphite brick model geometries](image)

(a) Key-way slot
Defect

(b) Key-way slot
Defect

Figure 8.1: The simplified graphite brick model geometries used for investigation of sub-surface defects (a) inclined crack and (b) straight slot.

The results in Figures 8.2 and 8.3 show a shift in amplitude towards the high-frequency end as the slot’s through-wall extent is increased, suggesting that the amplitude responses are more sensitive to the slot’s tip (when the slot’s tip is within the skin-depth of the EC at the given frequency). On the other hand, the phase responses are more sensitive around the low-frequency end of the frequency spectra, which suggests that the phase responses are mainly affected by the perturbation of the EC path due to the presence of the slot. In both cases the magnitudes of the amplitude and phase responses increased with increasing slot through-wall extent, which is to be expected as the larger the slot’s through-wall extent the more the EC path is perturbed.
According to the modelling results, the differences between inclined and straight slots of the same through-wall extent are negligible. This could be due to the size of the probe used for modelling. The circumferential extent of an inclined slot is much smaller than the area covered by the probe, and hence the effect of slot inclination may not be fully captured.

Figure 8. 2: The resultant phase and amplitude responses of the modelled inclined defects after subtraction of the reference data. (Note the slot sizes are measured after subtracting the size of the key-way radial extent from the brick edge).
Although the modelling work assumed a simplified case, the results from this work indicate the performance of the asymmetric gradiometer in detecting graphite sub-surface defects. According to the modelling results, keyway slots having a radial extent of at least 13 mm as measured from the key-way base can be detected, although a slot size smaller than 13 mm has not been investigated here. After understanding how the probe responses change when different sizes of sub-surface slot are created within the graphite brick model, it was necessary to carry out an experiment using sample graphite bricks. The following section describes the
experimental procedures and the results for the graphite sub-surface slots and cracks studied as part of this project.

8.3 Experimental Study of Graphite Sub-Surface Defects

In order to assess the capability of the new asymmetric gradiometer probe in detecting sub-surface cracks in the graphite bricks, the probe response was tested experimentally on a sample graphite brick containing different types of sub-surface defects. Throughout the experimental work two types of graphite sub-surface defects, namely machined slots and generated cracks were produced within a full-scale unirradiated graphite brick supplied by EDF Energy, and investigated experimentally. The remainder of this section now presents the methodology and results of these studies.

8.3.1 Graphite Sub-Surface Slots

Sub-surface cracks are normally expected to initiate from a key-way at present age of the reactors. Therefore, a saw cut radial sub-surface slot was created in a full-scale unirradiated graphite brick (with bulk electrical conductivity ≈92 kS/m), and assigned a circumferential position of 0°. The slot’s through-wall extent was then incrementally changed following every measurement, which was taken at 19 circumferential positions for every slot size (the same as in the modelling work). The data in this experiment were collected using a Solartron 1260 Impedance Analyser along with the new asymmetric gradiometer sensor, which was fitted onto a laboratory scanner (see Figure 8.6a for illustration). During the measurements, the scanner and the Solartron were controlled using a LabVIEW program to move the sensor to a series of discrete positions (between -90° and 90° with respect to the slot position at an increment of 10°), and multi-frequency measurements were taken at each sensor position. The process was repeated three times at each position to obtain three sets of multi-frequency data, and averaged over each frequency point to reduce random measurement errors. Following the experiments, the data were
processed by subtracting the positional averages of the multi-frequency data from the actual data, and plotted on a surface plot for visualisation of the defect location.

Figure 8.4: Images that show the smallest (6 mm left) and largest (40 mm right) graphite sub-surface slots investigated in the experiments

Figure 8.4 shows the smallest and the largest graphite slots investigated in the experimental work, while Figure 8.5 shows the amplitude and phase responses of the EC probe due to the different size slots.

8.3.2 Results and Discussion

As can be seen in Figure 8.5, the slot locations were more visible on the amplitude responses, although the phase responses started dominating this when the slot through-wall extent rose above 30 mm. The results from this study indicated that a slot of greater than 15 mm through-wall extent could be successfully located from the amplitude responses. Once the size of the slot was below 15 mm the amplitude responses tended to be contaminated showing no obvious indication of the slot location. This could be due to the response caused by the variations in the graphite conductivity within its volume (possibly created during the manufacturing process) being larger than the response due to the smaller slots (in this case a slot less than 10 mm in radial extent), and hence dominating the response of the EC probe.

As in the modelling results discussed in Section 8.2.1, the changes in amplitude and phase responses increased in magnitude as the through-wall extent of the slot increased, except for the phase responses of the 6 mm, 10 mm, 15 mm
and 20 mm through-wall slots. In these cases, the probe phase responses were more or less identical. This could be attributed to the changes caused by variations in graphite electrical conductivity being larger than the actual slot responses, which also suggests that the EC perturbations were still dominated by the variations in the graphite electrical conductivity values through its volume.
For the cases where the slot through-wall extents were greater than 30 mm, both the amplitude and the phase responses showed the strongest indications of the slot locations compared with all other cases. The increased changes in sensor response with increasing through-wall extent of the graphite slots are to be expected. This is because the larger the slot through-wall extent, the larger the amount of the EC signal perturbations, leading to the biggest changes in both phase and amplitude responses.

Visual comparisons of the above experimental results with the modelling results in Figure 8.3 show slightly different phase and amplitude responses. There may be many factors contributing to these differences. The first is that the model only simulated a simplified case of the brick omitting both the methane holes and the key-way slots, whereas the experimental data were acquired from a brick containing both key-way slots and methane holes. Secondly, the modelling work assumed constant bulk conductivity (100 kS/m), whilst the experimental brick sample is expected to have had some form of electrical conductivity variation throughout its volume. Finally, the origins of the modelled slots started from the outer edge of a brick with no key-ways, while the experimental slots were created.
from the key-way base. These simplifications of the model are expected to have resulted in different EC distributions, hence altering the phase and amplitude responses of the EC probe from those collected experimentally. Additionally, the model results essentially represent the ideal case where data noise and other factors which could contribute to signal degradation were assumed to be negligible in the FE model. For these reasons, it was possible in the modelling to detect the smallest slot with clear indications of its location in both the phase and the amplitude responses. On the other hand, the results from the experimental work suffered from various practical challenges including the presence of the methane holes and possible variations in brick electrical conductivity through its volume. These may have disguised some of the signal changes caused by smallest slot, although the results generally show signs of the presence of larger slots, which were not detectable using the 70 mm gradiometer.

The penetration depth within the graphite brick studied in this section is expected to be much lower than those within the AGR core due to its increased electrical conductivity. Since the electrical conductivity of irradiated graphite is lower (larger penetration depth), in principle this could benefit the detection of in-core sub-surface cracks. However, the variability of the graphite volumetric density (caused by radiolytic oxidation) could mask the EC response to a small sub-surface cracks. In addition, the stress in the core could cause a crack faces to make full or partial contact, and hence led to reduced or disguised response.

The tests on the graphite sub-surface slot described above have more or less demonstrated the advantage of the new EC sensor over the existing 70 mm gradiometer probe that only detected key-way originated slots with more than 30 mm through-wall extent [50]. However, none of the studied cases so far represent the expected types of graphite sub-surface cracks within the AGR core, which also means that cracks similar to those expected within the AGR core need to be studied.

8.3.3 Sub-Surface Cracks

In order to test the capability of the EC probe for detecting more realistic sub-surface graphite cracks, it was thought necessary to carry out further
experimental work using the types of graphite cracks normally expected within the AGR core.

![Diagram of experimental setup](image)

Figure 8.6: Some of the realistic graphite cracks generated during the experimental tests. (a) experimental setup (b) full through-wall crack without wedges on the key-way, (c) full through-wall crack with wedges on the key-way, (d) a 23 mm crack from the bore and (e) a 5 mm crack from the bore.

For this reason a series of tight sub-surface cracks were generated in one of the graphite key-way corners supplied by EDF Energy, and assigned $\approx 85^\circ$ with respect to probe position. Two different brick samples were used during this study: a virgin graphite brick that had a bulk electrical conductivity of approximately 92 kS/m and
another brick with an electrical conductivity of approximately 27 kS/m. The latter brick contained key-way slots but no methane holes, whereas the former brick had full methane holes and key-way slots (see Figure 8.6).

The cracks were generated using a simple wedge system (250 mm long) that locked into the key-way and increased the tensile force around the key-way corners, which eventually led to the generation of a crack starting from one of the key-way corners (see Figure 8.6). The crack through-wall extent was controlled by adjusting the force applied between the key-way sides. The wedge equipment was left in place during the subsequent scanning of the eddy current probe. In this respect the cracks were left slightly open, held apart by the loaded wedge. The crack depth was measured from the visual extent of the crack at the end of the brick, and assumed that the crack tip runs equally into the brick along the full length of the wedge tool.

The measurements were collected using similar procedure as in Section 8.3.1, but processed differently. The data from this experiment were processed using three different methods to determine the best at locating sub-surface cracks. These three methods were investigated with the main aim of improving the detection capability for realistic sub-surface cracks. The need for these improvements mainly arises due to the expectation of reduced detection capability as a result of the potential bridging effect within sub-surface cracks, which could still allow the EC to flow to some extent between the cracks.

**Method 1**

Method 1 was similar to the work presented by Fletcher et.al [6, 50] using the 70 mm symmetrical gradiometer probe and a sample graphite brick with sub-surface slots. This method does not require a reference brick. In other words, the slotted brick acts as its own reference brick in order to determine the location of the slot. This is also similar to the method adopted in Section 8.3.1. In this method, firstly positional multi-frequency phase and amplitude responses of the probe around the region of interest were obtained. Secondly, the positional averages of both amplitude and phase responses were calculated and subtracted from the original responses as shown in Figure 8.7.
Method 1

\[ A_{cr} \in \mathbb{R}^{f \times p} \leftarrow \text{Amplitude response} \]
\[ P_{cr} \in \mathbb{R}^{f \times p} \leftarrow \text{Phase response} \]
\[ A_{av} = (\text{mean}(A_{cr}^T))^T \]
\[ P_{av} = (\text{mean}(P_{cr}^T))^T \]

for \( k = 1:19; \)
\[ \Delta A(:, k) = A_{cr}(:, k) - A_{av}; \]
\[ \Delta P(:, k) = P_{cr}(:, k) - P_{av}; \]
end;

Figure 8.7: Crack data processing using Method 1.

Method 2

Method 2 uses a similar principle to Method 1 in that it uses the positional average information of both the amplitude and phase responses. However, this method also exploits an additional reference brick with an arbitrary bulk electrical conductivity value. This method is almost equivalent to subtracting the response of the reference brick from the cracked brick. The only reason that the positional average data is included in this method is to minimise a possible measurement errors. The reference brick in this case was mainly used to improve the quality of the detected signal so that the location of the slot could be better identified.

A preliminary study of this method has also indicated that the use of data measured using a reference brick with an arbitrary bulk electrical conductivity value only introduces an offset compared with data collected from a brick with the same bulk conductivity as the cracked brick. This could be useful when inspecting for in-core graphite cracks, as measured values from the calibration brick on the hoist could be used to process the in-core data.

In this method, first the positional multi-frequency phase and amplitude responses of the probe around the cracked and the reference bricks were measured. Positional averages of both the amplitude and phase responses for each brick were then calculated. The averaged positional responses were then subtracted from their
respective brick data at each position. This was then followed by the subtraction of the change in the reference brick data from the change in the cracked brick data as shown in Figure 8.8.

\[ \begin{align*}
A_{cr} & \in \mathbb{R}^{f \times p} \leftarrow \text{Amplitude response} \\
P_{cr} & \in \mathbb{R}^{f \times p} \leftarrow \text{Phase response} \\
A_{ref} & \in \mathbb{R}^{f \times p} \leftarrow \text{Amplitude response} \\
P_{ref} & \in \mathbb{R}^{f \times p} \leftarrow \text{Phase response} \\
\end{align*} \]

\[ \begin{align*}
A_{cr, av} &= (\text{mean}(A_{cr}^T))^T \\
P_{cr, av} &= (\text{mean}(P_{cr}^T))^T \\
A_{ref, av} &= (\text{mean}(A_{ref}^T))^T \\
P_{ref, av} &= (\text{mean}(P_{ref}^T))^T \\
\end{align*} \]

\textbf{Method 2}

\begin{verbatim}
for k = 1:19;
    \Delta A_{cr}(\cdot, k) = A_{cr}(\cdot, k) - A_{cr, av};
    \Delta P_{cr}(\cdot, k) = P_{cr}(\cdot, k) - P_{cr, av};
    \Delta A_{ref}(\cdot, k) = A_{ref}(\cdot, k) - A_{ref, av};
    \Delta P_{ref}(\cdot, k) = P_{ref}(\cdot, k) - P_{ref, av};
end;
\end{verbatim}

\[ \begin{align*}
\Delta(\Delta A_{cr}) &= \Delta A_{cr} - \Delta A_{ref}; \\
\Delta(\Delta P_{cr}) &= \Delta P_{cr} - \Delta P_{ref}; \\
\end{align*} \]

Figure 8. 8: Crack data processing using Method 2.

\textbf{Method 3}

The data processing technique adopted in Method 3 is also similar to the previous two methods. However, this method does not subtract the positional average of the response from the cracked brick; instead it subtracts the positional average of the reference brick from the cracked brick data as shown in Figure 8.9.
8.3.4 Results and Discussion

The results in Figures 8.10 and 8.11 show single-frequency positional scans of the graphite sub-surface cracks calculated using each of the above three methods. It should be noted that the reason that the phase and the amplitudes changes are plotted at different frequency points in Figures 8.10 and 8.11 is because the peak phase changes for the studied sub-surface cracks were found to occur around 400 kHz, whereas the peak amplitude changes extended to 1 kHz. The difference in frequency points where the peak phase and amplitude changes occurred may be attributed to the amplitude changes being more sensitive to the crack tips, whereas the phase changes were mainly responsive to the general perturbations of the EC flow.

Comparison between the results presented in Figures 8.10 and 8.11 clearly shows that Method 2 produced better indications of the sub-surface cracks than the other two methods investigated in this chapter. The positional variations for the phase and amplitude responses of a given crack calculated using Methods 1 and 3
were sometimes very wide, although the peak changes in amplitude and phase signals were still at the approximate locations of the cracks. However, as the crack distance increased from the graphite bore (in the cases of 23 mm and 17 mm away from the bore) there were no obvious indications of the cracks on either the amplitude or the phase responses when calculated using Methods 1 and 3. On the other hand, Method 2 produced amplitude and phase responses that had much narrower positional variations around the cracked region of the brick, even for the crack with the largest distance from the bore (Figures 8.10f and 8.11f). The reason that Method 2 produced better indications of the sub-surface cracks compared with the other two methods could be that this method may have reduced the measured data variations caused by possible measurement noise and non-uniform conductivity distributions within the sample brick. This may have been achieved during the final stage of the signal processing method where the changes in the data from the reference brick were subtracted from the changes in the cracked brick data.

Looking into the amplitude and the phase changes processed using the three methods when there is no crack within the sample brick (Figure 8.10g and Figure 8.11g) clearly shows certain level variations in the data when processed using M1 and M3. Ideally, at the absence of a crack within the sample brick the processed data should not show changes over the inspected positions as in the case seen when the data is processed using M2. The main reason that the data processed using M2 show no change over the inspected region is because for $\delta D_{cr} = \delta D_{ref}$ that is when the brick is defect free then $[D_{cr} - D_{cr,Av}] - [D_{ref} - D_{ref,Av}] = 0$, where $D_{cr}$ is the data measured from cracked brick, $D_{cr,Av}$ is the positional average of the cracked brick data, $D_{ref}$ is the data measured from reference brick and $D_{ref,Av}$ is the positional average of the reference brick data. This means that if we work out the amplitude and phase fractional changes caused by a crack with different through-wall extent (Table 8.2) we tend to get infinite value for the data processed using M2. In practical term this implies that the when the data is processed using M2, the full amount of the changes caused by the presence of the crack can be recovered. In contrast, the fractional changes caused by M1 and M3 tend to reduce as the
through-wall extent of the cracks reduces or as the changes caused by the cracks approaches the background changes (see Table 8.2).

Figure 8.12 shows the amplitude and phase responses when the crack extended fully across the graphite through-wall thickness and was approximately 225 mm long axially from the top of the brick. In this case the crack extended beyond the axial dimensions of the EC probe. For a fully extended radial crack with the plastic wedges still locked into the key-way (Figures 8.12a and 8.12c), all three methods seemed to produce more or less identical phase and amplitude responses with the exact location of the crack around 85°. However, this was with the exception of some small negative offsets seen in the curves calculated using Method 1.

The reason we see curves with similar positional variations around the cracked region could be the result of the crack being fully extended through the radial thickness of the brick (surface-breaking crack), whilst having an axial extent larger than the probe dimension. In other words, the EC were now fully perturbed by the presence of the defect; hence the signals detected by the receiver coils were large enough to indicate the location of the crack regardless of the data processing method used.

However, when the plastic wedges were removed from the key-way, the tensile force between the graphite key-way reduced, which might have led to a reduction in the crack gape (slightly closed crack), and potentially reduced the through-wall extent of the crack making it act as a sub-surface crack. Consequently, the signal response is expected to reduce in magnitude compared with the signal measured when the plastic wedges were still in place, and this was seen in (Figure 8.12). In this case the responses calculated using Method 1 and Method 3 tended to have slightly wider positional variations than the calculated curves using Method 2 (Figures 8.12b and 8.12d).
Figure 8.10: Phase responses of the probe due to cracks positioned ≈ 85° at 400 Hz against probe position for the three methods. Note the abbreviations M1, M2 and M3 on the legend denote the three methods discussed above.
Figure 8. 11: Amplitude responses of the probe due to cracks positioned ≈ 85° at 1 kHz against probe position for the three methods. Note the abbreviations M1, M2 and M3 on the legend denote to the three methods discussed above.
In general, the data processing technique presented in Method 2 indicated the locations of the studied sub-surface cracks with much better circumferential accuracy than the other two methods. Figure 8.13 shows the single-frequency phase and amplitude changes caused by each of the investigated sub-surface cracks, when processed using Method 2. The magnitudes of the phase and amplitude changes increased with the through-wall extent of the sub-surface cracks, showing the sensitivity of the probe to small changes in crack through-wall extent. However, the differences in both phase and amplitude responses between the cracks terminating at 7.5 mm and 5 mm from the graphite bore were negligible.

<table>
<thead>
<tr>
<th>Distance from Bore to crack tip (mm)</th>
<th>Phase and Amplitude change with respect to un-cracked brick (%)</th>
<th>Method 1 (M1)</th>
<th>Method 2 (M2)</th>
<th>Method 3 (M3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23 mm from bore</td>
<td></td>
<td>P = 33.31</td>
<td>P = inf</td>
<td>P = 39.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 33.44</td>
<td>A = inf</td>
<td>A = 38.92</td>
</tr>
<tr>
<td>17 mm from bore</td>
<td></td>
<td>P = 37.75</td>
<td>P = inf</td>
<td>P = 48.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 40.62</td>
<td>A = inf</td>
<td>A = 52.01</td>
</tr>
<tr>
<td>14 mm from bore</td>
<td></td>
<td>P = 49.34</td>
<td>P = inf</td>
<td>P = 63.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 51.48</td>
<td>A = inf</td>
<td>A = 65.93</td>
</tr>
<tr>
<td>12 mm from bore</td>
<td></td>
<td>P = 58.57</td>
<td>P = inf</td>
<td>P = 76.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 63.71</td>
<td>A = inf</td>
<td>A = 82.92</td>
</tr>
<tr>
<td>7 mm from bore</td>
<td></td>
<td>P = 69.79</td>
<td>P = inf</td>
<td>P = 93.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 78.91</td>
<td>A = inf</td>
<td>A = 103.80</td>
</tr>
<tr>
<td>5 mm from bore</td>
<td></td>
<td>P = 72.31</td>
<td>P = inf</td>
<td>P = 95.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 79.46</td>
<td>A = inf</td>
<td>A = 103.80</td>
</tr>
<tr>
<td>Full length without wedges</td>
<td></td>
<td>P = 178.98</td>
<td>P = inf</td>
<td>P = 209.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 200.58</td>
<td>A = inf</td>
<td>A = 257.78</td>
</tr>
<tr>
<td>Full length with wedges</td>
<td></td>
<td>P = 1.0518e+03</td>
<td>P = inf</td>
<td>P = 1.3016e+03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 654.32</td>
<td>A = inf</td>
<td>A = 857.02</td>
</tr>
</tbody>
</table>

Table 8.2: Changes in the Amplitude (A) and Phase (P) responses of the three methods with respect to un-cracked brick calculated as: \[
\frac{\text{Peak (ΔDcracked)}}{\text{ΔDun-cracked}} \times 100
\]. Note: for M2 the numerator is zero due to the processing method.
Figure 8.12: Amplitude and phase responses of the probe due to a fully through-wall crack positioned ≈ 85°, and plotted using the three methods:

(a) Phase change with wedges on key-way
(b) Phase change without wedges
(c) Amplitude change with wedges on key-way
(d) Amplitude change without wedges

Figure 8.13: Phase and amplitude response of the probe due to different sub-surface cracks calculated using Method 2.

One of the reasons for this could be that the 5 mm crack tip maintained contact up to a distance where it reached around 7.5 mm away from the bore, and hence produced similar responses. The crack at the largest distance from the bore (23 mm) produced a smooth phase response showing the approximate location with
relatively narrow positional variations. On the other hand, the amplitude response of the same crack was less smooth, and the location of the peak change was shifted to 100°, although it still indicated the approximate crack location.

Figures 8.14 and 8.15 show the multi-frequency surface plots of the phase and amplitude responses of the cracks as a function of frequency and probe position. The data in these plots were processed using Method 2, mainly due to its advantage in locating the crack location compared with the other two methods.
Figure 8.14: Multi-frequency amplitude and phase responses of the probe due to different sizes of sub-surface cracks positioned at \( \approx 85^\circ \).
As for the slotted brick test case in Section 8.3.1, the crack locations on the multi-frequency surface plots are more visible on the amplitude responses, although the phase responses show clear indications of the crack locations at discrete frequency points. Empirical comparisons between the changes in phase due to the graphite slots presented in Figure 8.5 and the results in Figure 8.14 also show better indications of the crack locations in Figure 8.14. This is mainly due to the different methods adopted to process the measured data. In other words, the sub-surface slot test results in Figure 8.5 were processed using a technique identical to Method 1, whereas the surface plots in Figure 8.14 were processed using Method 2. Therefore, these results suggest that Method 2 is much more robust in locating the graphite sub-surface cracks than the other methods considered in this chapter.

As discussed earlier, removal of the plastic wedges from the graphite key-way where the cracks were formed resulted in a reduction of the tensile forces between the key-way sides. This also meant that the natural compressional forces of the cylindrical graphite brick (mainly due to the cylindrical structure of the brick) probably acted to close the crack. In these situations, some parts of the cracked
regions may have maintained good contact between the crack faces allowing the ECs to pass through the cracked region. These in turn may have resulted in the reduced changes in both amplitude and phase responses which were seen when compared with the responses when the plastic wedges were still in place (when sufficient tensile forces were maintained between the key-way sides). The effects of the force applied on the key-way where the cracks originate from its corner can be clearly seen in Figure 8.15. In this case both amplitude and phase responses showed clear reductions in magnitude when the plastic wedges were removed from the graphite key-way. This result can be directly related to the likely behaviour of a possible crack within the AGR core, mainly due to the stresses within the core, which could act to either open or close a pre-existing crack, and hence could make sub-surface crack detection a more complex task.

The results presented up to this point clearly indicate the advantage of Method 2 over the other two methods investigated in this section. However, this way of processing the EC data may have a limitation for use in real applications, as it requires a reference brick ideally with an identical electrical conductivity distribution to the cracked brick under investigation.

To test the effect of using a reference brick that has a different bulk electrical conductivity to the cracked brick under investigation, each of the two graphite brick samples discussed previously were used as references for one another. That is: the reference brick with approximately 92 kS/m bulk electrical conductivity was used to locate a crack in the brick that had a bulk electrical conductivity of approximately 27 kS/m, and vice versa.

Due to the brittle nature of the low-conductivity sample brick (27 kS/m), it was not possible to carry out this study using different sizes of sub-surface cracks generated in this particular brick. Instead, the study was limited to a fully through-wall crack. However, it was still possible to carry out studies of sub-surface cracks generated in the high-conductivity brick (92 kS/m) using the brick having a bulk conductivity value of approximately one-third of the cracked brick (27 kS/m) as its reference.

Figures 8.16a and 8.16b show the differences between the calculated phase and amplitude responses of the three methods when a fully through-wall crack
generated in the low conductivity (27 kS/m) graphite was located using a reference brick that had a bulk electrical conductivity of 92 kS/m. It is clear to see from these plots that all three methods have successfully indicated the location of the crack in both the amplitude and phase responses. However, as mentioned previously, using a reference brick with a different bulk electrical conductivity value from the cracked brick introduced some positive offset to the calculated curves. This offset was mainly caused by the difference between the cracked and the reference brick bulk electrical conductivities.

The offsetting effect is also visible in Figures 8.17a and 8.17b, where data from a fully through-wall crack in the 92 kS/m graphite brick were processed using a reference brick that had 27 kS/m bulk electrical conductivity. However, in this case the offset was negative, whilst having similar magnitudes to those seen in Figure 8.16, proving that the offsets were mainly caused by the differences between the bricks’ electrical conductivities.

Methods 2 and 3 produced more or less similar amplitude and phase responses in both cases, indicating the approximate location of the crack. However, as the crack distance from the graphite bore increased, each of these methods failed to indicate the approximate location of the crack with narrow positional variations (see Figures 8.18a and 8.18b).
Figure 8.16: Phase and amplitude responses of crack generated in low-conductivity brick (27 kS/m) when processed using the three methods that used two different reference bricks that had 27 kS/m and 92 kS/m electrical conductivity values.
Figure 8.17: Phase and amplitude responses of crack generated in high-conductivity brick (92 kS/m) when processed using the three methods that used two different reference bricks that had 27 kS/m and 92 kS/m electrical conductivity values.
(a) Phase changes showing full plots (left) and zoomed plots (right)

(b) Amplitude changes showing full plots (left) and zoomed plot (right)

Figure 8.18: Phase and amplitude responses of 5 mm crack away from a high-conductivity brick (92 kS/m) bore when processed using the three methods that used two different reference bricks that had 27 kS/m and 92 kS/m electrical conductivity values.

The lack of narrow positional variations seen around the crack location in Figure 8.18 may be caused by the differences in the brick geometries between the cracked brick and the brick used as a reference (the absence of the methane holes in the reference brick). This difference may have partially disguised the responses of the sub-surface crack, causing less variation with position with no clear indication of the crack.
location. The reason that this effect was not visible for the fully through-wall crack could be due to the amount of signal change caused by the extent of the crack being much larger than the signal caused by the differences in the brick geometry.

The above study could be improved with the use of identical geometry graphite bricks, but with different electrical conductivity values. In this way, one of the brick samples could be used to generate a sub-surface crack and the other to act as the reference. This work would also benefit from further investigations of the above methods using graphite sub-surface slots instead of tight cracks to see to what extent the sub-surface slot locations could be identified when processed using a reference brick with an arbitrary bulk electrical conductivity value. Although sub-surface slots are far from being realistic cracks, in this way one could easily control both the axial and the through-wall extents of the slots, and hence distinguish the exact effects of using a reference brick with an arbitrary bulk electrical conductivity.

It is also worth attempting to determine the through-wall extents of the cracks. In this case only a limited number of realistic cracks and slots were investigated, and therefore it was not possible achieve this.

8.4 Summary

This chapter has presented experimental and modelling studies of graphite sub-surface slots and realistic cracks. The capability of the new EC sensor to detect more realistic graphite sub-surface cracks was tested. This was then followed by the investigation of three different experimental data processing methods. The overall results from these studies have indicated that the EC method is capable of detecting some deep sub-surface cracks, particularly those expected to initiate within AGR graphite bricks. A machined slot approximately 16 % (≈15 mm from the key-way base) of the entire graphite wall were successfully located, whereas the realistic crack was only detectable when the through-wall extent of the crack reached approximately 34 % of the entire graphite wall (≈32 mm from the key-way corner or 23 mm from the bore).
9 Conclusion and Future works

9.1 Conclusions

This thesis has presented three major aspects of the EC-based NDT techniques for inspection of the graphite bricks in the AGR core. The three major areas investigated in this thesis are as follows:

- The study of a custom-designed EC sensor
- The study and development of AGR brick conductivity profiling algorithm
- A feasibility study of detecting realistic key-way root sub-surface cracks

9.1.1 The Study of a custom designed EC sensor

During the study of a custom designed EC sensor, different sensor configurations and parameters were considered. The effects of sensor parameters on the sensitivity of the sensor to the graphite sample being inspected were studied, and a novel EC sensor configuration was proposed. The proposed EC sensor configuration was found to yield a significant improvement in sensitivity with depth compared with the existing EC probe (43 % improvement in sensitivity at the lowest inspection frequency).

The study of the new EC sensor configuration included consideration of the EC inspection PECIT tool, and the effects of the conducting components of the tool were fully investigated through successive FE simulations. The results from these studies showed that with the current configuration of the new EC sensor the effect of the tool stainless steel components on the probe sensitivity accounts for much less than a 1 %.

A prototype sensor was built and tested on graphite samples. The number of coil turns on the prototype sensor was selected such that the stray capacitance of the cable that connects the probe with host laptop (in the real AGR inspection
The study and design of the custom EC sensor for the AGR core inspection were executed successfully. Currently, the new EC probe is under consideration for inclusion within the new AGR core inspection system, which is being developed by EDF Energy and its partners.

9.1.2 The Study and Development of AGR brick Conductivity Profiling Algorithm

The graphite sample conductivity-profiling task in this thesis has extended the feasibility studies carried out in the previous projects related to the AGR core inspection system, but in this case using the new EC sensor. This task was approached by extending the use of the existing RGN algorithm through the implementation of new constraining techniques. The constraining techniques implemented were based on a priori information from a sample graphite brick machined with radial drilled holes to produce an electrical conductivity gradient. The new constraining techniques were mainly implemented to deal with the ill-posed nature of the inverse EC problem as well as to prevent the effects of model calibration errors. The results from this study produced a substantial improvement in the accuracy (a minimum global error of 1.3 % or 98.7 % accuracy) of the reconstructed electrical conductivity profile of the sample graphite brick, when inverted using experimentally-measured data. In addition to producing accurate profiles, the new constraining techniques reduced the computational time by an average of 60 % compared with the unconstrained solutions.

Studies concerned with different regularisation operator matrices, namely DF and NOSER operators, were carried out. The results showed that the DF operator performed slightly better than the NOSER approach for most of the reconstructed cases using both simulated and experimentally measured data. This improvement was mainly caused by the smoothness of the expected solutions, and most importantly was due to the features of the DF operator that allowed it to correlate neighbouring discrete conductivity values. This ensured a smooth transition between
neighbouring conductivities whilst preventing the divergence of the solution from the user-defined reference value.

The effects of different initial estimates on the accuracy of the reconstructed profiles of sample brick were investigated using 45 kS/m and 72 kS/m graphite bulk conductivities, as initial estimates. The results from these studies showed that for a forward model containing 2.8 % and 3.2 % calibration errors in the real and imaginary parts of the data, the solution produced profile errors of 13.6 % and 11.2 % for the DF operator and 16 % and 12.5 % for the NOSER operator, when initial bulk estimate of 45 kS/m and 72 kS/m respectively applied. These results clearly showed that selecting an initial estimate closer to the expected solution produced a better estimate of the true profile than one where the initial value was further away from the true solution. However, for well-posed problems, the choice between two different initial estimates should not affect the overall accuracies of the reconstructed profiles, except that small initial values may increase the computational time of the solution, provided that identical regularisation parameters and operator matrices are selected.

A method of selecting an optimum regularisation parameter based on the L-curve technique has been studied. This study was carried out using experimentally-measured data (using the new EC sensor) from a sample graphite brick machined with radial drilled holes to impose resistivity gradient. The results from this study showed that the approximate corner value corresponding to the optimum regularisation parameter could be extracted, which is in accord with the previous studies made using the symmetrical gradiometer sensor. The approximate regularisation parameter value extracted from the L-curve also agreed reasonably well with the value determined in a trial and error fashion through a single-step inversion. However, the latter is limited to cases where the approximate solution to a given problem is known. Nevertheless, reasonable agreement between the L-curve method and the value determined through a trial and error approach also implies that the L-curve method could be used to estimate the so-called optimum regularisation parameter for a given graphite inverse problem. The major drawback of this method is that it requires the solutions of successive inverse problems with different values of regularisation parameters.
A formulation of the RLM algorithm derived from MIT systems, for AGR graphite electrical conductivity reconstruction was presented and tested on actual data collected from two different AGR power stations. This method uses a damping parameter, which could be updated during the iteration, and hence works fairly well even with less optimal regularisation parameter. The reconstructed reactor profiles using both RLM and RGN algorithms were compared against resistivity measurements from trepanned samples taken out of the reactor core at the same locations where the data for reconstruction were collected. The RLM algorithm has produced reasonable estimates of the trepanned sample resistivity trends, with the mean profiler errors ranging between 1.5 % and 16.9 %. On the other hand, the RGN showed mean profile errors within the range of 19.1 % and 31.8 % compared with the trepanned sample measurements. These differences in the reconstructed profiles errors clearly indicated the advantage of RLM algorithm over the RGN. However, the RLM required slightly more time to converge than the RGN, mainly due to the trials it has to go through during the iterations. One of the main advantages of the RLM method is that the value of the regularisation parameter $\lambda$ does not need to be optimal. Instead, the lack of optimality in the chosen value of $\lambda$ could be compensated through the damping parameter $\gamma$.

As an additional validation the reconstructed profiles were also compared with the resistivity predictions from the PECIT system that is currently used by EDF Energy, and agreed reasonably well with the reconstructed near-bore profiles. The reactor data reconstruction results, particularly those validated against trepanned sample measurements and PECIT predictions were generally successful, suggesting that both algorithms operated, as they should. The results from this study also confirm the suitability of the new monotonic constraining technique applied within the algorithms to invert the reactor data from one of the stations.

The effects of forward model calibration errors on the reconstructed profiles have been studied using different error levels and distributions added to the simulated data. The study showed that the accuracies of the reconstructed profiles depended not only on the error levels; they also depended upon the distributions of the errors in the data. This study clearly indicated that in order to reconstruct the graphite brick conductivity/resistivity profile accurately the forward model needed
to be calibrated with the measurements from a complete inspection system using a brick with known electrical conductivity distribution. To overcome the effects of the calibration errors on the reconstructed profiles a new forward model calibration technique was employed. This calibration method took into account the frequency dependence of both the real and imaginary parts of the mutual inductance data. The profiles reconstructed using this method produced what is believed to be the expected profile trend, although these particular profiles have not yet been validated against the trepanned sample measurements.

9.1.3 A Feasibility Study of Detecting Realistic Key-way Root Cracks

With regard to the feasibility of detecting slots and realistic key-way corner originated sub-surface cracks using the new EC sensor; the results indicated that the EC method is capable of detecting some deep sub-surface cracks. A the machined slots approximately 16 % (≈15 mm from the key-way base) of the entire graphite wall were successfully located, whereas the realistic cracks were only detectable when the through-wall extent of the cracks reached approximately 34 % of the entire graphite wall (≈32 mm from the key-way corner or 23 mm from the bore). In contrast, a discussion with EDF Energy confirmed that the existing Ø70 mm PECIT was only able to detect a realistic crack with a through-wall extent of 10 mm away from the graphite bore (48 % of the entire graphite wall), confirming the advantage of the new asymmetric gradiometer sensor over the existing one. In addition, the new post-measurement data processing method presented in this thesis showed a significant improvement in identifying the locations of sub-surface cracks with a narrow positional peak compared with the existing method, although this method relied on obtaining measurements from a reference brick ideally with identical geometry to the brick under test.

9.2 Future Work

Although the majority of the work in this thesis was completed successfully, there is potential future work that could benefit the overall implementation of the
system for routine industrial use. The potential future work and improvements cover different aspects of the systems discussed in this thesis and are described in the following sections.

9.2.1 Measurement System

The EC system used for graphite conductivity profiling and sub-surface crack detection in this thesis is based on a commercial instrument known as Solartron Impedance Analyser. This system can only gather the multi-frequency data sequentially (a single frequency measurement at a time), and hence takes a long time to acquire the full set of multi-frequency measurements required for conductivity reconstruction and crack detection. From a practical point of view, a slow measurement system is directly associated with increased cost, as this could increase the overall inspection time of the reactor core. Therefore, the future work in this area could be an investigation of a suitable system that performs measurements using multi-frequency simultaneously, and incorporate this into the current measurement system.

9.2.2 Excitation Technique

The work presented in this thesis has only focused around the multi-frequency implementation of both the graphite conductivity profiling and sub-surface crack detection techniques. The multi-frequency method in this thesis was adopted because it was believed that sinusoidal excitation offers better signal-to-noise performance due to its reduced bandwidth. However, this was not compared with other excitation techniques. An alternative to multi-frequency excitation, such as a pulsed EC technique, could be applied to the methods developed in this thesis. Further investigations of the relative merits of the multi-frequency technique over pulsed EC methods could be undertaken to select the one that offers the best performance. This could benefit the overall implementations of the conductivity profiling and crack detection system, and could be included as future work.
9.2.3 Reconstruction Algorithm and Forward Model

The constraining technique adopted in this thesis mostly relies on priori knowledge about the expected profile trend. This means that the user needs to have some insights into the expected radial conductivity distributions of the graphite brick under investigation, prior to reconstruction. Generally speaking, this information can be obtained from the historic trepanned sample measurements from different reactors and it may be possible to incorporate this information into the reconstruction algorithm through an appropriate constraining technique. The suitability of the monotonic constraining technique for the reconstruction of the brick conductivity profiles of the HPB reactors was proved through comparison of the reconstructed profiles against the trepanned sample measurements and the PECIT predictions. The general assumption for the AGR brick radial density/conductivity variations is normally an increasing trend, which has semi-quadratic features from the graphite bore to the brick periphery, and hence the monotonic constraining technique is expected to work reasonably well. However, the variability between the graphite bricks from station to station coupled with the slightly different brick to brick mating arrangement and its influence on the pressure differential through the brick wall, some of the in-core graphite bricks were expected to have non-monotonic density/resistivity variations. Therefore, monotonic assumption is not always guaranteed. For this reasons the monotonic constraining technique may not be suitable in these limited cases.

Future work in this area could include further test the current reconstruction algorithm using a data from different reactors, particularly from those expected to have non-monotone density/resistivity variations and improve its performance accordingly. It may also be worth attempting to incorporate the historic trepanning measurements within the inversion algorithm. One ways of achieving this could be by building a look-up table from the historic resistivity data. This could allow the user to incorporate information from the look-up table into the reconstruction algorithm,
and use it as a constraining mechanism to penalise extreme variations from those available within the look-up table.

As discussed in Chapter 7 the RLM algorithm produced accurate conductivity profiles compared to RGN, but this algorithm took longer to converge due to the trials that it has to go through during the iterative process. The overall computational speed of the reconstruction process can be improved by simplification of the forward model, as well as implementation of conditions that avoid unnecessary computations during the reconstruction process. The latter method is already implemented in this thesis, which sets a limit on the amount of function reduction and step calculations through the use of tolerance values. However, the former could be considered as future work to improve the speed of the reconstruction process. The simplification of the forward model could start by removing the graphite model features that will not have a significant impact on the overall accuracy of the solution. One of these simplifications could be removing the key-way slots from the graphite forward model through a detailed sensitivity study to determine its effect on the reconstructed solution. Since the EC flow opposite to the probe location in the graphite forward model is negligible, reducing the graphite brick geometry to half-brick model can also make further simplification of the forward model geometry. This would mean that the semi-cylindrical section of the graphite brick could be further simplified with its quarter symmetry and would reduce the number of elements by a significant amount. In principle, this could greatly increase the reconstruction speed and should be considered in the future study.

9.2.4 Key-way Root Crack Detection

From the results presented in this thesis it is clear that the new EC sensor is capable of detecting realistic key-way originated sub-surface cracks with much better sensitivity than the existing probe. However, this work could benefit from further investigations of the new post-processing method using graphite sub-surface slots instead of tight cracks, to see to what extent the sub-surface slot locations can
be identified when processed using a reference brick with an arbitrary bulk electrical conductivity value. Although the sub-surface slots are far from being realistic cracks, in this way one can simply control both the axial and the through-wall extents of the cracks, and distinguish the effects of using different reference samples from those generated by variations in brick geometry and axial lengths of realistic cracks.

It is also worth attempting to determine the through-wall extents of the realistic key-way root cracks. In this thesis, only limited numbers of realistic cracks were investigated through the experiment, hence it was not possible to estimate the sizes of sub-surface cracks. However, if sufficient numbers of crack sizes were created in both reference and the brick under investigation, the crack sizes could be estimated by minimising the difference between the two datasets in the least square sense.

9.2.5 Reference Brick Preparation

The current reference brick used for laboratory test of the reconstruction algorithm relies on a brick with radially drilled holes to generate conductivity gradient across the brick radial thickness. The through-wall electrical conductivity of this brick was estimated based on the bore and periphery values, and interpolating these values linearly. Although this method has achieved the required conductivity gradient, the EC perturbations caused by the holes during the mutual inductance measurements of the EC probe may not fully represent those caused by graphite density variations. This is also true for the graphite sub-surface crack detection, since this study employed a brick with homogenous conductivity distributions.

The future work could consider looking at improving the current reference brick arrangement. Two approaches could be considered to improve the existing reference brick arrangement: radial hole drilling and thermal oxidation. If the radial hole drilling approach is adapted, which is the existing method then reducing the hole diameter and the distance between them could improve the arrangement of the reference brick. The thermal oxidation approach requires the brick to undergo a
heating process roughly above 600 °C to 800 °C to achieve the required level of graphite oxidation.

9.2.6 Extension to Wider NDT Applications

The work in this thesis has merely focused on the inspection of the graphite bricks in the AGR core, although the techniques presented could also be used for other industrial applications such as NDT of conducting coatings in many industrial components. Therefore, the work in this thesis could benefit from further investigations and feasibility studies to extend the conductivity profiling technique presented in this thesis to other industrial NDT applications.
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Appendix A: RLM Reconstruction Algorithm

% Load COMSOL interface
% Load forward Model
% Create Voxel grids for E field calculation within the brick volume
% Specify number of discretised domains: n ← 7

**Problem:** \( \sigma^* = \arg\min_{\sigma} \left\{ \frac{1}{2} \| F(\sigma) - M(\sigma) \|^2 + \frac{1}{2} \lambda \| L(\sigma - \sigma_{\text{ref}}) \|^2 \right\} \)

Extract reference data from homogenous conductivity profile: \( F(\sigma_{\text{ref}}) \)
Calculate the Jacobian from homogenous conductivity profile: \( J_0(\sigma_{\text{ref}}) \)

Regularisation operator: \( L = \begin{cases} 2 & \text{for } i = j \\ -1 & \text{for adjacent } i \text{ and } j \\ 0 & \text{otherwise} \end{cases} \)

Regularisation parameter: \( \lambda_0 = \max(\text{diag}(J_0^T J_0)) \) or \( 1 \times 10^{-28} \)
Initial damping parameter: \( \gamma_0 = \lambda_0 * 10e - 3 \)

Set: \( \varepsilon_1 = 1e - 21; \varepsilon_2 = 9e - 3; k_{\text{max}} = 15; \text{ and } \gamma(k) = \gamma_0; \)
Set: \( I = \text{eye}(7,7); \)

Load the measured data: \( M(\sigma) \)

Calculate residuals: \( r = F(\sigma_{\text{ref}}) - M(\sigma); \)
Calculate Gradient: \( G = J_0 r; \)
Initial estimate: \( \sigma(k) = \sigma_{\text{ref}}; \)

Calculate the initial Objective Function: \( g(k) = \left\{ \frac{1}{2} \| r \|^2 + \frac{1}{2} \lambda_0 \| L(\sigma_{\text{ref}}) \|^2 \right\} \)

while \( k < 15 \) and exit ≠ true

Calculate initial conductivity value using single step reconstruction:

\[
H = J_0^T I_0 + \gamma(k) I + \lambda_0 (L^T L)
\]
\[
d_{\text{RLM}} = -H^{-1} G
\]
\[
\sigma_{\text{new}} = d_{\text{RLM}} + \sigma(k)
\]
% Continued

% Constraints (Optional)
% non−negativity and non−zero constraints
for i = 1:n;
    if σ_{new} \leq 0;
        σ_{new} = σ_{ref}
    end
end

% Monotonicity constraints
A = diff(σ_{new})
for i = 1:n − 1;
    if σ_{new}(i + 1,:) \leq σ_{new}(i,:),
        σ_{new}(i + 1,:) = σ_{new}(i,:) + abs(A(i,:))
    end
end

% Solve forward problem using σ_{new}
Calculate residuals: \( r = F(σ_{new}) − M(σ) \)

Step and Function tolerance based Exit conditions
if \( \|σ_{new} − σ(k)\| < ε_2(\|σ(k)\| + ε_2) \)
    exit = true
end
if \( \|g(k + 1) − g(k)\| < ε_1(1 + \|g(k)\|) \)
    exit = true
end
else \( g(σ_{new}) = \left\{ \frac{1}{2} \|r\|^2 + \frac{1}{2} λ_0 \|L(σ_{new} − σ(k))\|^2 \right\} \)

Extract the E fields and Calculate the Jacobian: \( J(σ_{new}) \)

Calculate gain ratio: \( ρ = \frac{(g(σ_k) − g(σ_{new}))}{(M(0) − M(σ_{new} − σ_k))} \)
% Continued

Update the damping parameter:

\[
\text{if } \rho > 0; \\
\quad \sigma_{k+1} = \sigma_{\text{new}}; \\
\quad G = J(\sigma_{k+1})r + \lambda_0 L^T L(\sigma_{k+1} - \sigma_k) \\
\quad \gamma = \gamma \times \max(0.5, 1 - (2\rho - 1)^3); \\
\quad \eta = 2; \\
\text{else} \\
\quad \gamma = \gamma \times \eta; \\
\quad \eta = \eta \times 2; \\
\quad \text{if } \eta = 32; \\
\quad \quad \text{exit} = \text{true}; \\
\text{end} \\
\text{end} \\
\quad k = k + 1; \\
\text{end}
\]

Figure A. 1: The simplified RLM algorithm used to reconstruct graphite conductivity profile
Appendix B: The Extended RGN Reconstruction Algorithm

%d Load COMSOL interface
%d Load forward Model
%d Create Voxel grids for E field calculation within the brick volume
%d Specify number of discretised domains: n ← 7

Problem: \( \sigma^* = \arg \min_{\sigma} \left\{ \frac{1}{2} \| F(\sigma) - M(\sigma) \|^2 + \frac{1}{2} \lambda \| L(\sigma - \sigma_{\text{ref}}) \|^2 \right\} \)

Extract reference data from homoginous conductivity profile: \( F(\sigma_{\text{ref}}) \)
Calculate the Jacobian from homoginous conductivity profile: \( J_0(\sigma_{\text{ref}}) \)

Regularisation operator: \( L = \begin{cases} 2 & \text{for } i = j \\ -1 & \text{for adjacent } i \text{ and } j \\ 0 & \text{otherwise} \end{cases} \)

Regularisation parameter: \( \lambda_0 = \max(\text{diag}(J_0^T J_0)) \) or \( 1 \times 10^{-28} \)

Load the measured data: \( M(\sigma) \)

Calculate residuals: \( r = F(\sigma_{\text{ref}}) - M(\sigma) \)

Initial estimate: \( \sigma(k,:) = \sigma_{\text{ref}} \)

Calculate the initial Objective Function: \( g(k) = \left\{ \frac{1}{2} \| r \|^2 + \frac{1}{2} \lambda \| L\sigma_{\text{ref}} \|^2 \right\} \)

Calculate initial conductivity value using single step reconstruction:

\[
\sigma_{\text{new}} = (-J_0^T J_0 + \lambda_0 (L^T L)^{-1} J_0 r) + \sigma(k,:) \\
\sigma_{\text{new}} = \sigma_{\text{new}}^T
\]

while \( k < 15 \) and exit \( \neq \text{true} \)

% Constraints (Optional)
% non - negetivity
for \( i = 1: n; \)
    if \( \sigma_{\text{new}} \leq 0; \)
        \( \sigma_{\text{new}} = \sigma_{\text{ref}}; \)
    end
end

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Figure B. 1: The simplified RGN algorithm used to reconstruct graphite conductivity profile
Appendix C: Complete drawing of the proposed probe former

Figure C. 1: Complete drawings of the proposed probe former
Appendix D: Complete drawing of the Existing probe former

Figure D. 1: Complete drawing of the existing probe former, reproduced from [6]