Closed-Loop Identification for Model Predictive Control of HVAC Systems: From Input Design to Controller Synthesis

DOI:

[10.1109/TCST.2019.2917675](https://doi.org/10.1109/TCST.2019.2917675)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Valenzuela, P. E., Ebadat, A., Everitt, N., & Parisio, A. (2019). Closed-Loop Identification for Model Predictive Control of HVAC Systems: From Input Design to Controller Synthesis. *IEEE Transactions on Control Systems Technology*. <https://doi.org/10.1109/TCST.2019.2917675>

Published in:

IEEE Transactions on Control Systems Technology

Citing this paper

Please note that where the full-text provided on Manchester Research Explorer is the Author Accepted Manuscript or Proof version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version.

General rights

Copyright and moral rights for the publications made accessible in the Research Explorer are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Takedown policy

If you believe that this document breaches copyright please refer to the University of Manchester's Takedown Procedures [<http://man.ac.uk/04Y6Bo>] or contact uml.scholarlycommunications@manchester.ac.uk providing relevant details, so we can investigate your claim.

Closed-loop identification for model predictive control of HVAC systems: from input design to controller synthesis

Patricio E. Valenzuela *Member, IEEE*, Afroz Ebadat, Niklas Everitt and Alessandra Parisio *Senior Member, IEEE*

Abstract—Heating, ventilation and air conditioning (HVAC) systems are responsible for maintaining occupants’ thermal comfort and share a large portion of the overall building energy use. Hence, it is of great interest to improve the performance of HVAC control systems and thus the building energy efficiency. Model predictive control (MPC) has been proved to be a promising control strategy to be employed in this field. However, MPC implementation relies on the model of the system and inaccurate models can deteriorate the control performance whilst overly complicated models can lead to prohibitive computational burden. Because of this, existing models do not usually allow the MPC controller to adjust multiple setpoints (e.g., both temperature and flow rates) and do not include the dynamics of the heating and ventilation subsystems with their local controllers. In this work we address the challenge of developing more reliable HVAC models for MPC controllers based on experimental data. Data is obtained from an experiment designed using a graph theoretical technique, which guarantees maximum information content in the data. The resulting models are employed to design local controllers of the heating and ventilation subsystems, which are experimentally tested in a real HVAC testbed. A supervisory MPC controller that incorporates the closed-loop models of the heating and ventilation subsystems is then developed. This can lead to a control strategy able to more effectively adapt key HVAC setpoints based on weather conditions, occupancy, and actual thermal comfort, as shown by a numerical study based on data from the HVAC testbed.

Index Terms—Input design, Model predictive control, System identification, Controller synthesis, HVAC systems.

I. INTRODUCTION

Heating, ventilation and air conditioning (HVAC) systems play a crucial role in the development of sustainable buildings. The main goal is to achieve comfortable indoor climate and ventilation levels while minimizing the energy use. These goals require adjustments of multiple setpoints, primarily temperatures and flow rates. Today these setpoints are either kept constant or manipulated by simple reset rules [1]. Furthermore, existing control strategies typically consider only the heating demand, disregard time varying building conditions and do not account for the dynamics of the HVAC subsystems, such as the heating and ventilation subsystems [1], [2]. Several approaches for improving energy efficiency in buildings have been proposed in recent years (e.g. [3]–[5]). However, the

design of effective multivariable HVAC supervisory control that allows for simultaneously adjusting multiple key HVAC setpoints is still a challenge to be addressed [1].

Model predictive control (MPC) has been proved to be particularly effective in the supervisory control of buildings for its ability to integrate economic, social and environmental aspects, as well as predictions and technical and operational constraints [6]–[8]. MPC has been shown to be able to reduce the energy consumption of buildings [6], [9]. Relevant potential energy savings have been yielded in both simulations (e.g., [10]–[12]), and in experimental studies on actual buildings (e.g., [13]–[16]). Given a model of the system to be controlled, the MPC computes the control actions such that the predicted cost function (i.e., the thermal energy use in this study) is minimized while fulfilling the limitations of the actuation signals (e.g., heating and cooling dampers) and controlled variables (i.e., internal temperatures and CO₂ levels). The MPC performance can be considerably affected by the quality of the model being used, although uncertainty can be handled and properly incorporated into the MPC design to improve the robustness of the resulting control actions (e.g., [14], [17], [18]). In the area of building climate control, MPC performance can be improved if dynamical models of the HVAC subsystems, i.e., heating and ventilation subsystems, are incorporated into the MPC formulation. The authors in [19] show that an MPC strategy that does include the low-level controllers significantly improves its performance with respect to an MPC that does not include those controllers. However, existing MPC-based frameworks attempting to consider more accurate, and hence nonlinear, models have the drawback of convergence and computational complexity issues, e.g., [20]. Thus, adopting physical-based models can increase the computational burden and prevent the use of MPC strategies for real-world applications. A sensible solution is to employ data-driven techniques and system identification methods to identify the dynamics of the HVAC subsystems along with their local controllers, i.e., closed-loop dynamics, based on measured data, and then incorporate these closed-loop dynamics into the MPC design. By doing so, the model accuracy of the overall system to be controlled and, consequently, the control performance, can be improved [21], [22]. Closed-loop identification for MPC has been studied in different settings in [23], [24]. Some MPC formulations consider the possibility to simultaneously perform re-identification of the model during normal operations of the MPC controller, see e.g. [25] or [26].

On the other hand, modeling and identification are very costly in terms of time and resources and hence it is of significant importance to carefully design the identification ex-

This work was supported by the Swedish Research Council under contracts 621-2011-5890 and 621-2009-4017.

At the time of the research study, Patricio E. Valenzuela, Afroz Ebadat and Niklas Everitt were with the Department of Automatic Control and ACCESS Linnaeus Center, School of Electrical Engineering, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden. (e-mail: {pva, ebadat, neveritt}@kth.se.)

Alessandra Parisio is with the School of Electrical and Electronic Engineering, The University of Manchester, Manchester M13 9PL, United Kingdom. (e-mail: alessandra.parisio@manchester.ac.uk.)

periments using optimal input design techniques to minimize the aforementioned costs [27]. Furthermore, carefully designed identification experiments allow for the reduced complexity models to be near-optimal [28].

A. Statement of contributions

Most of the available control frameworks are not able to compute optimal values of multiple setpoints and do not consider uncertainties, occupancy and model mismatch, which significantly limits the potential energy savings practically achievable by applying advanced control [1], [9]. This article is motivated by the need to address the open challenge to design robust multivariable HVAC supervisory control that is able to compute optimal values of multiple setpoints, such as temperatures and air flow rates, and to dynamically adapt these key HVAC setpoints to the actual building conditions, based on weather and occupancy levels. We show that improved energy efficiency and control performance can be achieved by effectively coupling MPC with identification of suitable models from the HVAC data and still obtain control strategies that can be easily implemented in actual buildings. We employ a data-driven approach to identify the open-loop dynamics of the heating and ventilation subsystems [29] and design effective local feedback controllers. The resulting closed-loop dynamics of the controlled HVAC subsystems are then incorporated into a supervisory MPC framework without introducing extra complexity in the control strategy.

To identify the HVAC subsystem dynamics, we adopt a data-driven adaptive identification technique based on recent theoretical results on input design [30], [31] for obtaining information-rich feasible experiments. We extend the MPC strategy designed in [14], [32] to account for the closed-loop models of the HVAC subsystems. The MPC problem is kept linear, which allows the controller model to be as complex as necessary and produce accurate predictions without increasing the computational time of the optimization [33].

Experiments are carried out on a typical HVAC system, which is available at the Royal Institute of Technology (KTH) campus. Dynamical models of the heating and ventilation subsystems for open-loop operation are obtained based on the performed experiments. The obtained models are utilized to tune low-level controllers so as to track the heating and ventilation setpoints provided by an MPC controller. Lastly, dynamical models of the HVAC subsystems for closed-loop operation are derived and incorporated into the supervisory MPC framework, which improves the performance and the energy efficiency of the overall HVAC system.

We point out that the proposed methodology can be directly applied to implement MPC strategies in HVAC systems. Hence, the present contribution serves also as a guideline for practitioners interested in the design and implementation of MPC schemes for HVAC systems. Furthermore, the outcome of this study can be employed for modeling the dynamics of the subsystems comprised in a typical HVAC system not only for control design, but also for other purposes, e.g. simulation.

Structure of manuscript: The rest of this document is organized as follows. In Section II a description of the system

Fig. 1. Diagram of one radiator in the heating subsystem.

is presented. In Section III the proposed methodology is illustrated, while the experimental results obtained on the actual HVAC system available at the KTH campus are shown in Section IV. The results of the simulation study on the HVAC system using two MPC strategies are discussed in Section V. Finally, conclusions are presented in Section VI.

Notation: We denote by $\mathbb{R}$ the real set, by $\mathbb{R}^p$ the set of real p -dimensional vectors, and by $\mathbb{R}^{r \times s}$ the set of real $r \times s$ matrices. The time shift operator is denoted by q , i.e., $qx(t) = x(t+1)$. The expected value is denoted by $\mathbf{E}\{\cdot\}$. Finally, $\det$ and tr stand for the determinant and the trace functions respectively.

II. HEATING AND VENTILATION SUBSYSTEMS

This work is based on a prototype of an HVAC control system available at the KTH university campus. The current controller considers a scenario-based model predictive control (SMPC) to adjust the temperature and ventilation levels in a student laboratory [14]. The existing implementation relies on empirically tuned proportional-integral (PI) controllers for the heating and ventilation subsystems, which do not guarantee tracking performance and hence lead to undesired temperature values and a deterioration of tracking performance.

A. Heating subsystem

The heating subsystem consists of four radiators, each of them connected to the same piping network. The pipes contains hot water, whose temperature is regulated by an external system, and it cannot be controlled. Nevertheless, it is possible to control the flow of hot water to the radiators through an electro-valve. Figure 1 depicts the diagram of one of the radiators in the system. In that figure, $u^h(t)$ denotes the control signal of the electro-valve, which is given in opening percentage (i.e. $u^h(t) \in [0, 100]$), the red line represents the flow of hot water to the radiators, and the blue line indicates the flow of cold water from the radiators. In addition, we can measure the surface temperature (in Celsius degrees) of one radiator in four different zones, denoted by $y^h(t) \in \mathbb{R}^4$ in Figure 1.

The objective here is to tune a PI controller for the mean temperature of the radiator based on a model of the radiator dynamics.

B. Ventilation subsystem

The ventilation subsystem consists of a pipe circuit providing fresh air to the room, as is shown in Figure 2. The air flow

Fig. 2. Diagram of the ventilation and cooling system.

rate to the room can be controlled through an electro-valve, where $u^{\text{air}}(t) \in [0, 100]$ denotes the control signal defining the opening percentage of the dampers. Approximately 30% of the air flow rate can be cooled down by a cooling system. The cooling system is composed by a heat exchanger, where the air is cooled by cold water. The flow of cold water can also be controlled through an electro-valve, where $u^{\text{ac}}(t) \in [0, 100]$ denotes the control signal for the opening percentage of the valve. In addition, we can also measure the temperature (in Celsius degrees) of both the unconditioned and conditioned air entering the room, denoted by $y^{\text{air}}(t) \in \mathbb{R}$ and $y^{\text{ac}}(t) \in \mathbb{R}$ respectively.

The objective here is to tune a controller to regulate the temperature of the incoming air based on a model of the ventilation subsystem.

Remark 1: The temperature dynamics of the heating and ventilation subsystems are assumed to be independent of the occupancy level in the room. This is a reasonable assumption since these subsystems are built to operate independently of the room conditions (room occupancy, room temperature and room CO_2 concentration).

III. METHODOLOGY

A. System identification

The goal in system identification is to estimate a model based on the input-output data $\mathbf{Z}_N := \{\mathcal{U}_N, \mathcal{Y}_N\}$, where $\mathcal{U}_N := (u(1), \dots, u(N))$ ($u(t) \in \mathbb{R}^{n_u}$) are the inputs, and $\mathcal{Y}_N := (y(1), \dots, y(N))$ ($y(t) \in \mathbb{R}^{n_y}$) are the measured outputs. The estimated model parameters $\hat{\theta}_N \in \Theta \subseteq \mathbb{R}^{n_\theta}$ are chosen to minimize an identification cost function. In this work, we obtain $\hat{\theta}_N$ by using the prediction error method [34], where $\hat{\theta}_N \in \Theta$ is such that the cost function

$$V_N(\hat{\theta}_N) = \frac{1}{2N} \sum_{t=1}^N \varepsilon(t, \theta)^\top \varepsilon(t, \theta), \quad (1)$$

$$\varepsilon(t, \theta) := y(t) - \hat{y}(t|\theta), \quad (2)$$

is minimized, where

$$\hat{y}(t|\theta) := \mathbf{E}\{y(t) | \mathbf{Z}_{t-1}, \theta\} \quad (3)$$

is the optimal one-step ahead predictor given by θ , the actual model parameters.

In this work, we consider one-step ahead predictors associated with discrete-time, linear and time-invariant models described by

$$y(t) = G(q, \theta)u(t) + H(q, \theta)e(t), \quad (4)$$

where $G(q, \theta)$ and $H(q, \theta)$ are real rational functions in the time shift operator q , parameterized by θ , and $\{e(t)\}$ is a white noise process with zero mean and variance λ_e . Depending on the parameterization of $G(q, \theta)$ and $H(q, \theta)$, different models can be considered in (4). We refer to [34, Chapter 4] for more details on the model (4).

We assume that the elements in the model set

$$\mathcal{M} = \{(G(q, \theta), H(q, \theta)) : \theta \in \Theta\}, \quad (5)$$

define stable one-step ahead predictors (3) such that $\lim_{q \rightarrow \infty} H(q, \theta) = I$. In addition, we assume that there exists $\theta_0 \in \Theta$ such that (4) describes the true system when $\theta = \theta_0$ [34], i.e., there is no under-modeling (or, equivalently, there exists a model in $\mathcal{M}$ describing the true system).

Remark 2: The nonlinear behavior of the heating and ventilation subsystems of the HVAC system are approximated by linear models. Hence, the term “true system” refers to the dynamical system in the model set $\mathcal{M}$ that best represents the linearized dynamics of the heating and ventilation subsystems. To account for the model mismatch with respect to the nonlinear system when designing the controllers, we use loop shaping and $\mathcal{H}_\infty$ methods to ensure appropriate stability margins, as presented in the subsection III-C.

The system identification problem is summarized as:

Problem 1: Given $\mathbf{Z}_N = (\mathcal{U}_N, \mathcal{Y}_N)$, find $\hat{\theta}_N \in \Theta$ as

$$\hat{\theta}_N = \arg \min_{\theta \in \Theta} \frac{1}{2N} \sum_{t=1}^N \varepsilon(t, \theta)^\top \varepsilon(t, \theta), \quad (6)$$

where $\varepsilon(t, \theta)$ is given by (2).

Problem 1 is solved using the System Identification Toolbox available in Matlab [35].

The model sets used in this work are Box-Jenkins and Output-Error [35], which have been selected as they provide sufficient degrees of freedom for capturing the relevant dynamics of the heating and ventilation subsystems in linear models, hence suitable for the low-level control design of the heating and ventilation units (as it is shown in figures 6 and 9).

Remark 3: It might occur that the identified model obtained by solving Problem 1 is unstable. In such case, the resulting model is modified by replacing the unstable poles by its reflection into the open unit disc, while preserving the steady-state gain. This can be done as the identified heating and ventilation subsystems are inherently stable.

B. Experiment design

The objective in experiment design is to maximize the information content in $\mathbf{Z}_N$ about the system, quantified in terms of a scalar function associated with the intended model application. This step is relevant for ensuring that the data used for identifying the system can guarantee a prescribed accuracy of the estimated model. Here, we follow the approach in [30] and [31], where an input signal $\mathcal{U}_N$ is designed as a realization

of a stationary process. In this section, we restrict the system to be

$$y(t) = G_0(q)u(t) + e(t), \quad (7)$$

where G_0 is a real rational function in the time shift operator q , and $\{e(t)\}$ is a white noise process with zero mean and variance λ_e . The system (7) is estimated using the model

$$y(t) = G(q, \theta)u(t) + e(t), \quad (8)$$

where $G(q, \theta)$ is a real rational function in q , parameterized by $\theta \in \Theta$.

The objective is to estimate θ_0 with maximum accuracy as defined by a scalar function h of the Fisher information matrix $\mathcal{I}_F$ [34]. We define this scalar function as $h : \mathbb{R}^{n_\theta \times n_\theta} \rightarrow \mathbb{R}$. To obtain the desired results, h must be a concave, matrix nondecreasing function [36, pp. 108]. Different choices of h have been proposed in the literature [37]. Some examples are $h = \log \det$, and $h = -\text{tr}\{(\cdot)^{-1}\}$. In this article, $h = \log \det$ is used, which can be interpreted as a measure of the volume of the information in the experiment.

The Fisher information matrix $\mathcal{I}_F$ for the model (8) can be computed as

$$\mathcal{I}_F := \frac{1}{\lambda_e} \mathbf{E} \left\{ \sum_{t=1}^N \psi_{\theta_0}(t) \psi_{\theta_0}(t)^\top \right\}, \quad (9)$$

where

$$\psi_{\theta_0}(t) := \left. \frac{d\hat{y}(t|\theta)}{d\theta} \right|_{\theta=\theta_0}, \quad (10)$$

$$\hat{y}(t|\theta) = G(q, \theta)u(t). \quad (11)$$

The expected value in (9) is with respect to the realizations of $\mathcal{U}_N$.

Equation (11) does not depend on the noise realization. Therefore, we rewrite (9) as

$$\mathcal{I}_F = \frac{1}{\lambda_e} \int_{\mathcal{U}_N \in \mathbb{R}^N} \sum_{t=1}^N \psi_{\theta_0}(t) \psi_{\theta_0}(t)^\top dP(\mathcal{U}_N), \quad (12)$$

where $P(\mathcal{U}_N)$ is the cumulative distribution function of $\mathcal{U}_N$.

We note that (12) depends on $P(\mathcal{U}_N)$, the cumulative distribution function (cdf) of $\mathcal{U}_N$. Therefore, the input design problem is to find a cdf $P^{\text{opt}}(\mathcal{U}_N)$ which optimizes $\log \det(\mathcal{I}_F)$.

To obtain a tractable problem, we restrict $u(t)$ to a finite set $\mathcal{C}$ with c_{seq} elements. Based on this requirement, we optimize the probability mass function $p(\mathcal{U}_N)$ over the set of marginal distributions of stationary processes, given by

$$\mathcal{P}_{\mathcal{C}} := \left\{ f : \mathcal{C}^N \rightarrow \mathbb{R} \mid f(\mathbf{x}) \geq 0, \forall \mathbf{x} \in \mathcal{C}^N; \right. \\ \sum_{\mathbf{x} \in \mathcal{C}^N} f(\mathbf{x}) = 1; \\ \left. \sum_{v \in \mathcal{C}} f(v, \mathbf{z}) = \sum_{v \in \mathcal{C}} f(\mathbf{z}, v), \forall \mathbf{z} \in \mathcal{C}^{(N-1)} \right\}. \quad (13)$$

The last condition in (13) (with slight abuse of notation) guarantees that $p \in \mathcal{P}_{\mathcal{C}}$ is the marginal probability mass function of a stationary process [38].

The input design problem is then defined as follows:

Problem 2: Design an optimal input signal $\mathcal{U}_N^{\text{opt}} \in \mathcal{C}^N$ as a realization from $p^{\text{opt}}(\mathcal{U}_N)$, where

$$p^{\text{opt}}(\mathcal{U}_N) := \arg \max_{p \in \mathcal{P}_{\mathcal{C}}} \log \det(\mathcal{I}_F(p)), \quad (14)$$

with

$$\mathcal{I}_F(p) = \frac{1}{\lambda_e} \sum_{\mathcal{U}_N \in \mathcal{C}^N} \sum_{t=1}^N \psi_{\theta_0}(t) \psi_{\theta_0}(t)^\top p(\mathcal{U}_N), \quad (15)$$

and $\psi_{\theta_0}(t) \in \mathbb{R}^{n_\theta}$ given by (10).

The solution to Problem 2 is obtained using the graph theoretical approach in [30] and [31], and is implemented through a Markov chain having p^{opt} as stationary distribution. The main advantage of using the graph theoretical input design is its ability to cope with physical constraints imposed by the actuators through the choice of the input alphabet $\mathcal{C}$, while attaining a convex optimization problem for the design of the optimal input sequence. We refer to [30], [31], [39] for the implementation details.

Remark 4: Even though the dynamics of the heating and ventilation subsystems are generally nonlinear, they can be well approximated by their linearized models, which is the main reason for the input design method (14) to assume a linear model. In case the open-loop model is nonlinear, the input design technique (14) can still be adopted and the design of the low-level controllers has to account for the nonlinear models of the heating and ventilation subsystems. In addition, given an appropriate design of the local controllers of the HVAC subsystems, the closed-loop dynamics of the heating and ventilation subsystems, and then the relationship between the setpoint to the controlled variable and the actual variable (e.g., air temperature from the ventilation subsystem), can be assumed to be well approximated by linear models (as it can be seen in the comparison between the simulated and real responses shown in figures 6 and 9). Hence, the proposed MPC strategy can still be applied even considering nonlinear open-loop dynamics of the HVAC subsystems.

C. Controller design

1) Heater controller design: Once a model for the heating dynamics has been obtained, we aim to design a PI controller for the sample mean of the measured temperatures in the radiator. We consider the following model of the heater dynamics

$$y^h(t) = G^h(q, \theta) u^h(t) + e^h(t), \quad (16)$$

where $e^h(t)$ is white noise with zero mean and variance λ_e^h . Since we aim to design a PI controller for the mean temperature in the radiator, the PI controller is designed for

$$\hat{y}^h(t|\theta) = \frac{1}{4} \sum_{i=1}^4 G_i^h(q, \theta) u^h(t), \quad (17)$$

where $G_i^h(q, \theta)$ denotes the model for the heater dynamics associated with the i -th measurement in $y^h(t)$.

The discrete-time PI controller is implemented as

$$u^h(t) = C^h(q)(r^h(t) - y^h(t)), \quad (18)$$

where $r^h(t)$ is the desired radiator temperature, and

$$C^h(q) = \frac{\tau_I + \gamma q^{-1}}{1 - q^{-1}}.$$

The coefficients $\tau_I, \gamma \in \mathbb{R}$ are adjusted using the lead-lag design to obtain the desired stability margins [40].

We note that, due to physical limitations of the actuator, the input (18) might not be achieved by the actuator, leading to a saturation problem with the integral action in the PI controller. To prevent this issue, the PI controller (18) is modified to include an anti-windup scheme [41], which removes the integral action from $C^h(q)$ when the bounds of the actuator are reached.

2) *Ventilation unit controller design:* As for the heating controller, the controller design for the ventilation subsystem is based on a model of the ventilation subsystem. The model for the ventilation subsystem is given by

$$y^{\text{ac}}(t) = y^{\text{air}}(t) + G^{\text{ac}}(q, \theta)u^{\text{ac}}(t) + G^{\text{air}}(q, \theta)u^{\text{air}}(t) + H^{\text{ac}}(q, \theta)e^{\text{ac}}(t), \quad (19)$$

where $e^{\text{ac}}(t)$ is white noise with zero mean and variance λ_e^{ac} . By introducing $y^{\text{air}}(t)$ in (19) we only model the reduction in the temperature achieved in $y^{\text{ac}}(t)$ by the ventilation subsystem.

Since the input $u^{\text{air}}(t)$ is controlled directly by the MPC strategy (i.e., it is a decision variable in the MPC problem), the discrete-time controller for $y^{\text{ac}}(t)$ is implemented as

$$u^{\text{ac}}(t) = C^{\text{ac}}(q)(r^{\text{ac}}(t) - y^{\text{ac}}(t)), \quad (20)$$

where $r^{\text{ac}}(t)$ is the desired temperature of the cooled air. The controller $C^{\text{ac}}(q)$ is designed using the $\mathcal{H}_\infty$ control toolbox available in Matlab. The controller $C^{\text{ac}}(q)$ is obtained by solving

$$C^{\text{ac}}(q) = \arg \min_{C(q)} \left\| \begin{bmatrix} W(q) S_{z^{\text{ac}}x}(q) \\ S_{u^{\text{ac}}x}(q) \end{bmatrix} \right\|_\infty, \quad (21)$$

where $z^{\text{ac}} := r^{\text{ac}} - y^{\text{ac}}$, $x := [u^{\text{air}} \quad r^{\text{ac}}]^\top$, S_{yx} denotes the transfer function from x to y when the control law (20) is used, and W is a user-defined weighting factor, which is typically defined as the lower bound on the inverse of $|S_{z^{\text{ac}}r^{\text{ac}}}(e^{j\omega})|$, $\omega \in [-\pi, \pi]$ (cf. [42]). We note that the optimization (21) is over the set of stabilizing controllers. In addition, we recall that the infinity norm of a function F is given by

$$\|F(q)\|_\infty := \sup_{\omega \in [-\pi, \pi]} \bar{\sigma}(F(e^{j\omega})), \quad (22)$$

where $\bar{\sigma}(A)$ denotes the maximum singular value of a matrix $A \in \mathbb{R}^{n \times m}$.

In order to use the robust control toolbox in Matlab, we transform the discrete-time model (19) into a continuous-time one by using the bilinear transformation

$$z = \frac{1 + sT_s/2}{1 - sT_s/2}, \quad (23)$$

where T_s is the sampling period. After completing this transformation, we solve (21) in the continuous-time domain, and then we obtain $C^{\text{ac}}(q)$ by reversing the mapping (23).

From the equation (19) we see that $u^{\text{air}}(t)$ acts as a disturbance to the temperature of the cooled air $y^{\text{ac}}(t)$. Since we know the value of $u^{\text{air}}(t)$ for all t , we can compensate its effect by modifying (20) as

$$u^{\text{ac}}(t) = C^{\text{ac}}(q)(r^{\text{ac}}(t) - y^{\text{ac}}(t)) + G_{\text{ff}}(q, \theta)u^{\text{air}}(t), \quad (24)$$

where G_{ff} is a feedforward compensator given by

$$G_{\text{ff}}(q, \theta) := -G^{\text{ac}}(q, \theta)^{-1}G^{\text{air}}(q, \theta). \quad (25)$$

If the inverse of G^{ac} is unstable and/or improper, the feedforward law (25) can still be used, where the nonminimum phase zeros of G^{ac} are replaced by their stable reflection into the complex unit disc and by adding the required number of delays to achieve a proper function.

As with the heating controller, the control signal $u^{\text{ac}}(t)$ for the ventilation subsystem is also limited, which can introduce windup effects in the closed loop. To avoid this, the implementation of the controller (24) is done by including an anti-windup scheme, following the same procedure as for the heating subsystem.

We note that the cooling subsystem can also provide heating power when the room air temperature is lower than the incoming air temperature.

D. MPC design

The use of closed-loop identified models for online predictive controllers can be highly beneficial in terms of reducing disturbances and model errors, as well as being less disruptive to normal plant function.

In this section we describe the design of an MPC strategy for HVAC systems that incorporates the closed-loop dynamics of the heating and ventilation subsystems.

Since layering is an essential architectural feature of complex systems like buildings, an effective building control architecture is likely to be multilayer. Our proposed control architecture includes the low-level feedback controllers described in III-C and a supervisory MPC controller, which computes the setpoints to be tracked by the low-level controllers. This hierarchical control scheme has been implemented in the KTH testbed [14], which controls both the indoor temperature and the CO₂ concentration levels. The current implementation of MPC relies on empirically tuned PI controllers. In this section we extend the design of the supervisory MPC controller to incorporate the closed-loop dynamics of the HVAC subsystems.

Description of the control architecture: The control objective is to minimize the thermal energy use while keeping an acceptable indoor comfort level for occupants. The indoor temperature and the air CO₂ concentration levels (both to be considered as comfort indicators) are controlled through the ventilation unit and the radiators, which are both actuated using local PI controllers. The MPC controller computes, at each point in time, setpoints for the local controllers using new measurements and updated information about weather, occupancy patterns and building state.

At each time instant, the output of the MPC controller is a heating, cooling, and ventilation plan for the next N hours (N being the prediction horizon). According to the receding horizon philosophy, only the first step of this control plan is applied to the HVAC system. After that, the whole procedure is repeated based on new measurements and updated information about weather, occupancy patterns, and building state. This introduces feedback into the system, since the control action is a function of the system state and the acting disturbances.

At each point in time t , the MPC computes the setpoints for the local controllers, which are: *i*) mass air flow rate, $\dot{m}^{\text{air}}(t)$; *ii*) supply air temperature, $T^{\text{ac}}(t)$; and *iii*) radiators mean radiant temperature, $T^{\text{h}}(t)$.

The overall MPC problem is formulated as a cascade of two linear problems, as described in [14]. Since the CO_2 concentration dynamics are independent of the thermal ones and the CO_2 comfort has priority, two separated sub-problems can be addressed: *i*) the CO_2 -MPC problem, which aims at minimizing the energy use while keeping the CO_2 levels in given comfort bounds; *ii*) the T-MPC problem, controlling instead the indoor temperature and deciding the additional ventilation level to guarantee the thermal comfort.

Remark 5: We remark that posing the problem of minimizing the energy use while maintaining both indoor CO_2 and thermal comfort as one single problem to be solved would result in a non-convex problem and then increase the computational burden. This could make prohibitive the computational requirements for buildings comprising a large number of thermal zones. We then handle these nonlinearities so as the nonlinear terms in the modeling of the CO_2 and thermal dynamics can be equivalently rewritten as linear equations and formulate the overall control problem as a cascade of two linear problems.

The output of the CO_2 -MPC problem is a sequence of air flow rates, $\{\dot{m}_{\text{CO}_2}^{\text{air}}(t)\}_{t=0}^{N-1}$, from the ventilation subsystem over the whole prediction horizon, which is integrated into the T-MPC problem to account for the corresponding heat flow.

Note that the existing low-level PI controllers are designed through an empirical tuning technique, hence without any model identification, and that the currently implemented MPC neglects the dynamics of the heating and ventilation subsystems and those of their local controllers, hence assuming that the low-level controllers are able to quickly and accurately track the setpoints computed by the high-level MPC controller.

A detailed description of the CO_2 -model and the CO_2 -MPC problem can be found in [14]. In this study we focus on the T-MPC problem, modified to include the identified closed-loop dynamics of the heating and ventilation subsystems. We remark that the CO_2 -MPC problem can be similarly modified.

Control-oriented thermal model with closed-loop dynamics of HVAC subsystems: Here we extend the control-oriented model in [14], [32] to incorporate the closed-loop dynamics of the identified HVAC subsystems.

The physical model of the indoor thermal dynamics is based on the energy balance and consists of one lumped node for the room air and two lumped nodes for each wall slab (one for the outdoor wall surface and one for the indoor wall surface). Walls are modeled as two capacitances and three resistances,

which are placed between the air temperature, the internal wall surface temperature, the external wall surface temperature, and the room temperature. As for the outdoor walls, we account for the outdoor temperature and for the different radiation heat exchange due to the orientation of the outdoor walls. Solar gains are taken into account by projecting the direct radiation incident on a horizontal surface onto the glazed surfaces in the zone. The projection accounts for the building location (longitude and latitude), the solar position (hour and day of the year) and the window orientation. Details on the modeling of the internal and external wall temperatures can be found in [14], [32] and references therein.

Consider the dynamics of the room temperature, indicated with T^{room} . They are modeled as follows:

$$\begin{aligned} m^{\text{air}} c^{\text{pa}} \dot{T}^{\text{room}}(t) = & Q^{\text{air}}(t) + Q^{\text{int}}(t) + \sum_j Q^{\text{wall},j}(t) \\ & + \sum_j Q^{\text{win},j}(t) + Q^{\text{h}}(t), \end{aligned} \quad (26)$$

where c^{pa} is the specific heat of the dry air, m^{air} is the air mass in the room and j is the index corresponding to the various walls in the room. In (26), the left-hand term represents the heat stored in the room air, while Q^{air} is the heat flow due to ventilation and used to provide fresh air and cooling power. Q^{int} sums the internal gains, i.e., the heat flows that are due to equipment, lighting and occupancy. $Q^{\text{wall},j}$ and $Q^{\text{win},j}$ represent the heat flows exchanged between walls and windows in the j -th surface and room, which include the outside temperature and solar radiation. Q^{h} is the heating flow necessary to keep the room environment within the thermal comfort range. We focus on heat flows to be controlled for HVAC operation to derive a control-oriented model. These heat flows are

$$\begin{aligned} Q^{\text{air}}(t) &= \dot{m}^{\text{air}}(t) c^{\text{pa}} (T^{\text{ac}}(t) - T^{\text{room}}(t)), \\ Q^{\text{h}}(t) &= S^{\text{h}} c^{\text{h}} (T^{\text{h}}(t) - T^{\text{room}}(t)), \end{aligned} \quad (27)$$

where c^{h} is the heat transfer coefficient of the radiators in the heating subsystem and S^{h} is the emission area of the radiators. In order to derive an equivalent linear model of the thermal dynamics, we introduce the nonnegative variables $\Delta u^+(t)$ and $\Delta u^-(t)$ as inputs such that

$$\Delta u^+(t) - \Delta u^-(t) := \Delta \dot{m}^{\text{air}}(t) (T^{\text{ac}}(t) - T^{\text{room}}(t)), \quad (28a)$$

$$\Delta u^+(t) + \Delta u^-(t) := \Delta \dot{m}^{\text{air}}(t) |T^{\text{ac}}(t) - T^{\text{room}}(t)|, \quad (28b)$$

where $\Delta \dot{m}^{\text{air}}(t)$, defined as $\Delta \dot{m}^{\text{air}}(t) := \dot{m}^{\text{air}}(t) - \dot{m}_{\text{CO}_2}^{\text{air}}(t)$, is the additional air flow rate required for guaranteeing the thermal comfort. By introducing the inputs above, $Q^{\text{air}}(t)$ in (27) can be rewritten as

$$\begin{aligned} Q^{\text{air}}(t) &= \dot{m}_{\text{CO}_2}^{\text{air}}(t) c^{\text{pa}} (T^{\text{ac}}(t) - T^{\text{room}}(t)) \\ &\quad + c^{\text{pa}} (\Delta u^+(t) - \Delta u^-(t)), \end{aligned}$$

where $\dot{m}_{\text{CO}_2}^{\text{air}}$ is the optimal air flow rate obtained by the CO_2 -MPC problem.

In the following, we describe the new thermal model incorporated into the MPC formulation, which includes the closed-loop dynamics of the HVAC subsystems.

Thus we derive the following state-space descriptions from the closed-loop models of the heating and ventilation subsystems¹

$$\begin{aligned} x^h(t+1) &= A^h x^h(t) + B^h r^h(t) \\ y^h(t) &= C^h x^h(t) + D^h r^h(t), \end{aligned} \quad (29)$$

and

$$\begin{aligned} x^{ac}(t+1) &= A^{ac} x^{ac}(t) + B^{ac} u^{ac}(t) \\ y^{ac}(t) &= C^{ac} x^{ac}(t) + D^{ac} u^{ac}(t). \end{aligned} \quad (30)$$

The input, r^h , and the output, $y_c^h = T^h$, in (29) represent the setpoint to the radiator low-level controller and the mean radiator temperature respectively, as shown in Figure 1. The input, $u^{ac} = [r^{ac}, u^{air}]^\top$, and the output, $y^{ac} = T^{ac}$, in (30) represent the setpoints to the low-level controller of the ventilation subsystem and the supply air temperature respectively, as shown in Figure 2.

Hence, the dynamics of the indoor temperature, incorporating the closed-loop dynamics of the heating, ventilation and cooling subsystems, can be modeled with the following discrete-time LTI system

$$\begin{aligned} x_T(t+1) &= A_T(t)x_T(t) + B_T(t)u_T(t) + E_T w_T(t) \\ y_T(t) &= C_T x_T(t), \end{aligned} \quad (31)$$

where the output $y_T(t)$ is the indoor temperature at time t , $u_T(t) := [\Delta u^+(t), \Delta u^-(t), u^{ac}(t)^\top, r^h(t)]^\top$, and the matrices $A_T(t)$ and $B_T(t)$ are time-varying since they depend on $\dot{m}_{CO_2}^{air}(t)$. The state of the thermal model, x_T , is the vector of the temperatures of the room, the walls, the floor and the ceiling as well as the state variables describing the closed-loop dynamics of the HVAC subsystems, x^{ac} and x^h . The model disturbance vector, w_T , represents the outdoor temperature, the incident solar radiation, the internal gains, and the heat flows due to occupancy, equipments and lighting.

Define $x^{ai}(t)$ as the vector containing the temperatures of the room and of the inner and outer parts of the walls, and $u^{ai}(t) := [\Delta u^+(t), \Delta u^-(t)]^\top$. Then, $u_T(t)$ and $x_T(t)$ can be also written as $[u^{ai}(t)^\top, u^{ac}(t)^\top, r^h(t)]^\top$ and $[x^{ai}(t)^\top, x^{ac}(t)^\top, x^h(t)^\top]^\top$ respectively. The matrices $A_T(t)$, $B_T(t)$ can be expressed as follows:

$$\begin{aligned} A_T(t) &= \begin{bmatrix} A^{ai}(t) & C^{ac} & C^h \\ \mathbf{0} & A^{ac} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A^h \end{bmatrix}, \\ B_T(t) &= \begin{bmatrix} B^{ai}(t) & D^{ac} & D^h \\ \mathbf{0} & B^{ac} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & B^h \end{bmatrix}, \end{aligned}$$

where $A^{ai}(t)$ and $B^{ai}(t)$ are matrices of appropriate dimensions, whose elements are parameters describing the thermal dynamics of the temperatures of the room and the inner and outer parts of the walls [14].

MPC problem formulation: In this section, we formulate the novel MPC formulation incorporating the closed-loop dynamics of the HVAC subsystems.

Since the control objective is to minimize the energy use, we need to introduce nonnegative auxiliary variables, Γ^+ and Γ^- , to model the absolute value such that²

$$\begin{aligned} \max\{0, T^{ac}(t) - T^{room}(t)\} &\leq \Gamma^+(t) \\ \max\{0, T^{room}(t) - T^{ac}(t)\} &\leq \Gamma^-(t) \\ \Delta u^+(t) &\leq \Delta \dot{m}_{max}^{air}(t) \Gamma^+(t) \\ \Delta u^-(t) &\leq \Delta \dot{m}_{max}^{air}(t) \Gamma^-(t), \end{aligned} \quad (32)$$

where $\Delta \dot{m}_{max}^{air}(t) := \dot{m}_{max}^{air}(t) - \dot{m}_{CO_2}^{air}(t)$. The cost function is $J_T(x_T(0), u_T) = \sum_{t=0}^{N-1} (\rho_{air}(\Gamma^+(t) + \Gamma^-(t)) + c(t)^\top u_T(t)) \Delta t$, where Δt the sampling period, N is the prediction horizon, $c(t)$ is the cost vector at time t and ρ_{air} is a weighting factor on auxiliary variables modeling the absolute value $|T^{ac}(t) - T^{room}(t)|$.

We then define bounds on the inputs $u_T(t)$ of the form $u_T^{min} \leq u_T(t) \leq u_T^{max}$, and express comfort constraints on the indoor temperature as $y_T^{min}(t) \leq y_T(t) \leq y_T^{max}(t)$. Constraints (32) can be written in a compact form as mixed constraints on inputs and outputs.

Summarizing, the constraints on inputs, comfort constraints on outputs and mixed constraints on inputs and outputs can be expressed as polytopic constraints, respectively as $F_T u_T(t) \leq f_T$, $G_T y_T(t) \leq g_T$ and $M_T^y y_T(t) + M_T^u u_T(t) \leq m_T$, where F_T , G_T , M_T^y , M_T^u and f_T , g_T , m_T are matrices of appropriate dimensions.

Thus, the T-MPC problem can be formulated as follows:

$$\begin{aligned} \min_{u_T} \quad & J_T(x_T(0), u_T) \\ \text{s.t.} \quad & \text{T-model (31)} \\ & F_T u_T(t) \leq f_T \\ & G_T y_T(t) \leq g_T \\ & M_T^y y_T(t) + M_T^u u_T(t) \leq m_T, \end{aligned} \quad (33)$$

with $t = 0, \dots, N-1$.

We remark that the presented MPC formulation for HVAC operation control is general and can easily integrate different HVAC subsystems, which can be identified by following the experimental procedure outlined in Section III. Moreover, the models of the heating and ventilation subsystems are obtained by employing a novel input design technique, which can handle the physical constraints of the dampers.

Scenario-based MPC problem formulation: In order to account for the main sources of uncertainty affecting the building, i.e., weather and occupancy, we use probabilistic concepts. Since current standards allow comfort violations up to given levels, building climate control leads naturally to stochastic formulations with probabilistic constraints, i.e., chance-constrained formulations, which limit the probability of these violations. Unless the uncertainties follow specific distributions, e.g., Gaussian or log-concave, chance-constrained problems are generally non-convex and thus numerically difficult to be handled. Uncertainties like solar radiation and occupancy do not usually follow probability distributions that

¹The state space matrices are computed using the command `ss` in Matlab.

²We note that Equation (32) includes the constraints in (28).

allow to formulate equivalent deterministic problems and make MPC problems tractable. A possible solution for obtaining a tractable and distribution-free MPC problem is to apply the *scenario approach* [43], where scenarios are i.i.d. samples extracted from general probability distributions. The fundamental idea is to draw a specific large number S of samples $\hat{w}_T^i$, commonly called *scenarios*, from the uncertainty w_T according to the probability distribution $\mathbb{P}[\cdot]$. The number of samples is selected to guarantee the feasibility of the solution so that it has generalization properties, i.e., it satisfies with high probability also unseen scenarios.

We notice that, by using (31), $y_T(t)$ can be expressed as a function of the initial state $x_T(0)$, input $\{u_T(k)\}_{k=0}^{t-1}$, and disturbances $w_T(t)$. Thus we have uncertainty in the constraints $G_T y_T(w_T(t)) \leq g_T$ and $M_T^y y_T(w_T(t)) + M_T^u u_T(t) \leq m_T$ of the T-MPC problem (33). We can handle uncertainty in those constraints by requiring them to be fulfilled with a probability of at least $1 - \alpha$ for $\alpha \in (0, 1)$; this implies that *chance constraints* have to be formulated, i.e. $\mathbb{P}[G_T y_T(w_T(t)) \leq g_T] \geq 1 - \alpha$ and $\mathbb{P}[M_T^y y_T(w_T(t)) + M_T^u u_T(t) \leq m_T] \geq 1 - \alpha$.

By applying the scenario approach and by taking advantage of the linearity of the constraints, the chance-constrained T-MPC problem can be approximated with the following scenario-based MPC problem (SMPC):

$$\begin{aligned} \min_{u_T} \quad & J_T(x_T(0), u_T) \\ \text{s.t.} \quad & \text{T-model (31)} \\ & F_T u_T(t) \leq f_T \\ & G_T \max_i y_T(\hat{w}_T^i(t)) \leq g_T \\ & M_T^y \max_i y_T(\hat{w}_T^i(t)) + M_T^u u_T(t) \leq m_T. \end{aligned} \quad (34)$$

The scenario problem (34) is deterministic and linear and thus it can be solved efficiently by standard numerical algorithms.

In [44], the authors compute lower bounds on the sample size S guaranteeing that the optimal solution of the approximated scenario problem is a feasible solution for the original problem with high probability. In the aforementioned studies it is shown that the conservativeness of the scenario approach can be improved if the problem contains multiple chance constraints and a chance constraint belongs to a special class of constraints (e.g., linear or quadratic constraints); both assumptions hold in our study. In this work, the results in [44] are used to set a lower bound on the number of scenarios providing guarantees on the probability of constraint violation.

E. Proposed methodology for MPC control design of HVAC systems

The proposed methodology for modeling and designing an MPC strategy for HVAC systems is:

- 1) Choose the number of samples N_1 , and the bounds on the input signal $\underline{u}, \bar{u} \in \mathbb{R}$. Given the sampling period T_s , run an experiment with $\mathcal{U}_{N_1}$ as input sequence, where $u(t)$ is uniformly distributed on $[\underline{u}, \bar{u}]$. Save the results as $\mathbf{Z}_{N_1} = (\mathcal{U}_{N_1}, \mathcal{Y}_{N_1})$.
- 2) Based on $\mathbf{Z}_{N_1}$, obtain $\hat{\theta}_{N_1}$ by solving Problem 1.

(a) Input sequence, $\mathcal{U}_{N_1}$.

(b) One measured output in $\mathcal{Y}_{N_1}$.

Fig. 3. Experimental results, step 1.

- 3) Choose the number of samples N_2 . Given $\hat{\theta}_{N_1}$, design $\mathcal{U}_{N_2}^{\text{opt}}$ by solving Problem 2 based on [30] and [31]. Perform the experiment on the system based on $\mathcal{U}_{N_2}^{\text{opt}}$, and save the results as $\mathbf{Z}_{N_2} = (\mathcal{U}_{N_2}^{\text{opt}}, \mathcal{Y}_{N_2})$.
- 4) Based on $\mathbf{Z}_{N_2}$, obtain a second estimate $\hat{\theta}_{N_2}$ by solving Problem 1.
- 5) Design a discrete-time controller based on the model derived from $\hat{\theta}_{N_2}$.
- 6) Implement the MPC scheme (33) based on the identified closed-loop models of the heating and ventilation subsystems and the thermal model of the room.

Remark 6: The identification of the closed-loop dynamics of the individual HVAC subsystems and the tuning of the low-level controllers can be done offline, in the design phase. Once the models of the closed-loop subsystems are identified and the controller parameters are set (which can be performed simultaneously for all the subsystems), the computational requirements in the operational phase are basically just the ones needed for solving the linear MPC problem at each sampling time.

Remark 7: The proposed methodology assumes that it is possible to design local controllers for the heating and ventilation subsystems. However, it might be the case that the HVAC system has in-built local controllers for those subsystems and that they cannot be modified. In that case, the methodology described here can still be applied without considering the low-level controller synthesis, and employing the proposed experiment design to identify the closed-loop dynamics of the HVAC subsystems and for designing the supervisory MPC strategy.

TABLE I
log det($\mathcal{I}_F$) FOR DIFFERENT $\mathcal{U}_{N_2}$.

Input	Optimal	Uniform	Binary
log det($\mathcal{I}_F$)	55.23	48.88	53.51

IV. EXPERIMENTAL RESULTS

A. Heating subsystem

We use the method presented in Section III-E to identify the main dynamics in the heating subsystem, using a sampling time of $T_s = 600$ [s]. As first step, we generate $\mathcal{U}_{N_1}$ with $N_1 = 62$. The generated input is presented in Figure 3(a). During the first 30 minutes of the experiment the input is fixed to zero, and then a step of magnitude 50 is employed during the next 30 minutes. The objective of this initial signal is to reduce the effect of the initial conditions in the experiment. After the initial input, we generate the signal according to step 1) in Section III-E, where $\underline{u} = 10$, and $\bar{u} = 90$. One of the measured outputs is presented in Figure 3(b). From this figure we see that the response of the temperature depends dynamically on the applied input sequence. The experiment length is approximately 10 hours.

The collected data $\mathbf{Z}_{N_1}$ is employed to fit a first order output error model. Following [34, Chapter 14], before identifying a model, we shifted the input-output data in $\mathbf{Z}_{N_1}$ by its mean. By using the `oe` command available in Matlab System Identification Toolbox, we obtain

$$\hat{y}^h(t|\hat{\theta}_{N_1}) = G^h(q, \hat{\theta}_{N_1})u^h(t) = \begin{bmatrix} \frac{0.09q^{-1}}{1 - 0.39q^{-1}} \\ \frac{0.07q^{-1}}{1 - 0.62q^{-1}} \\ \frac{0.08q^{-1}}{1 - 0.55q^{-1}} \\ \frac{0.09q^{-1}}{1 - 0.40q^{-1}} \end{bmatrix} u^h(t). \quad (35)$$

The model (35) is used to design a second experiment $\mathcal{U}_{N_2}$ solving Problem (2). In this case, we consider an input sequence sampled from a pmf associated with a stationary process, using the method in [31] with $n_m = 3$ and $\mathcal{C} = \{10, 90\}$. The computational time for deriving the optimal pmf in a laptop Dell Latitude E6430, equipped with Intel Core i7 2.6 [GHz] processor, and 8 [GB] of RAM memory is approximately 2 seconds. As a reference, the optimal input sequence is compared with a white noise sequence uniformly distributed in $[10, 90]$, and a binary distributed white noise process with alphabet $\mathcal{C}$. The results are presented in Table I. From Table I, we conclude that the optimal input sequence ('Optimal' in Table I) retrieves more information from the system than the inputs defined as white noise processes uniformly distributed in $[10, 90]$ ('Uniform' in Table I), and Binary distributed with alphabet $\{10, 90\}$ ('Binary' in Table I).

The optimal input sequence is employed to obtain a second data set. For this experiment we set $N_2 = 162$, where the first 10 samples are the same initial input sequence described for

(a) Input sequence, $\mathcal{U}_{N_2}$.

(b) One measured output in $\mathcal{Y}_{N_2}$.

Fig. 4. Experimental results, step 3.

the first experiment. Part of the collected data is presented in Figure 4, where the measured output increases its value despite the fixed amplitude employed as input. This can be explained by the fact that the water temperature increases during the experimental time. The experiment length is approximately 26 hours.

The data set $\mathbf{Z}_{N_2}$ is employed to update the output error model. As in the first step, we shift the input-output data in $\mathbf{Z}_{N_2}$ by its mean. By using the `oe` command available in Matlab System Identification Toolbox with initial estimate $G^h(q, \hat{\theta}_{N_1})$, we obtain

$$\hat{y}^h(t|\hat{\theta}_{N_2}) = G^h(q, \hat{\theta}_{N_2})u^h(t) = \begin{bmatrix} \frac{0.07q^{-1}}{1 - 0.37q^{-1}} \\ \frac{0.06q^{-1}}{1 - 0.57q^{-1}} \\ \frac{0.07q^{-1}}{1 - 0.50q^{-1}} \\ \frac{0.07q^{-1}}{1 - 0.37q^{-1}} \end{bmatrix} u^h(t). \quad (36)$$

To analyze the model (36), we compute the sampled auto-covariance function of the model residuals associated with the i -th measurement, given by

$$\hat{R}_{\varepsilon_i^h}(\tau) := \frac{1}{N_2 - \tau} \sum_{t=1}^{N_2 - \tau} \varepsilon_i^h(t, \hat{\theta}_{N_2}) \varepsilon_i^h(t - \tau, \hat{\theta}_{N_2}), \quad (37)$$

$$\varepsilon_i^h(t, \hat{\theta}_{N_2}) := y_i^h(t) - \hat{y}_i^h(t|\hat{\theta}_{N_2}), \quad (38)$$

where $\varepsilon_i^h(t, \hat{\theta}_{N_2})$ denotes the one-step ahead prediction error

(a) Autocorrelation function for ε_1^h (top) and ε_2^h (bottom).

(b) Autocorrelation function for ε_3^h (top) and ε_4^h (bottom).

Fig. 5. Autocorrelation function for the model residuals.

at time t associated with the i -th output. Here we test the whiteness hypothesis for $\{\varepsilon(t, \hat{\theta}_{N_2})\}_{t=1}^{N_2}$. Figure 5 presents the autocorrelation function (i.e. the autocovariance function normalized by its sampled variance $\hat{R}_{\varepsilon_i^h}(0)$) for the model residuals for $\tau \in \{0, \dots, 24\}$, together with the 99% confidence region (in yellow). The results show that the whiteness assumption made for the measurement noise must be rejected. Thus, the identification results can be improved if a noise model is included. However, since the identification of the system is performed in open loop, the wrong assumption on the noise model does not add bias to the model estimate [34, Chapter 8]. Hence, the model (36) can be employed to tune a PI controller for the heating subsystem.

The model (36) is employed to tune the PI controller (18) to obtain the desired stability margins. The coefficients of the PI controller are $\tau_I = 6.93$, and $\gamma = -2.42$, which gives a phase margin of 68 degrees at $\omega_\phi = 9 \cdot 10^{-4}$ [rad/s], and a gain margin of 13.2 [dB] at $\omega_g = 5.2 \cdot 10^{-3}$ [rad/s].

To analyze the model, we perform an experiment in the heating subsystem and measure the step response of the mean temperature in the surface of the radiator. This is then compared with the step response of the resulting closed-

Fig. 6. Step response of the closed-loop system, heating subsystem.

loop model. Figure 6 shows the step response of the system ('Experiment' in Figure 6) and the simulated model ('Model' in Figure 6) when the reference is set to 28 [C]. From this figure we can see that the simulated response is close to the measured response, but it can still be improved by identifying the closed-loop system. Hence, we perform an experiment in the closed-loop system by defining the reference value $r^h(t)$ as a realization of a white noise process, uniformly distributed in [20, 34] [C], and saving 144 samples (equivalent to 24 hours of experimental data). The experimental data is divided into 106 samples for identification, and 37 samples for validation. A Box-Jenkins model of third order for both the process model and the noise filter is identified. The simulated response of the re-identified model is presented in Figure 6 (Re-identified model). In this case, the simulated response is closer to the measured response than the closed-loop model derived from the open-loop model. Therefore, the re-identified model is employed by the MPC to predict the dynamics of the mean temperature of the radiator. The closed-loop model is then given by

$$y^h(t) = \frac{0.51q^{-1}(1 + 0.4q^{-1} + 0.2q^{-2})}{(1 + 0.03q^{-1})(1 - 0.4q^{-1} + 0.18q^{-2})} r^h(t) + \frac{(1 - 0.89q^{-1})(1 + 0.74q^{-1} + 0.39q^{-2})}{(1 - 0.64q^{-1})(1 + 0.33q^{-1} + 0.19q^{-2})} e^h(t). \quad (39)$$

The model (39) is then converted into the state space representation (29) incorporated into the MPC formulation.

B. Ventilation subsystem

Due to practical considerations, we model the ventilation subsystem in the HVAC testbed by using the first two steps defined in Section III-E and we consider a sampling period of $T_s = 60$ [s]. In order to perform the first step of the method illustrated in Section III-E, we need to define the lower and upper bounds on the input signals $u^{ac}(t)$, and $u^{air}(t)$. For $u^{ac}(t)$ we consider the lower bound $\underline{u} = 34$ and the upper bound $\bar{u} = 100$, since the effect of the ventilation subsystem on the air temperature is negligible for $u^{ac}(t) \leq 34$. As for $u^{air}(t)$, we define the lower bound as $\underline{u} = 10$ and the upper bound as $\bar{u} = 100$. For the experiment on the air conditioning

Fig. 7. Experimental results for the ventilation subsystem.

(AC) system, $N_1 = 255$ samples are collected, equivalent to an experiment length of 4 hours and 15 minutes. The input-output data collected in this experiment is presented in Figure 7. From this figure, we observe that the control signals are set to their lower bounds for 15 minutes so as to turn off the ventilation subsystem before starting the experiment.

Based on the data set $\mathbf{Z}_{N_1}$, we identify the model (19) of the ventilation subsystem. We note that the temperature of the incoming air is preconditioned to 20.8 [C] (except for negligible variations), and thus we consider $y^{\text{air}}(t) = 20.8$ during all the experiment. Moreover, we remove the offset in $u^{\text{ac}}(t)$ by subtracting 34 to its samples, which corresponds to the offset imposed by the dead-band effect. In addition, the first 200 samples are employed for identification, and the last 55 ones are used for validation.

Fig. 8. Autocorrelation function for the model residuals, and cross correlation functions between the model residuals and the inputs.

After preconditioning the data set $\mathbf{Z}_{N_1}$, we use the `pem` command available in the Matlab System Identification Toolbox to identify the transfer functions in Equation (19), based on the first 200 samples. The resulting estimates are

$$\hat{G}^{\text{ac}} = \frac{-1.3 \cdot 10^{-3} q^{-1} (1 + 9.97 q^{-1}) (1 - 0.98 q^{-1})}{(1 - 0.02 q^{-1}) (1 - 0.60 q^{-1}) (1 - 0.98 q^{-1})}, \quad (40)$$

$$\hat{G}^{\text{air}} = \frac{-6.4 \cdot 10^{-4} q^{-1} (1 - 5.57 q^{-1}) (1 - 1.12 q^{-1})}{(1 - 0.02 q^{-1}) (1 - 0.60 q^{-1}) (1 - 0.98 q^{-1})}, \quad (41)$$

$$\hat{H}^{\text{ac}} = \frac{(1 - 0.99 q^{-1}) (1 - 0.42 q^{-1}) (1 + 0.36 q^{-1})}{(1 - 0.02 q^{-1}) (1 - 0.60 q^{-1}) (1 - 0.98 q^{-1})}, \quad (42)$$

where $\hat{G}^{\text{ac}}$ denotes $G^{\text{ac}}(q, \hat{\theta}_{N_1})$, and similar definitions hold for $\hat{G}^{\text{air}}$ and $\hat{H}^{\text{ac}}$.

We evaluate the accuracy of the estimated model by inspecting the autocorrelation function of the one-step ahead prediction error. Figure 8 presents the autocorrelation function of the model residuals, and the cross correlation function between the model residuals and the inputs (the normalized cross covariance function), where the cross covariance function is

$$\hat{R}_{\varepsilon^{\text{ac}} u^{\text{x}}}(\tau) := \frac{1}{N_1 - \tau} \sum_{t=1}^{N_1 - \tau} \varepsilon^{\text{ac}}(t, \hat{\theta}_{N_1}) u^{\text{x}}(t - \tau), \quad (43)$$

with $\text{x} \in \{\text{ac}, \text{air}\}$. We note that the correlation functions are computed over the validation data. We observe that the

Fig. 9. Step response of the closed-loop system, ventilation subsystem.

autocorrelation function does not have all its samples contained in the 99% confidence region, which implies that the whiteness assumption on the model residuals is rejected. This can occur if the estimated model does not describe completely the system dynamics. To analyze this, we observe in Figure 8 that the cross correlation function satisfies the uncorrelatedness assumption for the 99% confidence region, which suggests that there are no unmodeled dynamics in the model residuals. Hence, we can employ the estimates (40) and (41) for the controller design.

The estimates (40)-(41) are then employed to design a controller following the steps described in Section III-C2, with

$$W(q) = \frac{9.56(1 + 0.90q^{-1})}{1 - q^{-1}}. \quad (44)$$

The filter (44) forces the $\mathcal{H}_\infty$ optimization (21) to focus on the low-frequency range and to guarantee integral action in the optimal controller. The resulting controller is

$$C^{ac}(q) = \frac{-12.5(1 + q^{-1})(1 - 0.98q^{-1})}{(1 - q^{-1})(1 - 0.98q^{-1})} \times \frac{(1 - 0.60q^{-1})(1 - 0.02q^{-1})}{(1 + 0.14q^{-1} + 0.14q^{-2})}. \quad (45)$$

The controller (45) provides a phase margin of 64.2 degrees at $\omega_\phi = 4.7 \cdot 10^{-3}$ [rad/s], and a gain margin of 9.94 [dB] at $\omega_g = 1.57 \cdot 10^{-2}$ [rad/s].

The feedforward compensator (25) is given by

$$G_{ff}(q, \hat{\theta}_{N_1}) = -\frac{0.05q^{-1}(1 - 5.57q^{-1})(1 - 1.12q^{-1})}{(1 - 0.98q^{-1})(1 + 0.10q^{-1})}. \quad (46)$$

We analyze the designs (45)-(46) by implementing the compensators in the HVAC testbed and running the system with the proposed feedback scheme. Figure 9 presents the experimental results of tracking the reference $r^{ac}(t) = 17.5$ [C] for all t , for the cases with and without the feedforward compensator and $u^{air}(t) = 50$. In both cases we observe that the temperature of the air tries to follow the desired reference. To analyze the effect of the feedforward compensator (46) on the system, we set $u^{air}(t) = 100$ after 21 samples, and the results are shown in Figure 9. We can see that the feedforward compensator reduces the effect of changing the air flow damper in the conditioned

air temperature. Therefore, the final implementation of the controller takes the feedforward compensator into account.

To analyze the accuracy of the identified model, Figure 9 shows the one-step ahead prediction obtained from the model of the closed-loop dynamics, which is built by employing (40)-(42) and the compensators (45)-(46). We can see that the predicted output is close to the actual output. Therefore, the closed-loop model for the ventilation subsystem can be used for predictions in the MPC formulation. The closed-loop model for the ventilation subsystem is converted into the state space representation (30) with a sampling period of 10 minutes, which is used into the MPC formulation.

V. SIMULATION EVALUATION OF THE MPC CONTROLLER WITH CLOSED-LOOP DYNAMICS

We analyze the performance of the designed MPC framework incorporating the closed-loop dynamics of the heating and the ventilation subsystems and discuss simulation results obtained by the application of the MPC control strategies described in the previous sections to the HVAC system. Weather and occupancy data from March 20th, 2014 are used.

Three different controllers are considered:

- 1) MPC-no CL: MPC without the closed-loop dynamics of the heating and ventilation subsystems.
- 2) MPC-CL: MPC with the closed-loop dynamics of the heating and ventilation subsystems.
- 3) SMPC-CL: scenario-based MPC with the closed-loop dynamics of the heating and ventilation subsystems.

The control strategies are simulated using a sampling period of 10 minutes and a prediction horizon of 8 hours. In the case of the scenario-based MPC, at each time step it computes the control signals using 40^3 scenarios for the disturbances (wall temperatures, solar radiation, and occupancy) over the prediction horizon. The allowed comfort violation is 1% (i.e., $\alpha = 0.01$).

The simulation is performed in Matlab R2016b complemented with IBM CPLEX to solve the optimization problem associated with MPC. The computational time required for the execution of one simulation step for all the MPC strategies is less than one second in a computer model Dell XPS 13, equipped with Intel Core(TM) i7-5500U 2.40 [GHz] processor, and 8.00 [GB] RAM memory. Hence, the proposed strategies are suitable for real-world applications.

For all the MPC strategies considered in the simulation, the heating and ventilation subsystems are controlled by the low-level controllers designed in this article. In addition, we assume that the CO₂-MPC controller delivers a constant air flow rate equal to $\dot{m}_{CO_2}^{air}(t) = 0.10$ [Kg/s]. Thus, the difference in performance is only due to the inclusion of the closed-loop models of the HVAC subsystems into the MPC formulation.

Remark 8: In practice the final setpoints for the air flow rate (and then the damper positions of the ventilation subsystem) are the sum of the optimal values calculated by both control problems, the CO₂-MPC problem and the T-MPC problem. In this simulation study, without loss of generality and for the sake of simplicity, we focus on the T-MPC problem and consider a constant air flow rate equal to the standard lower

Fig. 10. Comparison among MPC-no CL, MPC-CL and SMPC-CL. Top: AC temperature. Middle: additional air inlet mass flow. Bottom: Room occupancy.

bound as resulting from the CO_2 -MPC problem, which has been proved to be sufficient to guarantee the CO_2 requirements, given the considered occupancy pattern. A time-varying air flow rate can also be considered in the CO_2 -MPC problem.

The room temperature profiles resulting from the application of the three control strategies are quite similar and the thermal comfort is never violated, although the control signals are different, as shown in Figure 10. In fact, we observe that including the closed-loop dynamics of the HVAC subsystem helps reduce the requirements on the cooling system, which are unnecessarily high in case of the MPC-no CL. The control requirements on the supplied air temperature and the additional air flow rate obtained by MPC-no CL are more aggressive than those imposed by the MPC formulations including the closed-loop dynamics of the HVAC subsystems. We can see that the AC temperature is at its minimum value for MPC-no CL, whilst both MPC-CL and SMPC-CL result in less demanding actuation levels. The air flow rate control setpoints obtained by the three compared strategies are also different, as shown in the bottom plot of Figure 10, however the control inputs computed by the MPC controllers incorporating the closed-loop dynamics are very similar. The different control requirements on the AC temperature and on the additional air flow rate resulting from the MPC-CL and the SMPC-CL lead to a reduction in the energy use. When the room occupancy increases, the MPC with the closed-loop dynamics of the heating and ventilation subsystems compensates faster for the effect of changes in occupancy while keeping the resulting indoor temperature within the bounds without any unnecessary energy transferred, thus improving the energy efficiency. In fact, the MPC-CL and the SMPC-CL lead to energy saving of 78% and 82% respectively compared to the energy used by the MPC-no CL.

In summary, including the closed-loop dynamics in the MPC strategy helps improve significantly the energy efficiency, since it results in less energy use and better compensation of the effects of changes in occupancy levels without violating the indoor thermal comfort.

Remark 9: In the specific case study performed in this section, the improvement achieved by SMPC-CL compared to MPC-CL appears minor (just 4%). However, the SMPC-CL

computes its control inputs based on a worst-case scenario approach, which leads to a more robust behavior against unknown disturbances. As a consequence, the SMPC-CL usually yields less violations with respect to the deterministic MPC-CL and this can come at a cost of a slightly higher energy use. The advantages of the SMPC will be evident when occupancy is significantly higher than the one considered in this study. When the occupancy is low, SMPC-CL can be quite conservative and then the MPC-CL might be a better choice. In any case, the conservativeness of the SMPC-CL can be mitigated by an improved training of the scenarios and a different tuning of parameter α . Finally, we also notice that an energy saving of 4% in one room and over 7 hours might be significant when scaling up to multiple rooms and longer temporal horizons.

Remark 10: The complexity and the size of the SMPC problem stays the same as the one of the MPC problem, since the worst-case scenario is considered in the scenario-based approach and the problem is still linear. Then the computational requirements of the SMPC-CL is the same as the one for the MPC-CL. An additional, however quite limited, effort might be required for generating the scenarios online but, if enough historical data is available, it can be used directly without any online scenario generation [43], [44].

Remark 11: The proposed approach is suitable for HVAC systems with a very large number of devices. In fact, the average computational time of the MPC problems (including the SMPC ones) is lower than one second and only linear problems are solved, for which very efficient and standard solving algorithms exist and solvers are available (both commercial, like CPLEX, and open-source, like lpsolve³ and glpk⁴). Furthermore, the proposed MPC optimization problem can be easily extended to include a large number of devices and also multiple thermal zones (if required). In that case, the solving algorithm can take significant advantage of the sparsity of the optimization problem and decrease even further the computational effort. The structure of the MPC problems would also make possible to resort to well-established distributed approaches to solve these problems, such as dual decomposition and alternating direction method of multipliers [45]. Showing this, however, is outside the scope of this article and left as future work.

VI. CONCLUSIONS

A methodology to model the dynamics of the heating and ventilation subsystems and design controllers for them is presented. A supervisory MPC strategy incorporating the closed-loop dynamics of the heating and ventilation subsystems is designed in order to improve the control performance and the building energy efficiency. The illustrated methodology considers experiment design, identification and controller design, which is analyzed in the HVAC testbed available in a KTH university building. The proposed approach provides a guideline to practitioners interested in implementing energy-efficient MPC strategies for HVAC systems. The final design

³<http://lpsolve.sourceforge.net/5.5/>

⁴<https://www.gnu.org/software/glpk/>

for the low-level controllers is implemented and experimentally evaluated in the actual testbed, while the supervisory MPC controller is assessed through a simulation study to better analyze the improvement in the control performance.

REFERENCES

- [1] F. Lamnabhi-Lagarigue, A. Annaswamy, S. Engell, A. Isaksson, P. Khargonekar, R.M. Murray, H. Nijmeijer, T. Samad, D. Tilbury, and P.M.J. Van den Hof, "Systems & control for the future of humanity, research agenda: Current and future roles, impact and grand challenges," *Annual Reviews in Control*, vol. 43, pp. 1–64, 2017.
- [2] Y. Wang, J. Kuckelkorn, and Y. Liu, "A state of art review on methodologies for control strategies in low energy buildings in the period from 2006 to 2016," *Energy and Buildings*, vol. 147, pp. 27–40, 2017.
- [3] F. Oldewurtel, A. Parisio, C.N. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari, "Use of model predictive control and weather forecasts for energy efficient building climate control," *Energy and Buildings*, vol. 45, pp. 15–27, 2012.
- [4] G.T. Costanzo, S. Iacovella, F. Ruelens, T. Leurs, and B.J. Claessens, "Experimental analysis of data-driven control for a building heating system," *Sustainable Energy, Grids and Networks*, vol. 6, pp. 81–90, 2016.
- [5] J. Zhao, K.P. Lam, B.E. Ydstie, and V. Loftness, "Occupant-oriented mixed-mode EnergyPlus predictive control simulation," *Energy and Buildings*, vol. 117, pp. 362–371, 2016.
- [6] E. Khanmirza, A. Esmaeilzadeh, and A.H.D. Markazi, "Design and experimental evaluation of model predictive control vs. intelligent methods for domestic heating systems," *Energy and Buildings*, vol. 150, pp. 52–70, 2017.
- [7] Z. Wang and L. Wang, "Intelligent control of ventilation system for energy-efficient buildings With CO₂ predictive model," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 686–693, June 2013.
- [8] A. Afram and F. Janabi-Sharifi, "Theory and applications of HVAC control systems—A review of model predictive control (MPC)," *Building and Environment*, vol. 72, pp. 343–355, Feb. 2014.
- [9] H. Thieblemont, F. Haghighat, R. Ooka, and A. Moreau, "Predictive control strategies based on weather forecast in buildings with energy storage system: A review of the state-of-the-art," *Energy and Buildings*, vol. 153, pp. 485–500, 2017.
- [10] S. Treado and Y. Chen, "Saving Building Energy through Advanced Control Strategies," *Energies*, vol. 6, no. 9, pp. 4769–4785, Sept. 2013.
- [11] M. Fadzli Haniff, H. Selamat, R. Yusof, S. Buyamin, and F. Sham Ismail, "Review of HVAC scheduling techniques for buildings towards energy-efficient and cost-effective operations," *Renewable and Sustainable Energy Reviews*, vol. 27, pp. 94–103, Nov. 2013.
- [12] M. Fiorentini, J. Wall, Z. Ma, J.H. Braslavsky, and P. Cooper, "Hybrid model predictive control of a residential HVAC system with on-site thermal energy generation and storage," *Applied Energy*, vol. 187, pp. 465–479, 2017.
- [13] D. Sturzenegger, D. Gyalistras, M. Morari, and R.S. Smith, "Model Predictive Climate Control of a Swiss Office Building: Implementation, Results, and Cost-Benefit Analysis," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 1, pp. 1–12, 2016.
- [14] A. Parisio, D. Varagnolo, M. Molinari, G. Pattarello, L. Fabbietti, and K.H. Johansson, "Implementation of a Scenario-based MPC for HVAC Systems: an Experimental Case Study," in *19th IFAC World Congress*, Aug. 2014.
- [15] Z. Vána, J. Cigler, J. Šíroky, E. Žáčeková, and L. Ferkl, "Model-based energy efficient control applied to an office building," *Journal of Process Control*, vol. 24, no. 6, pp. 790–797, Jun. 2014.
- [16] M. Aftab, C. Chen, C.K. Chau, and T. Rahwan, "Automatic HVAC Control with Real-time Occupancy Recognition and Simulation-guided Model Predictive Control in Low-cost Embedded System," *Energy and Buildings*, vol. 154, pp. 141–156, 2017.
- [17] J. Drgona, M. Kvasnica, M. Klauco, and M. Fikar, "Explicit Stochastic MPC Approach to Building Temperature Control," in *2013 IEEE 52nd IEEE Conference on Decision and Control*, 2013.
- [18] Y. Ma, J. Matuško, and F. Borrelli, "Stochastic model predictive control for building HVAC systems: complexity and conservatism," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 1, pp. 101–116, Jan 2015.
- [19] K.C.S. Yu, R. Bourdais, H. Gueguen, and D. Dumur, "Integration of low level controller constraints in supervisory control of buildings," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 12062–12067, 2014.
- [20] A. Afram, F. Janabi-Sharifi, A.S. Fung, and K. Raahemifar, "Artificial neural network (ANN) based model predictive control (MPC) and optimization of HVAC systems: A state of the art review and case study of a residential HVAC system," *Energy and Buildings*, vol. 141, pp. 96–113, 2017.
- [21] D.R. Saffer and F.J. Doyle, "Closed-loop identification with MPC for an industrial scale CD-control problem," *IEE Proceedings-Control Theory and Applications*, vol. 149, no. 5, pp. 448–456, 2002.
- [22] A. Esmaili, J. Li, J. Xie, and J.D. Isom, "Closed-loop identification for plants under model predictive control," *Control Engineering Practice*, vol. 72, pp. 206–218, 2018.
- [23] Y. Zhu and F. Butoyi, "Case studies on closed-loop identification for MPC," *Control Engineering Practice*, vol. 10, no. 4, pp. 403–417, 2002, Mechatronics.
- [24] Y. Zhu, "Multivariable process identification for MPC: the asymptotic method and its applications," *Journal of Process Control*, vol. 8, no. 2, pp. 101–115, 1998.
- [25] O.A.Z. Sotomayor, D. Odloak, and L.F.L. Moro, "Closed-loop model re-identification of processes under MPC with zone control," *Control Engineering Practice*, vol. 17, no. 5, pp. 551–563, 2009.
- [26] C.A. Larsson, C.R. Rojas, X. Bombois, and H. Hjalmarsson, "Experimental evaluation of model predictive control with excitation (MPC-X) on an industrial depropanizer," *Journal of Process Control*, vol. 31, pp. 1–16, 2015.
- [27] G.C. Goodwin and R.L. Payne, *Dynamic System Identification: Experimental Design and Data Analysis*, Academic Press, New York, 1977.
- [28] H. Hjalmarsson, "From experiment design to closed-loop control," *Automatica*, vol. 41, no. 3, pp. 393–438, 2005.
- [29] G. Goddard, J. Klose, and S. Backhaus, "Model development and identification for fast demand response in commercial HVAC systems," *IEEE Transactions on Smart Grid*, vol. 5, no. 4, pp. 2084–2092, July 2014.
- [30] P.E. Valenzuela, C.R. Rojas, and H. Hjalmarsson, "Optimal input design for non-linear dynamic systems: a graph theory approach," in *Proceedings of the 52th IEEE Conference on Decision and Control*, Florence, Italy, Dec. 2013.
- [31] P.E. Valenzuela, C.R. Rojas, and H. Hjalmarsson, "A graph theoretical approach to input design for identification of nonlinear dynamical models," *Automatica*, vol. 51, pp. 233–242, 2015.
- [32] A. Parisio, M. Molinari, D. Varagnolo, and K.H. Johansson, "Energy Management Systems for Intelligent Buildings in Smart Grids," in *Intelligent Building Control Systems. A Survey of Modern Building Control and Sensing Strategies*, J.T. Wen and S. Mishra, Eds., chapter 10. Springer International Publishing, 2018.
- [33] D. Picard, J. Drgoňa, M. Kvasnica, and L. Helsen, "Impact of the controller model complexity on model predictive control performance for buildings," *Energy and Buildings*, vol. 152, no. Supplement C, pp. 739–751, 2017.
- [34] L. Ljung, *System Identification. Theory for the User*, 2nd ed., Upper Saddle River, NJ: Prentice-Hall, 1999.
- [35] L. Ljung, "System identification toolbox for use with Matlab," 2007.
- [36] S.P. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.
- [37] C.R. Rojas, J.S. Welsh, G.C. Goodwin, and A. Feuer, "Robust optimal experiment design for system identification," *Automatica*, vol. 43, no. 6, pp. 993–1008, Jun. 2007.
- [38] A. Zaman, "Stationarity on finite strings and shift register sequences," *The Annals of Probability*, vol. 11, no. 3, pp. 678–684, Aug. 1983.
- [39] P.E. Valenzuela, *On risk-coherent input design and Bayesian methods for nonlinear system identification*, Ph.D. thesis, KTH, School of Electrical Engineering, 2016.
- [40] J.M. Maciejowski, *Multivariable Feedback Design*, Addison Wesley, 1989.
- [41] G.C. Goodwin, S. Graebe, and M.E. Salgado, *Control System Design*, Prentice-Hall, 2000.
- [42] S. Skogestad and I. Postlethwaite, *Multivariable feedback control: analysis and design*, 2nd Edition, New York: Wiley, 2007.
- [43] G. Calafiore and M. Campi, "The Scenario Approach to Robust Control Design," *IEEE Transactions on Automatic Control*, vol. 51, no. 5, pp. 742–753, 2008.
- [44] G. Schildbach, L. Fagiano, C. Frei, and M. Morari, "The scenario approach for Stochastic Model Predictive Control with bounds on closed-loop constraint violations," *Automatica*, vol. 50, no. 12, pp. 3009–3018, 2014.
- [45] S.P. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, *Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers*, now, 2011.