THE AC-MOT COLD ATOM ELECTRON SOURCE

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Contents

List of Tables 6
List of Figures 7
Abstract 11
Declaration 12
Copyright 13
Acknowledgements 14

1 Introduction 15
   1.1 Electron beam applications ......................... 16
   1.2 Beam quality metrics ............................... 17
       1.2.1 Emittance .................................... 18
       1.2.2 Brightness .................................. 21
       1.2.3 Spatial coherence .............................. 22
       1.2.4 Energy spread ................................ 24
   1.3 Producing high quality electron beams ................. 25
       1.3.1 Thermionic electron sources .................... 26
       1.3.2 Photoinjectors ................................ 28
       1.3.3 Field emission electron sources ............... 30
       1.3.4 Cold neutral plasma electron sources ........... 32
   1.4 Uses of CAESs ..................................... 35
   1.5 Thesis outline ...................................... 36

2 The electron source 38
   2.1 Cooling and trapping atoms .......................... 38
       2.1.1 Doppler cooling ............................... 38
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1.2</td>
<td>The Zeeman effect</td>
<td>41</td>
</tr>
<tr>
<td>2.1.3</td>
<td>The one dimensional MOT</td>
<td>42</td>
</tr>
<tr>
<td>2.1.4</td>
<td>Approximating a two-level atom</td>
<td>44</td>
</tr>
<tr>
<td>2.1.5</td>
<td>The three dimensional rubidium MOT</td>
<td>47</td>
</tr>
<tr>
<td>2.1.6</td>
<td>Laser locking system</td>
<td>47</td>
</tr>
<tr>
<td>2.1.7</td>
<td>Rubidium source</td>
<td>50</td>
</tr>
<tr>
<td>2.2</td>
<td>The alternating current MOT</td>
<td>57</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Fast zeroing of magnetic fields</td>
<td>60</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Trapping coils</td>
<td>60</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Laser light polarisation in an AC-MOT</td>
<td>64</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Trapping optics</td>
<td>66</td>
</tr>
<tr>
<td>2.2.5</td>
<td>Imaging the MOT</td>
<td>69</td>
</tr>
<tr>
<td>2.2.6</td>
<td>Estimating the MOT population</td>
<td>71</td>
</tr>
<tr>
<td>2.2.7</td>
<td>Vacuum system</td>
<td>72</td>
</tr>
<tr>
<td>2.2.8</td>
<td>Converting between the AC-MOT and the DC-MOT</td>
<td>74</td>
</tr>
<tr>
<td>2.3</td>
<td>Producing cold electrons from a MOT</td>
<td>74</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Photoionisation channels</td>
<td>74</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Ionising the trapped atoms</td>
<td>77</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Optical chopper</td>
<td>80</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Experimental cycle</td>
<td>84</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Trigger generation</td>
<td>85</td>
</tr>
<tr>
<td>3</td>
<td>Heating mechanisms in a cold neutral plasma</td>
<td>89</td>
</tr>
<tr>
<td>3.1</td>
<td>Excess energy</td>
<td>89</td>
</tr>
<tr>
<td>3.2</td>
<td>Three-body recombination</td>
<td>95</td>
</tr>
<tr>
<td>3.3</td>
<td>Disorder-induced heating</td>
<td>96</td>
</tr>
<tr>
<td>3.4</td>
<td>Space-charge expansion</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>The electron beamline</td>
<td>105</td>
</tr>
<tr>
<td>4.1</td>
<td>Emittance from a waist scan</td>
<td>106</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Calculating the A and B coefficients</td>
<td>108</td>
</tr>
<tr>
<td>4.2</td>
<td>The electron beamline</td>
<td>110</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Beamline design constraints</td>
<td>113</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Beamline design</td>
<td>117</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Limitations of the beamline</td>
<td>128</td>
</tr>
<tr>
<td>4.3</td>
<td>The effects of the magnetic trapping fields</td>
<td>131</td>
</tr>
</tbody>
</table>
List of Tables

5.5.1 AC vs DC source parameters .......................... 186
List of Figures

1.2.1 Emittance as defined by the effective trace-space volume . . . . . 19
1.2.2 Coherence length vs electron temperature . . . . . . . . . . . 23
1.2.3 Energy spread vs electron temperature . . . . . . . . . . . . . 25
1.3.1 Schematic of a thermionic electron source . . . . . . . . . . . 27
1.3.2 Typical photoinjector design . . . . . . . . . . . . . . . . . . . . 28
1.3.3 Schematic of a field emission electron source . . . . . . . . . . . 31
1.3.4 Design of the original Eindhoven CAES . . . . . . . . . . . . . 33
1.3.5 Diagram of McCulloch et al’s 2013 CAES . . . . . . . . . . . . 34
2.1.1 Doppler Cooling . . . . . . . . . . . . . . . . . . . . . . . . . . . . 39
2.1.2 1D optical molasses . . . . . . . . . . . . . . . . . . . . . . . . . . 42
2.1.3 Energy level diagram of a 1D MOT . . . . . . . . . . . . . . . . . 43
2.1.4 Rubidium 85 $D_2$ line hyperfine structure . . . . . . . . . . . 45
2.1.5 Optical pumping of $^{85}$Rb . . . . . . . . . . . . . . . . . . . . . 46
2.1.6 An isometric view of a MOT . . . . . . . . . . . . . . . . . . . . . 48
2.1.7 Rubidium absorption spectrum . . . . . . . . . . . . . . . . . . . . 49
2.1.8 Design of the locking system optics . . . . . . . . . . . . . . . . 51
2.1.9 Doppler free saturated absorption spectrum . . . . . . . . . . . . 52
2.1.10 Circuit diagram for the locking system photo diodes . . . . . . . 53
2.1.11 Locking system solenoid power supply circuit diagram . . . . . . 54
2.1.12 Locking system solenoid design . . . . . . . . . . . . . . . . . . . 55
2.1.13 The magnetic field of locking system solenoid . . . . . . . . . . . 56
2.1.14 Laser stability due to the locking system . . . . . . . . . . . . . 56
2.2.1 The directions in which electrons can be extracted from a MOT . . 58
2.2.2 Emittance growth due to vertical or horizontal extraction beam . . 58
2.2.3 Emittance growth as a function of MOT solenoid radius . . . . . . 59
2.2.4 Induced magnetic eddy fields as a function of AC switch off phase 61
2.2.5 2D trapping fields . . . . . . . . . . . . . . . . . . . . . . . . . . . . 63
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.6</td>
<td>Gradient of the trapping fields along the z axis</td>
<td>64</td>
</tr>
<tr>
<td>2.2.7</td>
<td>HV high speed switching circuit for EOMs</td>
<td>65</td>
</tr>
<tr>
<td>2.2.8</td>
<td>Trapping optics</td>
<td>66</td>
</tr>
<tr>
<td>2.2.9</td>
<td>Diagram of the vertical trapping beam</td>
<td>68</td>
</tr>
<tr>
<td>2.2.10</td>
<td>Location of MOT cameras</td>
<td>70</td>
</tr>
<tr>
<td>2.2.11</td>
<td>Diagram of the vacuum chambers</td>
<td>73</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Energy level diagrams showing different photoionisation schemes</td>
<td>76</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Two colour photoionisation energy level diagram</td>
<td>77</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Excitation and ionisation optics</td>
<td>79</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Ionisation laser pulse shape</td>
<td>80</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Chopper extinction time</td>
<td>81</td>
</tr>
<tr>
<td>2.3.6</td>
<td>Circuit to drive the optical chopper</td>
<td>82</td>
</tr>
<tr>
<td>2.3.7</td>
<td>Chopper voltage profile</td>
<td>83</td>
</tr>
<tr>
<td>2.3.8</td>
<td>Experimental cycle timing diagram</td>
<td>86</td>
</tr>
<tr>
<td>2.3.9</td>
<td>Trigger box waveforms</td>
<td>87</td>
</tr>
<tr>
<td>3.1.1</td>
<td>The Coulomb-Stark potential of a single ion in an electric field</td>
<td>91</td>
</tr>
<tr>
<td>3.1.2</td>
<td>Distribution of emission angle vs final momentum</td>
<td>92</td>
</tr>
<tr>
<td>3.1.3</td>
<td>Source temperature as a function of extraction energy and excess ionisation energy</td>
<td>93</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Effect of disorder-induced heating on the brightness of ion beam</td>
<td>97</td>
</tr>
<tr>
<td>3.3.2</td>
<td>The effect of disorder-induced heating on brightness of electron beam</td>
<td>98</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Ion temperature at different optical lattice filling factors</td>
<td>99</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Phase space distribution after space charge expansion</td>
<td>102</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Bunch shaping leading to a reduction in emittance growth</td>
<td>103</td>
</tr>
<tr>
<td>4.1.1</td>
<td>The action of a waist scan</td>
<td>107</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Superposition of electrostatic field maps</td>
<td>109</td>
</tr>
<tr>
<td>4.1.3</td>
<td>Typical $A$ and $B$ coefficients</td>
<td>111</td>
</tr>
<tr>
<td>4.2.1</td>
<td>The equipotentials of an Einzel lens</td>
<td>112</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Optical access for the horizontal trapping beams</td>
<td>114</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Optical access for the vertical trapping beams</td>
<td>115</td>
</tr>
<tr>
<td>4.2.4</td>
<td>The equipotentials of the extraction and correction field</td>
<td>116</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Electron beamline</td>
<td>118</td>
</tr>
<tr>
<td>4.2.6</td>
<td>The extraction field map</td>
<td>119</td>
</tr>
<tr>
<td>4.2.7</td>
<td>The combined extraction and correction lens field map</td>
<td>120</td>
</tr>
</tbody>
</table>
4.2.8 Effect of the correction lens on the beam .................................. 121
4.2.9 Effect of the correction lens at various bias ratios ....................... 122
4.2.10 The Einzel lens field .............................................................. 123
4.2.11 Standard deviation of $B$ as a function of Einzel lens position .... 124
4.2.12 2D vs 3D Einzel lens maps .................................................... 124
4.2.13 The position of the steering coils in relation to the beamline ....... 126
4.2.14 Magnetic steering field ............................................................ 127
4.2.15 $A$ and $B$ as a function of $z$, bias map .................................. 129
4.2.16 Measured temperature for a given temperature electron bunch ... 130
4.2.17 Linear regime as defined by the $\chi^2$ of the linear fits for the A
coefficients ................................................................. 131
4.3.1 2D magnetic trapping field map .............................................. 132
4.3.2 Non-linear portion of the solenoid lens ..................................... 133
4.3.3 The non-linearity of the beamline at 10 eV ................................ 134
4.3.4 Emittance of beam on the AC and DC beamlines ...................... 135
4.4.1 Diagram of the beamline and its support structure ..................... 136
5.1.1 The operation of an MCP ......................................................... 142
5.1.2 MCP camera mounting ........................................................... 144
5.1.3 Electron beam cross-section background removal .................... 146
5.1.4 Electron beam cross section background offset removal .......... 147
5.2.1 Electron counting using threshold method ................................ 149
5.2.2 Electron intensity distribution using threshold method ............ 150
5.2.3 Electron intensity distribution using the threshold-cropped, method 151
5.2.4 Example of a single image in the process of being tagged ........ 154
5.2.5 An original image, a ‘perfect’ image calculated by the neural net-
work, and the residual noise ...................................................... 155
5.2.6 An original image, a ‘perfect’ image calculated by the neural net-
work, and the residual noise ...................................................... 156
5.2.7 Single electron intensity distribution ....................................... 157
5.3.1 Example of a TOF measurement ............................................. 159
5.3.2 Time of flight map ................................................................. 161
5.3.3 Electron time of flight as a function of extraction bias .............. 162
5.3.4 Electron pulse length as a function of beam energy ................... 162
5.4.1 $A$ and $B$ coefficient map ....................................................... 164
5.4.2 Correlation between the amplitude and width of the single electron
spots ................................................................. 165
5.4.3 Distributions of noise in electron images ........................................... 166
5.4.4 Simulated electron bunch and the resultant simulated electron cross
section ................................................................................................. 167
5.4.5 Calculated charge vs input charge for simulated data ...................... 168
5.4.6 Estimation of the rotation of a 2D Gaussian ..................................... 169
5.4.7 Example of fitted Gaussian a simulated electron distribution .......... 170
5.4.8 Width of the fitted Gaussian compared to the underlying distribution 171
5.4.9 The Gaussians fitted to bunches of varying bunch charge ............... 172
5.4.10 Width of the fitted Gaussians as a function of bunch charge after a
Gaussian filter was applied to the image .............................................. 173
5.4.11 Waist scans of electron bunches with varying bunch charge .......... 175
5.4.12 The TOF measurements for the AC-MOT CAES ......................... 177
5.4.13 A and B coefficients for AC-MOT case ........................................ 178
5.4.14 Temperature and source size measurement of the electron beam
from an AC-MOT ............................................................................. 179
5.4.15 Cross sections and 2D Gaussian fits for the AC-MOT case ............ 180
5.4.16 A and B coefficients for DC-MOT case ...................................... 182
5.4.17 Cross sections and 2D Gaussian fits for the DC-MOT case .......... 183
5.4.18 Temperature and source size measurement of the electron beam
from an DC-MOT ............................................................................. 184
5.5.1 AC versus DC temperature ............................................................ 185
5.6.1 Low energy AC-MOT CAES waist scan ........................................ 187
5.6.2 Low energy DC-MOT CAES waist scan ....................................... 188
Abstract

This thesis presents details of a new type of cold atom electron source (CAES) that uses an alternating current magneto-optical trap (AC-MOT). Cold atom electron sources produce electron bunches by threshold photoionisation from a population of laser-cooled atoms. The electrons extracted from the atoms are made into a beam which has an ultra-low transverse velocity distribution. The low velocity distribution increases the brightness and reduces the energy spread of the beam, making CAESs potential sources for future electron diffraction and electron spectroscopy studies.

The newly commissioned source detailed here produces focused electron bunches with characteristic temperatures as low as 12.6 ± 0.8 K, at a rate of up to 10 Hz, and with a bunch charge of 2.2 ± 0.2 fC, comparable to other state-of-the-art cold atom electron sources. The AC-MOT CAES can also produce cold bunches with energies <200 eV, which is lower than has previously been reported for this type of source.

The electrons originate from a population of laser-cooled $^{85}$Rb atoms held in an AC-MOT. The AC-MOT CAES design allows the magnetic trapping fields to be switched to zero in 32 µs, more than ∼175 times faster than is possible using a comparable DC-MOT source. This means that electron bunches can be created and extracted in a magnetic field-free environment without any reduction in trap density and hence bunch charge, as occurs in conventional DC-MOT sources.

The design, development, characterisation, and operation of the systems needed for the new AC-MOT CAES are detailed in this thesis. This includes details of the atom trapping system, the photoionisation system, the electron extraction system and the various timing systems for creation and characterisation of the resultant cold electron bunches.

Finally, the characterisation of the AC-MOT CAES itself is presented, and the measured beam metrics are used to compare the new AC-MOT CAES with other CAES designs which are under development.
Declaration

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Chapter 1

Introduction

North West England has a long and varied history in the field of accelerator physics. Research carried out in the north west includes Rutherford’s atom scattering experiments in 1911 [1], important work on the LHC [2, 3], and the development of the VELA[4], ALICE [5, 6], and CLARA [7] accelerators at the Cockcroft Institute.

Manchester also has a rich history in the study of atomic and molecular physics. Novel work on superelastic electron scattering [8], (e, 2e) spectroscopy [9], and developments in atomic cooling and trapping [10] have all taken place within the University of Manchester Atomic and Molecular Physics Group.

In light of the wealth of expertise available, a cross-disciplinary project was proposed: the design and development of a cold atom electron source (CAES) in Manchester. The source would act as a testing bed and a prototype for future inter-disciplinary work which could develop the CAES technology for use in other accelerator and atomic physics projects.

A CAES is a relatively novel type of electron source in which cold atoms are ionised and the resulting free electrons extracted and focused into a bunched beam [11]. The CAES presented in this thesis has a new design where the traditional DC magneto-optical trap (MOT) used in many other CAESs [12, 13] is replaced with an AC-MOT [10], which allows the magnetic fields used in a MOT to be rapidly extinguished, $\sim 175$ times faster than in a comparable DC-MOT. Rapidly eliminating the magnetic fields allows the extraction of the free electrons under magnetic field-free conditions without significant loss of trap density, which, under certain conditions, yields significant benefits to the quality of the resulting electron beam.

This thesis describes the operation of an AC-MOT CAES, compares the quality of the electron beam produced by the AC-MOT CAES with the beam produced by an
1.1 Electron beam applications

Electron beams have wide-ranging applications and the parameters of the beams used in each application vary significantly. Most potential applications can be split into four main categories: imaging, spectroscopy, fabrication, and healthcare.

The first microscopic imaging performed using an electron beam was carried out in 1931 by Ernst Ruska and Max Knoll [14]. The design of the transmission electron microscope (TEM) they built has since been developed into electron microscopes with resolutions below 0.5 Å, sufficient to image single atoms [15, 16]. Along with such significant increases in spatial resolution have been similar increases in temporal resolution with ultrafast electron diffraction imaging of crystalline samples performed at temporal resolutions of <200 fs, sufficient to resolve structural changes on atomic length scales [17, 18].

Electron beams are also used in scanning electron microscopes (SEM). In a SEM an electron beam is raster scanned across a sample, and the back-scattered electrons, secondary electrons, or X-rays emitted by the sample are detected and used to produce an image. Modern SEMs have been shown to have resolutions of ∼2 nm when imaging biological samples [19].

In addition to these more common imaging methods, there are also low energy electron diffraction (LEED) techniques, which use backscattered, low energy electrons to image surfaces [20, 21], often in conjunction with electron spectroscopy techniques.

In electron spectroscopy, an electron beam with a well-defined energy is used to excite electrons in an atom, releasing photons which are then detected, revealing the electronic structure of the material. The electronic structures of samples can also be studied through electron loss spectroscopy; examining the beam current after it passes through the sample to measure the sample’s absorption spectrum [22].

Electron beams are also widely used in the fabrication of nanostructures. Electron beam lithography uses focused electron beams to cure nanoscale sections of an electron sensitive resist, selectively. The cured resist is then developed in a solvent, enabling selective removal of either the cured or uncured portions of the resist [23]. Gas assisted, focused electron beam induced deposition uses an electron beam to decompose a beam of gaseous molecules into its constituents, some of which are then deposited into a
Finally, electron beams have medical therapeutic uses. Electrons, accelerated in a linac to between 1 MeV to 30 MeV, can treat cancers close to the surface of the skin, since electrons have a low penetration depth. The low penetration has the benefit that any healthy tissue behind the lesion only receives a relatively small dose [25].

To describe the type of electron beam required for each use case, and to characterise the beams themselves, requires a set of values that quantitatively encapsulate the beam; a set of beam quality metrics.

### 1.2 Beam quality metrics

A charged particle beam can be described using many different parameters which can then be used to both compare different electron sources and give insight into the specific source and its potential applications.

First and foremost is the beam’s energy. The energy of electron beams range from $<0.1$ eV [26] up to the 100 GeV beams produced in the Large Electron Positron collider [27]. The electron energy sets the electron’s de Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} \quad (1.2.1)$$

where $h$ is Planck’s constant, $p$ is the electron’s momentum, $m_e$ is the mass of an electron, and $v$ is the electron’s velocity. The de Broglie wavelength is the wavelength of the matter wave associated with a particle of momentum $p$ and governs the wave-like properties of the particle.

The second headline figure of merit is the beam current. Highly monochromated cw (continuous wave) beams can have currents of $<1$ nA [28], whilst the beams used in electron beam welding can have currents of up to 10 mA [29]. Pulsed beams tend to have far lower average beam currents since the average current depends on both the duty cycle of the beam and the charge in each of the electron bunches which make up the beam. The peak current in bunched beams, however, can reach values of $>130$ A for bunch charges of 700 pC using ultrafast bunches [30], and ultrafast electron diffraction studies have been carried out with bunches as short as 102 fs, with bunch charges of 60 fC [17].

Whilst the beam energy, bunch charge, and temporal shape can describe the beam macroscopically, the quality of an electron beam is highly dependent on the motion of
the individual electrons which make up the beam. That motion can be parameterised using the beam’s emittance, brightness, coherence length, and energy spread, each of which is discussed in detail below.

1.2.1 Emittance

The emittance of a beam characterises the position and momentum distribution of the beam [31]. The emittance of a beam is closely related to the three position and three momentum coordinates of the particles making up the beam, which together fill a volume in the six-dimensional position-momentum phase space. Emittances are measured in meter radians and tend to be in the range of 0.1 nm rad to 1000 nm rad [31], with ‘low’ emittance beams being highly collimated with a small cross-sectional area whilst ‘high’ emittance beams have a high divergence and a large cross-sectional area.

The emittance can be defined using the moments of the particle distribution in the space of position, \( x \), and divergence, \( x' = dx/dz \), known as \( x - x' \) space or trace space. The moments, \( \langle x^2 \rangle, \langle x'^2 \rangle \), and \( \langle xx' \rangle^2 \), together combine to give the rms emittance:

\[
\bar{\varepsilon}_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}. \tag{1.2.2}
\]

The rms emittance contains the relevant information about the beam’s rms width and total transverse kinetic energy through the \( \langle x^2 \rangle \langle x'^2 \rangle \) term, less the kinetic energy which correlates with position, represented by \( \langle xx' \rangle^2 \). This leaves only the random or thermal kinetic energy.

It is instructive to examine the emittance in a uniform density, elliptical beam, where all the forces affecting the beam particles are a linear function of the position as, in that case, the second moments can be calculated analytically. The boundary of such a distribution in the \( x - x' \) plane is described by an ellipse

\[
A_x = \gamma x^2 + 2\alpha xx' + \beta x'^2, \tag{1.2.3}
\]

where \( \gamma, \alpha, \) and \( \beta \) are arbitrary constants defining the ellipse. In the case where \( \alpha = 0 \) and \( \langle xx' \rangle^2 = 0 \) the other second moments of the distribution will be

\[
\langle x^2 \rangle = \frac{\beta A_x}{4} = \frac{x_{\text{max}}^2}{4}, \tag{1.2.4}
\]
CHAPTER 1. INTRODUCTION

Figure 1.2.1 – The beam, with its initial trace-space distribution shown on the left, undergoes a non-linear transformation, ending with the trace-space distribution shown on the right. Despite the reduction in beam quality the total trace-space area, outlined in purple, does not change, but the effective emittance, represented by the orange ellipse, does increase: continuing to reflect the quality of the electron beam.

and

\[
\langle x'^2 \rangle = \frac{\gamma A_x}{4} = \frac{x'_{\max}^2}{4}.
\]  

(1.2.5)

where \(x_{\max}\) is the maximum \(x\) position of any particle in the beam and \(x'_{\max}\) is the maximum \(x\) divergence of any particle in the beam. Hence the beam’s total area in trace space is

\[
A_x = x_{\max}x'_{\max}\pi = 4 \langle x^2 \rangle \langle x'^2 \rangle \pi = 4\tilde{\epsilon}_x\pi,
\]

(1.2.6)

four-times the rms emittance. It is meaningful to use the four-times rms emittance to describe real beams since in practice few particles are found outside that area. \(\epsilon_x = 4\tilde{\epsilon}_x\) is referred to as the effective emittance in the remainder of this thesis. The benefit of using the effective emittance is demonstrated in figure 1.2.1.

As well as providing a useful description of the beam at a single point along the beamline, the emittance remains constant whilst:

- There is no acceleration so \(\beta\gamma = \text{const}\), where \(\gamma = v/c, \beta = \frac{1}{\sqrt{1-v^2/c^2}}\), \(c\) is the speed of light, and \(v\) is the group velocity of the beam.

- The beam isn’t affected by any forces which are non-linear in position or momentum, so trace-space area remains elliptical.

- There is no coupling between the \(x\)-motion and motion in the other (\(y\) and \(z\)) directions.

The invariance of the emittance under these conditions arises from the fact that the trace space area, and hence the emittance, can be written as a projection of the phase space.
volume onto the $x - p_x$ plane,

$$A_x = \frac{1}{p} \iint dx d p_x = \frac{1}{\gamma \beta mc} \iint dx d p_x,$$  \hspace{1cm} (1.2.7)

and, from Liouville’s Theorem, $\iint dx d p_x = \text{const}$. In equation 1.2.7 $p_x$ is the momentum of the particle in the $x$-direction, $p$ is the average momentum of the beam, and $m$ is the mass of the particles which make up the beam.

From equation 1.2.7 a fully invariant form of the emittance can be constructed, the *normalised emittance*:

$$\varepsilon_n = \beta \gamma \varepsilon = 4 \beta \gamma \bar{\varepsilon}$$  \hspace{1cm} (1.2.8)

which will remain constant with respect to acceleration of the beam and can be used to fairly compare beams at different energies.

We can also use equation 1.2.7 to recast equation 1.2.2 in terms of the transverse position, $x$, and transverse momentum, $p_x$, of the beam:

$$\varepsilon_{n,x} = \frac{1}{mc \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle xp_x \rangle^2}}.$$  \hspace{1cm} (1.2.9)

Then, at a beam waist where there is no correlation between $x$ and $p_x$, the emittance reduces to

$$\varepsilon_x = \frac{1}{mc} \sigma_x \sigma_{p_x},$$  \hspace{1cm} (1.2.10)

where $\sigma_x = \sqrt{\langle x^2 \rangle}$ is the rms beam size and $\sigma_{p_x} = \sqrt{\langle p_x^2 \rangle}$ is the rms transverse momentum spread. Since the transverse momentum spread for a beam with a thermal velocity distribution can be characterised as

$$\sigma_{p_x} = \sqrt{mk_B T},$$  \hspace{1cm} (1.2.11)

where $k_B$ is Boltzmann’s constant, and $T$ is the beam’s temperature in Kelvin, then by substituting equation 1.2.11 back into equation 1.2.10, the relationship between the beam’s emittance and its temperature at a beam waist can be written as:

$$\varepsilon_x = \sigma_x \sqrt{\frac{k_B T}{mc^2}}.$$  \hspace{1cm} (1.2.12)

The electron source itself is a beam waist since at the source there is no correlation between $x$ and $p_x$. Equation 1.2.12 therefore relates the source parameters to the beam emittance; a smaller or lower temperature source will produce a lower emittance beam.
Though the normalised emittance is invariant under acceleration or the action of linear forces, non-linear forces which act on the beam can cause growth of the emittance by distorting the beam’s phase space volume and continuity. For example, a non-linear electrostatic lens which distorts the phase space of a beam will increase the emittance despite not altering the total volume occupied by the beam in phase space. Once the emittance of a beam has grown, it is difficult to reduce. Emittance reductions are usually achieved by removing particles from the beam [31].

Since the emittance can be reduced arbitrarily by removing particles from the beam, the emittance alone is not sufficient to characterise the quality of a charged particle beam. As the beam current is critical to most electron beam applications the critical metric is the number of particles with a given emittance, which is known as the brightness of the beam.

### 1.2.2 Brightness

The brightness is defined as the current density per unit solid angle along the axis of the beamline,

\[
\mathcal{B}_\perp = \frac{J}{d\Omega} = \frac{dI}{dSd\Omega}
\]

(1.2.13)

where \(J\) is the current density, \(\Omega\) is the solid angle subtended by the beam, \(I\) is the beam current, and \(S\) is the cross sectional area of the beam [31].

Since \(\mathcal{B}_\perp\) can vary across the beam, an average brightness is more useful for most applications. The average brightness is defined as the average beam current confined by a four-dimensional volume in trace space \(V_4\),

\[
\overline{\mathcal{B}}_\perp = \frac{I}{V_4}.
\]

(1.2.14)

where \(V_4 = \int \int dSd\Omega\) is the integral of the trace space volume. The trace space volume is not necessarily the same as the effective trace space volume used to define the emittance. If the particle distribution, however, occupies an ellipsoid in trace space then the average brightness can be written in terms of the emittance as

\[
\overline{\mathcal{B}}_\perp = \frac{I}{4\pi^2\varepsilon_x\varepsilon_y},
\]

(1.2.15)

where \(\varepsilon_x\) and \(\varepsilon_y\) are the rms emittances in the \(x\) and \(y\) axes, since in that case the volume \(V_4\) is identical to the volume used in the trace space formulation of the emittance,
equation 1.2.2. $\overline{B}_\perp$ can also be normalised to give the invariant normalised transverse brightness,

$$\overline{B}_{n\perp} = \frac{\overline{B}_\perp}{(\beta\gamma)^2}.$$  \hspace{1cm} (1.2.16)

As with the normalised emittance, normalised brightnesses are useful not only because of their invariance with respect to the energy of the beam, but also because they can be used to compare electron sources operating at different energies.

As with the emittance, the brightness can also be related to the thermal distribution of the electrons in the beam. Substituting equation 1.2.12, relating the emittance along one axis to the temperature of a thermal electron distribution, into equation 1.2.15 gives

$$\overline{B}_\perp = \frac{I}{4\pi^2\sigma_x\sigma_y} \frac{mc^2}{k_B T}$$  \hspace{1cm} (1.2.17)

$$= \frac{J mc^2}{4\pi k_B T}$$  \hspace{1cm} (1.2.18)

where $J = I/\pi\sigma_x\sigma_y$ is the current density. The brightness therefore usefully encapsulates the quality of an electron beam and equation 1.2.18 shows how a reduction in the source temperature will result in an increase in the beam’s brightness.

Even with a high brightness beam, however, producing electron diffraction images requires the beam to be spatially phase coherent across the transverse area of the sample which is to be imaged.

### 1.2.3 Spatial coherence

The coherence length of a beam is the length scale over which the phase of the beam’s constituent particles is correlated. For diffraction imaging, the coherence length sets the maximum size of the unit cell spacing for which interference will be observed. The coherence length can be written as,

$$L_c = \frac{\lambda}{2\pi\sigma_\theta},$$  \hspace{1cm} (1.2.19)

where $\lambda$ is the de Broglie wavelength of the particles in the beam and $\sigma_\theta$ is the rms angular spread of the source.

A high coherence length beam is usually acquired by propagating the beam over a large distance and then using an aperture to isolate a small fraction of the beam. Isolating a portion of the beam, however, reduces the total flux and therefore reduces
Figure 1.2.2 – The relationship between the coherence length of an electron beam and the beam’s temperature.

the resulting image’s signal to noise ratio. Ideally, the source itself would produce a high coherence length beam, allowing the total beam flux to be used in imaging.

From equation 1.2.19, a long coherence length beam would have a small rms angular spread at the source. For a paraxial beam, where $v_z \gg v_{x,y}$, the angular spread of the beam can be related to the beam’s rms momentum spread as

$$\sigma_{p_x} = \hbar \sigma_\theta |k| = \hbar \sigma_\theta \frac{2\pi}{\lambda},$$

(1.2.20)

where $k$ is the wavevector of the beam and $\hbar$ is the reduced Planck constant. The rms momentum spread for a thermal distribution of particles is given by equation 1.2.11, allowing the coherence length to be related to the temperature for a thermal beam of electrons:

$$\mathcal{L}_c = \frac{\hbar}{\sqrt{m_e k_B T}},$$

(1.2.21)

where $m_e$ is the mass of an electron. Since the coherence length is proportional to one over the square root of the temperature, low temperature beams, such as those produced by a CAES, will have long coherence lengths. Figure 1.2.2 illustrates the relationship between temperature and coherence length.
1.2.4 Energy spread

As well as the emittance, brightness, and coherence length, the beam’s energy spread is an important parameter for a range of applications of electron beams. The effects of a large energy spread are most apparent when the beamline includes lens elements whose actions are energy dependent, with particles at different energies deflected by the lens to a different degree. Such chromatic aberrations increase the beam’s spot size if the beam has a large energy spread.

Monochromatic electron beams, as well as being less susceptible to chromatic aberrations, are important in material analysis and nanofabrication. Techniques such as high resolution electron loss spectroscopy, electron-beam milling, and inverse photoemission spectroscopy each require electron beams with energy spreads of $<0.3 \text{ eV}$ [28, 32–38].

The energy spread of an electron beam has two components: the stochastic distribution of electron momenta, and any systematic variation in the electron energies arising from the range of potentials at which the electrons were produced.

The energy distribution is most usefully defined in the co-moving frame: the frame of reference moving at the group velocity of the beam. In the co-moving frame the energy distribution of thermal electrons can be determined from the temperature of the bunch. The energy distribution, $f_E(E)$, for electrons at temperature $T$ is:

$$f_E(E)dE = \frac{2}{\sqrt{\pi}} \left( \frac{E}{k_B T} \right)^{1/2} \exp\left( -\frac{E}{k_B T} \right) d\left( \frac{E}{k_B T} \right), \quad (1.2.22)$$

where $k_B$ is the Boltzmann constant. Finding the mean of the energy distribution is equivalent to determining $1/2m_eV_{\text{rms}}^2$, where $V$ is the velocity distribution of the electrons in the beam [39]. Since thermal particles have a Gaussian velocity distribution the standard deviation of the velocity is given by $V_{\text{rms}}^2$. The mean of the energy distribution, $\bar{E}$, therefore gives the energy spread of the electrons:

$$\bar{E} = \Delta E = \int_0^\infty E f(E)dE = \frac{2k_B T}{\sqrt{\pi}} \int_0^\infty \left( \frac{E}{k_B T} \right)^3 \exp\left( -\frac{E}{k_B T} \right) d\left( \frac{E}{k_B T} \right), \quad (1.2.23)$$

which evaluates to give the energy spread as $\bar{E} = 3/2k_B T$. Figure 1.2.3 shows the energy probability distribution as a function of temperature along with the mean of the distribution, and hence the energy spread, at each temperature and shows how reducing the electron temperature of a beam has a linear effect on the energy spread of the beam. Only with electron temperatures $<10 \text{ K}$ can one achieve the ultra-low energy spreads.
Figure 1.2.3 – The relationship between electron temperature and the electron energy probability distribution. The red dashed line shows the mean of the distribution, and hence the energy spread, as a function of the electron temperature.

needed by high resolution electron loss spectroscopy.

### 1.3 Producing high quality electron beams

Each of the beam quality metrics discussed above, the emittance

$$\varepsilon_x = \sigma_x \sqrt{\frac{k_B T}{m c^2}},$$

(1.3.1)

the brightness

$$\mathcal{B}_\perp = \frac{I}{8\pi^2 \sigma_x \sigma_y mc^2 k_B T},$$

(1.3.2)

the coherence length

$$\mathcal{L}_c = \frac{\hbar}{\sqrt{m_e k_B T}},$$

(1.3.3)

and the energy spread

$$\Delta E = \frac{3}{2} k_B T,$$

(1.3.4)

are characterised by the spatial extent of the source, \(\sigma_{x,y,z}\), and the source temperature, \(T\). By targeting those more fundamental parameters when designing an electron source,
the quality of the resulting electron beam can be controlled. Equally important is the beam current $I$, and the bunch length. The beam current governs the brightness of a beam and the signal to noise ratio for any measurements made using an electron beam, whilst the bunch length is vital for any temporally resolved measurements made using the beam.

How the beam parameters are controlled in each of the most widespread methods of producing an electron beam; photoinjector, thermionic, and field emission, is discussed below, and typical beam parameters for each of those source types are given.

1.3.1 Thermionic electron sources

Thermionic electron sources are the most widespread source of electron beams. In most applications, the source consists of a tungsten filament which is resistively heated in an electric field to produce electrons via thermionic emission.

The same principle can, however, be used to produce high-quality electron beams, with a schematic of a typical thermionic source shown in figure 1.3.1. Such high-quality electron thermionic sources usually consist of an LaB$_6$ crystal cathode heated sufficiently to emit electrons. An LaB$_6$ crystal is used due to its low work function. The electrons are accelerated towards an anode by a large bias applied to the cathode, on the order of $100$ kV, and focused into a beam by the electric fields produced by a negatively biased Wehnelt cylinder, shown in figure 1.3.1. The Wehnelt cylinder also acts to suppress electron emission from areas of the cathode other than at the tip.

Thermionic electron sources are excellent for producing high current density, cw electron beams, with long operating lifetimes [40]. They can also be used to produce high current pulsed beams of $>2$ µs by pulsing a retarding electrode placed along the beamline [41].

The diameter, $\sigma_x$, and the divergence angle, $\theta \approx \sigma_x'$, and hence the transverse emittance of the beam produced by a thermionic source is defined at the crossover point, which is shown in figure 1.3.1. Thermionic electron sources with normalised transverse emittances as low as 1 nmrad have been reported [42]. The normalised brightness is dependent on the current density and the emittance, with typical values for LaB$_6$ based sources of $4.8 \times 10^8$ Asr$^{-1}$mm$^{-2}$ [43].

The energy spread of a thermionic source is dependent on the stability of the DC power supplies and the thermal energy of the electrons. Typical thermionic sources have energy spreads of $\sim 1.5$ eV. Improved energy spreads can be achieved using monochromators in exchange for a loss of beam current.
Figure 1.3.1 – A schematic of the thermionic electron source. The LaB$_6$ crystal is heated by a current until it reaches sufficient temperature to emit electrons. The cw beam of electrons, in green, is accelerated by the bias applied to the cathode. The Wehnelt cylinder is negatively biased with respect to the cathode to produce a field, with the equipotentials marked with dashed blue lines, to focus the beam to the cross over - the point marked by two red arrows. The beam width, $\sigma_x$, and angular divergence, $\theta$ at the cross over point sets the source size and angular divergence.
Due to the high source temperature, the transverse coherence length of thermionic sources is dependent on the angle subtended by the source at the sample, $\alpha$, and the de Broglie wavelength of the electrons, $\lambda$,

$$L_c = \frac{\lambda}{2\alpha}. \quad (1.3.5)$$

Inserting typical values for thermionic sources into equation 1.3.5 gives a coherence length of $\sim$1 nm. Hence the desired coherence length is usually achieved by placing an aperture along the beamline [40].

1.3.2 Photoinjectors

The electron beam from a photoinjector is produced by firing a high intensity, short duration laser pulse at a surface, the photocathode. The photons hit the cathode and cause electrons to be emitted via the photoelectric effect. The emitted electrons are then accelerated by either rf or DC electric fields into a beam. A schematic of a typical photoinjector is shown in figure 1.3.2.

In rf photoinjectors, the rf fields used typically have a frequency on the order of 1 GHz. $\sim$1 GHz is chosen as a trade off between maximising the accelerating gradient and the complexity of the system. The gradient is dependent upon the wavelength
of the rf field, but higher frequency accelerating fields present greater engineering complexity in rf generation, cavity design, manufacture, and operation. At ∼1 GHz the accelerating cavities are ∼150 mm long and are relatively straightforward to manufacture, and ∼1 GHz klystrons with which the rf is generated are mature, well understood, technology.

In order for the electrons to be accelerated in the correct direction, and to minimise the energy spread of the beam, the laser pulses used to photoionise the electrons in the cathode have to be very short so that electrons are only produced during a few degrees of phase of the rf cycle [31].

To produce high brightness beams from a photoinjector, many varied techniques are used which fall into three main categories; increasing the bunch charge, decreasing the spatial size of the source, and decreasing the angular divergence of the source.

By increasing the bunch charge whilst maintaining the beam’s emittance, brighter beams are produced due to the higher current densities. The current density is dependent on the quantum efficiency of the photocathode material, the intensity of the laser pulse, and the strength of the accelerating field [44]. Electron bunches with charges of 3 nC have been produced using high quantum efficiency (QE), multi-alkali photocathodes [45].

By reducing the spatial size of the area illuminated by the laser pulse, the transverse emittance of the electron bunch can be reduced [46]. The minimum laser spot size is dependent on the wavelength of light used, and the focal length of the lens used to focus the beam onto the cathode. Reducing the laser spot size, however, increases the rate at which the surface of the photocathode is damaged, reducing the QE of the cathode [47], therefore reducing the lifetime of the source.

The angular divergence of the beam can also be controlled. As the angular divergence is dependent on the difference between the cathode work function and the photon energy, reducing the wavelength of the laser pulse such that the photon energy is closer to the work function of the cathode has been shown to reduce the beam emittance, whilst the laser spot size remains the same [48].

The energy spread of the beam from a photoinjector is dependent on the phase range of the accelerating field over which the bunch is ionised. There are additional contributions from space charge forces due to the self-field of the bunch, and the field from the positive charge built up in the cathode surface; the mirror charge, as the electrons are extracted [44].

Photoinjectors, such as the one installed on the VELA machine at the Cockcroft
Institute, can produce electron beams with bunch charges of up to 250 pC with a normalised emittance of 4 µm rad and a 5% energy spread [49].

1.3.3 Field emission electron sources

In a field emission source, a large electric field is applied to a surface, inducing the release of electrons from the surface via quantum tunnelling. To maximise the beam current the electric field at that surface must be maximised. For a pair of electrodes with a potential difference between them of \( V \), the maximum surface field is found with an ultra sharp point, or tip, since

\[
E = \frac{V}{r}
\]

(1.3.6)

where \( E \) is the surface field, and \( r \) is the radius of the (spherical) tip. The tips used in field emission sources regularly have radii <100 nm giving field strengths of \( 10^{10} \text{ V m}^{-1} \) at the tip for a 1 kV bias. The huge electric fields result in large stresses on the tips, so they must be mechanically strong as well as nanoscopically sharp [40].

To enable field emission, the surfaces must not be contaminated or oxidised. The tips are therefore either heated, in which case they are called ‘Schottky emitters’, or operated as ‘cold tips’ at room temperature under UHV conditions [40]. The thermal energy of the electrons in ‘Schottky emitters’ is sufficient that they do not typically need to tunnel out of the material. Both Schottky emitters and cold tips are typically fashioned from tungsten, but Schottky emitters undergo additional surface treatments to improve beam quality.

The tips act as the cathode in a field emission gun, with the electrons formed into a beam with the help of two anodes. The first anode produces the field to extract the electrons from the tip, and the second accelerates the electrons up to the required energy. The arrangement is shown in figure 1.3.3. The gun also frequently features an additional magnetic lens to increase the flexibility of beam parameters. [43].

Tungsten field emission sources can produce electron beams with brightnesses in excess of \( 10^6 \text{ Asr}^{-1}\text{mm}^{-2} \) and energy spreads of 0.3 eV. Field emission sources using carbon nanotubes as a cathode have also been manufactured, producing beams with some of the highest brightnesses ever recorded at \( 10^{12} \text{ Asr}^{-1}\text{m}^{-2}\text{V}^{-1} \) [50, 51]. The brightness of such sources is limited by the maximum current that can be extracted from the tiny, 1.9 nm² emitting area before the Coulomb forces between the electrons in the beam, or space charge forces, damage the beam quality.

Field emission sources are typically used in scanning electron microscopy where
Figure 1.3.3 – The electron beam is shown in green with the beam axis in orange. Electrons are extracted from the field emission tip by strong electric fields produced by Anode 1. The electrons are then accelerated by the bias applied to Anode 2.
tiny probe sizes can be achieved by the high brightness, low emittance cw beams.

### 1.3.4 Cold neutral plasma electron sources

In 2005 a novel design for an electron source was proposed by Claessens et al. [52]. In their design, electrons were produced by threshold photoionisation of atoms which had been cooled and held in a magneto-optical trap (MOT) at temperatures on the order of 10 µK. The creation of such a cold neutral plasma was first reported in 1999 from the near threshold photoionisation of meta-stable Xenon [53] with the aim of designing a tool that could be used in the study of ‘strongly coupled plasma’, such as the plasma found in astrophysical systems [54].

Claessens et al. suggested using a pulsed electric field applied across the plasma to extract the electrons into a low-temperature beam, with the predicted three-orders of magnitude reduction in the temperature of the electrons compared to contemporary sources resulting in significant gains in the brightness and the coherence length of the beam. They also noted that such a source would not suffer from ageing since the trapped atoms, from which the electrons would be extracted, could be continually replaced [52].

Claessens et al. realised their design in 2007 [11]. Their source comprised of a rubidium MOT surrounded by four rods placed at the corners of a square 20 mm to a side, as shown in figure 1.3.4. Two of the rods were held at ground whilst a voltage pulse was applied to the other two to produce a pulsed accelerating field of up to 300 V cm\(^{-1}\) across the MOT. A two colour photoionisation scheme was used with a blue pulsed dye laser exciting the population of atoms placed in the \(5p^2P_{3/2}(F = 4)\) state by the trapping beams used in the MOT into high lying Rydberg states. The Rydberg states were then ionised through collisions with other Rydberg atoms. The free electrons were extracted by the pulsed field, and the resulting electron beam was directed at an MCP/phosphor assembly where it was amplified and the image formed on the phosphor was recorded by a charge coupled device (CCD).

The source produced 200 ns long electron bunches of charge 1.2 pC, with a maximum characteristic temperature of 500 ± 400 K and a calculated normalised emittance of 0.3 mm mrad. The low bunch charge meant that despite the low emittance the brightness of the beam was still comparatively low. Claessens et al. also showed that the expansion of the beam was dominated by space charge.

To mitigate the effects of space charge driven expansion McCulloch et al. designed a source that could shape the electron distribution of the beam, shown in figure 1.3.5 [55]. By giving the electrons a uniform, elliptical distribution the space charge forces in
Figure 1.3.4 – Schematic view of the setup built by Claessens *et al.* [11] at the Eindhoven University of Technology. The atom cloud (A) is trapped and cooled by the trapping laser beams (T) and the trapping solenoids (B). An ionisation laser beam (I) ionises the atoms, and the pulsed electric field provided by four biased rods (R) accelerates the electrons towards the MCP/phosphor assembly. The electron bunch is recorded with a CCD and a digital oscilloscope. Reproduced with permission from [11].
Figure 1.3.5 – In McCulloch et al’s design the atoms are trapped in a pseudo-mirror MOT before a shaped excitation beam and ribbon photoionisation beam produce a shaped cold neutral plasma. The electrons are accelerated by the biased plates (labelled $-2000 \text{ V}$ and GND) and are then imaged using an MCP. The insert shows how the bunches retain the shape imposed upon them by the shaped excitation beam due to their low electron temperature, with the image degrading as the electrons are provided with more and more initial excess energy by reducing the ionisation laser wavelength. Adapted with permission from [12].

the bunch remain linear, making the space charge driven expansion reversible [56–58].

Their source consisted of a pseudo-mirror MOT formed between two annular electrodes. The first, held at $-2 \text{ keV}$, was made of optically transparent indium tin oxide (ITO) coated glass, giving optical access for two of the trapping beams whilst also being conducting. The second, held at ground, was made of gold allowing it to act as an infrared (IR) mirror, reflecting two of the other trapping beams into the centre of the trapping region.

Before each electron pulse was produced, the trapping beams were switched off and the trapping fields allowed to decay. A separate excitation beam had its phase distribution altered, using a spatial light modulator, such that the beam’s intensity
distribution at the trap produced a population of excited atoms that was uniform along the axis of the excitation beam. A ‘ribbon’ ionisation beam, orthogonal to the excitation beam, was used to ionise a slice of the excited population, resulting in an electron bunch with the required distribution to limit space charge driven emittance growth [59].

CAESs are also suitable for producing ultrafast electron bunches for use in Ultrafast electron diffraction (UED). Taban et al. [60], building on the work carried out in the same group by Claessens et al., designed and built a source capable of applying a 30 kV pulsed electric field across a MOT with a <30 ns rise time [60]. Taban et al. used their apparatus to produce low temperature, ~10 K electron bunches with calculated temporal lengths of order 50 ps using pulsed field ionisation of Rydberg atoms [61].

The same apparatus was later used to produce 10 K, 25 ps FWHM bunches [62, 63]. Engelen et al. and later Franssen et al. applied a static electric field across the atom cloud and used a 780 nm cw laser to excite the atoms into the $5p^2P_{3/2}(F = 4)$ state. The excited atoms were then either further excited into high lying Rydberg states and field ionised, or photoionised directly to the continuum, by a 58 fs FWHM long, 0.46 mJ laser pulse originating from an optical parametric amplifier.

Contemporaneously, McCulloch et al. used a 5 ns ionisation pulse and a 110 fs excitation pulse to produce 150 ps long cold electron bunches, extracted by a static electric field from a population of laser cooled atoms [12]. The ultrafast electron bunches McCulloch et al. produced had a normalised rms emittance of $538 \pm 26 \text{mm} \cdot \text{mmrad}$, measured by imaging a pepper-pop pattern of beam-lets onto the cold atom population using the bunch shaping techniques they had previously developed [64].

As well as demonstrating emittances as low as 1 mm·mm rad for a bunch of rms width 30 µm [65], CAESs have also demonstrated the long coherence lengths possible due to the low electron temperatures. A 10 K electron bunch from a ~35 µm source would give relative transverse coherence lengths, $C_\perp = \mathcal{L}_c/\sigma_{c_x}$, of $C_\perp = 2.5 \times 10^{-4}$, sufficient to image protein micro-crystals [63].

1.4 Uses of CAESs

Whilst cold atom electron source technology is still relatively new there have been several demonstrations of its suitability for ultrafast electron diffraction and single shot electron diffraction.

UED using a beam from a CAES was first demonstrated by Mourik et al. [65] in 2014. Mourik et al. produced electrons via two colour photoionisation of cold $^{85}\text{Rb}$
atoms held in an MOT. The atoms were first excited into the 5P state, and then a 100 fs ionisation pulse was used to produce several hundred electrons in each shot.

The electron source had a transverse size of $\sigma_x = 30 \mu m$ and $\sigma_y = 50 \mu m$, with the electrons accelerated to 10 keV before being focused by two magnetic lenses onto either the sample plane or the detector plane. The beam imaged a 13 nm to 20 nm thick monocrystalline graphite sample held on a 200 mesh copper TEM grid. Each image was made up of 1000 shots. The group recorded diffraction images using both 250 K and 10 K electrons, demonstrating the improvement in image quality due to the reduced source temperature.

Single shot electron diffraction was first demonstrated by Speirs et al. [66] in 2015 using cold $^{85}$Rb atoms held in a MOT in a two colour scheme but with a 5 ns ionisation pulse and a much larger, $\sigma_z = 425 \mu m$, ionisation volume. Speirs et al. produced approximately 500000 electrons (80 fC) at 8 keV which were focused through a sample of gold foil to the detector using a magnetic lens.

Due to the high signal to noise ratio of the single shot images, Speirs et al. used image registering to align the single shot images, reducing the effect of beam instabilities on the images. The same group also demonstrated single shot, reflection high-energy electron diffraction of a silicon wafer.

1.5 Thesis outline

A CAES relies upon the combination of cold atom physics, plasma physics, and electron optics. In this thesis, each of the contributions from these fields to the operation of a CAES is addressed before the characterisation of the resulting electron beam from the new AC-MOT CAES is presented.

First, the principles by which the AC-MOT CAES produced cold electrons are discussed in chapter 2. The chapter describes the operation of a MOT, the laser locking system we built to frequency stabilise the trapping lasers, the adaptations required for the AC-MOT, and the photoionisation process which produces the cold neutral plasma from the trapped cold atom population. The chapter concludes by describing the experimental process and the timing hardware developed to control the source.

The heating mechanisms which affect the cold electrons in the cold neutral plasma are described in chapter 3. The processes of disorder-induced heating, three-body recombination, space charge expansion, and the electron temperature due to the excess energy given to the electron in the ionisation process are described, and each process’s
effect on the final source temperature is discussed.

The source temperature was measured by extracting the cold electrons into a beam using the electron beamline presented in chapter 4. Details of the beamline’s design and manufacturing process are then given. Chapter 4 also describes the method by which the beamline was used to measure the source temperature and presents simulations showing that the rapid elimination of the trapping magnetic fields made possible by the AC-MOT benefits the quality of the electron beam most in a low energy regime.

Chapter 5 discusses the techniques by which the bunch charge, beam energy, bunch temperature, and source size were measured and presents the results of the beam characterisation as the relevant combination of those beam parameters. Chapter 5 then compares a \( \sim 1 \) keV beam from the AC-MOT to a similar beam from the DC-MOT and other CAESs. The AC-MOT CAES was able to produce an electron beam with a source temperature of \( 12.6 \pm 0.8 \) K, which is comparable to other state of the art CAESs [12, 62].

Section 5.6 describes similar experiments at a lower beam energy where the rapid elimination of the magnetic trapping fields is more beneficial. Experimental results suggest that the electron beam temperature is reduced in the AC-MOT CAES, compared to the DC-MOT CAES at such low energies.

Finally, chapter 6 summarises the work presented in this thesis and presents possible avenues for further investigation.
Chapter 2

The electron source

As described in section 1.3.4, in a cold atom electron source (CAES) atoms are laser cooled and then photoionised to produce a cold neutral plasma. The free electrons in the plasma are then extracted by an electric field and accelerated into a focused beam.

This chapter describes the techniques we used to produce a cold neutral plasma. First, section 2.1 details the cooling and trapping of atoms in a magneto optical trap (MOT). Section 2.2 then describes the alterations to the traditional MOT design used to quickly extinguish the magnetic trapping fields in order to better control the electron extraction. Finally, section 2.3 gives the details of the photoionisation process and the experimental cycle.

2.1 Cooling and trapping atoms

The electrons in a CAES are extracted from a cold neutral plasma generated by ionising the atoms in an ultracold atomic gas. An effective way of producing an ultracold atom cloud of sufficient density for the CAES is with a magneto optical trap (MOT). A MOT combines a laser field with a spatially varying magnetic field to both cool and trap atoms [67, 68].

2.1.1 Doppler cooling

The cooling process in a MOT can be best understood by considering a two-level atom in a resonant laser field, shown schematically in figure 2.1.1. A photon incident upon a two-level atoms in the lower state will excite the atom into the upper state. Upon absorption, the photon will also impart its momentum, \( p_{\text{photon}} = k\hbar \) to the atom, where
Figure 2.1.1 – A photon of momentum $p_{\text{photon}}$ is incident upon an atom with some momentum $p_{\text{atom}}$. As the photon is resonant with the Doppler shifted transition of the atom, it is absorbed. The atom’s momentum is therefore reduced by $p_{\text{photon}}$. Some time later the atom undergoes spontaneous emission in some random direction releasing a photon with momentum $p'_{\text{photon}}$, leaving the atom with momentum $p_{\text{atom}} + p_{\text{photon}} - p'_{\text{photon}}$. The direction of the emitted photon is random, so over many cooling cycles its contribution to the atom’s momentum sums to zero meaning, on average, the atom loses $p_{\text{photon}}$ per scattering event.
**CHAPTER 2. THE ELECTRON SOURCE**

$k$ is the photon’s wavevector and $\hbar$ is the reduced Planck constant. If the photons are red detuned with respect to the two-level transition, only those atoms which are heading toward the laser field, brought into resonance with the field via the Doppler effect, will absorb the photon. However, when the excited atom decays at some time later via spontaneous emission it undergoes a second momentum change with the same magnitude as the first, but in an arbitrary direction.

Over many of these cycles the momentum change due to the emitted photons averages to zero, so the net affect on the atom is a reduction of its momentum in the direction opposite to the laser field.

For a two-level atom with a ground state $|g\rangle$ and excited state $|e\rangle$, the absorption and then spontaneous emission of a photon results in a force, $F$, equal to the momentum transfer per scattered photon multiplied by the scattering rate,

$$F = \Delta p_{\text{photon}} \gamma_{\text{scat}}$$  \hspace{1cm} (2.1.1)

$$= \hbar k \gamma_{ee},$$  \hspace{1cm} (2.1.2)

where $\gamma = 1/\tau$ is the spontaneous emission rate, $\tau$ is the lifetime of the excited state, and $\rho_{ee}$ is the relative excited state population.

The excited state population itself can be parameterised as [69]

$$\rho_{ee} = \frac{s_0}{2 \left( 1 + s_0 + \left( \frac{2\delta}{\gamma} \right)^2 \right)},$$  \hspace{1cm} (2.1.3)

where $s_0 = I/I_s$ is the resonance saturation parameter, $I$ is the intensity of the laser field, $I_s$ is the saturation intensity, and $\delta$ is the detuning from resonance of the laser field as an angular frequency. The resultant ‘spontaneous scattering force’ can therefore be written as [69]

$$F = \frac{\hbar k \gamma_{s_0}}{2 \left( 1 + s_0 + \left( \frac{2\delta}{\gamma} \right)^2 \right)}.$$  \hspace{1cm} (2.1.4)

Therefore the retarding force from a laser field, red detuned from resonance by $\delta_r$, acting upon an atom with a Doppler shift of $\delta_d = \mathbf{v} \cdot \mathbf{k}$ can be written as:

$$F = \frac{\hbar k \gamma_{s_0}}{2 \left( 1 + s_0 + \left( \frac{2(\delta_d + \delta_r)}{\gamma} \right)^2 \right)}.$$  \hspace{1cm} (2.1.5)
where \( v \) is the atom’s velocity and \( k \) is the wave vector of the laser field. The force parameterised in equation 2.1.5, however, only slows atoms. It does not trap them because, as the atom’s velocity approaches zero, the force on the atom also approaches zero, meaning the atoms leave the beams via a random walk. To trap the atoms the force must be supplemented with a spatially varying trapping force. The trapping force in a MOT is provided by applying a spatially varying magnetic field which affects the atoms via the Zeeman effect, shifting the atomic energy levels into resonance with the laser field as they leave the trapping region [70].

### 2.1.2 The Zeeman effect

The Zeeman effect describes how an atom’s energy levels will shift in an external magnetic field of strength \( B \) by the Zeeman energy shift,

\[
\Delta E = \mu_M B
\]

\[
= \mu_B g_F M_F B,
\]

where \( \mu_M \) is the magnetic moment of the atom, \( \mu_B \) is the Bohr magneton, \( M_F \) is the projection of the atomic angular momentum along \( B \), and \( g_F \) is the Landé g-factor given by [71]

\[
g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)},
\]

where \( F \) is the total atomic angular momentum,

\[
g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)},
\]

\( J \) is the total electronic angular momentum, \( L \) is the orbital angular momentum, and \( S \) is the spin angular momentum.

The Zeeman frequency shift between two energy sub levels \( |E_1\rangle \) and \( |E_2\rangle \) is therefore

\[
\Delta f = \frac{\Delta E}{\hbar}
\]

\[
= \frac{1}{\hbar} (\Delta E_2 - \Delta E_1)
\]

\[
= \frac{\mu_B B}{\hbar} (g_2 M_{F'} - g_1 M_F),
\]

where \( g_{1,2} \) are the Landé g-factors of the two states, and \( M_F \) and \( M_{F'} \) are the angular momenta of the ground and excited magnetic substates.
CHAPTER 2. THE ELECTRON SOURCE

42

(a) The laser field (in red), is red detuned from resonance by δ, and therefore has only a small probability of interacting with the stationary atom.

(b) Giving the atom some velocity \( v_a \) causes the laser fields to be Doppler shifted in the rest frame of the atom. Photons coming from the right are now much closer to resonance with the atom, so are far more likely to be scattered and change the atom’s momentum.

Figure 2.1.2 – The principle of a one-dimensional optical molasses

Combining the Zeeman shift and Doppler cooling in the simplest one-dimensional case demonstrates the principle of operation of a MOT.

2.1.3 The one dimensional MOT

In a 1D MOT a pair of red detuned counter-propagating laser beams produces a one dimensional retarding force which is dependent on the velocity component of the atoms along that axis, as shown in figure 2.1.2.

For low intensities, the force on a particular atom can be written as

\[
F_{ss} \approx F_+ + F_-
\]

\[
= \frac{\hbar k \gamma s_0}{2 \left(1 + s_0 + \frac{\left(2 (\delta_r - \delta_d) \gamma \right)^2}{\gamma} \right)} - \frac{\hbar k \gamma s_0}{2 \left(1 + s_0 + \frac{\left(2 (\delta_r + \delta_d) \gamma \right)^2}{\gamma} \right)}
\]

using equation 2.1.5 where the effects of stimulated emission are ignored. Expanding as a Maclaurin series in \( v \),

\[
F_{ss} \approx F_{ss}(0) + v \frac{d}{dv} F_{ss}(0) + \ldots
\]
Figure 2.1.3 – The effects of the magnetic field on an atom’s sub-levels in a 1D magneto optical trap. For an atom at position \( z' \), the atom will couple more strongly to the \( \sigma^- \) beam because of the angular momentum selection rules between the ground state and the excited state. The horizontal dashed lines show the detuning of the \( M_{F'} = -1 \) and \( M_{F'} = +1 \) sub-levels with respect to the frequency of the laser field, \( \omega_L \).

and truncating to first order gives

\[
F_{ss} \approx -\frac{8\delta\hbar k s_0 v \cdot k}{\gamma \left( 1 + s_0 + \left[ 2\delta r / \gamma \right]^2 \right)^2} = -\beta v
\]  

(2.1.16)

a damping force proportional to the atom’s velocity with a damping constant of \( \beta \). Such an arrangement is referred to as a one dimension (1D) optical molasses.

The trapping force in a MOT can be understood as an extension of the 1D optical molasses model where the two-level atom has been replaced with a four-level atom and an external magnetic field has been introduced. The energy level diagram for such a system is shown in figure 2.1.3.

The four-level atom has a single \( F = 0 \) ground state along with an \( F = 1 \) excited state containing three magnetic sub-levels; \( M_{F'} = +1, 0, -1 \). In a magnetic field the atomic transitions will be Zeeman shifted by \( \Delta f \), defined in equation 2.1.12. Since the ground state has no magnetic sub-levels, the Zeeman shift reduces to

\[
\Delta \omega = \frac{\mu_B B}{\hbar} g_2 M_{F'},
\]  

(2.1.17)
when written as an angular frequency.

In a linear magnetic field \( B(z) = Az \), where the \( z \) axis is defined by the incident laser beams and \( A \) is the magnetic field gradient, the force on a particular atom, given by equation 2.1.16, is modified such that the detuning term is a function of both position and velocity

\[
F_{\text{MOT}} = \frac{\hbar k \gamma s_0}{2 \left( 1 + s_0 + \left( \frac{2 \delta}{\gamma} \right)^2 \right)} - \frac{\hbar k \gamma s_0}{2 \left( 1 + s_0 + \left( \frac{2 \delta}{\gamma} \right)^2 \right)}
\]  \hspace{1cm} (2.1.18)

where \( \delta = \delta_r \pm k \cdot v \pm \mu' Az / \hbar \), and \( \mu' = g_2 M_F \mu_B \).

For small variations of \( z \) and \( v \), equation 2.1.18 can again be expanded as a Maclaurin series and truncated to first order to give

\[
F_{\text{MOT}} \approx -\beta v - \kappa z,
\]  \hspace{1cm} (2.1.19)

where the damping force constant, \( \beta \), and the restoring force constant, \( \kappa \), are given by

\[
\beta = \frac{8 \delta \hbar k^2 s_0}{\gamma (1 + s_0 + [2 \delta / \gamma]^2)^2},
\]  \hspace{1cm} (2.1.20)

\[
\kappa = \frac{\mu' A}{\hbar k} \beta.
\]  \hspace{1cm} (2.1.21)

An atom of mass \( m \) experiencing such a force undergoes damped harmonic motion about the trap centre with characteristic angular frequency \( \omega_{\text{MOT}} = \sqrt{\kappa / m} \) and a damping time constant \( \Gamma_{\text{MOT}} = \beta / m \) [69]. The damping in a MOT is capable of reducing the speed of the trapped population from tens of ms\(^{-1}\) to sub mm s\(^{-1}\).

### 2.1.4 Approximating a two-level atom

For the work described in this thesis, rubidium 85 is cooled and trapped to be used as a source of electrons since its \( D_2 \) line at 780.24 nm is easily accessible with readily available low bandwidth laser systems.

In reality, the \(^{85}\text{Rb} \) \( D_2 \) transition has 6 hyperfine states, as shown in figure 2.1.4, and 35 magnetic sub-levels, as shown in figure 2.1.5. The numerous possible states present a problem for the cooling and trapping of \(^{85}\text{Rb} \) as many thousands of cycles are required to slow and then trap an atom for a prolonged time. A further complication arises since an excited atom could fall out of resonance with the trapping laser fields by
Figure 2.1.4 – Rubidium 85 $D_2$ line hyperfine structure, with frequency splittings between the hyperfine energy levels, reproduced from [72].
decaying into the $F = 2$ ground state, which is separated by 3 GHz from the cooling transition [69]. To cool and trap the maximum number of atoms for as long as possible two techniques are used; exploitation of the selection rules to approximate a two-level system, and radiation at a second frequency to ‘re-pump’ those atoms that have fallen into the $F = 2$ ground state back into the cooling cycle.

Any photonic excitation or decay of an atom must obey the selection rule $\Delta M_F = 0, \pm 1$, so for rubidium atoms excited into the $F' = 4, M_{F'} = \pm 4$ states, transitions to ground states other than $F = 3, M_F = \pm 3$ are dipole forbidden as $\Delta F \neq 1$, meaning they cannot occur as a dipole transition and must take place as higher order transitions with far lower probabilities. The set of allowed transitions between the $F = 3$ and $F' = 4$ states are shown in figure 2.1.5 (red lines). By using circularly polarised light in which the photons have angular momentum number $L = \pm 1$, the atom in the $M_F = \pm 3$ state will be returned to the $M_{F'} = \pm 4$ state, resulting in a closed two state system. The action of the circularly polarised light is shown in figure 2.1.5 as the solid red lines.

The same circularly polarised light will also initially pump the atoms into the $F = 3, M_F = \pm 3$ state in a process called optical pumping [73]. The optical pumping
schematic is shown schematically in figure 2.1.5. Quarter wave plates are used to turn the linearly polarised light from the laser into circularly polarised light.

Excited rubidium atoms in the $F' = 4$ state can also decay into the $F = 2$ state, split by 3.035 GHz from the $F = 3$ state, although this transition is dipole forbidden. As the transition has a linewidth of 6.06 MHz and the laser light which is provided by a Sirah Matisse TX Titanium:Sapphire laser has a bandwidth of $\sim$ 10 kHz, any atom that relaxes into the $F = 2$ ground state is lost to the cooling and trapping cycle.

To return these atoms into the cooling cycle a re-pump laser beam is added, detuned by 2.9525 GHz with respect to the trapping frequency. The action of the repump light is shown in figure 2.1.5 (blue lines). The method by which the repump frequency is introduced to the trapping laser beams is described in appendix A.

### 2.1.5 The three dimensional rubidium MOT

In three dimensions, the trapping magnetic field can be produced using two identical solenoids of radius $r$ separated by distance $d$ in an anti-Helmholtz configuration, where the field generated by one solenoid opposes the field generated by the other. If the separation of the coils is $d = \sqrt{3}r$, then the magnetic field gradient in the trapping region will be approximately linear [74].

The anti-Helmholtz coils along with three pairs of counterpropagating trapping laser beams produce a 3D MOT, as shown in figure 2.1.6. Each of the beams is circularly polarised. In a 3D MOT the interactions in the trap become more complicated than described above, but the principle of operation remains the same.

Maintaining a large trapped atom population is only possible if the laser field remains resonant with the cooling transition. Since the natural linewidth of the D$_2$ line in rubidium is only 6.065 MHz, to efficiently cool and trap rubidium atoms in a MOT requires the trapping laser field to be frequency stabilised with sub-MHz accuracy for several hours at a time.

### 2.1.6 Laser locking system

The trapping laser is frequency stabilised using an error signal derived from a Zeeman shifted, Doppler free, saturated absorption spectrum of rubidium vapour. The absorption spectrum of room temperature rubidium is Doppler broadened by $\sim$ 200 MHz, as seen in figure 2.1.7. To provide a sharp feature with which to lock the laser frequency, the Doppler broadening is removed with a pump/probe saturated absorption scheme using
Figure 2.1.6 – An isometric view of a MOT, with the main features labelled. The polarisation of each of the beams is indicated as either right hand circular ($\sigma^+$), or left hand circular ($\sigma^-$).
rubidium vapour held in a glass cell. A diagram of the pump-probe set-up is shown in figure 2.1.8.

In the pump/probe scheme two counter-propagating laser beams at the same frequency pass through the vapour cell. One, the pump beam, has an intensity much greater than the saturation intensity of the vapour (1.699 mW cm\(^{-2}\)) \([72]\) whilst the other, the probe beam, is as low intensity as possible, and is incident on a photodiode after passing through the vapour cell. The beams counter-propagate through the vapour cell at the same frequency and are therefore resonant with the two populations of atoms which have Doppler shifts of the same magnitude. These two populations are different when the laser frequency is far from resonance with the transition, but when the laser frequency is on resonance both beams interact with the same population of atoms; those with no velocity component along the axis of the beams, \(v_z = 0\). The transition in that population is saturated by the pump beam leaving fewer atoms with which the probe beam can interact so more probe beam photons are able to pass through the vapour.

The measured intensity of the probe beam therefore increases when the laser frequency is on resonance and decreases when the beam is far from resonance. Measuring the probe beam’s intensity as the laser frequency is varied gives the Doppler free saturated absorption profile shown in figure 2.1.9.

A second probe beam also passes through the cell offset from the pump beam to

---

**Figure 2.1.7** – The absorption spectrum of room temperature rubidium vapour. The ‘detuning’ is with respect to the \(F = 3\) to \(F' = 4\) transition of the \(D_2\) line marked by the blue vertical line. All of the absorption lines are labelled with the corresponding transitions.
CHAPTER 2. THE ELECTRON SOURCE

record the Doppler-broadened spectrum, using a second photodiode. The recorded
intensity is scaled and subtracted from the saturated absorption spectrum to give a
Doppler-free saturated absorption spectrum as seen in figure 2.1.9. The circuit diagram
for the two photo-diodes is shown in figure 2.1.10.

To derive an error signal from the Doppler-free saturated absorption spectrum with
which to frequency lock the laser, a 1 kHz AC magnetic field is applied to the vapour
cell. The magnetic field Zeeman shifts the $F = 3, M_F = 3 \rightarrow F' = 4, M_{F'} = 4$ transitions
in the rubidium atoms by $\pm 20$ MHz scanning the transitions back and forth across the
laser frequency. This causes the measured intensity of the probe beam to vary at double
the frequency of the magnetic field when on resonance. The measured intensity of the
probe beam and a measurement of the AC current passing through the solenoid used to
drive the magnetic field are sent to a Brookdeal 9503-SC lock-in amplifier. The lock-in
amplifier calculates the error signal from the frequencies and phase offset of the two
input signals. The error signal is fed to a software proportional-integral-derivative (PID)
controller which controls the scan piezo on the Matisse TX laser.

The magnetic field is produced by passing current through a solenoid wrapped
around the vapour cell. To minimise the inductance of the solenoid whilst providing a
uniform field across the entire vapour cell, as shown in figure 2.1.13, the solenoid was
wound on a 3D printed former shown in figure 2.1.12.

The system is able to lock the laser at frequencies offset up to 50 MHz from the
locking transition’s natural frequency by applying a DC Zeeman shift to the transition
using an additional DC magnetic field. The offset sinusoidal bias needed to drive the
magnetic field is produced by the circuit shown in figure 2.1.11.

Measuring the frequency of the laser light using a High finesse WS-UL shows the
additional long-term stability of the laser that is provided by the locking system, as seen
in figure 2.1.14. The system also provides an absolute reference for the trapping laser
light which will never experience any drift, allowing the MOT to consistently cool and
trap rubidium for several hours at a time.

2.1.7 Rubidium source

The rubidium vapour which loads the MOT is produced by a rubidium SAES Alkali
Metal Dispenser [75]. The dispensers contain a rubidium salt. To emit rubidium vapour
the dispensers are resistively heated to 700°C by a current of 5.5A, supplied by a
constant current power supply. As the thermal velocity distribution of the vapour is
very large the MOT only traps $\sim 1\%$ of the rubidium released by the dispenser. That is
**Figure 2.1.8** – A schematic of the laser locking system. A portion of the trapping beam is picked off the main beam with a 10% pick off. The beam is directed at mirror M1 which redirects the beam to M2. M2 sends the beam through the glass block, which produces the two probe beams. The probe beams pass through a quarter-wave plate ($\lambda/4$), producing circularly polarised light, and enters the rubidium cell (Rb cell). The rubidium cell is held inside a solenoid (not shown) which produces the sinusoidal magnetic field. After passing through the cell the twin probe beams are measured by a pair of photodiodes, after the beams pass through another quarter-wave plate ($\lambda/4$) and a polarising beam-splitter (PBS). The PBS and second quarter wave plate are required by the pump beam. M3 directs the pump beam through a half-wave plate ($\lambda/4$) which, together with the PBS gives control over the power in the pump beam which passes through the rubidium cell. The PBS directs the pump beam along the path of one of the two probe beams, allowing total overlap of the two beams over the entire length of the cell, increasing the size of the saturated absorption signal.
Figure 2.1.9 – The Doppler free saturated absorption spectrum of room temperature rubidium vapour showing the sharp features which can be used to lock the frequency of the trapping laser. The ‘detuning’ is with respect to the $F = 3$ to $F' = 4$ transition of the D$_2$ line, marked by the blue vertical line. The other saturated absorption peaks and cross over resonances - where the pump and probe beams are resonant with different Doppler shifted transitions - are also marked with the initial and final hyperfine states of the corresponding transition.
Figure 2.1.10 – Circuit diagram for the probe and background photodiodes. The photodiodes are each reverse biased and their output signals are independently amplified by a pair of OPA228s. The amplified signals are then fed to an OPA228 configured as a difference amplifier. The difference between the two signals is output to form part of the error signal sent to the lock-in amplifier. The 50k potentiometers are used to balance the signals from the two photodiodes.
Figure 2.1.11 – The offset sinusoidal current driven through the locking system solenoid is produced in three stages: Circuit A produces the 1 kHz sinusoidal signal, $V_{AC}$, using an AD9850 signal generator controlled by an ATTINY85 microprocessor running the code linked in appendix B.3. Circuit B produces the offset bias, $V_{DC}$, using a 5V reference bias from a REF02 and a voltage divider. Circuit C takes the two signals, $V_{AC}$ and $V_{DC}$, sums and amplifies them using an OPA4227, before using a power MOSFET (a STP45N65M5) to drive the current through the locking system solenoid. The output signal, $V_{B \text{ phase}}$, is also measured and sent to the lock-in amplifier.
Figure 2.1.12 – A schematic of the solenoid used to produce the offset sinusoidal magnetic field. The solenoid is wound onto a former that also houses the rubidium cell. The plug helps maintain the temperature inside the former to avoid any rubidium vapour condensing upon the face of the cell and creating a reflective surface. The solenoid has 2 layers of windings and 237 windings per layer. The ID of the solenoid winding is 18 mm.
Figure 2.1.13 – The magnetic field strength as a function of the distance from the centre of the solenoid (blue). Also shown is the variation in the detuning of the $F = 3$ to $F' = 4$ $D_2$ transition over the length of the cell. As the variation contributes to the broadening of the saturated absorption line it is beneficial to keep it as small as possible.

Figure 2.1.14 – The stability of the trapping laser frequency, with respect to the $F = 3$ to $F' = 4$ transition of the $D_2$ line in rubidium, when the locking system is enabled (blue) compared to when the laser only uses its internal stabilisation systems (orange).
CHAPTER 2. **THE ELECTRON SOURCE**

sufficient to load the MOT in $\sim 6$ s.

Before the dispensers can be used they must be ‘outgassed’ to remove any contaminants in the rubidium salt by holding them at their outgassing current of 3A for $\sim 5$ minutes. After the chamber pressure returned to $\sim 5$ times the initial pressure, the heating current was slowly increased to $\sim 5.5$ A to heat the dispenser to its emission temperature. Each time the vacuum chamber was opened the outgassing procedure had to be repeated.

Three dispensers are mounted across four copper feedthroughs which are in turn mounted on a CF70 flange. The dispensers are mounted in line of sight of the MOT. Each dispenser contains $\sim 3$ mg of Rubidium, sufficient to load the MOT for several hundred hours.

Whilst loading the MOT using the dispensers leads to background pressures of around $\sim 5 \times 10^{-9}$ Torr, higher than from other methods of loading a MOT, the dispensers are simple and inexpensive. For applications of the CAES, such as a particle accelerator, that require pressures below $1 \times 10^{-10}$ Torr a different but similarly compact method of loading the MOT would be required. Alternatively a differential pumping scheme between the source and the accelerator would need to be employed.

### 2.2 The alternating current MOT

Any charged particle bunches which are extracted from a MOT must pass through the MOT’s magnetic fields. The fields distort the bunch’s phase space and increase the emittance of the electron bunch.

The size of the emittance growth due to the trapping fields is, to a significant extent, dependent on the direction in which the electrons are extracted. The two options, extracting the electrons ‘vertically’ or ‘horizontally’ are illustrated in figure 2.2.1. If the electrons are extracted horizontally the effects of the magnetic field are significant, as the electrons do not experience a cylindrically symmetric field during extraction. If the electrons are extracted vertically, through the centre of one of the solenoids, however, they are only affected by a cylindrically symmetric solenoid field.

Using General Particle Tracer (GPT) [76], a charged particle tracking code, the difference in emittance growth for a charged particle bunch extracted vertically versus horizontally from the MOT can be quantified. The results from GPT simulations of 10 K, 5000-electron electron bunches are shown in figure 2.2.2.

For high energy electron bunches, the effect of the solenoid field for vertically
**CHAPTER 2. THE ELECTRON SOURCE**

**Figure 2.2.1** – A schematic of a MOT with the possible directions of extraction shown by the straight, light green arrow. Electrons can either be extracted along the axis of the solenoids (left), which is defined as extracting the electrons vertically, or perpendicular to the axis of the solenoids (right), which is defined as extracting the electrons horizontally.

**Figure 2.2.2** – The normalised rms emittance as a function of the position along the beamline for vertical vs horizontal extraction through the trapping solenoids. Extracting the electrons horizontally has significant negative consequences for the beam quality.
extracted electrons is limited. For low energy bunches, however, the effects can be significant, depending on the size of the trapping solenoids used, as the GPT simulation results shown in figure 2.2.3 demonstrate. The large effect at low energy results from the bunch expanding into the non-linear region of the solenoid field, where the affect of the lens on an electron is not a linear function of the radial position of that electron, leading to a distortion of the bunches phase space and hence an emittance growth, as described in section 1.2.1.

To avoid having any emittance growth due to the trapping fields, the current through the solenoid can be switched off and any eddy fields induced by the switch off allowed to decay before the electrons are extracted. Whilst the eddy fields decay, which typically takes $\sim 10$ ms, atoms which were previously trapped no longer experience a trapping force. The atoms therefore leave the trapping region which significantly reduces the source density.

The dynamics of extracting electrons through the magnetic trapping fields are explored more fully in section 4.3.1. Allowing the magnetic fields to decay before every extraction also severely limits the repetition rate of the experiment as the trap would have to be refilled between every cycle.
CHAPTER 2. THE ELECTRON SOURCE

2.2.1 Fast zeroing of magnetic fields

A solution to the problem of extracting electrons through the trapping magnetic field was developed in Manchester in 2008 by Harvey & Murray [10]: the alternating current magneto-optical trap (AC-MOT). In an AC-MOT the DC magnetic field is replaced by a 5 kHz sinusoidal magnetic field, allowing the trapping magnetic fields, as well as any induced eddy magnetic fields, to be zeroed in < 20\(\mu\)s.

Such a rapid zeroing time is a consequence of two factors. Firstly, since the solenoid is driven with a ‘pure’ sine wave, the relationship between the driving voltage, the induced current and the induced magnetic field is a simple phase delay. The drive bias waveform can therefore be adjusted to zero the current through the solenoid as quickly as possible. Fine tuning the drive voltage signal for a fast switch off of a DC current is very difficult, by comparison, as each harmonic of the switch-off has its own voltage-current relationship that must be considered.

Secondly, since each half-cycle of the sinusoidal magnetic field induces a current of equal magnitude but opposite direction in any inductor coupled to the field, any induced eddy current is immediately cancelled by an induced current in the opposite direction. So long as there is an even number of half wave cycles used between each zeroing, there will be no net eddy fields that need time to decay before electrons are extracted.

For sinusoidal magnetic fields with a frequency above 5 kHz, there are only small differences in trap population and temperature between the AC and DC-MOTs - with the AC-MOT tending to have a slightly higher temperature and a slightly lower population [10].

The 5 kHz sinusoidal drive signal is produced by an NI PCIe-6321 DAQ card. The signal is amplified by a Behringer Europower EP4000, a high power audio amplifier. The particular switch off phase for the drive signal that zeros the magnetic fields most quickly was determined experimentally by measuring the size of induced eddy magnetic fields and the current through the solenoid for a range of drive voltage switch off phases, as shown in figure 2.2.4. A switch-off phase of 79° zeroed the current and magnetic fields most quickly, in 34 ± 2\(\mu\)s. Hence this switch off phase was used for the relevant work presented in this thesis.

2.2.2 Trapping coils

To produce the magnetic field used in trapping the rubidium atoms a pair of anti-Helmholtz coils are required which can produce a linear field gradient of \(\sim 250\ G\text{mm}^{-1}\)}
Figure 2.2.4 – The time taken for the magnetic field to decay to 2.5% of its maximum value as a function of the switch-off phase of the drive voltage. The switch off signal is shown in black. The colours relate the current profiles (top) to the decay times (bottom).
at the centre of the trap.

The coils and the formers upon which they were mounted needed to satisfy some important conditions. The coils had to be low enough inductance that the available power supplies could drive a 5 kHz sine wave through them. They also had to be designed such that the 5 kHz AC magnetic field the coils produced would not couple strongly enough to nearby conductors that they inductively heated the conductors which would lead to an increase in the vacuum pressure. The coils also had to allow optical access for the trapping beams, as well as for the two beams used to ionise the trapped atom population in order to produce the electron beam.

To make the inductance of the coils low, and to ensure they did not couple to the tank, we chose small coils mounted inside the vacuum chamber. Using small internal coils added the additional constraint that the coils would have to be taken into account when designing the electron optics for the extraction and focusing of the electron beam. The former upon which the coils were mounted were also split to stop any current loops forming and minimise the inductive heating effect.

Using a custom Python code, the magnetic field produced by coils of various radii, heights, and turn counts with various wire thicknesses were calculated by summing the contributions at each point from a series of wire loops. The calculations used a $\rho, z$ cylindrical co-ordinate system with the $z$-axis passing through the centre of the two solenoids, perpendicular to the wire loops. The contribution from a single wire loop of radius $R$ centred at $z = z_0$ is given by [77]:

\[
B_z = \frac{\mu_0 I}{2\pi} \frac{1}{\left[(R + \rho)^2 + (z - z_0)^2\right]^{\frac{3}{2}}} \left[K(k^2) + \frac{R^2 - \rho^2 - (z - z_0)^2}{(R - \rho)^2 + (z - z_0)^2}E(k^2)\right], \tag{2.2.1}
\]

\[
B_\rho = \frac{\mu_0 I}{2\pi} \frac{1}{\left[(R + \rho)^2 + (z - z_0)^2\right]^{\frac{3}{2}}} \left[-K(k^2) + \frac{R^2 + \rho^2 + (z - z_0)^2}{(R - \rho)^2 + (z - z_0)^2}E(k^2)\right], \tag{2.2.2}
\]

where $\rho$ is the radial displacement from the centre of the current loop, $z$ is the on-axis displacement from the centre of the loop, and

\[
k^2 = \frac{4R\rho}{(R + \rho)^2 + (z - z_0)^2}, \tag{2.2.3}
\]

is the argument of the complete elliptical integrals $E$ and $K$ [78].

Based on these calculations and the available materials, an inner coil diameter of 36 mm was chosen to be wound with PTFE coated copper wire, core diameter 0.9 mm
Figure 2.2.5 – The magnitude of the magnetic field generated by the anti-Helmholtz coils in the $\rho - z$ plane, which passes through the centre of the solenoids. The concentration of contour lines show the location of the wires which make up the solenoid. As the electron beam will be extracted vertically through the centre of the upper solenoid under DC operation the beam will experience a cylindrically symmetric field.

and sheath diameter 1.1 mm. The coils were wound in 12 layers of 12 rows of wire for a total of 144 turns. The coils were mounted such that the centres of the coils are 42 mm apart.

Figure 2.2.5 shows the strength of the cylindrically symmetric magnetic field, $|B|$ over a plane perpendicular to the coils, centred at $z = 0$ and $\rho = 0$. At the centre of the trap, the field is zero.

Figure 2.2.6 shows that the trapping field gradients, $\delta B_z / \delta \rho$ and $\delta B_\rho / \delta z$, remain approximately constant over the trapping volume at values of $130.4 \pm 0.3 \text{ Gmm}^{-1} \text{ A}^{-1}$ and $256.1 \pm 3.6 \text{ Gmm}^{-1} \text{ A}^{-1}$ respectively, as required for the linear trapping force of the MOT. Since $\delta B_z / \delta \rho$ is twice as large in the trapping volume compared to $\delta B_\rho / \delta z$, the MOT will be compressed vertically.

The solenoid formers were made from oxygen-free high conductivity copper and
are split to reduce inductive heating from the AC magnetic field driving eddy currents in the copper. The mounting of the formers and their operation in the electron beam line is discussed in section 4.2.

Producing an AC-MOT is not as simple as replacing the DC magnetic fields with AC magnetic fields, however. As the polarity of the trapping magnetic field is switched, circularly polarised light that previously produced a spatially restoring force on the trapped atoms now pushes those atoms out of the trap. For an AC-MOT to trap atoms efficiently the polarisation of the trapping beams must also be reversed in phase with the sinusoidal magnetic field.

### 2.2.3 Laser light polarisation in an AC-MOT

The polarity of the trapping light is switched between right-hand circular and left-hand circular by passing the trapping beams through a Leysop EM400A electro-optic modulator (EOM), which acts as a variable half-wave plate. The EOM is switched between the half wave condition and full wave condition by applying $V_{\lambda/2} = -68.2\text{ V}$ and $V_0 = 466.1\text{ V}$, respectively, across its crystals in phase with the AC magnetic field.

To produce the 5 kHz square wave between $V_{\lambda/2}$ and $V_0$, the circuit shown in figure 2.2.7 is used. The square wave signal produced by the NI DAQ card is sent via a
CHAPTER 2. THE ELECTRON SOURCE

Figure 2.2.7 – The circuit diagram for the circuit which produces the 5 kHz, $\sim 500$ V, square wave to drive the polarisation switching EOMs. A description of the operation of the circuit is included in the text.

The half-bridge driver controls the gates on two STP45N65M5 MOSFETs, ensuring that only one of them can be conducting at any time. Connected to the source of the ‘top’ MOSFET is $V_{\lambda/2}$ and the drain of the ‘bottom’ MOSFET is connected to $V_0$. The EOM is connected between the two MOSFETs. When the input signal is ‘high’, the top MOSFET is conducting and the bottom open, putting a bias of $V_{\lambda/2}$ across the EOM. When the input signal is ‘low’, the bottom MOSFET is conducting and the top is open, putting a bias of $V_0$ across the EOM.

The switch (marked SW in figure 2.2.7) controls the behaviour of the voltage drive. The switch has three positions: ‘high’, which sends a ‘high’ signal to the driver; ‘low’, which sends a ‘low’ signal to the driver, and ‘ext’ which passes the incident square wave signal to the driver.

$V_0$ can be adjusted between 0 V and $-600$ V, whilst $V_{\lambda/2}$ can be adjusted between the value of $V_0$ and $V_0 + 600$ V, ensuring the MOSFETs cannot be reverse biased. Both biases are adjusted using an external, 5 k$\Omega$, 10 turn potentiometer to find the half wave and full wave conditions.

The resulting square wave applied to the EOM has a rise and fall time of $10 \pm 1$ ns, more than sufficient for the operation of the AC-MOT. It can also produce a continuous DC voltage at either $V_{\lambda/2}$ or $V_0$, controlled by a front panel switch.
Figure 2.2.8 – The optics used to produce the three pairs of counter-propagating trapping beams and their paths to the trapping region between the two trapping solenoids. The mounts for the optics are not shown. Labelled in the diagram are the non-polarising beam splitters (NPBS), and the quarter wave plates (\(\lambda/4\)). The vertical trapping beam is labelled ‘\(V\)’, whilst the two horizontal trapping beams are labelled ‘\(H1\)’ and ‘\(H2\)’. The coordinate axes used in the description of the source are also shown, with \(z\) in the vertical direction.

### 2.2.4 Trapping optics

As discussed in section 2.1, a rubidium MOT requires six counterpropagating trapping laser beams all with the same-handed circular polarisation, frequency locked at 384.231571 THz with megahertz accuracy, and additional repump laser light blue detuned from the primary trapping frequency by 2.9525 GHz.

The AC-MOT further requires that the trapping light flip from left to right-handed circular polarisation in phase with the trapping magnetic field. Finally, for the CAES the trapping light must also be quickly extinguished before the atoms are ionised into a cold neutral plasma. The optics necessary to provide the trapping beams are described below, and are shown schematically in figure 2.2.8.

The trapping beams are produced by a Sirah Matisse TX Titanium:Sapphire tunable
CW ring laser, pumped by a Nd:YAG laser at 1064 nm, frequency doubled to 532 nm. After a portion of the beam is picked off by a 10% pick off for the laser locking system, discussed in section 2.1.6, the main beam is transported from the laser table to the CAES table via a beam pipe.

On the CAES table, the beam passes through an IR coated lens with a focal length of 500 mm to collimate the beam. The beam then passes through a half wave plate and then a polarising beam splitter, which together send 400 ± 50 mW into the excitation beam optics (not shown in figure 2.2.8) and 300 ± 50 mW into the trapping beam optics. The ‘trapping’ beam is reflected by a silvered mirror before coming to a focus where the optical chopper described in section 2.3.3 is positioned (not shown in figure 2.2.8). After the focus the beam passes through the repump EOM, described in appendix A and then the phase switching EOM described in section 2.2.3.

After the phase switching EOM, three separate trapping beams are produced from the main beam. The ‘vertical’ beam, labelled ‘V’ in figure 2.2.8, is picked off first by a non-polarising beam splitter (NPBS). A second NPBS produces the remaining two horizontal trapping beams, labelled ‘H1’ and ‘H2’ in figure 2.2.8. The other three beams required by the MOT are produced by retroreflecting each beam back along itself. A retroreflection scheme is used as it only requires one phase switching EOM - producing six independent trapping-beams requires more than one EOM to preserve the switching polarizations for the AC-MOT whilst balancing the powers of the beams.

A $\lambda/4$ waveplate is placed before each retroreflecting mirror. The beam’s two passes of the waveplate impart a $\lambda/2$ retardation to the phase along one axis, changing the light’s left-hand circular polarization to right-hand circular polarization and vice versa.

The vertical trapping beam, with a total power of $150 \pm 20$ mW, is expanded up and re-collimated using a beam telescope to a diameter of $17.4 \pm 0.2$ mm, giving the beam a power density of $11.0 \pm 0.2$ mWcm$^{-2}$. The beam then passes through an aperture, reducing it to $11.4 \pm 0.2$ mm. The vertical beam is then directed upwards, offset 6.4° from vertical so it can pass through the centre of the trapping region whilst avoiding the microchannel plate (MCP) positioned at the end of the electron beamline, as shown in figure 2.2.9. The trapping beam exits the vacuum chamber through a CF70 window fitted to one port of the 5-way top flange. The trapping beam passes through a $\lambda/4$ waveplate and is then retroreflected back into the chamber and back through the trapping region by a 25 mm mirror.

The first horizontal beam, H1, with power $68 \pm 8$ mW, is expanded up by a beam telescope to a diameter of $11.3 \pm 0.2$ mm, giving it an average power density of
Figure 2.2.9 – Showing how optical access is provided for the vertical trapping laser. The insert shows the view along the trapping beam from the position marked by the eye.
11.5 ± 0.2 mW cm\(^{-2}\). The beam is translated vertically by a beam periscope and directed through the centre of a CF70 window into the main trapping chamber. It passes through the centre of the trapping region before exiting the other side through a second CF70 window. As with the vertical beam, the horizontal beam then passes through a \(\lambda/4\) waveplate and is retroreflected back along itself into the trapping region.

The second horizontal beam, H2, is expanded up to a diameter of 12.6 ± 0.2 mm, with power 78 ± 9 mW and hence has an average intensity of 10.6 ± 0.2 mW cm\(^{-2}\). H2 is directed into the trapping region orthogonal to H1.

### 2.2.5 Imaging the MOT

To record the position, size, shape and atomic population of the MOT, two cameras were used positioned to give orthogonal views of the MOT as shown in figure 2.2.10. The \(xy\) camera, positioned below the trapping chamber, observed the MOT via a mirror positioned directly below the MOT, and recorded the MOT’s \(x\) and \(y\) sizes and position, as defined by the coordinate axes shown in figure 2.2.8. The \(yz\) camera looked directly at the MOT and recorded its \(z\) position. The total intensity of the MOT recorded by this camera was used to make an estimate of the MOT population, as discussed in 2.2.6.

Using LabVIEW, the live feed from the cameras was transmitted to a computer and displayed. The images were analysed to determine the MOT’s size, position, and population.

The size and position of the MOT were determined by fitting a 2D Gaussian to the image of the MOT cloud, using the trapping coils as a reference to both scale the images and provide an origin for the position measurements. The standard deviation of the Gaussian is recorded as the size of the MOT and the centroid of the Gaussian is recorded as the MOT’s position.

\(z = 0\) was set as the halfway point between the two trapping coil formers, as determined by positioning cursors on the edges of the formers in the images from the ‘yz’ camera, while the \(z\) scale was calculated from the difference in the \(z\) position of the two cursors, given the coils were 19.35 mm apart. \(x = 0\) and \(y = 0\), and the \(x\) and \(y\) scales were determined similarly with images from the ‘xy’ camera, using the inside edges of the former’s central hole as a reference.

As the cameras record at 20 Hz, the parameters of the 2D Gaussians must be determined quickly. The algorithm to determine an initial estimate for the fit is carried out by a LabView Virtual Instrument (VI).

First, the VI puts the image through a threshold filter, with the cut off for the filter
Figure 2.2.10 – A diagram of the positions of the MOT cameras with respect to the MOT. To avoid interfering with the vertical trapping beam the xy camera observes the MOT through a mirror. The elements of the diagram are to scale but the distances between them are not.
set at 50% of the maximum value in the image. Any pixel below the threshold is set to zero and any pixel above the threshold is set to 255. Once thresholded the centre of mass of the resulting image is determined, and cross-sections of the original image are taken at the coordinates of the centre of mass.

1D Gaussians are fitted to each cross-section of the original image, with the FWHM of the cross-section used to estimate $2.355\sigma$. The centroid of the centre of mass of the threshold filtered image used for the initial estimate of the mean and the the pixel value at the centroid used for the initial estimate of the amplitude. The output parameters from the 1D Gaussians are then used as the initial estimates for the fitting of the 2D Gaussian via a Levenberg-Marquardt algorithm [79, 80].

The position and size of the 2D Gaussian fit are recorded as measurements of the position and size of the MOT. The cameras are uEye UI214xSE-M R3s [81], fitted with Navitar MVL25M23 f/1.4 [82] macro lenses.

### 2.2.6 Estimating the MOT population

The MOT population can be estimated from the fluorescence emitted by the trapped atoms as measured by the yz camera observing the MOT.

The sensitivity of the cameras to 780 nm light as used in the MOT was measured by illuminating the camera’s CCD with attenuated laser beams at various known powers. The CCD has a linear response to intensity so the sum of the charge collected on every pixel is directly proportional to the power in the illuminating beam, as long as the pixels are not saturated.

The macro lens fitted to the camera has a 17.8 mm aperture and is positioned 320 mm from the centre of the trap, as shown in figure 2.2.10. The camera collects a fraction of the photons emitted by the MOT equal to the solid angle subtended by the camera’s aperture, $\Omega_c = 1.99 \times 10^{-6}$. Dividing the total collected intensity during one exposure by the exposure time gives $P_c = P_{\text{MOT}} \Omega_c$, where $P_{\text{MOT}}$ is the total power emitted from the MOT, assuming the MOT emits photons isotropically.

Knowing the total power, $P_{\text{MOT}}$, and the frequency of the trapping laser photons, $\omega$, the MOT’s population can be estimated from

$$N = \frac{P_{\text{MOT}}}{\gamma_{\text{scat}} \frac{h}{\omega}}.$$ (2.2.4)
where
\[
\gamma_{\text{scat}} = \frac{\gamma_{s0}}{2 \left[ 1 + s_0 + \left( \frac{2\delta}{\gamma} \right)^2 \right]},
\]
(2.2.5)
is the scattering rate of atoms which have been pumped into a two level system, as discussed in section 2.1. In equation 2.2.5, \(1/\gamma\) is the lifetime of the excited state in the two state system, \(s_0\) is the intensity of the trapping laser field, and \(\delta\) is the detuning of the trapping laser photons from the two state transition.

The trapped MOT population was highly dependent on the precise alignment of the trapping beams, but an estimate of the trapped population was made for all of the experimental runs.

### 2.2.7 Vacuum system

The lifetime of atomic traps is highly dependent on the background pressure. To achieve the very low pressures required to effectively cool and trap atoms from which to create a cold neutral plasma, a vacuum chamber which could reach ultra-high vacuum (UHV) conditions was required.

Whilst many atom trapping experiments use small glass cells the intention of the CAES is to create charged particle beams precluding anything but metallic chambers. The main chamber into which the CAES was installed is shown in figure 2.2.11.

To minimise the magnetic fields produced by the chambers, they are made of 316L stainless steel which has an extremely low magnetic permeability and good outgassing properties. All the nuts and bolts are A4 stainless steel. The chambers are bolted to one another using Conflat knife-edge flanges and copper gaskets.

The trapping chamber, containing the anti-Helmholtz coils used in the MOT and the start of the electron beamline, is a 6-way, CF160 cross, with 4 additional CF70 ports fitted with CF70 windows to provide optical access for the horizontal trapping beams.

Above the trapping chamber is a 331.7 mm long CF160 straight section which contains the beamline. At the end of the straight there is a CF160 flange with 5 CF70 ports. Three of the CF70 ports have windows whilst the remaining two are used for electrical feedthroughs.

On the bottom and on one side of the trapping chamber are CF160 windows. The outward facing side of the trapping chamber has a second 5 way CF70 flange fitted with the rubidium dispensers discussed in section 2.1.7, an ion gauge, and three CF70 windows for optical access. The remaining two sides of the trapping chamber are fitted with pipes leading to the vacuum pumps.
Figure 2.2.11 – A diagram showing the trapping and beamline vacuum chambers. Windows are displayed in light blue and provide optical access for both the trapping beams, the positions of which are shown with orange arrows, and the cameras, which are not shown.
CHAPTER 2. THE ELECTRON SOURCE

Vacuum pumps

Two pumps are used to decrease the pressure in the vacuum chambers from ambient to UHV; a Pfeiffer Hipace 700 turbopump and an Agilent scroll backing pump.

The turbopump is mounted on a pipe leading to the chamber, whilst the scroll pump backs onto the turbopump. The scroll pump is used to reduce the chamber pressure to $<1 \times 10^{-1}$ Torr, at which point it is safe to engage the turbopump. The turbopump, along with a bake-out procedure, can reduce the chamber pressure to $<1 \times 10^{-9}$ Torr, sufficient for atom trapping experiments.

Pressure gauge

To measure the pressure in the trapping chamber a Kurt J. Lesker Company 354 Series Ion Gauge is used. The ion gauge is sensitive between $5 \times 10^{-2}$ Torr and $1 \times 10^{-9}$ Torr, calibrated using air. It can be controlled by a computer using the RS485 connection.

2.2.8 Converting between the AC-MOT and the DC-MOT

To change from the AC-MOT to the DC-MOT required replacing the audio amplifier with a constant current power supply, and switching the polarisation switching EOM to the $\lambda/2$ condition.

Due to the imperfect non-polarising beam splitters required to provide all the trapping beams, discussed in section 2.2.4, the average power in each trapping beam varied when changing between the AC and DC-MOTs. The change in the trapping beam powers meant a small realignment of the trapping beams was required each time the MOT configuration was switched.

2.3 Producing cold electrons from a MOT

To produce an electron beam from the MOT the trapped atoms must be ionised and the free electrons extracted and focused into a beam. This section describes the ionisation process, which is enacted in a pulsed two colour photoionisation scheme, and the timing of the overall experimental cycle.

2.3.1 Photoionisation channels

There are several different schemes which can ionise the cold atoms held in a MOT.
CHAPTER 2. THE ELECTRON SOURCE

Direct photoionisation

The simplest way to photoionise an atom is with a single photon that has enough energy to excite an electron from the ground state to beyond the ionisation threshold as shown in figure 2.3.1a. For a photon with energy $\hbar \omega$ incident on an atom $A$ with ionisation potential $\phi \leq \hbar \omega$ then

$$A + \hbar \omega \rightarrow A^+ + e^- \quad (2.3.1)$$

where the ion and electron have excess energy $K = \hbar \omega - \phi$.

Rubidium, however, exhibits a Cooper minimum in its photoionisation cross section near the ionisation threshold [83]. Cooper minima are local minima in the photoionisation cross-section at specific photon energies caused by a sign change in the bound-free transition dipole of a particular angular momentum channel. The Cooper minimum reduces the ionisation efficiency significantly.

As well as having a small cross section the direct photoionisation of rubidium Rb requires 296 nm light [72]. The operation of laser systems in the ultraviolet requires significant technical and safety complications compared to the other photoionisation channels.

Multiphoton ionisation

Again using only a single frequency but with a very high photon flux, multiple photons with energy $\hbar \omega < \phi$ can promote a single electron beyond the ionisation threshold as shown in figure 2.3.1b. Explicitly such multi photon ionisation events can be written as

$$A + n \cdot \hbar \omega \rightarrow A^+ + e^- \quad (2.3.2)$$

The photon scattering rate $R(\omega)$ is proportional to the photon flux, $\tau$, raised to the power of the number of photons required to ionise the atom, $n$,

$$R(\omega) \propto \sigma_n(\omega) \tau^n \quad (2.3.3)$$

where $\sigma_n$ is the generalised absorption cross-section, calculated using time dependent perturbation theory [84].

If the intermediate virtual states are close to a resonant state of the atom then the multiphoton excitation process is resonantly enhanced as in 2.3.1c. Such resonantly enhanced multiphoton ionisations (REMPI) can be described as the laser field coupling the ground state to an intermediate state with $n$ photons, followed by ionisation by a
CHAPTER 2. THE ELECTRON SOURCE

Figure 2.3.1 – The energy level diagrams of different photoionisation schemes each with different residual energies, $K$.

Further $m$ photons,

$$A + n \cdot \hbar \omega \rightarrow A^+ + m \cdot \hbar \omega \rightarrow A^+ + e^-$$  \hspace{1cm} (2.3.4)$$

where $A^+$ is the resonant excited state. Since the number of photons required to ionise $A^+$ is fewer than are required to ionise $A$, the ionisation rate is greatly increased [85].

Along with the cross section of each process, the excess energy $K$ imparted to the free electrons is important as it is related to the final electron temperature. For REMPI, despite its large cross-section the resulting free electrons have variable excess energy and thus a high electron temperature [86–88].

Two colour photoionisation

A very low electron energy spread can be achieved using a two colour photoionisation scheme. One laser field is used to excite the atoms to an intermediate state and a second is used to ionise that intermediate state to energies very close to threshold, or to a high Rydberg state, as shown in figure 2.3.2. Using a two colour scheme gives fine control of the final excess energy by adjusting the ionisation laser frequency whilst avoiding the Cooper minima present in direct photoionisation from the ground state. The process
\begin{align*}
A^+ \rightarrow h \omega_r &\rightarrow A^+ + h \omega_b \rightarrow A^+ + e^-.
\end{align*}

The combination of fine control of the initial excess energy of the electrons and a high photoionisation cross section make a two colour photoionisation scheme ideal for producing an dense ultracold neutral plasma from which to extract high charge cold electron bunches.

A two colour process can also promote electrons into high Rydberg states from which they either auto-ionise through collisions with other excited atoms in the MOT or are field ionised by an external electric field. All the work presented in this thesis uses a two colour process to produce free electrons.

### 2.3.2 Ionising the trapped atoms

Before the atoms in the trap are ionised using the two colour photoionisation scheme, the trapping beams are blocked by the electromechanical chopper described in section 2.3.3 and any excited atoms are allowed to decay into the ground state.

Once the atoms are all in the ground state an ‘excitation’ laser pulse excites the atoms into the $5^2D_{3/2}F = 3$ state, followed by an ‘ionisation’ laser pulse which ionises the excited atomic population. Since all the atoms start in the ground state, and the cross sections for REMPI and NREMPI are relatively low, the size of the electron source is defined by the overlap between the two photoionisation beams.

The two beams are aligned orthogonally with respect to one another, intersecting...
inside the cloud of trapped atoms at the centre of the MOT. Only the atoms in the overlap region are ionised as those are the only atoms that will be in the excited state when the ionisation pulse arrives. The excitation and ionisation beams were focused at the overlap region to minimise the initial source size and hence minimise the resultant beam’s emittance. The electron source density can be controlled by adjusting the atomic population density in the overlap region and varying the power of the excitation beam.

Excitation beam

The excitation beam itself is produced as an offshoot of the main trapping beam by passing the main beam through a half-wave plate followed by a polarising beam splitter, as shown by the solid red line in figure 2.3.3. A second half-wave plate and polarising beam splitter follows, used to control the power in the excitation beam independently of the power in the trapping beams.

The excitation beam then passes through a Gooch & Housego Acousto Optics M080-1B-GH2/E acousto-optical modulator (AOM), which is used to switch the excitation beam on with a rise time of $3.3 \pm 0.3 \mu s$ and a fall time of $4.5 \pm 0.4 \mu s$.

The AOM detunes the excitation beam by $+80 \text{ MHz}$. In addition to the $-18 \text{ MHz}$ detuning from resonance of the trapping beams, the total detuning of the excitation beam is $+62 \text{ MHz}$ from the cooling transition. As there are no higher energy hyperfine states to couple to, the detuning just reduces the efficiency of the excitation process by $\sim 50\%$ at the typical $\sim 500 \text{ mWcm}^{-2}$ intensity of the excitation beam.

The excitation beam passes through an aperture to block the $0^{th}$ order beam from the AOM, followed by three lenses which together focus it at the position of the MOT inside the chamber. The beam passes through the vacuum chamber and is then blocked after exiting the chamber through a window. The spectra of the ionisation laser pulses were recorded at each ionisation wavelength using a Solar Laser Systems SHR Spectrometer.

Ionisation beam

The Ionisation beam is produced by a GWU Versascan optical parametric oscillator (OPO) pumped by a Continuum Surelite SLIII-10. The beam consists of $3.81 \pm 0.35 \text{ ns}, 1.44 \pm 0.03 \text{ mJ}$, ionisation pulses. The beam has a repetition rate of up to $10 \text{ Hz}$. The OPO can produce laser pulses with a huge range of wavelengths, between 410 nm and 709 nm. However, as the temperature of cold atom electron sources is lowest when the atoms are ionised close to the ionisation threshold, the OPO was only used to vary the ionisation wavelength between 478 nm and 484 nm.
Figure 2.3.3 – The excitation (red) and ionisation (blue) beamlines. The red dashed line shows the path of the 0th order beam from the AOM. The beamlines are overlaid on the trapping beamlines to show the relative position of all the laser light used to produce cold electrons.
The ionisation laser beam pulse’s temporal length was measured by directing the
beam at a sheet of white paper and measuring the light scattered onto an FCI-125G-
010HRL photodiode positioned close to the paper. The photodiode was reverse biased
with 9V, and the output, terminated by 50Ω, was measured using a LeCroy HDO6104A
1 GHz oscilloscope. The average pulse shape is shown in figure 2.3.4.

The beam produced by the OPO is highly astigmatic, so a series of cylindrical lenses,
shown in figure 2.3.3, are used to keep the beam focused before a final spherical lens is
used to focus the beam at the centre of the MOT. The beam passes through a CF160
window into the trapping chamber. The beam passes through the vacuum chamber and
is then blocked after exiting the chamber through a second CF160 window.

### 2.3.3 Optical chopper

The electromechanical chopper used to block the trapping beams before the cold atoms
are ionised was built from a hard disk drive. The chopper can completely block the
trapping beams in $19.6 \pm 2.0\,\mu$s upon receiving a trigger pulse and unblock them again
in $46.2 \pm 2.0\,\mu$s, as shown in figure 2.3.5.

Designed after the work of Maguire et al. [89], a small graphite disc was glued to
**CHAPTER 2. THE ELECTRON SOURCE**

**Figure 2.3.5** – The optical chopper can extinguish the trapping beams in $19.6 \pm 2.0 \mu s$, comparable with the rate at which the magnetic fields are zeroed in the AC-MOT. The insert shows the long delay, $\sim 7.7 \text{ ms}$ between the chopper trigger signal and the blocking of the beams.

The end of the read arm of a hard disc drive. Driving a current through the voice coil on the other end of the read arm moves the graphite disc into the path of the beam to block it. Driving a current in the other direction moves the disc back out of the path of the beam.

To move the read arm of the hard disc drive quickly a large current must be applied to the voice coil. Since the coil has a relatively large impedance a 35 V bias is required to supply sufficient current to move the arm quickly. The resistive impedance of the coil, however, is only 4 $\Omega$ and driving 8.75 A through the coil would quickly damage it. To provide a large initial voltage spike to move the coil, followed by a small holding current to keep the read arm in place once it has moved without damaging the coil the circuit shown in figure 2.3.6 is used. The circuit produces the desired voltage profile, shown in figure 2.3.7, from a single 5V positive trigger pulse of length $t_0$. Adjusting $R$ changes the length of the voltage spike used to move the voice coil whilst the time the beams are blocked for is set by $t_0$.

The trigger pulse is initially buffered then inverted before being used to control two H-bridges which are labelled A and B in figure 2.3.6. When the trigger is high the ‘forward directions’ of two H-Bridges are conducting. When the trigger is low the reverse directions are conducting.
Figure 2.3.6 – The circuit used to drive the voice coil on the optical chopper arm. A description of the operation of the circuit is included in the text.
Figure 2.3.7 – Timing diagram of the incident trigger pulse, the signal sent to the enable pin of H-bridge A, and the output voltage profile.
The rising and falling edges of the trigger pulse are used to trigger the 555 timer, which produces a pulse of a length determined by \( R \) at both the start and end of the trigger pulse. The signal from the 555 timer is then inverted.

The non-inverted 555 signal controls the enable pin of H-bridge A, whilst the inverted signal controls the enable pin of H-bridge B.

When H-Bridge A is enabled, and the trigger is high, 35 V is supplied across the voice coil causing it to move the graphite disc into the "blocked" position. Once the arm has moved, H-Bridge A is disabled and H-Bridge B is enabled and used to hold the chopper in the "blocked" position for a time-dependent on \( t_0 \). After \( t_0 \), H-bridge A is once again enabled, but now the trigger pulse is low and the read arm is driven in the reverse direction and the chopper unblocks the beam.

Extinguishing the trapping beams before ionisation allows the ionisation region, and hence source size, to be determined by the overlap of the ionisation beams. Since the overlap of the ionisation beams can be smaller than the extent of the trap population, using the overlap to set the source size results in a lower emittance beam.

### 2.3.4 Experimental cycle

The timing diagram of a single experimental cycle is shown in figure 2.3.8. A pulsed operation, with a single electron bunch produced per experimental cycle, takes greatest advantage of the zeroing of the magnetic trapping fields possible with the AC-MOT.

Each experimental cycle begins by loading the trap with rubidium atoms for up to 500 ms. Once the trap is loaded the trapping beams are blocked by the electromechanical chopper described in section 2.3.3, and in the AC-MOT case, the magnetic trapping fields are zeroed. After the trapping beams are extinguished and the fields zeroed the excitation beam is introduced, followed 5 µs later by the ionisation laser pulse.

The free electrons are accelerated by the electrostatic field across the ionisation region and along the beamline, described in chapter 4, in a bunch. The bunch is focused by an Einzel lens and hits an MCP at the end of the beamline. The MCP, described in section 5.1.1, amplifies the electron bunch and accelerates it onto a phosphor screen, illuminating a section of the phosphor.

The image on the phosphor screen produced by the amplified electron bunch is then imaged onto a CCD, described in section 5.1.2, with an exposure time of 50 µs. The image on the CCD is recorded by a computer.

The experiment can operate at up to \( \sim 2 \) Hz, limited by the rate at which the image on the CCD can be recorded, and the CCD cleared of charge. The timing of the events
which make up each cycle is controlled using an NI-DAQ card NI PCIe-6321.

2.3.5 Trigger generation

Two channels on the NI-DAQ are used to produce the 5 kHz sine wave and square waves required by the AC-MOT, whilst two more channels produce a pair of independent pulse trains to trigger the 10 Hz trigger for the flashlamp on the pump laser and the between 1 Hz and 10 Hz triggers for the camera, the optical chopper, the on-resonance beam, and the Q-switch on the pump laser.

The buffer on the NI DAQ card can only store a 10 ms long sequence at the 500 kHz sample rate required for the 5 kHz sine wave. So any signals produced by the NI-DAQ card will be repeated every 10 ms, giving the signals out of the card a minimum frequency of 100 Hz. To produce the $\leq 10$ Hz triggers for the experiment from the 100 Hz output from the NI-DAQ card requires two circuits to condition the two trigger pulse trains and allow only every tenth pulse past whilst blocking the others.

The ‘flashlamp’ pulse train was sent to both the flashlamp in the Continuum Surelite pump laser and a ‘trigger box’. The other ‘chopper’ train was only sent to the trigger box.

The trigger box allows the frequency of firing of the camera, chopper, and Q-switch, to be set anywhere between 0 and 10 Hz and is able to produce individual trigger pulses on demand. Inside the trigger box is an Arduino Nano running the code linked in appendix B.1. The Nano controls the signal sent to the ‘A’ inputs of two AND gates, while the ‘B’ input on AND gate 1 receives the flashlamp pulse train and the ‘B’ input on AND gate 2 receives the chopper pulse train. In the default state, the ‘A’ inputs are set low and the pulse trains are gated out by the two AND gates. When a set of conditions are met, depending on the mode the box has been set to, the Nano sets the top AND inputs high and the next trigger pulse on each channel is allowed through. Figure 2.3.9 shows the input and output wave-forms from the trigger box.

There are two modes in which the trigger box can operate which set the conditions under which the trigger pulses are allowed to pass. In ‘single shot’ mode a button on the box is pressed and upon its release then on the falling edge of the next flashlamp pulse, the A inputs are set to high. The next set of pulses are gated through, and on the falling edge of that next flashlamp pulse the A inputs are set back to low, so only one set of trigger pulses can pass through the box.

In ‘pulsed’ mode a similar scheme is used, but when the button is pressed and released, the Nano starts counting incident flashlamp pulses, gating every Nth pulse,
Figure 2.3.8 – A diagram showing the timing of events in each experimental cycle. The voltage drive for the AC-MOT is driven to zero at $t = 0$. Before each ionisation the MOT is loaded for up to 500 ms.
Figure 2.3.9 – The trigger pulses produced by the trigger box when operating in pulsed mode. The gate opens on the falling edge of the Nth pulse from the flashlamp trigger-train, allows the N+1th trigger to pass through, and closes again on the falling edge of the N+1th trigger.
where N is set using a rotary encoder. If the button is pressed and released again the Nano returns to blocking every pulse.

The push button on the rotary encoder is used to switch between modes. The Nano also updates a display with the value of N and the current mode of the trigger box.

The conditioned chopper pulse train is used to trigger the chopper directly. The conditioned flashlamp pulse train, on the other hand, is sent to three channels of a delay generator. Channel ‘A’ produces the trigger for the Q switch on the pump laser, channel ‘B’ the trigger for the camera, and channel ‘D’ the trigger for the on resonance beam.

Once the cold atoms have been ionised the physics of cold neutral plasmas take over. The charged particle dynamics in a cold neutral plasma, along with the initial excess ionisation energy, ultimately determine the temperature, and hence emittance, of the electron beam from the CAES. Each of the mechanisms which contribute towards increasing the temperature of the electrons is described in chapter 3.
Chapter 3

Heating mechanisms in a cold neutral plasma

The low transverse emittance beams produced by a CAES are ultimately the result of the low momentum distribution of the electrons and hence the low electron temperature. The temperature of the electrons is a consequence of two primary factors; the excess energy which the electrons are given during ionisation, and any heating processes that occur after ionisation.

This chapter discusses those heating processes and the techniques which have been proposed or demonstrated to suppress the heating those processes entail. The dependence of the source temperature on the excess energy provided in the ionisation process is discussed in section 3.1, the heating from three-body recombination is described in section 3.2, whilst sections 3.3 and 3.4 describe the microscopic and macroscopic effects, respectively, of space charge on the bunch temperature and the beam emittance.

3.1 Excess energy

As discussed in section 1.3.4, most existing CAESs photoionise atoms from an excited state to either the continuum or to high lying Rydberg states, with an external electric field applied across the source during ionisation. The Rydberg states then either auto-ionise or are ionised by collisions with other Rydberg states or with free electrons. Whilst the exact ionisation processes employed in a CAES are discussed in more detail in 2.3.1, the excess energy given to the electrons during ionisation, $\Delta E$, can be written
as the difference between the Stark shifted ionisation threshold

$$E_{0F} = \hbar c \left( \frac{1}{\lambda_0} - 4\text{Ry} \sqrt{\frac{F}{F_0}} \right) \tag{3.1.1}$$

and the energy of the incident photon, $E_\lambda = \hbar c / \lambda$

$$\Delta E = \hbar c \left[ \frac{1}{\lambda} - \left( \frac{1}{\lambda_0} - 4\text{Ry} \sqrt{\frac{F}{F_0}} \right) \right] \tag{3.1.2}$$

where $\lambda$ is the wavelength of light used to ionise the cold atoms, $\text{Ry}$ is the Rydberg energy, $\lambda_0$ is the wavelength required for field free ionisation, $F$ is the magnitude of the external electric field, and $F_0 = 5.142206 \times 10^{11} \text{Vm}^{-1}$ is the atomic unit for the electric field.

Whilst the excess energy does contribute to the final electron temperature, processes occurring during ionisation reduce the width of the transverse momentum distribution at the expense of increasing the longitudinal momentum distribution [90].

The process by which the electrons exchange transverse momentum for longitudinal momentum is best described by considering the trajectory of a single electron with excess energy $E_{\text{exc}}$ exiting the combined Coulomb-Stark potential of a single ion and an applied electric field of strength $|F|$, as shown in figure 3.1.1. The ion is placed at the origin and the z-axis lies along the vector of the applied electric field.

For small emission angles with respect to the z-axis, $\beta$, the electron will follow a simple trajectory out of the potential, only experiencing a small force towards the z-axis. For larger emission angles, however, the electron will follow an increasingly complicated trajectory around the ion, eventually escaping through the saddle point at $z = -\text{sgn}(F) / \sqrt{|F|}$ having exchanged some of its transverse momentum for longitudinal momentum in the field of the ion. Above some angle, $\beta_c$, the trajectories are bound and the electron does not escape the ion. The distribution of final transverse electron momenta is shown in figure 3.1.2.

Using the electron momenta distribution, a ‘transverse’ temperature distribution can be calculated as a function of the extraction field strength, along with the ‘expected’ temperature of isotropically emitted electrons with excess energy $\Delta E$: 
Figure 3.1.1 – The classical electron trajectories (orange/red) in the combined Coulomb-Stark potential for an ion in a constant electric field of strength $-0.13 \text{ MV m}^{-1}$. With greater excess energy ($E_{\text{exc}}$), electrons which are emitted at larger angles with respect to the $z$ axis are able to escape from the potential. The colour of the trajectory in the figure depends on the emission angle [90, 91].
Figure 3.1.2 – The asymptotic transverse momentum probability distributions, \( P(p_r) \), for electrons with excess energies \( E_{\text{exc}} = 3 \) meV (blue), 9 meV (orange), and 15 meV (green) [90].

\[
\sigma^2_{v_x} = \frac{1}{2} \int_0^{\beta_c} v_r(\beta)^2 \sin \beta \, d\beta \sqrt{\int_0^{\beta_c} \sin \beta' \, d\beta'} \tag{3.1.3}
\]

\[
= \frac{1}{2\left[1 - \cos \beta_c \right]} \int_0^{\beta_c} v_r(\beta)^2 \sin \beta \, d\beta, \tag{3.1.4}
\]

where \( \beta \) is the emission angle, \( \sigma^2_{v_x} \) is the standard deviation of the velocity transverse to the electric field, and \( v_r \) is the radial velocity. Figure 3.1.3 shows that for low excess energies the shape of the potential around the ion significantly reduces the temperature of the electron bunches that a CAES can produce.

The electron temperature can be further reduced by controlling the angular electron emission distribution, which is determined by the polarisation of the ionisation laser pulse [13].

Electron bunches produced using ultrafast, bandwidth limited laser pulses have far lower temperatures than would be expected if only the energy spread of the laser pulse was considered. The lower temperatures are due to the exchange of their transverse momentum spread with their longitudinal momentum spread [12, 90].

An additional energy spread originates from the size of the volume from which the
Figure 3.1.3 – The effective source temperature as a function of the excess ionisation energy $E_{exc}$ for various Stark field strengths. The dashed purple line shows the expected effective source temperature for isotropically emitted electrons with excess energy $E_{exc}$, if the effect of the ion field was ignored. The orange line shows the expected temperature of electrons with excess energy $E_{\lambda} = E_{exc} - E_F$, where $E_F$ is the Stark shift of the ionisation threshold caused by an applied $-0.1 \text{MV m}^{-1}$ electric field [90].
electrons are ionised and the strength of the extraction field. For an ionisation volume of width along the beam axis of $\sigma_z$, and an extraction field of strength $F_e$ the energy spread will be

$$\Delta E = F_e \sigma_z e,$$

(3.1.5)

where $e$ is the elementary charge.

The minimum possible size of the ionisation volume is set by how tight a focus can be achieved by the laser beams which ionise the atoms in a CAES. The size of the focus for a laser beam with wavelength $\lambda$, off a lens of focal length $f$ is

$$w_0 \approx \frac{4\lambda f}{\pi W},$$

(3.1.6)

where $W$ is the size of the beam at the lens. Typical beam and lens parameters give a spot size of $>2\mu$m which, in a $50\text{ kV}\text{m}^{-1}$ field, would give an energy spread of $\sim0.1$ eV. Because the energy spread is directly dependent upon the electric field across the extraction volume, at lower extraction fields, lower energy spreads could be achieved. For example, with a $500\text{ V}\text{m}^{-1}$ field the contribution to the energy spread would only be $1\text{ meV}$, significantly below the energy spread due to the low temperature.

A cold atom electron source is not, however, a single atom emitter. There are therefore additional heating effects arising from the interaction of the ensemble of electrons with both the ions and the other electrons.

As the Coulomb interactions in the cold neutral plasma (CNP) are stronger than the thermal energy of the plasma constituents, the electrons and ions that make up the CNP cannot be described using the mechanisms of traditional plasma physics. This interaction strength is quantified by the Coulomb coupling parameter:

$$\Gamma = \frac{e^2}{4\pi\varepsilon_0 ak_BT},$$

(3.1.7)

where $\varepsilon_0$ is the permittivity of free space, $T$ is the plasma temperature and

$$a = \left[\frac{3}{4\pi\rho}\right]^{1/3}$$

(3.1.8)

is the Wigner-Seitz radius for particle density $\rho$ [54]. When $\Gamma > 1$ the plasma is described as 'strongly coupled' and much work has been carried out to understand and describe the dynamics within such plasma [54, 92–96].
3.2 Three-body recombination

An important mechanism which increases the electron/ion temperatures is three-body recombination (3BR), where a pair of free electrons interact with an ion to form an excited atom and an energetic electron:

\[ A^+ + 2e^- \rightarrow A^* + e^- \]  

(3.2.1)

The 3BR rate in ultracold plasmas is proportional to the electron temperature \( T_e \) as [97]

\[ R_{3BR} = CT_e^{-9/2} \int n_e^2 n_i dV, \]  

(3.2.2)

where \( C \) is a constant calculated by Mansbach & Keck [98], \( n_e \) is the electron density, and \( n_i \) is the ion density. The \( T_e^{-9/2} \) dependence demonstrates that the electrons in an ultracold plasma remain weakly coupled, as if the electrons were strongly coupled the rate would be proportional to \( T_e^{-1} \) [99].

The electrons remain in the weakly coupled regime precisely because of 3BR: as the electron temperature decreases the 3BR rate rapidly increases due to the \( T_e^{-9/2} \) dependence. The high 3BR rate raises the electron temperature by adding energetic electrons into the plasma, therefore reducing the electron gas’s Coulomb coupling constant, \( \Gamma_e \). \( \Gamma_e \) will eventually plateau as the 3BR rate also depends on the ion and electron density, and each 3BR event removes an ion and an electron from the plasma. Simulations performed by Robicheaux & Hanson [100] showed that \( \Gamma_e \) plateaus at \( \sim 1/5 \) due to the interplay of the 3BR rate and the electron temperature. Hence the electrons should reach

\[ k_B T_e \sim 5e^2/4\pi\varepsilon_0 a, \]  

(3.2.3)

from the definition of \( \Gamma \) in equation 3.1.7. Whilst higher temperature electrons are expected to reach their 3BR equilibrium temperature in time scales on the order of 10 µs, lower energy electrons result in much higher 3BR rates and hence reach the plateau condition in \( \sim 1 \mu s \) [100].

As well as increasing the temperature of the remaining electrons, each 3BR event also removes an electron from the bunch, meaning its effect on the ultimate brightness of the beam is twofold. Minimising the number of 3BR events by separating the ions and electrons as quickly as possible is therefore important to increasing electron beam brightness.
3.3 Disorder-induced heating

Another source of heating arises from the spatially disordered distribution of ions and electrons within the plasma, producing a non-uniform potential between the constituent charged particles. The electrons’ and ions’ random distribution of potential energy is converted to kinetic energy as they oscillate around dynamic local minima in the potential. The oscillations result in an increase of the electron and ion temperature in a process known as disorder-induced heating (DIH), on time scales on the order of one plasma period $2\pi/\omega$, with the plasma frequency $\omega$ defined as

$$\omega = \sqrt{\rho e^2/\varepsilon_0 m_i, e},$$  \hspace{1cm} (3.3.1)

where $m_{i,e}$ is the ion or electron mass [101–103]. The electrons in the cold neutral plasma will therefore have a shorter thermalisation time compared to ions.

The importance of DIH is made apparent by considering the brightness and energy spread of a beam from a CAES as a function of the beam’s current. Whilst the brightness is expected to increase as the beam current increases, charged particle tracking simulations performed by van der Geer et al. [104] showed the opposite for an ion beam. As they increased the beam current, disorder-induced heating increased the emittance sufficiently that the brightness decreased, as shown in figure 3.3.1. Figure 3.3.1 also shows the increase in energy spread that results from the disorder-induced heating. Van der Geer et al. found that increasing the current of the ion beam from 1 pA to 100 pA reduced the brightness of the beam from $3 \times 10^5$ Am$^{-2}$sr$^{-1}$V$^{-1}$ to $2 \times 10^4$ Am$^{-2}$sr$^{-1}$V$^{-1}$[104].

We performed complimentary simulations in GPT using electron bunches. The simulation results are shown in figure 3.3.2. The simulations revealed that after an initial increase in brightness by increasing the bunch charge from 0.1 fC to 0.5 fC, the brightness begins to plateau at $\sim 4.6 \times 10^6$ Am$^{-2}$sr$^{-1}$V$^{-1}$ as the higher electron density resulted in more DIH occurring, which increased the electron temperature and the beam’s emittance. As with ions the higher beam emittance driven by DIH means increasing the bunch charge does not increase the brightness of the beam. For even higher electron bunch charges the pattern seen by van der Geer et al. [104] is likely to be repeated.

There have been several techniques proposed to reduce the effect of DIH, each of which impose some additional order upon where the electrons are emitted. The additional order reduces the variation in the resulting Coulomb potential and hence
Figure 3.3.1 – Particle tracking simulations performed by van der Geer et al. [104] showing that increasing the beam current of a cw ion beam leads to an increase of the beam’s energy spread (crosses, top) and a reduction of the beam’s brightness (crosses, bottom). The dot-dash line shows the fundamental limit of the brightness and the open circles show the reduced brightness as calculated using the transverse beam temperatures. Reproduced with permission from [104].
Figure 3.3.2 – Electron bunches with a Gaussian spatial distribution of width $\sigma_{x,y} = 50 \, \mu m$, zero ionisation energy, and a range of bunch charges were accelerated up to 1300 eV in simulations using GPT. The emittance (top) and brightness (bottom) of the bunches after drifting for 0.5 m is shown as a function of the bunch charge.
CHAPTER 3. HEATING MECHANISMS IN A COLD NEUTRAL PLASMA

Figure 3.3.3 – (a) Two-dimensional atomic lattices with the same density but different filling factors. The solid circles represent filled lattice sites and the open circles the vacant lattice sites. (b) Simulations of the time evolution of the ion temperature in a body-centred-cubic lattice with filling factor $f$ but identical atomic densities. Higher filling factors result in more spatial correlations between the ions and therefore a smaller disorder-induced heating effect and a lower final ion temperature. Reproduced with permission from [103].

reduces the variation in the electron’s final kinetic energy [105].

One such technique is to ionise atoms held in an optical lattice, where the interference of high-intensity laser beams is used to hold atoms in a crystal-like structure [106]. A well-ordered crystal structure is achieved at high filling factors ($f > 0.99$), but it is experimentally challenging to reach filling factors of $f > 0.5$ due to collisional losses from multiply occupied sites [107]. However, simulations carried out by Murphy and Sparkes [103] showed that even at lower filling factors there is a reduction in the final ion temperature by ionising atoms held in a lattice, as shown in figure 3.3.3.

Alternatively, order can be imposed upon the plasma by exciting the atoms into Rydberg states with low angular momentum. Such Rydberg states perturb the Rydberg states of any atom in the ‘blockade radius’ $r_b = |C_6/v|^{1/6}$ of the excited atom, where $C_6$ is the van der Waals’s coefficient, and $v$ is the power broadened linewidth of the
state. The perturbation prevents the adjacent atoms from being excited by a laser field resonant with the unperturbed Rydberg state [103, 108].

The blockade radius for the Rydberg state of $^{85}\text{Rb}$ with $n = 60$ and $S_{1/2}$ is 6 µm, larger than the Wigner-Seitz radius and hence larger than the average inter-particle spacing in the plasma. Exciting atoms into such Rydberg states will, therefore, impose additional order on the plasma by setting a minimum distance between ionisation sites. Murphy et al. [103] modelled the effect of using the Rydberg blockade to suppress disorder-induced heating and found with an experimentally achieved blockade radius of $r_b/a = 1.2$ [109], the emittance of the resultant ion beam would reduce by a factor of 2.6, resulting in a seven-fold increase in brightness.

### 3.4 Space-charge expansion

Whilst the plasma dynamics set the initial temperature of the electrons, higher charge electron bunches will end up with higher temperatures due to macroscopic Coulomb expansion, or space-charge expansion.

The macroscopic effects of the space charge distribution become more important than the microscopic particle-particle interactions when the Debye length, defined for non-relativistic, thermal beams as

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e}\right)^{1/2},$$

(3.4.1)
is much lower than the electron beam’s radius. Beyond $\lambda_D$ the microscopic effect of an individual charge is screened by the redistribution of the other charges in the bunch in a process called ‘Debye shielding’ [31].

To understand space charge expansion, the bunch potential must be calculated from the bunch’s charge distribution using Poisson’s equation for electrostatics:

$$\nabla^2 \phi(r) = -\frac{\rho(r)}{\varepsilon_0}.$$  \hspace{1cm} (3.4.2)

Whilst equation 3.4.2 is difficult to solve, even with computational methods, physically relevant analytic solutions exist in the form of 3D ellipsoids [31]. Such 3D ellipsoids have linear space-charge forces, meaning that whilst the bunch will change volume under space charge expansion, the distribution of charges within the bunch will remain uniform. As the distribution of charges remains uniform, the space-charge
expansion remains linear and can, therefore, be reversed by linear charged particle optics [56, 57].

The reversibility of the space-charge forces in a uniform ellipsoid is illustrated by considering the phase space evolution of bunches with different initial distributions. Figure 3.4.1 shows the phase space distributions of various different initial charge distributions after undergoing space-charge driven expansion. Whilst the total phase space occupied cannot change as space charge forces are conservative, as discussed in section 1.2.1, the change in the effective space charge volume occupied is distinct between the different charge distributions.

For the ellipsoid bunch, the transverse momentum of an electron remains linearly dependent on the position of the particle so the effective phase space volume, and hence the emittance, remains constant. For the flat-top and Gaussian distribution, the non-linear space charge forces distort the phase space distribution, increasing the effective phase space volume and therefore increasing the emittance.

3D ellipsoid bunches evolve under space charge expansion from initial distributions of the form
\[
\rho(r, z) = \delta(z)\rho_0 \sqrt{1 - \left(\frac{r}{R_0}\right)^2},
\]  

(3.4.3)

where \(\delta(z)\) is the Dirac delta function and \(R_0\) is the bunch radius [110]. Such "pancake" distributions have been produced from photocathodes by shaping the photionisation laser pulse shape [111, 112].

As the distribution of charges produced by a CAES can be defined by the overlap between the excitation and ionisation laser beams and the cloud of cold atoms, the charge distribution can be defined by controlling the profile of either of the laser beams, taking the distribution of atoms in the MOT into account. Such shaping experiments, first performed by Mcculloch et al. [113], have produced bunches with arbitrary shapes. They also used the techniques they developed to determine the emittance and coherence length of the electron bunches from a CAES [55].

The various initial bunch charge distributions were produced using a spatial light modulator to alter the phase distribution of a 500 ns excitation beam [59, 114]. A reduction in the emittance growth of an ion bunch has been demonstrated via bunch shaping [59], as shown in figure 3.4.2. Producing bunches with reversible space charge expansion would allow the space charge limit to be overcome in electron and ion microscopy [115, 116], high energy particle injection [117], and UED [118].

The result of each of the heating mechanisms is an electron temperature which is
Figure 3.4.1 – 5000-electron electron bunches with various density profiles (left) underwent space charge expansion in GPT simulations. The phase space distributions recorded after 1 ns (right) show how the initial distribution of electrons impacts the final phase space distribution, and hence emittance of the beam, for even low charge bunches.
Figure 3.4.2 – (a) The experimentally measured transverse ion beam density profiles for various excitation beam profiles: half-spherical (HS), Gaussian (GS), flat-top (FT) and conical (CN), for ion numbers $N = 2000$ and $N = 71000$. (b) Radial expansion factors against ion number for each shape individually, with circle, plus, times, and square corresponding to the transverse radii containing 50%, 75%, 90%, and 95% of the bunch charge, respectively, compared to their initial values as derived from the laser intensity profiles used to produced the bunches. Bunches with linear space charge forces undergo self-similar expansion, so the radial expansion factors for such bunches should be similar, as is prevalent in the HS and FT bunches. Reproduced with permission from [59].
not entirely dependent on the excess energy the electrons are given during the ionisation process. To determine the electron temperature and ultimately characterise the beam, the free electrons must be extracted from the cold neutral plasma into a beam, and the parameters of that beam must be measured.
Chapter 4

The electron beamline

Whilst the earliest example of a CAES used pulsed electric fields to extract electrons from the cold neutral plasma [11], most existing CAESs, including the CAES presented in this thesis, use shaped electrostatic fields [61, 113]. There is also a design based around an rf cavity which would use rf fields to extract the electrons [119].

Although the beam energy, bunch charge, and temporal profile of the beam are all relatively straightforward to measure, with the methods by which they were measured in this work presented in chapter 5, determining the emittance of the beam is more complicated.

The most common method of measuring a beam’s emittance is to direct the beam through a pepper-pot like grid of small apertures [31]. The particles passing through the apertures form narrow beamlets each with small divergence. After some drift region, the divergence of each beamlet can be deduced from the relative size of the particle distribution to the size of the initial aperture. Plotting the divergences of the beamlets against the position of the apertures and the current density of the beamlets yields a map of the trace space volume from which the emittance can be directly determined.

The pepper-pot method, however, requires placing a pepper-pot grid of apertures in the beamline which would then need to be removed to carry out measurements of other beam parameters. Removing and reintroducing the aperture involves an expensive and complicated x-y translator, or the breaking of the vacuum. McCulloch et al. [12] avoided such complications by imaging a pepper-pot pattern directly onto the cold atom population using a spatial light modulator. They used the resultant beamlets to carry out the emittance measurements as described above. McCulloch et al. also directly measured the temperature of the beam from the blurring of sharp-edged features imaged onto the cold atom population using the spatial light modulator [64].
An alternative technique used to measure the emittance of a beam is a waist scan [61]. In a waist scan, the size of the beam in the plane of a detector is varied by either adjusting the strength of a focusing element in the beamline or by moving the plane of the detector. The emittance can then be deduced from the set of beam sizes. As no additional equipment was required, a waist scan was the method employed to measure the beam emittance in the work presented in this thesis.

In this chapter, details on how the emittance is recovered from a waist scan, and the beamline built to carry out the waist scan, are presented in section 4.1. Section 4.2 explains the design process for the beamline, discusses the constraints under which the beamline was designed, and presents the parameters of the final beamline along with the consequences of those parameters for the temperature measurement. Section 4.3 describes the effect of the magnetic trapping fields on the electron temperature and demonstrates that the ability of the AC-MOT to quickly switch the trapping magnetic fields to zero is most important for low energy beams. Finally, section 4.4 gives the details of the support structure of the beamline and the logic behind its design.

4.1 Emittance from a waist scan

The temperature, source size and hence emittance of an electron beam can be determined using a waist scan. In a waist scan, the position of the beam waist is scanned through the plane of a detector by either changing the position of the detector or by altering the strength of focusing elements in the beamline. Both actions change the size of the electron beam at the detector. The size of the beam at the detector is the result of the initial source size and divergence undergoing some transformation due to the action of the beamline. Scanning the beam waist and measuring the beam size at the detector therefore yields a range of data points from which the initial source size and divergence can be derived, provided the action of the beamline is understood.

If the transformation is linear, i.e. the final size and divergence of the beam is a linear function of the beam’s initial size and divergence, then the transformation can be written using a transfer matrix:

$$
\begin{pmatrix}
\sigma_f \\
\sigma'_f
\end{pmatrix} = \begin{pmatrix}
A & B \\
C & D
\end{pmatrix} \begin{pmatrix}
\sigma_i \\
\sigma'_i
\end{pmatrix},
$$

(4.1.1)

where $\sigma_i$ and $\sigma'_i$ are the initial position and divergence of the beam, respectively; and $\sigma_f$ and $\sigma'_f$ are the final position and divergence of the beam, respectively. The transfer
Figure 4.1.1 – As the strength of the lens element is increased the position of the beam waist changes and hence the measured beam size at the plane of the detector changes. The beam envelope at three points along the beamline is shown (inserts) demonstrating the action of an increasing lens strength, with the plane of the lens element marked with a black line and the plane of the detector marked with an orange line.

The matrix, $M_T = ABCD$, is a function of the electro- and magneto-static fields that make up the beamline. The final size of the beam can therefore be written as

$$\sigma_f = A\sigma_i + B\sigma'_i.$$ \hspace{1cm} (4.1.2)

The $A$ and $B$ coefficients can be varied by changing the strength of the lenses that make up the beamline. An increase in lens strength reduces the focal length of the lens; bringing the beam waist closer to the lens. Reducing the strength results in the opposite effect; reducing the divergence applied to particles passing through the lens and increasing the focal length, pushing the beam waist further from the lens. As the position of the beam waist changes the size of the beam measured in the stationary detector plane will also change, as shown in figure 4.1.1.

The resulting range of $\sigma_f$ can be fitted with equation 4.1.2 to give $\sigma_i$ and $\sigma'_i$ if $A$ and
are known. Since $\sigma_f'$ can be written as a function of temperature as

$$
\varepsilon_x = \sigma_x \sqrt{\frac{k_B T}{mc^2}},
$$

from section 1.2.1, then from a series of $\sigma_f$ both the initial source size and temperature can be determined, provided the transformation between the initial and final sizes and divergences is linear.

The two most important sources of non-linearity in the beamline are space charge expansion and non-linear lensing fields. The non-linear lensing fields can be excluded by careful beamline design, whilst the effects of non-linear space charge expansion are explored in more detail in section 3.4.

### 4.1.1 Calculating the $A$ and $B$ coefficients

To determine the $A$ and $B$ coefficients for a given geometry and a range of lens biases, the beamline geometry was first imported into Poisson Superfish, a 2D electromagnetic field solver [120]. Poisson Superfish takes a 2D beamline geometry from a series of input parameters, computes a mesh based on the geometry and a mesh spacing, and solves the Poisson equation for the input geometry at every point on that mesh. The code outputs the solution as a 2D field map.

The field maps produced by each charged element were calculated independently, with a bias of 1 V, assuming cylindrical symmetry. The calculated fields were sampled with a radial sample interval of $\Delta r = 19.6 \mu m$ and a longitudinal sample interval of $\Delta z = 12.7 \mu m$ to produce a series of field maps.

The field maps were imported into GPT, a charged particle tracer code, and the field strengths in the field maps were scaled by an appropriate factor. Since electric fields are linear, the difference between the field maps produced as a superposition of field maps calculated by Superfish and the full field maps calculated directly in Superfish is negligible, as shown in figure 4.1.2. As the superposition principle holds and differences are negligible, at less than one part in $2 \times 10^4$, a range of different beamline biases can be tested without having to recalculate the fields using Superfish. GPT outputs the particle positions and momenta using a Cartesian coordinate system with the $z$ axis lying along the centre of the beamline.

With the field maps imported, scaled and superimposed upon one another, the $A$ and $B$ coefficients can be calculated. For the $A$ coefficients, a line of particles was produced in GPT, each particle having a different position along the $x$-axis, $x_i$, but with zero
Figure 4.1.2 – The top row shows, from left to right, an electric field map calculated from a complete beamline geometry using Superfish, $|E_f|$, as a sum of the fields for separate elements, $|E_s|$, and the field map produced by including each of the separate elements in GPT, $|E_g|$. The middle row shows the difference between the field map shown on the top line and $|E_f|$, whilst the bottom row shows the relative size of the difference map to the field map in question.
transverse velocity. The tracer code is executed, and the final positions of the particles are plotted against their initial positions. The $A$ coefficient is found from the gradient of the linear fit between the initial and final positions using equation 4.1.2, setting $\sigma_f^\prime = 0$. Once the $A$ coefficient is recorded the field map for the lens which is performing the waist scan is re-scaled and the process repeated. The process is repeated until the range over which the lens bias can be scanned is covered.

The $B$ coefficient was calculated similarly. All the particles are started at $x_i = 0$, along with the centre axis of the beamline, with a range of transverse momenta. The initial ‘divergence’ of the particles is defined as the particle’s transverse momentum divided by the particle’s final velocity. The $B$ coefficient is found from the gradient of the linear fit between the final positions of the particles and the initial divergence of the particles, now using equation 4.1.2 but now setting $\sigma_f = 0$. Again, the process is repeated to cover the scan range of the lens biases.

The relevant scales of the $A$ and $B$ coefficients determine the accuracy of the temperature measurements which can be made using the beamline. The larger the $B$ coefficient and the smaller the $A$ coefficient, the more the beam’s size at the end of the beamline depends upon the source temperature, making the resultant measurement more accurate.

This process of calculating the $A$ and $B$ coefficients was used to assess the design at every stage during the development of the electron beamline. The $A$ and $B$ coefficients of a typical beamline are shown in figure 4.1.3.

### 4.2 The electron beamline

The electron beamline itself is made up of three main components, as shown in figure 4.2.5: The extraction region, where electrons are extracted from the cold neutral plasma; a lens element to perform the waist scan; and an MCP/phosphor positioned at the end of the beamline to measure the size of the beam.

To perform the waist scans required to measure the temperature we chose an Einzel lens. An Einzel lens is a three element electrostatic lens that can change the focus of a charged particle beam without changing the final energy of the beam [121]. The first and last elements of the lens are held at the same potential, and the strength of the lens is changed by varying the bias of the middle element. Due to the expected cylindrical symmetry of the beam, a cylindrical Einzel lens is most suitable.

The equipotentials of a cylindrical Einzel lens are shown in figure 4.2.1, with
Figure 4.1.3 – The $A$ and $B$ coefficients (top) of a beamline which uses an Einzel lens to perform the waist scans, and (bottom) simulated waist scans performed using that beamline for a source of $\sigma = 170 \, \mu m$, and temperatures 25 K (blue), 90 K (orange), and 125 K (green)
Figure 4.2.1 – The equipotentials of an Einzel lens, where the first and final element are set to zero volts, and the middle electrode is held at some bias, are shown in purple, with the electrodes in black and the biased electrode coloured red.

the action of the lens occurring at the gaps between the electrodes. With the middle electrodes biased positively with respect to the end electrodes, as an electron beam passes between the first and second electrodes the electrons are retarded axially, and the beam is expanded radially. The reverse occurs when the electrons pass between the middle and final electrodes. The lensing results from the reduced speed of the electrons as they are compressed by the fields between the middle and final lens element.

An Einzel lens can perform the required waist scans without twisting the beam, as would occur with a solenoid lens, and without producing electron bunches of different energy and charge, as would result from varying the extraction bias.

The minimum radius of the Einzel lens is set by the desired filling factor - the fraction of the lens over which the electrostatic fields are linear with respect to the radius. The minimum size of the linear region needed is therefore determined by the expected beam parameters. The expected beam size was estimated from the maximum length of the beamline, as set by the available vacuum chambers to $\sim 0.5\,\text{m}$, an expected source size of $100\,\mu\text{m}$, and an expected source temperature of $< 150\,\text{K}$. With those parameters a low energy, 100 eV electron beam would expand to $\sim 6\,\text{mm}$ over the
length of the beamline. For a filling factor of below 10%, in order to limit spherical aberrations, the lens diameter must be > 60 mm. The middle element of the Einzel lens is recommended to be the same length as the diameter of the lens, whilst the lengths of the first and last element only have to be long compared to the diameter of the middle lens element [121].

There were also a series of important constraints for the design, primarily from the requirements of the AC-MOT, as well as available lab space, power supplies, and vacuum chambers, which are stated below.

4.2.1 Beamline design constraints

Trapping field solenoids

Since internal MOT coils were to be used to counter the problems with induction heating described in section 2.2.2, the MOT coils had to be included in the beamline design. The coils, as well as the entire structure that supported them and anything else that might be coupled to by the AC trapping field, also needed to be split to prevent currents from circulating as the source would otherwise suffer from the same induction heating problem as the external coils.

Trapping beams

Three pairs of anti-parallel trapping laser beams are used for the AC-MOT, all of which need optical access to the trapping region, which lay on the axis of the beamline. Optical access for the two pairs of horizontal beams was achieved by leaving the space between the two internal solenoids open, as shown in figure 4.2.2. Providing optical access for the vertical beam was more challenging as if the beam was positioned orthogonally to the horizontal trapping beams, it would hit the MCP at the end of the beamline.

The ‘vertical’ beam was therefore offset from vertical by 6.4°. The offset allowed it to pass through the centre of the trapping region and out of a window at the other end of the beamline where it could be retro-reflected, whilst missing the MCP.

Ideally, the electron beam would be shielded along its entire length from any insulating surface which might charge up and interfere with the beamline’s optics. To minimise the amount of the beamline exposed, a steel beampipe was included in the design over the length of the beamline. As can be seen in figure 4.2.3, however, the ‘vertical’ trapping beam’s new trajectory required the beam to go from ‘inside’ the beamline to ‘outside’ the beamline, so ellipsoidal holes were cut in the grounded beampipe to
Figure 4.2.2 – The optical access for the horizontal trapping beams (H1, H2) is provided by keeping the space between the solenoid formers open. The excitation and ionisation beams (not shown) lie in the same plane as the four horizontal trapping beams.

provide the required access. The holes constrained the position of the Einzel lens; it could either be positioned before the ellipsoid holes or after them.

Cylindrical symmetry

To maintain the cylindrical symmetry of the electron beamline with both the internal solenoids and the required optical access, the electrons are extracted vertically through the centre of the top trapping solenoid.

Extracting the beam vertically means the electrons are ionised from the cold atoms in the middle of a cylindrically symmetric but non-uniform electric field. Whilst the field extracts the electrons from the ion population it also causes the beam to expand. To correct the expansion and keep the electron bunches inside the beamline a lens was added immediately after the top solenoid. The equipotentials of the combined extraction and correction field are shown in figure 4.2.4. An aperture lens was chosen as it could correct the divergence of the electron beam before the beam expanded into the non-linear portion of the beamline whilst maintaining optical access for the vertical trapping laser beam.
Figure 4.2.3 – Showing how optical access is provided for the vertical trapping laser. The insert shows the view along the trapping beam from the position marked by the eye.
Figure 4.2.4 – The equipotentials of the combined extraction and correction field are shown in purple, with the electrodes outlined in black. The negatively biased electrode is filled in blue and the positively biased electrode in red. The position of the atom trap is shown in yellow, and the path of the electron beam is shown by the green arrow.
4.2.2 Beamline design

Subject to the constraints described above, the beamline was designed in several stages. First, Superfish was used to calculate 2D, cylindrically symmetric, electrostatic field maps for a range of element lengths and radii for the proposed geometry. The field maps were then imported into GPT where the $A$ and $B$ coefficients were calculated, as described in section 4.1.1. Once the final beamline design was chosen, it was replicated in OpenSCAD [122] and Autodesk Fusion 360, so the support structure could be designed around it. The beamline is defined using a polar coordinate system, with $z$ along the axis of the beamline and $z = 0$ halfway between the MOT coils, and $r$ as the radial position.

The entire beamline, in both isometric and cross-section, is shown in figure 4.2.5. Figure 4.2.5 will be referenced for the remainder of this section.

Extraction region

As the most critical constraints were on the region containing the AC-MOT at the start of the beamline those were the first design parameters considered.

An inner diameter for the internal MOT coils of 36 mm was chosen. This diameter satisfied the constraints of the required magnetic field, as discussed in section 2.1, the induction issue described in section 4.2.1, and the optical access required. To fit the 144 turns required by the parameters of the trapping fields the solenoid had to be 16 mm high.

The former upon which the solenoid was wound has a wall thickness of 2 mm, so the first aperture of the beam line is 32 mm in diameter and 20 mm long. The flange on each end of the formers is 78 mm in diameter and 2 mm thick. Both solenoid formers were machined identically from a single block of copper, and a 1mm slot was cut into them. The solenoid formers are labelled ‘A’ and ‘B’ in figure 4.2.5. Each former was mounted on a steel plate, and the gap between the two plates is 20 mm, in order to satisfy the requirements for the magnetic trapping field.

The electrons exit the extraction region with a large divergence due to the shape of the extraction field which is shown in figure 4.2.6. To collimate the beam and avoid the beam becoming unmanageably large further down the beamline a correction lens is placed immediately after the extraction region resulting in the combined field shown in figure 4.2.7. The correction lens is labelled ‘C’ in figure 4.2.5. The effect of the correction lens on the particle trajectories is shown in 4.2.8.
Figure 4.2.5 – The electron beamline in cross-section (left) and isometric view (right). In the cross-section view the extraction region is made up of ‘A’ and ‘B’, which label the trapping solenoids, and ‘C’ which labels the correction lens. The lens element used to perform the waist scan consists of ‘D’ the drift region, and ‘E’ the Einzel lens. Finally ‘F’ labels the front plate of the MCP/phosphor. The position of the atom trap is also labelled. In the isometric view, the three pairs of trapping beams are displayed in red, the path of the excitation beam in orange and the path of the ionisation beam in blue.
Figure 4.2.6 – The magnitude of the extraction field for each volt of bias applied to the bottom extraction electrode. Contours are positioned every $3 \text{ V m}^{-1} \text{ V}^{-1}$. The lens elements themselves are at a much larger radius so are not shown.
Figure 4.2.7 – The magnitude of the combined extraction and correction lens field with 2700 V applied to the bottom MOT solenoid former and 5000 V applied to the middle element of the correction lens. Contours are positioned every 10 kV m$^{-1}$. The lens elements themselves are at a much larger radius so are not shown.
The correction lens is essentially an aperture Einzel lens, where the first element is the top solenoid former, the middle element is a 2 mm thick steel plate with an aperture, and the third element is an 11.5 mm thick steel plate that also forms part of the support structure of the beamline and is held at ground. The aperture size is set by the aperture of the solenoid formers to 32 mm.

As the electrons are born essentially stationary in the centre of the defocusing extraction field but reach full beam velocity by the time they reach the correction lens, the correction lens must be far stronger than the extraction field’s lensing effect to refocus the beam.

To collimate the electron beam using the correction lens therefore requires 2.07 V for every volt applied to the extraction electrode as shown in figure 4.2.9. To bring the beam to a focus at the end of the beamline with the correction lens requires 2.28 V to be applied to the correction lens for every volt applied to the extraction electrode.

**Einzel lens**

Positioned after the correction lens is a drift tube, where the holes for the vertical trapping beam are placed, followed by the Einzel lens, labelled ‘D’ and ‘E’, respectively,
**Figure 4.2.9** – As the ratio between the extraction bias $V_{ext}$ and the correction lens bias increases the beam becomes more and more focused. The ratio of the beam’s final radius, $\sigma_f$, to its initial radius, $\sigma_i$, passes through unity at $V_c/V_{ext} = 2.07$, and through zero at $V_c/V_{ext} = 2.28$.

in figure 4.2.5. The map of the electric field produced by the Einzel lens is shown in figure 4.2.10.

The drift tube and the Einzel lens are made from 47.8 mm ID steel tube; hence the length of the centre element of the Einzel lens was set to be 46.8 mm, with a gap of 4 mm between elements. The $z$ position of the Einzel lens, $z_e$, was chosen to maximise the variation in $B$ over the waist scan, giving a larger signal from which the source temperature could be determined, whilst minimising the impact of the trapping beam holes on the lensing field. The standard deviation of $B$ as a function of the Einzel lens position is shown in figure 4.2.11.

With the centre of the Einzel lens at $z = 354.4$ mm, the effect on the lens field of the holes in the drift tube that gave optical access for the trapping beams had to be considered. To calculate the size of the effect, the Einzel lens model was imported into CST, and a full 3d evaluation of the lens field was made.

A comparison of the 2D and 3D beamline models showed there was less than 1% difference in the field strength between them across the central 5 mm diameter cylindrical volume of the beamline, as shown in figure 4.2.12, with a similar difference in the $A$ and $B$ coefficients calculated using the 2D vs 3D field maps.
Figure 4.2.10 – The magnitude of the Einzel lens field for each volt of bias applied to the middle element of the lens. Contours are positioned every 3 V m$^{-1}$. The lens elements themselves are at a much larger radius, so they are not shown.
CHAPTER 4. THE ELECTRON BEAMLINE

Figure 4.2.11 – As the Einzel lens is positioned further down the beamline the variation in the $B$ coefficient over the range of possible Einzel lens biases remains stable before coming to a maximum at $z = 354.4$ mm and then decreasing afterwards.

Figure 4.2.12 – The fractional difference between the 2D field map of the Einzel lens and the 3D field map of the Einzel lens.
MCP shroud

After the Einzel lens, there is a bracing ‘top plate’, after which the beampipe continues with the slightly larger diameter of 53.4 mm. The final section of the beam pipe is called the ‘MCP shroud’, and shields the beam from the electrostatic fields produced by the MCP which might otherwise distort the beam.

Beam energy

The ratio between the correction lens bias and the extraction electrode bias limits the maximum beam energy as the SHV feedthroughs used are only rated to 5 kV, and the power supplies available can only produce 5 kV.

For a beam that is focused at the plane of the MCP using the correction lens, the beam energy is limited to ≈ 1.2 keV, as the electrons are born halfway between the two extraction electrodes.

Steering coils

The background magnetic field in the area of the beamline, as measured with a Bartington MAG-03 MC magnetometer is (-5.2, 16.8, 29.6) µT, quoted along the Cartesian axes, with the z axis aligned with the z axis of the beamline. Under such a field, a 2.5 keV electron beam would be displaced by 25 mm perpendicular to the z axis, which would place it outside both the linear regime of the beamline and the edge of the MCP used to image the beam.

To counter the effect of the earth’s magnetic field and any other static magnetic fields that affect the path of the electron beam, two pairs of external steering coils were added outside the beamline, as shown in figure 4.2.13. The field produced by the steering coils, shown in figure 4.2.14, was calculated from the Biot-Savart law for a series of line elements corresponding to the coils shown in figure 4.2.13.

Over the length and breadth of the beamline, the variance in the magnetic field produced by the coils is only 2.14 mG, as can be seen in figure 4.2.14. Due to the near constant magnetic field provided by the steering coils, there is a negligible lensing effect on the beam due to the steering field.

Without the steering coils, the electron beam does not reach the MCP. Using them, it is possible to steer the beam through the centre of the Einzel lens and onto the centre of the MCP.
Figure 4.2.13 – The steering coils (brown) are shown with respect to the beamline.
Figure 4.2.14 – The magnetic field produced by the steering coils. Top: the magnitude of the magnetic field, $|\mathbf{B}|$, on cross sections along the $y$, $x$ and $z$ axes, respectively. Bottom: The difference between the steering magnetic field and the earth’s magnetic field at the position of the beamline. The earth’s field was measured by a Bartington MAG-03 MC.
The current supply to the steering coils is provided by a benchtop power supply. Sub-Hertz instabilities in the supply current cause the beam to move slightly between experimental cycles.

The limitations of the proposed beamline could now be investigated using GPT simulations.

4.2.3 Limitations of the beamline

The first source of inaccuracy arising from the beamline results from variation in the $z$ position of the electron source. Not only do changes to $z_0$ affect the lens properties of the extraction region, but they also change the overall beam energy which has an impact on how other lenses along the beamline focus the beam.

To explore the size of the variation due to the source position along the $z$ axis, $A$ and $B$ were calculated in the same manner as described above for particle beams with various $z_0$. Figure 4.2.15 shows the $A$ and $B$ coefficients as well as typical waist scans for beams with a range $z_0$, demonstrating how important knowing $z_0$ is to making accurate source temperature measurements.

Also important in making accurate temperature measurements is the definition of the initial divergence in the optical transfer model. Defining the initial divergence of the beam as the initial transverse momentum divided by the final beam velocity imparts a scaling error on the calculated $B$ coefficients. The scaling error is a result of the electrons being born stationary in an electric field after which they are accelerated up to their final velocities.

To determine the scale factor, simulations of electron bunches with a range of temperatures were carried out using GPT. Electron bunches were produced with a thermal distribution of some characteristic temperature, and allowed to propagate down the beamline model. For each bunch temperature, a simulated waist scan was carried out by varying the einzel lens bias and recording the bunch size at the location of the MCP as a function of the einzel lens bias. The temperature was derived from the waist scan as described in section 4.1.

The initial temperatures of the bunches as set in the simulations were compared to the temperatures derived from the waist scan. The results are plotted in figure 4.2.16. A linear fit between the set and measured temperature gave a scale factor of $0.78 \pm 0.01$, meaning the temperatures recovered from the waist scans are scaled by $0.78 \pm 0.01$ compared to the actual source temperature. The scale factor was carried over to the results presented in section 5.4. For temperatures above $\sim 200$ K the linear model breaks
Figure 4.2.15 – The \( A \) and \( B \) coefficients for bunches starting at a \( z_0 \) positions of \(-5\) mm to \(5\) mm. Higher initial \( z_0 \) are shown in yellow and \( z_0 = 0 \) is in black. The lower plot shows typical waist scans of a 25 K beam using a beam line with these \( A \) and \( B \) coefficients.
Figure 4.2.16 – The relationship between the temperature of the bunch set in simulation and the temperature recovered from the fits to the simulated waist scans. The linear fit between the data is shown in orange.

down as the beam is no longer contained in the linear regime of the beamline.

To determine the range of beam parameters over which the linear model holds we can look, in simulation, at the linearity of the relationship between the initial and final positions and divergences of particles that travel along the beamline. If we fit a linear model to the final positions of the particles as a function of their initial positions and divergences, any deviations from that linear model indicate that a beam containing particles with those initial parameters will leave the linear regime of the beamline. A beam which leaves the linear regime will not be accurately modelled by the linear transfer model, and therefore the emittance could not be accurately measured using a waist scan. To quantify the deviation from the linear model we can use Pearson’s $\chi^2$ test.

Pearson’s $\chi^2$ test evaluates the differences between the observations and the expected values as

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$  \hspace{1cm} (4.2.1)

where, for particles with given initial positions or divergences, $O_i$ are the observations: the final positions of the particles, and $E_i$ are the expectations: the value of the linear model we fitted to the data for those initial positions or divergences. Larger $\chi^2$ values correspond to particles undergoing a more non-linear transfer between the initial particle
CHAPTER 4. THE ELECTRON BEAMLINEN 131

Figure 4.2.17 – The $\chi^2$ goodness of fit value for the linear fits between the set of initial and final positions of the particles. Across the entire range the $\chi^2$ value is below $4 \times 10^{-4}$, showing that for beams within the displayed parameter space the electrostatic optics of the beamline are linear.

positions and divergences and final particle positions and reveal a break down of the linear model.

In figure 4.2.17 the $\chi^2$ values of linear fits between the initial position $x_i$ and the final position $x_f$, as well as between the initial divergence $x'_i$, expressed as a temperature, and $x_f$, are plotted as a function of the maximum initial particle position and divergence. The $\chi^2$ values show that beams with an initial size below $\sim 1000 \mu m$ and zero divergence, and a temperature below $\sim 40$ K, will remain well inside the linear regime of the beamline, with a trade-off between the two parameters.

As well as determining the linear regime of the beamline, similar techniques can be used to calculate the non-linear effects of the magnetic fields on the electron beam and demonstrate under which parameters the ability of the AC-MOT to quickly zero the trapping fields is beneficial.

4.3 The effects of the magnetic trapping fields

The benefit of zeroing the magnetic trapping fields before electron extraction can be determined by calculating the effect of the trapping fields on the electron bunches. The effect of the trapping fields can be determined using particle tracking simulations,
Figure 4.3.1 – A map of the trapping field’s magnitude. The clusters of contour lines show the locations of the current carrying wires making up the solenoid.

The electrons are extracted axially through the top trapping solenoid, so the field they experience is essentially cylindrically symmetric. The height and depth of the windings are also small compared to the radius of the solenoid meaning the non-linearities in the magnetic field, which would drive emittance growth, will be small. To account for as many of the non-linearities as possible, a detailed field map must be produced.

### 4.3.1 2D field maps

Cylindrically symmetric 2D magnetic field maps can be calculated using the equations in section 2.2.2, with the resulting field maps calculated using a Python code. The resulting map is re-displayed in figure 4.3.1. The Python code calculates the field due to a series of 144 turns, wound in 12 layers. Each layer consisting of 12 rows of wire with 1.1 mm between the centres of adjacent wires. The 2D field maps are sufficient to determine any non-linearities that arise due to the spatial extent of the solenoid winding.
GPT can be used to determine the range of initial particle positions for which the lensing field produced by the solenoids is linear. In GPT a series of particles were produced at the centre of the trapping region with velocity $20 \times 10^6 \text{ms}^{-1}$ in the $z$ direction, equivalent to 1 keV electrons, each at different $r$ positions. The electrons were allowed to travel 0.5 m and their final distance from the $z$ axis was recorded.

Again the $\chi^2$ values of the linear fits between the initial and final positions were calculated and are shown in 4.3.2. The figure shows that for $\sim$1 keV electrons the trapping fields do not significantly affect the linearity of the beamline.

However, repeating the simulations at lower beam energies shows the non-linearities in the trapping field become more important at low energy. Figure 4.3.3 shows the $\chi^2$ treatment for a 10 eV beam, demonstrating the order of magnitude increase in non-linearity due to the trapping fields on such low energy beams.

These results are borne out by simulations of cold electron bunches travelling along the two beamlines. For a 10 K bunch, the normalised emittances as a function of beam energy, for both the AC and DC beamline, are shown in figure 4.3.4. The simulation results show that extinguishing the trapping fields before extraction could result in a reduction in the beam emittance of 20%, for low energy ($< 20 \text{eV}$) beams.

With the proposed beamline well understood, the design process turned to the manufacture of the beamline and the support structure surrounding it.
Figure 4.3.3 – The $\chi^2$ goodness of fit parameter for the linear fits between the initial and final particle positions for a range of source sizes and temperatures for a 10 eV beam. Top: The AC beamline, with the trapping fields zeroed before extraction. Bottom: The DC beamline showing an order of magnitude increase in the non-linearity over the AC beamline.
Figure 4.3.4 – A 10 K bunch was simulated travelling down the two beamlines, and the final emittance of the bunch was recorded at the end of the beamline. Below 100 eV, the DC trapping fields begin to affect the beam significantly.

4.4 Beamline support structure

To design the support structure, OpenSCAD, along with a custom dimensioning library, was used to produce the required technical drawings. The model was also imported into Fusion360. The steel components are all 316L stainless steel, and the fastenings are A4 stainless steel.

The diagram of the support structure in figure 4.4.1 shows how the entire assembly can be removed in one piece to be modified or repaired since the whole assembly is mounted vertically, hanging from the top steel vacuum flange, shown in green. Three of the five ports on the top flange have windows mounted upon them. The central window is used to view the phosphor screen mounted on the back of the MCP, whilst the other two provide optical access for the vertical trapping laser beam. The other two ports have electrical feedthroughs mounted on them; one port has a 4-way, BNC type feedthrough, the other has both a 2-way SHV and a 4-way SHV feedthrough mounted on it using a T-pipe. The BNCs are connected to the MOT solenoids whilst the SHVs are connected to the elements of the beamline and the MCP assembly.

The beamline itself is supported by four, 12 mm OD steel rods with tapped M6 holes at both ends. M6 studding is screwed into the holes. At the top, the studding screws into holes in the top flange, whilst at the bottom of the rods a 11.5 mm thick
Figure 4.4.1 – The beamline support structure, with the top flange shown in green, the MCP assembly in red, the beamline itself in blue, with the support ceramics and PEEK spacers in yellow, and the MOT assembly in purple, with the PEEK spacers and bolts again shown in yellow.
steel ‘mounting plate’ is held in place by M6 nuts. Three pairs of the nuts are stood-off from the studding by ceramic spacers to cut the current loop for any induced magnetic fields, whilst grounding the plate.

**MOT assembly**

Hanging from the mounting plate by four 5.5 mm OD steel rods is the ‘MOT assembly’, shown in purple in figure 4.4.1, consisting of the MOT solenoids, their formers, and the correction lens.

At the bottom of the MOT assembly is the base plate upon which the bottom solenoid’s copper former is mounted. As the former is biased, it is mounted on four M4 PEEK screws with PEEK spacers to set the height of the former away from the base plate. On top of the former, mounted on four M4 screws, is the 2 mm thick annular steel ‘bottom extraction plate’, with an OD of 108 mm and an ID of 32 mm.

The top solenoid’s copper former is mounted on the 2 mm thick steel ‘top extraction plate’. The extraction plate also has an ID of 32 mm but has an OD of 142 mm so that it can be mounted on the 5.5 mm steel rods hanging from the mounting plate. The top extraction plate is held in place by eight steel spacers, slid over the four 5.5 mm steel rods, two on each side of the plate. The spacers keep the MOT assembly square, ensuring the extraction electrodes and the solenoids are both correctly spaced and parallel to one another.

Above the top solenoid is the middle element of the correction lens, which is biased at up to 5 kV. Made from a 2 mm thick steel plate, the element is another annulus with an ID of 32 mm and an OD of 108 mm. The element is mounted off the mounting plate using four M4 PEEK screws and PEEK spacers to provide electrical isolation. The mounting plate is hung from the top flange using 4 mm long steel rods, which have M6 studding extending from the end, onto which the mounting plate is secured using M6 nuts and ceramic spacers. The ceramic spacers prevent a current loop from forming in the steel of the beamline which could be driven by the AC magnetic fields of the AC-MOT.

**Einzel Lens**

Above the mounting plate, which also forms the third element for the correction lens, is the drift tube followed by the Einzel lens, shown in blue in figure 4.4.1. Both the drift tube and the Einzel lens have an ID of 47.8 mm and are held in place by four ceramic
rods held between the mounting plate at the bottom and the ‘clamping plate’ at the top. The ceramic rods are used to align the elements of the beamline whilst keeping them electrically isolated from one another. The rods run up the side of the beamline, and a series of PEEK spacers are slid down the rods to space each element correctly and to electrically isolate the elements from one another.

Immediately above the mounting plate, isolated by 1 mm PEEK spacers is the last split element of the beam pipe. Following the second set of 1 mm PEEK spacers is the first element of the Einzel lens which includes the elliptic holes for the vertical trapping laser beams.

After a set of four 4 mm PEEK spacers is the middle element of the Einzel lens and following a further set of four 4 mm PEEK spacers is the final element of the Einzel lens.

The clamping plate at the top of the beamline is an annulus of ID 46.8 mm and OD 142 mm made of 5 mm thick steel plate. As well as holding the ceramic rods in place it also has two holes cut into it for the trapping laser beam to pass through and is in turn held in place on the 8 mm steel rods by eight 5 mm long spacers, secured in place by M3 bolts screwed into holes drilled and tapped in the side of the spacers. The spacers along with the nuts at the bottom of the 8 mm steel rods were used to align the beamline. The advance (a copper-nickel alloy also known as constantan) MCP shield is spot welded to the top of the mounting plate. Each of the elements of the beamline is grounded, apart from the bottom extraction plate, the middle correction lens element and the middle Einzel lens element, which are connected to the feedthroughs on the top flange.

**MCP assembly**

The MCP assembly is mounted off the top flange separately from the rest of the beamline on a water cut steel ring. Three M3 screws, stood off from the biased plates of the MCP and phosphor screen with ceramic spacers, hold the assembly in place. The steel ring is supported by four lengths of M6 studding which screw into tapped holes in the top flange.

Finally, to block light from windows that provide optical access for the vertical trapping beam from reaching the phosphor screen, an advance shroud was added to the support structure for the MCP.
4.5 High voltage power supplies

To produce the voltages applied to the biased elements of the beamline five Spellman MP5 5 kV power supplies, four positive and one negative, were purchased and an interface was designed allowing them to be controlled from both a front panel rotary encoder and a computer.

The MP5s have an RS-485 interface built in, allowing them to be controlled from a computer using the RS-485 protocol. So the supplies could also be controlled locally and the status of the supplies output on an LCD screen, an Arduino MEGA was used as an intermediary between the PC and the supplies.

In ‘PC’ mode the Arduino, connected to a PC via USB, communicates with a LabView program using the serial protocol. The serial messages sent by the PC are interpreted, and the appropriate message is sent to the relevant supply via RS-485. Any response from the power supplies is then relayed to the PC via the serial USB line.

In ‘Front panel’ mode, adjusting the rotary encoders on the front panel of the power supply causes the Arduino to send the appropriate messages to the appropriate MP5 and update the LCD. The rotary encoders are debounced in software. A three-throw switch is used to change the coarseness of the control between 0.1, 1, and 10 V.

The power supply can also be operated in ‘Both’ mode, allowing the supply to respond to changes initialised from both the PC and the front panel rotary encoders. A second three-throw switch controls the mode of the supply. The voltages are output on five SHV connectors on the back of the supply.

The code running on the Arduino MEGA is linked in appendix B.2.

A LabView program running on a PC is used to interface with the power supply. The program allows the user to set the voltages on the supplies directly or set a ramp which varies the output voltages of the supply over time and reports the currents and voltages measured internally by the MP5s.

There is a small scaling factor between the voltage measured internally by each of the supplies and the voltage output as measured using an HV probe. The scaling factor was taken into account when calculating the electrostatic fields produced by the beamline.

Whilst the beamline can perform a waist scan, that in itself is not sufficient to measure the emittance of the beam. To measure the emittance requires the cross section of the beam to be measured and the measurement must also be cross-checked against other measurements of the beam parameters. Additionally, to fully characterise the electron beam requires the beam energy, bunch charge, and the temporal profile of
the beam to be measured. How the waist scans were recorded, how the emittance measurements were cross-checked, and how the measurements of the other beam parameters were made forms the basis of chapter 5.
Chapter 5

Characterising the electron beam

The quality of an electron source can be determined by characterising the electron beam produced from the source. The beam can be characterised by measuring the beam’s emittance, bunch charge, beam energy, and the temporal shape of the electron bunches. This chapter describes the method by which each of these measurements were made.

The results of the measurements presented in this chapter show that the Manchester AC-MOT CAES has a source temperature of $12.6 \pm 0.8 \text{ K}$, and can produce beams with a bunch charge of $2.2 \pm 0.4 \text{ fC}$, with an emittance of $8.41 \pm 0.30 \text{ nm rad}$ for a source size of $182.4 \pm 3.8 \text{ \(\mu\text{m}\)}$, which is comparable to other existing CAESs.

5.1 Acquiring cross-section images of the beam

The technique common to making each of the measurements of the beam parameters is the acquisition of cross-section images of the electron beam. The cross-sections were acquired in several stages. First, the beam was directed at an MCP/Phosphor (microchannel plate/phosphor screen) assembly. The MCP amplifies the electron bunches and the amplified bunches are accelerated onto the phosphor screen. When the amplified bunches hit the phosphor screen the screen fluoresces. That fluorescence is captured by a charge coupled device (CCD), producing the required cross-section images of the beam.

5.1.1 Microchannel plate

The MCP/phosphor assembly is positioned at the end of the beamline, $0.492 \text{ m}$ from the centre of the trapping region. It is held by a custom designed mount attached to the
Figure 5.1.1 – Electrons in the incident electron beam enter the channels of the MCP. As the channels are not parallel to the applied electric field supplied by $V_{MCP}$, the incident electrons collide with the walls of the channels. Upon hitting the walls, the electrons scatter secondary electrons from the glass from which the MCP is made. The secondary electrons are accelerated by the applied bias $V_{MCP}$ and go on to produce additional secondary electrons when they hit the channel walls.

top flange.

The MCP is a series of tens of thousands of $10\mu m$ wide glass tubes arrayed in a grid, as seen in figure 5.1.1. When an electron bunch hits the plate, a portion of the bunch electrons that enter the channels hit the channel wall and scatter multiple other electrons in the glass from which the channels are made. The secondary electrons are accelerated by an electric field produced by a $1kV$ bias applied between the front and back of the MCP. Since the channels are angled at $10^\circ$ with respect to the electric field applied across the plate each of the accelerated secondary electrons will also hit the channel wall. Upon hitting the channel wall they scatter further electrons. These electrons are also accelerated and go on to scatter multiple additional electrons. The resulting exponential growth of the electron count down the channel means a single incident electron can result in $\sim 1 \times 10^4$ electrons exiting the back of the MCP [123].

Behind the MCP, biased at $5kV$ with respect to the front plate of the MCP, is a P24 phosphor screen. When struck by the amplified electron bunch the phosphor fluoresces and the light it produces is picked up by the camera described in section 5.1.2. The phosphor has a decay time of $22\mu s$ and fluoresces at $505\text{ nm}$.  

The MCP/Phosphor assembly used in making the charge measurements along with measurements of the other beam characteristics is a Photonis APD 1 PS 40/12/10 1 60:1
5.1.2 MCP camera

A camera is used to record the cross-section images of the beam produced on the phosphor screen. The camera used must be sufficiently sensitive that it can image the light produced when a single electron is amplified by the MCP and the amplified pulse hits the phosphor. The camera must also have the ability to be triggered using an external pulse so the image capture could be synchronised with the incident electron bunches. To satisfy these conditions we used an iXon3 888 [124]. The iXon3 has a $1024 \times 1024$ pixel sensor and each 13 µm pixel has a 14-bit resolution. The position of the camera with respect to the electron beamline is shown in figure 5.1.2.

The camera was used in ‘External Trigger’ mode. In External Trigger mode, once the previous image is read from the CCD, the camera goes into a ‘clearing cycle’, where one row, then one column of the CCD is cleared of charge. The clearing cycle is repeated until the entire sensor has been cleared of charge. The camera is then ready to accept a new trigger signal and continues performing clearing cycles until a trigger signal arrives. When the camera receives a new trigger signal the exposure begins. The exposure length was set in software to 50 s. Once the new image has been captured, it is read off the CCD, and the process repeats for the next image in the series. The image read-out from the CCD is sent through an output amplifier and an analogue/digital converter which digitises the signal before it is sent to the computer.

The sensor in the camera is cooled to $-80 \degree C$ to minimise thermal noise, meaning the primary source of noise from the camera is ‘read noise’. Read noise is mainly produced during the readout process, but also has contributions from the digitisation electronics, and increases with increased readout speed. At 10 MHz readout speed, the read noise was not sufficient to obscure the fluorescence on the phosphor resulting from the electron pulse due to single electrons being amplified by the MCP.

The images of the electron bunches are recorded on a computer running the Andor Solis software and are then exported to numbered ASCII files. For each shot in a series, the biases on each electrode are recorded along with the MOT parameters on a separate computer, with the trigger box described in section 2.3.5 used to synchronise the two data series by assigning each shot a ‘shot number’ which matches the numbered ASCII files. The two data sets are merged according to the shared shot number.

The camera is fitted with a Samyang 85mm f/1.4 lens and a set of bellows, together forming a fast, macro lens. The lens system has a magnification of 0.7, with each
Figure 5.1.2 – Schematic of the camera and lens system used to image the phosphor screen and its location with respect to the MCP/phosphor assembly and the electron beam. The grey shows the portion of the cross-section of the vacuum chamber which is steel and the yellow shows the light proof shroud used to minimise the external light entering the camera.
recorded pixel representing an area of $18 \, \mu m \times 18 \, \mu m$ on the phosphor screen. Therefore the minimum spot size that can be recorded is $90 \, \mu m$. The resolution was determined by recording an image of a $1 \, mm \times 1 \, mm$ grid placed in the object plane of the camera. The field of view of the camera was limited by the CF70 window through which it imaged the phosphor screen.

The camera was placed directly above the MCP on a 3D printed, light-tight shroud, imaging the MCP through a CF70 window, as illustrated in figure 5.1.2. There is also an internal shroud that shields the camera from scatter from the vertical trapping beam. Despite these attempts to exclude external sources of light from the images there remained a non-uniform background signal.

5.1.3 Background removal

Before the cross-section images could be used to derive the beam parameters the background offset which remained in the image had to be removed. For each experimental run, a background image was taken comprising of the sum of 200 shots recorded while the magnetic trapping fields were switched off. Switching off the magnetic trapping fields meant there was no atomic population from which electrons would be produced but any background signal due to laser light would be included. Figure 5.1.3 shows a typical background image, along with a typical bunch cross-section before and after the background signal was removed.

After the removal of the background, the residual noise is flat; it maintains the same mean value across the image, but it is not necessarily centred around zero, as seen in figure 5.1.3 c. To remove the residual offset, first the offset is determined from the mean value of an ‘empty’ portion of the image, one in which there are no electrons. The measured offset, typically around $-3$ units of pixel intensity, is then subtracted from every pixel in the image. Figure 5.1.4 shows the effect of the offset removal on an image along with how the total image intensity is affected.

5.1.4 Web electrons

Along with bunch electrons that are directly amplified by the MCP after entering one of its channels, some fraction of the electrons hit the ‘webbing’ of the MCP - the portion of the MCP’s structure between the channels. Any electron which hits the webbing can scatter an electron to the vacuum adjacent to the MCP.

Depending on the energy of the secondary ‘web’ electron, it can end up being
Figure 5.1.3 – a) (left) A typical image of an electron bunch where the colour shows the intensity of each pixel. (right) The mean value of each column of pixels in the image of the electron bunch. b) (left) The background image corresponding to the experimental run during which the image of the electron bunch was taken and (right) a plot of the mean pixel values in each column of the background, overlaid on the mean column pixel values from the electron image. c) (left) The residual image after subtracting the background and (right) the mean column pixel values after subtracting the background.
amplified by a channel of the MCP after travelling a short distance away from the initial scattering location. Effectively, the web electrons produce a halo around the central spot produced by the electron beam that must be considered during analysis of the images.

The intensity of the web electron halo is a function of the bias across the MCP and the electric field in front of the MCP. By reducing the bias across the MCP to 900 V the effects of the halo electrons were minimised whilst the signal-to-noise ratio of the images was still sufficient to detect single electrons hitting the MCP, despite the reduction in the gain of the MCP due to the reduced MCP bias. An MCP bias of 900 V was used for all the experiments presented.

### 5.2 Bunch charge

To measure the brightness of the beam and therefore allow a comparison to other electron sources we must determine the amount of charge which can be extracted from the source into each electron bunch.

The bunch charge, $Q$ was determined from the total image intensity of the electron beam cross-section images in two stages. First, the distribution of total pixel intensity associated with a single electron event was found, giving a mean single electron intensity. As there is a linear relationship between the total image intensity, the sum of all the pixel values making up the image after the background is subtracted, and the bunch charge, dividing the total image intensity by the single electron intensity yields the number of electrons making up the image. Determining the electron count of the bunch gives a
measure for the bunch’s charge. To determine the mean single electron (1e) intensity, the MCP was used to amplify and image a series of low charge, low electron density bunches, with typically only a few hundred electrons in each image, in which the spots produced by individual electrons could be distinguished. By isolating the spots the total intensity in the image due to each electron could be calculated as the sum of the values of the pixels that made up the spot. The mean value for the intensity of a single electron was determined from the distribution of all the total intensities associated with a single electron. To isolate the electrons in a single image and calculate the single electron intensity distribution several different methods were tried, and are described in the following sections.

5.2.1 1e distribution from a threshold method

In the simplest method, a background signal for the images was first subtracted from the image, and the image was normalised. The image was then ‘thresholded’; any pixel with a value below a threshold value was set to 0, and then the image was passed to the `ndimage.measurements()` function [125]. The function identifies any ‘object’ in the image, defined as any continuous region where the pixel values are not 0. Figure 5.2.1 shows the thresholded image next to the original image with all the distinct ‘objects’ highlighted.

For each object, the values of the pixels within a circle of radius 5 pixels of the object’s centre of mass were summed to give a value for the total intensity of a single electron. The electron intensities from a series of 250 images were plotted on a histogram shown in figure 5.2.2. If each object corresponds to a single electron, there will be a peak in the histogram centred around the mean summed intensity of a single electron, with the width of that peak related to the standard deviation of the single electron intensity.

The threshold method, however, combines multiple electrons into single objects if they are close enough that there is no region of zero-valued pixels dividing them, confusing the object intensity histogram. Whilst increasing the threshold value separates more of the overlapped electrons it also entirely removes some of the lower intensity electrons. The threshold value chosen in figure 5.2.2 results in a mean single electron intensity of $283 \pm 68$. Using this value to calculate the MCP gain using the method described in section 5.2.3, gives a gain of $1.89 \pm 0.23 \times 10^5$, which is about ten times as large as the figure found in the literature [123].

To avoid the overlap problem and make the threshold method more accurate the
images were cropped to remove as many of the overlapped electrons as possible. The cropped images, however, contained relatively few electrons from which to derive the average electron intensity whilst still containing many objects that did not resemble single electrons, as shown in figure 5.2.3.

The mean intensity from the cropped images is drastically different to the mean intensity from the un-cropped images and both methods gave values for the gain that were inconsistent with the literature. A better method of identifying the underlying 1e intensity distribution was therefore required.

The problem stems from a combination of overlapping electrons and the relatively high level of background noise in the images. Removing those features from the images and isolating single electrons would result in a more accurate single electron intensity distribution. Removing the noise from the image would also allow the low-intensity electrons excluded by the threshold method to contribute to the intensity distribution.

### 5.2.2 1e distribution from machine learning techniques

Machine learning techniques have shown great efficacy in removing the noise from or ‘denoising’ images [126, 127]. Therefore to first denoise, and then easily isolate the individual electrons in the images, a convolutional neural network (CNN) [128, 129]
Figure 5.2.2 – A thresholded image (top left) and the original image with the location of the identified electrons (top right). The orange circles denote which pixels were summed to give the total intensity for each electron. (Bottom) The distribution of total intensities (blue) and a double Gaussian fitted to the distribution. The mean of the higher amplitude Gaussian (283.4) is taken as the mean total intensity of a single electron.
Figure 5.2.3 – The intensity distribution (bottom) created only using electrons landing inside the region denoted by the white box imposed upon the images (top), excluding the noisier region to the bottom left of the image. The intensity distribution is shown in blue, and the two fitted Gaussian functions are in orange. The mean of the higher amplitude Gaussian is used as the mean of the single electron distribution.
was trained to turn the noisy images into ‘perfect’ images which could be used to produce the single electron intensity distribution.

The CNN determines the perfect images from the input images by first taking a 17x17 pixel area around each pixel of the image, called the ‘receptive field’, and assigning each of the pixel values in the receptive field to a node in the first layer of the neural network. Each node of a neural network holds one value, called its activation $a_i$, and each node is connected to nodes in the second layer of the network by an edge with some weight $w_i$. The activation of the nodes in the second layer of the network is the sum of

$$a_i = \sigma(w_i a_i - b_i) \quad (5.2.1)$$

over all the connections to the node in question. $b_i$ is an additional bias associated with the node and $\sigma(x) = \max(0,x)$ is the ReLU function \[130\], which compresses the activation between zero and one. Each node in the layer is connected to nodes in the previous layer, and so on until the final output layer is reached, which contains a single node. The final node’s activation sets the value of the corresponding pixel in the output ‘perfect’ image. The weights and biases make up the set of variables which are determined during the ‘training’ routine, discussed below.

In a fully connected neural network, every node on one layer of the network is connected to every node on the next layer of the network. The activations of each node on one layer are therefore determined as the sum over all the nodes and weights from the previous layer, along with the biases associated with the node itself. For multi-layer neural networks, the number of variables quickly becomes unmanageable, particularly when dealing with large 2D or 3D data.

The increase in the number of variables to manage when dealing with 2D data is solved in a CNN by exploiting the fact that the value of an individual pixel in an image is likely to be dependent on the value of nearby pixels and unrelated to the value of a distant pixel. The 2D data can, therefore, be subdivided into smaller regions and each region fed to copies of the same node, reducing the number of weights and biases which need to be managed. Another innovation in CNNs is the use of ‘max-pooling’ layers. The exact location of an identified feature in an image is far less critical than the fact that the feature exists - the presence of a single electron is far more important than where that electron is exactly located. Therefore taking the maximum of features over small regions in the output of a previous layer is an effective way to reduce the number of variables required by the network whilst retaining the ability of the network to recognise features.
The CNN used to denoise the electron images has five layers. The first is a convolutional layer with ten nodes. Copies of each of the ten nodes each take a 5-pixel region of the input 17-pixel ‘receptive field’ and process them according to the trained weights and biases. The output is then sent to a max-pooling layer which finds the maximum values from the outputs of the first layer, pooled in a $2 \times 2$ window. The max-pooled activations are then sent onto a second convolutional layer with 20 nodes and, again, a 5-pixel kernel. The output from the second convolutional layer is fed into a fully connected layer with ten nodes which feeds into the final single node. The final node stores the final output value of the pixel in the ‘perfect’ image.

The CNN only produces ‘good’ values in the output image if the weights and biases for each of the edges and nodes are set appropriately. The set of weights and biases are determined by ‘training’ the CNN using a set of training data the reflects the desired output of the CNN.

The training data needed to train the CNN to denoise the electron images consists of a set of input ‘noisy’ images with a corresponding set of output ‘perfect’ images. To produce the training data 1054 of the single electron spots in the first 50 images in the series were tagged using a web app. The app, tagger.j2, is a Flask server [131] which uses the jinja2 templater [132] to convert an HTML template into a web-page which presents each image in the series to the user, one at a time. The user can then use the cursor to select regions of the image containing a single electron to ‘tag’ them.

An example of an image in the process of being ‘tagged’ is shown in figure 5.2.4. The locations of each of the tagged regions and which image they belong to are saved to a sqlite3 database. Once $\sim 1000$ electrons were tagged, a Python script fitted a 2D Gaussian to each of the tagged regions. The set of fitted Gaussians from each image were then recombined into a single ‘perfect’ image. An example is shown in figure 5.2.5.

The noisy and perfect images together make a set of training data to train the CNN. The CNN is trained using a quasi-Newton stochastic gradient descent (SGD) called ADAM [133]. Essentially, an initial set of weights and biases for the network are chosen using a Xavier initialisation routine [134], and then a randomly selected subset of the training data is passed through the network. The sums of the squares of the differences between the ‘perfect’ pixel values and the output of the network over the selected subset of the training data make a loss function that can be used to judge the performance of that set of weights and biases. The gradient of the loss function, $\nabla C$, at the initial point is measured, and a step is taken in the direction of $-\nabla C$. After many iterations of the
**Figure 5.2.4** – Example of an image (labelled ‘quadrant/25’ in the data set) in the process of being tagged using the JavaScript app. The dark regions are electrons that have already been tagged.
Figure 5.2.5 – The original image and the image created by fitting a 2D Gaussian to each of the tagged regions, with the residual noise.

...gradient descent algorithm, a local minima of the loss function is reliably reached [133], resulting in a neural network which can predict the pixel values in the perfect output images from the pixel values in the input images.

It is important to note that since only a randomly selected subset of the training data is used to approximate the gradient of the loss function in each step of the SGD algorithm, the approach to a local minima of the loss function is stochastic. We chose a SGD algorithm over a direct gradient descent algorithm, which would use the entire set of training data, because the SGD algorithm is computationally much faster. The stochastic nature of the SDG algorithm also reduces the chances of the descent being trapped in a basin with a local minima far above that of the global minima of the loss function. The network is implemented using PyTorch [135].

The trained neural network then generated a full set of 'perfect' images based on the full set of noisy input images. Examples of the input and corresponding output images along with the residual noise are shown in figure 5.2.6. Every single electron in the output images can be easily separated from one another by a threshold function as the noise in the images is so low. The total intensity due to a single electron can then be calculated and a histogram built from those intensities to give the intensity distribution of single electrons, shown in figure 5.2.7.

Comparing the single electron intensity distributions produced by the threshold and neural network approaches shows how using the neural network to remove any overlapping electrons, and a large degree of the noise from the images, results in a more accurate single electron intensity distribution. Not only are more low-intensity electrons found and counted; only 40 electrons had a total intensity below ten according...
Figure 5.2.6 – A series of original images and the corresponding perfect images as calculated by the neural network. The residual noise after subtracting the perfect image from the original image is also shown. Electron signals with significant overlap with one another are ignored by the neural network as they were also ignored during the tagging process and are therefore not present in the ‘perfect’ set of images used to train the network. As they are ignored, the indistinct electrons remain in the ‘residual noise’ images.
CHAPTER 5. CHARACTERISING THE ELECTRON BEAM

Figure 5.2.7 – The single electron intensity distribution calculated by applying the same threshold method previously used on the original images to the perfect, noise-free, overlap-free images. Because of the low noise, the threshold level can be set very low meaning low-intensity electrons will contribute towards the distribution.

to the ‘cropped’ distribution, whereas 160 were found using the neural network; but the long, high intensity, tail seen in the intensity map produced from the cropped images is absent.

The mean of the intensity distribution gives a value for the intensity of a single electron of 19.8 ± 0.1 which we can use to derive the bunch charge of the electron beam from the beam cross-section images.

5.2.3 Determining the gain of the MCP

Finding the single electron intensity allows the gain of the MCP to be determined from the ratio of charge in the incident electron bunch, determined from the bunch cross-section images, to the total charge produced by the MCP, as determined from the integral of the current passing through the MCP.

The gain of the MCP was determined in two stages. First, the MCP was used to amplify a series of ‘high charge’ electron bunches which were then imaged by the phosphor screen and camera. Plotting the total image intensity (after background removal) against the integral of the current that flowed out of the MCP and through a 50 Ω resistor for a range of bunch charges reveals a linear relationship between image
(a) Comparing charge from the MCP to the sum of the pixel values recorded by the camera for 100 electron bunches at varying bunch charges shows the linear relationship between the two values.

(b) The bunch charge as determined from the total image intensity as a function of the amplified bunch charge measured by the MCP.

Figure 5.2.8 – The gain of the MCP can be determined from the relationship between the single electron intensity distribution, the total image intensity, and the bunch charge after being amplified by the MCP.

Dividing the total image intensity of an image of an electron bunch by the mean single electron intensity gives a value for the bunch charge of the bunch that made the image. Plotting the total amplified charge from the MCP against the intensity derived figure for the bunch charge gives a value for the gain of the MCP of $9.15 \pm 0.11 \times 10^3$, which is consistent with the figure from the literature of $\sim 1 \times 10^4$ [123]. The consistency of the gain calculation with the literature figure reinforces the efficacy of using the CNN technique to determine the single electron intensity distribution. Along with the bunch charge, both the temporal length of the electron bunch and the beam energy are required to calculate the brightness of the electron beam.

5.3 Beam energy

The beam energy is set by the position along the z-axis at which the electrons are born, $z_0$, and the applied extraction voltage, $V_{\text{ext.}}$. $z_0$ was determined by measuring the time of flight (TOF) of the electrons for different extraction voltages and comparing the measurements with a time of flight map calculated using particle tracking simulations in GPT.
5.3.1 Time of flight measurement

To measure the time of flight the time difference between the ionisation of the electrons and the time at which the electrons hit the MCP must be measured. To measure the ionisation time a reversed biased FCI-125G-010HRL photodiode was placed in the path of the ionisation laser beam behind a sheet of white paper. Voltage pulses from the photodiode caused by the ionisation laser pulse were measured - on a 50Ω terminated line - by a Rigol DS1104B digital oscilloscope. A second channel on the scope was linked to the grounded front plate of the MCP using a second 50Ω terminated line. When the electron bunches hit the MCP, they are amplified by the MCP. The MCP recharges through the 50Ω terminated line and a voltage pulse is measured on the oscilloscope giving the time at which the electrons hit the MCP. An example pulse as recorded by the oscilloscope is shown in figure 5.3.1.

The oscilloscope was triggered by the pulse from the photodiode. It then recorded the arrival of the pulse from the MCP. Both data series were transmitted over USB and recorded on a computer.

For each shot, two 1D Gaussians were fitted to the pulse from the photodiode and the pulse from the MCP. The differences between the peaks were calculated to give $\Delta t$. $\Delta t$, however, is not the time of flight of the electron bunch. The measurements
described above only give a time related to the time of flight of the electron bunches, $t_0$, as

$$\Delta t = t_0 (V_{\text{ext}}, z_0) + t_d,$$  \hspace{1cm} (5.3.1)

where $V_{\text{ext}}$ is the bias across the extraction region and $t_d$ is the sum of the delays to the measurement of the ionisation laser pulse and the electron pulse compared to the time the atoms in the trap are ionised and the time the electrons reach the MCP. As $t_d$ is a function of the electrical and optical set up of the measurement it does not vary with $V_{\text{ext}}$. Therefore from measurements of $\Delta t$ for a series of $V_{\text{ext}}$, $z_0$ and $t_d$ can be determined and hence the beam energy can be determined.

Deriving the beam energy directly from the length of the beamline and the $t_0$ data ignores the acceleration of the beam along the beamline. Instead, to determine the beam energy and hence the $z_0$ position of the beam the relationship between $z_0$ and $t_0$ was found and is shown in figure 5.3.2. The relationship was found in much the same way as the $A$ and $B$ coefficients were calculated: a series of particles starting at various $z_0$ positions and extraction biases, $V_{\text{ext}}$, was simulated travelling along the beamline, with the time of flight of the particles between their initial position and the MCP was recorded. The effect of varying the initial radial position of the particles, and hence changing the particle trajectory, on the simulated times of flight was less than the uncertainty on $t_d$, so the effect was ignored. The resultant $t_0 (V_{\text{ext}}, z_0)$ map was used to fit equation 5.3.1 to the collected $\Delta t$ data.

Figure 5.3.3 shows the results of the fit. The value for $z_0 = 148 \pm 80 \mu m$ is consistent with the $z_0$ position as measured by the MOT cameras. From the $z_0$ measurement the beam energy as a function of the extraction bias is $0.47 \pm 0.01 \text{ eV V}^{-1}$.

An estimate of the temporal shape of the bunch can also be taken from the length of the electron pulse measured by the MCP. The length of the pulse is plotted against the beam energy in figure 5.3.4. As the measured pulse is a result of the current ‘recharging’ the glass of the MCP after it is depleted by the amplification of the incident electron pulse, the length of the measured pulse gives an upper-limit for the temporal length of the electron bunch. The bunches were ionised to energies well above the ionisation potential, so electron-ion interactions which might affect the bunch length - such as auto-ionising Stark states - were minimised.

The electron pulse data suggests an upper bunch temporal length of $2.95 \pm 0.02 \text{ ns}$ for a bunch with energy 1200 eV. Along with the measurements of the beam energy, bunch charge, and bunch temporal shape, measurements of the source size and the source temperature were required to derive the brightness of the beam.
Figure 5.3.2 – The relationship between the extraction bias, z position, and the final beam energy (top). The time of flight of the electrons between ionisation and the MCP as a function of the source’s z position and the extraction bias (bottom). Both maps were calculated from simulations carried out in GPT.
Figure 5.3.3 – The time of flight of an electron beam as a function of the extraction bias. Each point is the mean of 11 TOF measurements, with the uncertainty on the measurements negligible at this scale.

Figure 5.3.4 – The pulse length of the electron signal as a function of the beam energy.
CHAPTER 5. CHARACTERISING THE ELECTRON BEAM

5.4 Electron temperature and beam emittance

Ensuring accurate temperatures can be recovered from the images of the electron beam requires a full simulation of the experimental process, with cross-checks at each stage. The simulations can also be used to determine the ideal settings for the beamline biases. The test consists of producing a set of simulated electron beam cross-section images from simulation data which together make up a waist scan. The widths of the cross-sections are then calculated, and the series of widths are used to fit for the temperature and source size. The simulated cross sections can also be used as an opportunity to test the method of measuring the bunch charge.

First, however, the ideal conditions for the beamline biases can be determined by calculating the $A$ and $B$ coefficients, defined in equation 4.1.2, of the beamline. To calculate the $A$ and $B$ coefficients the 2D electrostatic field maps calculated using Superfish and the magneto-static field maps calculated using the code described in section 4.3 were imported into GPT. GPT was then used to calculate the $A$ and $B$ coefficients for the beamline at a range of Einzel lens and extraction biases with a constant correction lens bias of 4880 V. The ideal set of biases are those that allow a waist scan to be carried out whilst maximising both the beam energy and the relative size of the $B$ coefficients compared to the $A$ coefficients. High beam energy will result in reduced space charge effects while the $A:B$ ratio informs the accuracy of the electron temperature measurement. Figure 5.4.1 shows the resultant maps for the $A$ and $B$ coefficients.

In general, the $A$ coefficients are larger than the corresponding $B$ coefficients, and at the upper and lower end of the range of extraction biases, varying the Einzel lens over the available range does not bring the beam waist into the plane of the MCP. Based on the $A$ and $B$ maps an extraction bias of 2650 V was chosen as a good compromise of beam energy and $A:B$ ratio.

To perform the simulated experimental runs, first, a series of 1000-electron electron bunches, with a range of source sizes and temperatures, were created around $z_0 = 0$ and allowed to propagate along the beamline for different Einzel lens and extraction biases. The $x$ and $y$ positions of the electron bunches was recorded at $z_{\text{MCP}} = 0.492$ m, the location of the MCP on the beamline.

From the $x$ and $y$ final position data, a series of images was created by assigning a Gaussian spot to each electron. The Gaussians took their range of widths and amplitudes from the distributions found in the charge calibration analysis presented in section 5.2.3. Gaussian noise was also added to the images, scaled to reflect the residual noise found in
Figure 5.4.1 – The $A$ and $B$ coefficients for a correction lens bias of 4880 V and a range of extraction and Einzel lens biases.
CHAPTER 5. CHARACTERISING THE ELECTRON BEAM 165

Figure 5.4.2 – The width of the signal due to single electrons being imaged by the MCP-phosphor-camera plotted as a function of their amplitude. The minimal correlation means any relationship between the amplitude and width can be ignored for the purposes of this work.

The charge calibration images along with a non-uniform background offset that reflected that found in the real images.

The distributions used to produce the range of widths, amplitudes and backgrounds are shown in figure 5.4.3. The correlation between the amplitude and width of single electron spots is shown in figure 5.4.2. As the Pearson correlation coefficient, which describes the correlation between two variables with a value of ±1 describing directly proportional variables and a value of 0 describing uncorrelated variables, was -0.0953 the correlation was ignored.

Figure 5.4.4 shows the final locations of the electrons from the GPT simulations and the image created from the electron locations.

The code saved the series of images as ASCII-array text files along with a metadata file which included the parameters which created each of the beam cross sections along with the statistical width of the underlying simulated electron distribution, as extracted directly from GPT. The ASCII-array text files and associated meta-data file are similar to the output of the real experiment, with the additional benefit that any figure derived from the data can be compared to the underlying ‘real’ values. Using the simulated data, an efficient method of calculating the Gaussian widths of the beam cross sections was developed.
CHAPTER 5. CHARACTERISING THE ELECTRON BEAM

Figure 5.4.3 – The distributions from which the range of amplitudes and widths of the Gaussians assigned to each electron are drawn, along with the distribution of background noise added to the images. 1D splines were fit to each distribution (orange).

5.4.1 Calculating the cross section Gaussian width

The ASCII-files were read into a NumPy array [136], and the non-uniform background offset was removed using the associated background file. The total charge in the image was then calculated according to the model described in section 5.2.3. The correlation between the calculated charges and the ‘real’ charges is shown in figure 5.4.5.

Each image was normalised to the highest pixel value in the image, and an initial estimate of the parameters of the Gaussian was made. As the image will be cropped based on this initial estimate a Gaussian filter was first applied to the image, ensuring that no cross sections are lost due to irregularities in the image. The amplitude was estimated as unity, since the image had been normalised to the largest pixel value. The centroid of the cross section was then estimated by applying a threshold filter to the image, with a threshold value of 0.5, and finding the centre of mass of the filtered image. From the centroid, $x_{ig}$, $y_{ig}$, the rotation and width of the Gaussian was determined by summing the pixel values along the lines

\begin{align}
  x &= \cos(\theta) + x_{ig} \\
  y &= \sin(\theta) + y_{ig}
\end{align}

(5.4.1)

(5.4.2)
Figure 5.4.4 – (Top left) The $xy$ position of each of the electrons in a simulated bunch when they reach the MCP is shown, with the image that electron bunch would produce (top right). (Bottom) A histogram of the $x$ positions for electrons with $|y| < 100 \mu m$ (blue) plotted along with the cross section (orange) of the image shown above.
Figure 5.4.5 – The electron counts derived from the simulated images as a function of the number of electrons used to produce the simulated image. The linear fit between the data, which has a gradient of $1.02 \pm 0.01$, is shown in orange.

for $0 < \theta < 2\pi$. A sine function was fitted to the summed pixel values plotted against $\theta$ and the fitted phase offset, $\phi_{ig}$, used as the estimate of the rotation of the 2D Gaussian. Figure 5.4.6 shows a typical example.

Estimates for the Gaussian’s major and minor axes, $\sigma_{maj}$ and $\sigma_{min}$ were determined by substituting $\phi_{ig}$ and $\phi_{ig} + \pi/2$ for $\theta$ in equation 5.4.2. The number of pixels along those lines was counted to give a value of the FWHM $\approx 2.355\sigma_{maj, min}$, respectively.

The images are then cropped at $2\sigma_{maj}$ each side of the centroid. The initial parameter estimation algorithm is then run on the cropped image, but no Gaussian filter is applied.

The initial estimates are used to create a set of upper and lower bounds for each parameter, which are altogether used to start a least-squares fit using a Trust Region Reflective algorithm [137], implemented by the Python library `scipy.optimize`. The fitted parameters are then used to plot a series of cross sections and 2D heat-map/contour images, typical examples of which are shown in figure 5.4.7.

The width of the fitted Gaussians, the standard deviation of the electron distribution used to produce the images, and the width as predicted by the $A$ and $B$ coefficients all show good agreement with one-another. As can be seen in figure 5.4.8 the Gaussian fit widths are closely related to the standard deviations with a gradient and the width as predicted by the $A$ and $B$ coefficients.
5.4.2 Low density electron bunches

For low-density electron beams, however, the fitting algorithm is far worse at recovering the width of the underlying distribution. Examples of Gaussians fitted to beam cross sections at \( z_{\text{MCP}} \) for a source of fixed size, \( \sigma = 250 \mu\text{m} \), temperature, \( T = 10 \text{K} \), and beamline biases \( V_{\text{ext}} = 2700 \text{ V} \), \( V_{\text{correction}} = 5000 \text{ V} \), \( V_{\text{Einzel}} = 1000 \text{ V} \) are shown in figure 5.4.9. At low bunch charges, the difference between the fitted Gaussian’s width and the width of the underlying distribution gets larger as the number of electrons in the bunches undersamples the underlying distribution.

By applying a Gaussian filter to the cross sections a better estimate of the width of the underlying distribution, \( \sigma_{\text{under}} \), can be made as the filter blurs out arbitrary subsections of the image created by stochastic variation in the electron density which can confuse the fitting algorithm. The filtered image is a convolution of the underlying distribution and a Gaussian with a given width, \( \sigma_{\text{filter}} \) and as the underlying distribution is assumed to be Gaussian the width of the Gaussian function fitted to the filtered image will be [138]

\[
\sigma_{\text{under} \otimes \text{filter}} = \sqrt{\sigma_{\text{under}}^2 + \sigma_{\text{filter}}^2}. \tag{5.4.3}
\]
CHAPTER 5. CHARACTERISING THE ELECTRON BEAM

Figure 5.4.7 – A typical example of the result of fitting a 2D Gaussian to the electron cross section. The fitted Gaussian is shown in orange in each plot. The histograms on the left (2D at the top, then histograms along the major and minor axes in blue below) show the underlying electron distribution as output by GPT. The image (top right) shows the simulated result of that electron distribution after it had been amplified and imaged, with cross sections along the major and minor axes shown below in green.
CHAPTER 5. CHARACTERISING THE ELECTRON BEAM

Figure 5.4.8 – The standard deviation of the underlying distribution (blue), the width of the Gaussian fits (orange), and the width predicted by the $A$ and $B$ coefficients (green) are shown as a function of the Einzel lens bias across the range of a waist scan.

In figure 5.4.10 the widths of the underlying distributions recovered from the filtered images using equation 5.4.3 are shown as a function of bunch charge.

To determine whether an image needs to go through the additional process after the initial threshold filter is applied, the algorithm counts the number of distinct regions above the filter value. The object count above which the additional filter step was included was varied, and a maximum distinct object count of five was found to catch the bulk of the cross sections which would otherwise confound the fitting algorithm.

5.4.3 Source size measurement

While a waist scan can be used to determine both the source size and source temperature, the source size can be measured independently and used to collaborate the waist scan measurements by determining the size of the excitation and ionisation beams at the source.

In each experimental cycle, the trapping beams are extinguished, and the atoms are allowed to decay from the excited state before either the excitation or ionisation beams are introduced. Therefore only atoms that are in the path of the excitation beam will be in the excited state when the ionisation beam is introduced. As discussed in 2.3.1, the only significant source of free electrons are atoms that are ionised from the excited
Figure 5.4.9 – The simulated image and the Gaussian fit to that image for a range of electron counts (shown above each image). At lower electron counts the Gaussian fit becomes an unreliable and inaccurate method of measuring the underlying distribution.
CHAPTER 5. CHARACTERISING THE ELECTRON BEAM

Figure 5.4.10 – The Gaussian width of fits to un-blurred images (blue) and fits to images with an additional Gaussian filter applied (orange) as a function of electron count for an electron bunch of width 238.5 µm (shown as the line in red). At low electron counts, the images with an additional Gaussian filter applied give a more accurate measure of the width of the underlying distribution.

state, so the spatial extent of the electron source is defined by the overlap between the ionisation beam, the excitation beam and the trap’s atomic population.

To measure the size of the excitation beam the power in the beam was reduced, and a CCD camera was placed perpendicular to the beam, in the plane of the centre of the atom trap. The cross section of the beam was recorded at that position, scaled, and fitted with a 2D Gaussian to give a beam size of \( \sigma_{\text{exc}} = 260 \pm 40 \text{ µm} \).

The size of the ionisation beam was measured by recording an image of the beam hitting a sheet of white paper placed in the plane of the atom trap. The image was recorded using a uEye camera fitted with a macro lens and an ND100 filter, which reduced the laser intensity by a factor of 100, to protect the CCD in the camera from the intense laser light. A series of 100 images were taken, summed, and a 1D Gaussian was fitted to cross sections of the laser beam taken at the level of the trap. From these 1D Gaussian, the beam size was measured as \( \sigma_{\text{ion}} = 370 \pm 30 \text{ µm} \).

The spatial extent of the trapped atom population was recorded over the duration of every experiment run using the process described in section 2.2.5. The mean of the spatial extent was found to be \( \sigma_{\text{trap}} = 1200 \pm 230 \text{ µm} \). The large uncertainty reflects the variation in the trap size between experimental runs. The variation necessitates an
alternate method to set the source size which is provided by the overlap of the excitation and ionisation region. Despite the large variation, the trap was always significantly larger than the excitation or ionisation beam.

5.4.4 Electron temperature measurements

The experimental electron temperature measurements were carried out in the same way as the simulated temperature measurements:

- The $A$ and $B$ coefficients of the beamline were calculated.
- A set of beam cross-sections were recorded for a range of Einzel lens biases.
- The cross sections were fit with 2D Gaussians using the algorithm described in section 5.4.1.
- The 2D Gaussian widths are fitted with

$$\sigma_f = A\sigma_i + B\sigma'_i$$

(5.4.4)

using the calculated $A$ and $B$ coefficients, where $\sigma'_i = \sqrt{\frac{kT}{2\pi U}}$ is the electron temperature $T$, as a function of the beam energy, $U$, and $\sigma_i$ is the source size.

As the electron temperature measurements would be confounded by space charge expansion, the impact of increasing bunch charge on the waist scans had to be determined and a bunch charge found under which the effects of space charge expansion were negligible.

5.4.5 Space charge expansion

The first waist scans performed all used the same ionisation wavelength, 483.53 nm, but with different excitation laser intensities. Varying the excitation laser intensity varied the number of atoms in the excited state. A higher excited state population, for the same intensity of ionisation laser wavelength and intensity, results in a higher charge electron bunch.

In figure 5.4.11 the Gaussian width of the cross sections is plotted against the Einzel lens bias for bunch charges of between 1.2 fC to 2.7 fC. The size of the cross-section remains similar across the range of bunch charges so for bunches of fewer than
Figure 5.4.11 – A series of waist scans, with the fitted temperature and source sizes displayed in the legend, performed with different excitation beam powers and hence different bunch charges. As the size of the beam at the MCP is not affected by the increasing bunch charge the space charge forces must be negligible for similar energy beams (1350 keV) with bunch charges below $\sim 2.7 \text{ fC}$. 
∼ 17000 electrons - bunch charges of less than ∼2.7 fC - the space charge expansion was negligible.

With a limit on the bunch charge below which the temperature measurements would not be affected by space charge expansion, we could perform a series of waist scans of the electron beam that would not be impacted by space charge by restricting the bunch charge to below that threshold.

The waist scans used a range of ionisation beam wavelengths from 478.5 nm to 483.5 nm for both the AC and DC-MOT. The ionisation beam wavelength was measured using a spectrometer as described in section 2.3.2. The energy in the ionisation pulses was kept constant at 1.44 ± 0.20 mJ. The ionisation pulse length FWHM was 3.81 ns, measured as described in section 2.3.2.

The MOT population for each experiment were set to ∼(12.8 ± 1.2) × 10^6 by adjusting the trapping beams, and was measured using the procedure described in section 2.2.6. The MOT population, however, was not a critical factor since the relevant parameter for the beam characterisation was the bunch charge, which was set by adjusting the excitation laser power.

The experimental cycle proceeded as described in section 2.3.4.

5.4.6 AC-MOT temperature measurements

For the AC-MOT case, the A and B coefficients for the beamline were calculated and are shown in figure 5.4.13. The biases used were \( V_{\text{ext}} = -2635.4 \) V and \( V_c = 4880.7 \) V, as proposed in section 5.4. The \( z \) position of the source was \( z = 0.52 ± 0.12 \) mm, hence the beam energy was 1196 ± 26 eV, as confirmed by TOF measurements shown in figure 5.4.12.

The set of cross sections were recorded for a waist scan of \( V_e = 1000 \) V to 4750 V, with example cross sections shown, along with the 2D Gaussian fits shown in figure 5.4.15.

The widths of the fitted 2D Gaussians were plotted as a function of the Einzel lens bias and are displayed in figure 5.4.14, with equation 5.4.4 fitted to the widths as described in 5.4.1. The source sizes derived from the waist scans are consistent with those predicted from the ionisation and excitation beam cross sections measured in section 5.4.3.

The uncertainties on the temperatures and source sizes were determined from the trace of the covariance matrix of the fit of equation 5.4.4 to the waist scans combined with the uncertainty on the beam energy. Hence the final uncertainties in the temperature
and source size measurements result from uncertainty in the source’s position along the beamline, inaccuracies in the beamline model, and any non-linearities in the beamline which undermine the linear transfer model used to derive the source parameters from the final beam size.

A calibrated electron source could be used to cross-check the beamline model and account for any inaccuracies or non-linearities. A more effective and less experimentally complicated approach would be to measure the beam emittance instead using a pepper-pot, technique as described at the beginning of chapter 4, which does not require a strictly linear beamline.

Decreasing the ionisation laser wavelength, and hence increasing the initial electron energy, resulted in a higher temperature and therefore higher emittance beam, consistent with the results from other existing CAESs.

The minimum temperature recovered from the waist scans was $12.6 \pm 0.8$ K for a source size of $176 \pm 10 \mu$m and hence an rms emittance of $8.41 \pm 0.10$ nm rad. The maximum derived coherence length of a beam produced by the source is therefore $8.38 \pm 0.53$ nm, calculated using equation 1.2.19. The brightness of the AC-MOT CAES beam was calculated as $0.19 \pm 0.04$ nA nm$^{-2}$ rad$^{-2}$ using equation 1.2.15. The uncertainties in the temperature, source size, bunch charge, and bunch length were propagated to the uncertainties in the emittance, coherence length, and brightness.
Figure 5.4.13 – For the AC-MOT case these are the calculated $A$ and $B$ coefficients as a function of the Einzel lens bias for an initial $z$ position of $z_0 = 0$. Also shown is a typical waist scan using the calculated parameters for a source of $\sigma = 170 \, \mu m$, and temperatures 25 K (blue), 90 K (orange), and 125 K (green).
Figure 5.4.14 – A series of waist scans at various ionisation energies using the AC-MOT, with the fitted source size and temperature for each ionisation energy displayed in the legend. From the fits, $z_0 = 51 \pm 12 \, \mu m$, $\bar{\sigma} = 182.4 \pm 3.8 \, \mu m$. The maximum recorded bunch charge was $2.3 \pm 0.4 \, fC$. 

<table>
<thead>
<tr>
<th>Energy</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>478.5 nm</td>
<td>82.1 ± 6.2 K</td>
<td>185.0 ± 7.9 µm</td>
</tr>
<tr>
<td>479.5 nm</td>
<td>52.7 ± 1.5 K</td>
<td>185.0 ± 2.6 µm</td>
</tr>
<tr>
<td>480.5 nm</td>
<td>31.1 ± 1.3 K</td>
<td>185.0 ± 2.8 µm</td>
</tr>
<tr>
<td>481.5 nm</td>
<td>25.4 ± 0.9 K</td>
<td>185.0 ± 2.9 µm</td>
</tr>
<tr>
<td>482.5 nm</td>
<td>20.6 ± 0.7 K</td>
<td>178.0 ± 2.5 µm</td>
</tr>
<tr>
<td>483.5 nm</td>
<td>12.6 ± 0.8 K</td>
<td>176.2 ± 10.0 µm</td>
</tr>
</tbody>
</table>
Figure 5.4.15 – A set of typical 2D Gaussian fits (orange) to the electron beam cross-section at a range of Einzel lens biases (shown above each image) for the AC-MOT case. The cross sections were recorded for an ionisation beam wavelength of $478.45 \pm 0.20 \text{ nm}$.
5.4.7 DC-MOT temperature measurements

The same steps were repeated, with the same beamline biases, using the DC-MOT instead of the AC-MOT. The $A$ and $B$ coefficients for the DC-MOT case are shown in figure 5.4.16, a sample of the cross sections and the associated 2D Gaussian fits are shown in figure 5.4.17, and the final temperature measurements for the DC case are shown in figure 5.4.18. Figure 5.4.18 shows two separate data series to cover the range of ionisation beam wavelengths. The two series have different source sizes due to a change in the overlap of the excitation and ionisation beams between experimental runs resulting in the ionisation region changing size. As described in section 5.4.3, the ionisation region size determines the source size. As with the AC source, the position of the DC source for both data series was cross-checked using TOF measurements.

A similar set of results was ultimately found in the DC case as in the AC case as the electron beam’s parameters are within the limits that keep the beam inside the linear portion of the beamline, as defined in section 4.3.1.

The minimum temperature recovered from the waist scans in the DC case was $17.4 \pm 1.1$ K for a source size of $238 \pm 18 \mu$m. Hence the minimum rms emittance of a beam from a DC-MOT was $13.2 \pm 1.2$ nmrad. The other beam parameters can be calculated for the DC-MOT CAES as they were for the AC-MOT CAES: the maximum coherence length found from the source is $7.13 \pm 0.53$ nm and the brightness of the beam from the DC-MOT was $0.09 \pm 0.03$ nAm$^{-2}$rad$^{-2}$.

5.5 Source parameters, AC vs DC

A direct comparison of the electron temperatures from the AC and DC CAESs as a function of the excess ionisation energy is shown in figure 5.5.1, along with the electron temperature due to the excess ionisation energy model described in section 3.1. The model follows the experimental data with a $14 \pm 3$ K offset. The offset is a result of the other heating mechanisms discussed in chapter 3; disorder-induced heating and three-body recombination.

Electron bunches were also produced with excess energies below the Stark-shifted ionisation threshold. As the bandwidth of the ionisation laser leads to an energy spread of only $0.8$ meV, the bunches produced from ionisation to below the Stark shifted ionisation threshold can only be the result of Rydberg-Rydberg interactions leading to ionisation. Despite the fact the electrons in these bunches were created with minimal excess energy, the bunches had a non-zero temperature due to the additional heating
Figure 5.4.16 – For the DC-MOT case these are the calculated $A$ and $B$ coefficients as a function of the Einzel lens bias for an initial $z$ position of $z_0 = 0$. Also shown is a typical waist scan using the calculated parameters for a source of $\sigma = 170 \, \mu m$, and temperatures 25 K (blue), 90 K (orange), and 125 K (green)
Figure 5.4.17 – A set of typical 2D Gaussian fits (orange) to the beam cross-section at a range of Einzel lens biases (shown above each image) for the DC-MOT case. The cross sections were recorded for an ionisation beam wavelength of $481.53 \pm 0.20$ nm
Figure 5.4.18 – A series of waist scans at various ionisation energies using the DC-MOT, with the fitted source size and temperature for each ionisation energy displayed in the legend. Two data series are plotted together for completeness. The first consists of the waist scans at 478.5 nm, 482.5 nm, and 484.5 nm with $z_0=3.6 \pm 1.3 \mu m$, $\sigma=238 \pm 54 \mu m$. The second consists of scans at 479.5 nm, and 480.5 nm with $z_0=0 \pm 15 \mu m$, $\sigma=315 \pm 25 \mu m$. The maximum recorded bunch charge was $2.5 \pm 0.4 fC$. 

- 478.5 nm, 91.7 $\pm$ 1.6 K, 241.9 $\pm$ 6.2 $\mu m$,
- 479.5 nm, 46.4 $\pm$ 5.6 K, 306.6 $\pm$ 21.6 $\mu m$,
- 480.5 nm, 23.6 $\pm$ 1.9 K, 323.5 $\pm$ 29.2 $\mu m$,
- 482.5 nm, 16.8 $\pm$ 6.8 K, 230.0 $\pm$ 42.0 $\mu m$,
- 484.5 nm, 17.4 $\pm$ 1.1 K, 244.7 $\pm$ 17.5 $\mu m$, 

![Graph showing beam size at MCP vs. Einzel lens bias (V)](image-url)
effects of DIH and 3BR, discussed in chapter 3.

The lack of a clear trend in the difference in temperature between the two cases suggests that at \( \sim 1 \text{ keV} \) the benefits from the ability to quickly zero the magnetic fields is limited. Equally cold electrons can be extracted from the DC-MOT with the trapping fields switched on as from the AC-MOT with the trapping fields switched off. The higher minimum temperature recovered in the DC case is likely a result of stochastic variations in the experimental process such as the MOT density distribution and the alignment of the excitation and ionisation beams.

The various source parameters for the AC and DC-MOT CAESs are collated in table 5.5.1 along with the source parameters of CAESs from other groups. The table shows that the source presented in this thesis is capable of producing electron bunches at the ultra-low temperatures exhibited by other existing CAESs, with the temperature similarly limited by the effects of disorder-induced heating and three body recombination [65, 66].

The comparatively high emittance of the beams presented in this thesis is ultimately due to the relatively large source size. Reducing the source size would require the laser beams used to set the ionisation volume to be focused more tightly than they were for in these experiments. As the size of the beam focus is dependent on the focal length of the lenses used, bringing the beam to a very small focal spot size would require
Table 5.5.1 – A table comparing the beam characteristics of the AC and DC-MOT CAESs built in Manchester to other CAESs.

<table>
<thead>
<tr>
<th>Source</th>
<th>$T$ (K)</th>
<th>$\sigma_i$ (µm)</th>
<th>$Q$ (fC)</th>
<th>$\varepsilon_{rms}$ (nm rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC-MOT</td>
<td>12.6</td>
<td>182</td>
<td>2.2</td>
<td>8.41</td>
</tr>
<tr>
<td>DC-MOT</td>
<td>17.4</td>
<td>238</td>
<td>2.5</td>
<td>12.9</td>
</tr>
<tr>
<td>[65]</td>
<td>10</td>
<td>32</td>
<td>0.016</td>
<td>1.3</td>
</tr>
<tr>
<td>[66]</td>
<td>10</td>
<td>425</td>
<td>80</td>
<td>141</td>
</tr>
</tbody>
</table>

lenses mounted as close as possible to the atom trap itself. The current lenses are placed immediately outside the chamber, so smaller focal spot sizes would necessarily require lenses to be placed inside the vacuum chamber. The reduced source size would, however, reduce the maximum possible bunch charge due to the smaller atomic population in the smaller ionisation region. The threshold above which space charge expansion becomes relevant to the expansion of the beam would also be reduced due to the increased electron density for equivalent bunch charges.

Reducing the source size would reduce the emittance and make the AC-MOT source suitable to perform electron diffraction studies due to its long coherence length and high brightness originating from the source’s low electron temperature. With a reduced source size electron diffraction studies similar to those described in section 1.4 could be carried out using the AC-MOT CAES.

From the table it is clear that whilst there may be some small benefit at beam energies of $\sim 1$ keV from the ability to zero the trapping fields, these experiments and other CAESs which use DC-MOT sources show the emittance growth due to the trapping fields to be negligible compared to the other mechanisms of temperature increase described in chapter 3. The limited impact of the trapping fields originates from both their relative weakness, with field strengths of only $\sim 25$ G/mm, and their linearity with position over the volume of the beamline the electron beam occupies. At higher beam energies the benefit of zeroing the trapping fields before extraction would likely be reduced further.

As suggested by the simulation results presented in section 4.3.1, however, for low-temperature low energy electron beams there should be an advantage to zeroing the trapping fields before extraction.
Figure 5.6.1 – A waist scan of a 200 eV beam produced using an AC-MOT.

5.6 Low energy cold electrons from an AC-MOT

Low energy, monochromatic electron beams are used in techniques such as low energy electron diffraction imaging [139, 140], low energy electron transmission imaging [30], energy loss spectroscopy [141], and electron stimulated desorption [142]. Experiments we conducted with lower energy electrons (196 ± 2 eV) show a significant reduction in the source temperature in the AC-MOT case compared to the DC-MOT case, suggesting that the AC-MOT CAES could find applications as a low energy, monochromatic electron source.

The experiments consisted of a pair of waist scans, one using an AC-MOT, shown in figure 5.6.1, and the other a DC-MOT shown in figure 5.6.2. The temperature of the beam from the AC-MOT was 17.8 ± 0.1 K compared to 78 ± 21 K for the beam from the DC-MOT. The large uncertainty on the temperature from the DC-MOT, and the large difference in the fitted source size between the AC and DC cases likely reflect the breakdown of the linear transfer model used by the waist scan measurement in the DC case. Such a breakdown suggests that non-linear trapping fields are distorting the electron beam in the DC case.

Both beams were produced using a 483.5 ± 0.1 nm ionisation beam and had average bunch charges of 0.75 ± 0.02 fC in the AC case and 0.65 ± 0.02 fC in the DC case. Whilst the bunch charges are below the level at which space charge forces were
negligible for 1.3 keV beams, as demonstrated in 3.4, the impacts of the bunch charge on low energy bunches were not characterised experimentally.

Below 200 eV the beam produced using the DC-MOT became too astigmatic to reliably extract its emittance using a waist scan. Whilst lower energy beams were produced, with electron beams at 11 eV transported from the AC source to the MCP, they were not investigated in any detail and are therefore not presented here.

Whilst the spread in the energy of the electrons in the bunch due to the temperature of the source is minimal, ∼5 meV, along with the thermal energy spread the beam has an additional energy spread dependent on the extent of the source along the beam axis, $z$, and the extraction field strength.

With the centre of the source at $z = 0$, and the extraction field applied such that the electrons are extracted in the positive $z$-direction, electrons born at $z < 0$ start higher in the potential, with more potential energy, than those born at $z > 0$. After being accelerated by the extraction field the electrons will end up with an energy spread dependent on the extent of the bunch in $z$, $\sigma_z$, and the strength of the extraction field:

$$\Delta U = \sigma_z E_{\text{ext}} e,$$  \hspace{1cm} (5.6.1)

where $e$ is the charge of the electron, as discussed in section 3.1. For the beam presented in figures 5.6.1 and 5.6.2, the energy spread due to the spatial extent of the source would
be $\sim 7$ eV, or $\sim 3.5\%$, assuming the source is approximately spherical. Such an energy spread would exclude the source from any low energy, monochromatic applications. The only approach that could reduce the relative energy spread would be to reduce the spatial extent of the source along the $z$ axis by using a flat, ribbon-like, extraction or ionisation beam. A ribbon-like beam maximises the population in the ionisation region, and hence the bunch charge, whilst minimising the $z$ extent of the source. Such a beam can be produced using a cylindrical beam telescope to compress the beam along one transverse axis, whilst leaving the other transverse axis unaltered.

In comparison, state of the art monochromatic electron sources have energy resolutions on the order of 100 meV [143], with novel technologies producing electrons with energy spreads of 12.5 meV [144]. Monochromators, which use perpendicular electrostatic and magnetic fields along with a series of apertures to precisely select electrons by their energy, can also be used to reduce the energy spread of electron beams at the expense of beam current. Monochromators have been used to produce beams with energy spreads of 50 meV, though the beam current was reduced to 0.2 nA [34].

Moufarez et al. [145] have suggested alternative ways to use atom cooling and trapping technology to produce monochromatic electrons. In their proposal a 2D MOT arrangement is used to reduce the transverse momentum spread of an atomic beam. The atoms in the beam are then excited into a field ionising Rydberg state before entering a region with a shaped electric field. The electric field is shaped such that the field-ionising states chosen ionise over only a few microns or less. Moufarez et al. have predicted energy resolutions better than 1 part in $10^5$, along with high beam currents, if a suitable field ionising state can be found.
Chapter 6

Summary

Over the course of the project, I developed a cold atom electron source based on an AC-MOT, which allows the trapping fields used in the atom trap at the heart of the CAES to be rapidly extinguished. The newly commissioned source based on that novel design has produced electron beams with characteristic temperatures as low as $12.6 \pm 0.8$ K, comparable to other state of the art CAESs [130].

Temperature measurements at beam energies lower than previously presented from a cold atom source suggest the AC-MOT design has benefits for the emittance of the electron beam under low energy regimes.

Building the source required the development and interconnected operation of several different systems, including a vacuum system, trapping optics, excitation and ionisation optics, an electron beamline, and measurement instrumentation.

To better understand the electron beam, I investigated the source using detailed particle tracking simulations and determined that the benefits derived from the AC-MOT would be most significant under a low energy regime.

I used similar particle tracking simulations to test the measurement techniques I intended to use in characterising the beam. I then characterised the beam produced by the CAES using those measurement techniques. I used the measured beam metrics to compare the novel AC-MOT CAES to an equivalent DC-MOT CAES and found that whilst above 1 keV the benefit of the AC-MOT is limited, below 200 eV the benefits of extinguishing the trapping fields before extraction could be significant.

The AC-MOT CAES could, therefore, supply the high quality, low energy electron beams used in low energy electron diffraction and low energy electron spectroscopy experiments, if further study reinforced the findings presented in section 5.6.

The remainder of this summary chapter reiterates the principal points of each of the
previous chapters and suggests some areas of future study that would be appropriate given the results of the project.

6.1 Introduction

The first chapter introduced the various beam metrics and demonstrated the dependence of the beam metrics on the source parameters; the source temperature, the source size, the bunch charge, and the temporal shape of the bunch.

An exploration of the various existing methods of producing high-quality electron beams was then given, discussing thermionic sources, photoinjectors, field emission sources, and cold atom sources, along with typical values of the beam metrics for each source. Finally, the current state of cold atom electron source technology was presented to contextualise this work and its position within the field.

6.2 The electron source

Chapter 2 presented the cold atom source I built. It explained how a cold atom population is accumulated in a MOT and describes the two colour photoionisation scheme the source uses to produce cold electrons from the cold atoms.

To explain the operation of the MOT, laser Doppler cooling was introduced, and an explanation was given for how the Doppler cooling effect can be supplemented by the Zeeman effect to yield a cooling and restoring force when two opposing laser beams are incident on a four-level atom. The methods used to approximate a four-level atom with rubidium-85 are described, along with the laser locking system I designed and built to frequency stabilise the trapping laser.

The chapter then describes the modifications I made to the standard MOT needed to rapidly extinguish the trapping magnetic fields, describing the AC magnetic field used instead of the DC field used in a standard MOT, and the polarisation switching optics required by the AC-MOT. The systems used to measure the position and population of the cold atom trap are also presented.

The two sets of optics I built for the two colour photoionisation scheme are described next, and the process and benefits of the two colour scheme compared to other photoionisation schemes is discussed. The chapter also presents the optical chopper which I designed and built to extinguish the trapping beams in $19.6 \pm 0.2 \mu s$, before ionisation.
Finally, the details of the experimental cycle and the equipment which generates the trigger signals for the various parts of the experiment are presented.

6.3 Electron heating mechanisms in a cold neutral plasma

Once the electrons are free of their parent atoms, the physics of cold neutral plasmas take over. Chapter 3 describes the primary mechanisms which determine the source temperature of a CAES.

The chapter starts by explaining how excess energy given to the electrons during ionisation impacts the final electron temperature and describes how electron-ion interactions impact the final electron velocity distribution which determines the electron temperature.

The process of three-body recombination (3BR), where electrons recombine with ions in the presence of a third electron, is then explained, along with a discussion of 3BR’s contribution to the final electron temperature.

The effects of space charge forces on the electron temperature are then explained. First, the details of the microscopic effect of disorder-induced heating, where the distribution of free electrons within the cold neutral plasma results in a temperature increase are given, followed by a description of non-linear space charge expansion and its impact on a beam’s emittance.

6.4 The electron beamline

The emittance of the beam was measured using a waist scan performed by the beamline presented in chapter 4.

The chapter explains the principle of a waist scan and how it can be used to determine the emittance of a charged particle beam. The underlying model is presented, and the link between the limitations of the model and the limitations of the waist scan method is demonstrated.

Taking those limitations into account, the design process for the beamline is presented along with a description of the beamline which was ultimately built for the AC-MOT CAES.

The impact of the magnetic trapping fields upon the emittance of the electron beam
from an MOT are explored, and I showed that the novel AC-MOT CAES design is of most significant benefit in a low energy regime.

Finally, the chapter gives details of the support structure surrounding the electron beamline, along with the design logic underlying the structure.

### 6.5 Characterising the electron beam

Chapter 5 presents the instrumentation used to record the waist scan along with a description of how the other beam parameters were measured.

The chapter opens with a description of the MCP/phosphor which amplifies and images the bunch, with the bunch cross-section images recorded using a camera.

The newly applied machine learning techniques I used to calculate the single electron intensity are presented and how I used the distribution to determine the bunch charge from the total intensity of the beam cross-section images is explained. The charge measurements were cross-calibrated by using them to measure the gain of the MCP, with the value derived agreeing with values found in the literature.

Next, how the beam energy was derived from measurements of the electron bunch’s time of flight between the source and the MCP was presented, along with how the electron source’s position along the $z$ axis was cross-checked with the position of the MOT.

Finally, the chapter explains how the entire measurement process was tested by producing a set of simulated data, and how the insights from those tests were brought into the final measurements of the electron beam’s metrics. A comparison between the new AC-MOT CAES to a more typical DC-MOT CAESs for 1182 eV beams is presented along with the preliminary findings using lower energy, $196 \pm 2$ eV beams. A beam with a low electron temperature of $12.6 \pm 0.8$ K was produced using the AC-MOT CAES.

Whilst the results show there may be marginal benefits of the AC-MOT CAES at $\sim 1$ keV the benefits are likely to be amplified at 200 eV and below. The chapter concludes by speculating on areas in which a low energy electron source could find applications, such as in low energy electron diffraction and low energy electron spectroscopy.
6.6 Further study

There are numerous avenues of interest for the next stage of the project, and areas in which the expertise and knowledge developed over the course of the project could be applied to develop new electron source technology.

6.6.1 Low energy cold electrons

The first avenue for further study is a complete characterisation of the electron beam from the AC-MOT at low energies. Along with the emittance and brightness of the beam, a measure of the beam’s energy spread would be important in determining the beam’s applicability to the low energy applications mentioned in section 5.6.

To allow a comprehensive measurement of the emittance of the electron beam at a full range of beam energies in both the AC and DC cases a pepper-pot technique, as described at the beginning of chapter 4 would be required. The pepper-pot technique is not dependent on the linear transfer model described in 4.1.1, so would be able to reliably determine the beam emittance independent of the linearity of the beamline and for a broader range of beam parameters.

The energy spread of the beam could be measured by reverse-biasing the Einzel lens, in which case it would act as an energy pass filter. Only electrons with sufficient energy to overcome the bias on the middle element of the lens would make it to the MCP. Plotting the charge making it to the MCP against the Einzel lens bias would give the cumulative electron energy distribution, from which the underlying electron energy distribution could be derived.

Retarding the beam sufficiently to use the Einzel lens as an energy filter, however, might distort the beam sufficiently that significant amounts of charge would hit the walls of the beamline, increasing the uncertainty on the measurement.

A better way to measure the energy spread of the beam would be with a Wein filter [116]. A Wein filter consists of perpendicular electric and magnetic fields tuned such that they are balanced for particles with a specific energy. Particles at that energy pass through the filter un-deflected, whilst particles at other energies are deflected. Coupling the filter with an aperture that removes any particles which are deflected allows the energy spread of a beam to be measured by measuring the charge passing through the aperture for a range of pass energies.

Whilst the absolute energy spread of the beam from an MOT based CAES can be reduced by reducing the strength of the extraction fields, in order to reduce the relative
energy spread the $z$ extent of the ionisation region must be reduced. Such a reduction in size would result in an increase of the space charge forces for similarly charged bunches as the same number of electrons are extracted from a smaller region. To maintain the same average beam current whilst reducing the bunch charge the repetition rate of the electron source could be significantly increased from its current value of 1 Hz to 10 kHz by extracting an electron bunch every time the trapping field is driven through zero by the AC drive voltage. Even at 10 eV, the electrons travel the 0.5 m length of the beamline in 380 ns, and are therefore able to escape the trapping region whilst the trapping fields are still effectively zero.

Such an endeavour would require the trapping and photoionisation optics to be re-designed to enable the comparatively high repetition rate.

It would be equally interesting to explore whether the non-linear effects of the trapping fields can be mitigated by simply using much larger trapping coils. The results from other CAESs which use DC trapping fields [62], and the simulation results presented in figure 2.2.3, suggest using large coils might be sufficient.

The result of the low energy program would be to use the low energy beam from an AC-MOT to carry out a low energy electron diffraction study, or a low energy electron spectroscopy study.

### 6.6.2 A CAES as the electron source for a particle accelerator

An alternate path would be to exploit the unique resources available in the North West of England in the form of the Cockcroft Institute’s electron beam facilities. With the knowledge and expertise developed over the course of this project a newly designed CAES could be developed that could potentially take the place of the photocathode electron gun currently installed on the VELA line [146].

Such a source would need to mirror the source parameters of the ALPHA-X photoinjector [147] which currently produces the electron beam for the facility. The source would also need to be portable and designed in concert with an experimental plan so the source could be rapidly installed on the beamline, tested, and removed with minimal downtime.

Due to the $\sim$6.5 MeV beam energies needed to couple into the VELA line, a DC-MOT would be suitable for the design. The high background pressures produced by the rubidium salt dispenser would, however, pose a significant problem. Either a compact Zeeman slower [148] or a differential pumping scheme would be required to keep the beam-line pressures at acceptable levels.
The required trapping laser light could be provided by a rack mounted, fibre coupled lasers, such as the LDL: Littrow "desmo" lasers built by MOGlabs. The ionisation scheme, however, would necessarily take advantage of the ultrafast Ti:Sapphire laser in-situ at the facility.

The beam diagnostics already in place could then be used to characterise the 6D phase space of the resulting electron beam fully, so long as the bunch charge is within the 10 pC to 250 pC operating range of the diagnostic suite [146].

Due to the high profile of the project the design would necessarily include the advances in CAES technology developed by other groups. For example, the techniques proposed by Murphy et al. [103] to reduce disorder-induced heating, and the bunch shaping demonstrated by Thompson et al. [59] which suppresses space charge driven emittance growth.

6.6.3 Cold atom ion beams

One of the most promising areas of cold atom/charged particle research is the development of cold atom ion beams [149, 150]. Due to the increased mass of the ions compared to electrons, the heating effects described in chapter 3 have a smaller impact on the beam quality.

Most of the applications of focused ion beams require cw beams, so the favoured approach is to use a 2-dimensional MOT to compress an atomic beam transversely. The atoms are then photoionised and accelerated [151].

As the low emittance of the cold atom ion beams originates from the small divergence of the source as opposed to a small source size, ion sources with larger source sizes, compared to other ion source designs, can be constructed. The larger source size means a higher ion current can be extracted from the source before space charge forces begin to drive emittance growth in the beam. Cold atom ion sources are therefore an ideal candidate for a next-generation ultra-bright, high-current ion beam.

The low transverse velocity distribution possible using atom cooling techniques corresponds to a brightness comparable with, and in some proposals better than, industry standard Ga$^+$ liquid metal ion sources [152].

The expertise and knowledge developed over the course of the AC-MOT CAES project would be very relevant to the development of a cold atom focused ion beam source in Manchester.
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Appendix A

Repump EOM

Re-pump light tuned to the $F = 2$ to $F' = 3$ transition is introduced as a sideband to the trapping and cooling beams using a New Focus 4431 Visible Phase Modulator Electro-Optic Modulator (EOM).

The EOM contains a crystal whose refractive index is dependent on the electric field applied across it. The crystal is placed in an RF cavity and introduces two sidebands to the light passing through it by applying an RF electric field across the crystal. The RF electric field changes the refractive index resulting in a change to the phase delay introduced to the light by the crystal. The change in the phase delay can be written as

$$ Ae^{i\omega t + i\phi}, \quad (A.1.1) $$

where $A$ is the amplitude of the light, $\omega$ is the frequency of the light and $\phi$ is the phase delay. The phase delay can be varied with time by varying the applied electric field with time. Choosing a sinusoidally varying electric field of angular frequency $k$ and amplitude $B$ gives

$$ Ae^{i\omega t + iB\sin(kt)}, \quad (A.1.2) $$

which, for small $B$, can be rewritten as

$$ Ae^{i\omega t} \left\{ 1 + \frac{B}{2} e^{ikt} - \frac{B}{2} e^{-ikt} \right\}, \quad (A.1.3) $$

$$ = A \left\{ e^{i\omega t} + \frac{B}{2} e^{i(\omega+k)t} - \frac{B}{2} e^{i(\omega-k)t} \right\}, \quad (A.1.4) $$

using a Taylor expansion and a sine identity.
Equation A.1.4 represents laser light at angular frequency $\omega$ in addition to two sidebands offset by $\pm k$.

The re-pump light must be tuned 2.9525 GHz higher than the main trapping and cooling frequency to be resonant with the $F = 2$ to $F = 3$ transition so it can efficiently return the ‘dark’ atoms to the cooling cycle. A TTI TGR6000 signal generator is used to produce the required RF signal which is amplified by a Mini Circuits ZHL-42 RF amplifier before being transmitted into the EOM.

The EOM contains a tunable RF cavity. The cavity is used to build up a large electric field so sufficient energy is transferred into the sidebands that the losses of atoms into the ‘dark’ state are overcome. $\sim 5\%$ of the power in the trapping frequency is transferred into each of the sidebands, measured using a scanning etalon. Only the blue shifted sideband is used to repump atoms which have fallen dark to the primary trapping beams back into the cooling cycle.
Appendix B

Code

B.1 Trigger box

The code can be found at https://github.com/mikeedjones/Trigger-Box

B.2 HV supply

The code can be found at https://github.com/mikeedjones/HighVoltageSupply

B.3 Laser locking system PSU

The code can be found at https://github.com/mikeedjones/LLS_signal_generation